已知渡河总时长不变，则有

$$\frac{S_1}{V*\cos\alpha_1}+\frac{S_2}{V*\cos\alpha_2}+\ldots+\frac{S_n}{V*\cos\alpha_n}=T$$

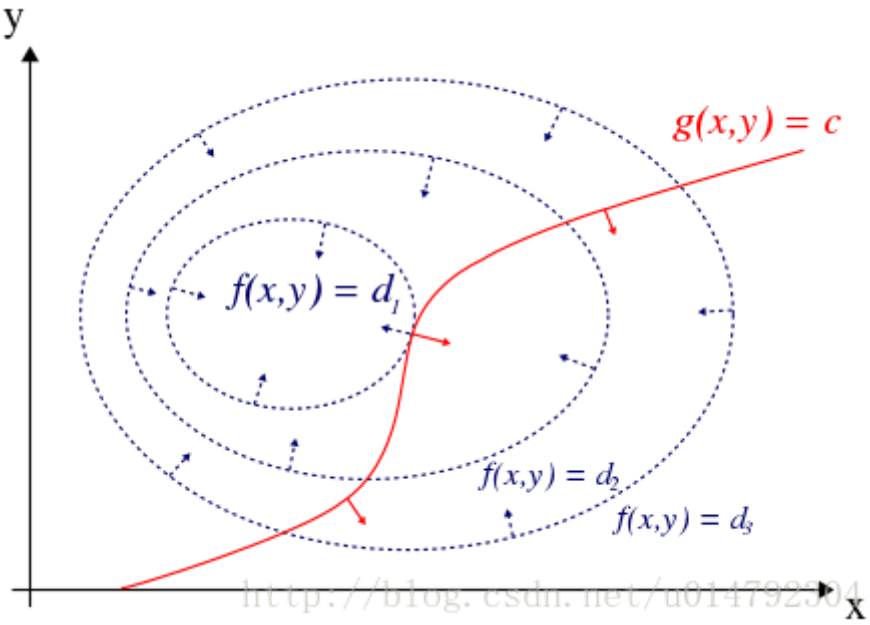
要求 dH 如下式的极小值

$$dH=\frac{S_1}{V*\cos\alpha_1}*(V*sin\alpha_1+V_1)+\frac{S_2}{V*\cos\alpha_2}*(V*sin\alpha_2+V_2)+\ldots+\frac{S_n}{V*\cos\alpha_n}*(V*sin\alpha_n+V_n)$$

可看做是如下的优化问题

$min_x f(x)$

s.t. $g(\boldsymbol{x})$=0



令$T(n)=\frac{S_1}{V_1*\cos\alpha_1}+\frac{S_2}{V_2*\cos\alpha_2}+\ldots+\frac{S_n}{V_n*\cos\alpha_n 1}$

该极值点处满足

$$\begin{cases}\bigtriangledown dH=\lambda\bigtriangledown T(n),\\ T(n)=T\end{cases}$$

求梯度如下

$$\begin{cases}\frac{S_1\sin\alpha_1\lambda}{V\cos{\alpha_1}^2}=\frac{S_1(V+V_1*\sin\alpha_1)}{V\cos^2\alpha_1},\\ \frac{S_2\sin\alpha_2\lambda}{V\cos{\alpha_2}^2}=\frac{S_2(V+V_2*\sin\alpha_2)}{V\cos^2\alpha_2},\\ \ldots,\\ \frac{S_n\sin\alpha_n\lambda}{V\cos{\alpha_n}^2}=\frac{S_n(V+V_n*\sin\alpha_n)}{V\cos^2\alpha_n},\end{cases}$$

约去公因式，可得

$$\begin{cases}\sin\alpha_1=\frac{V}{\lambda-V_1},\\ \sin\alpha_2=\frac{V}{\lambda-V_2},\\ \ldots,\\ \sin\alpha_n=\frac{V}{\lambda-V_n}\end{cases}$$

带入时间不变的等式，解得λ

进而求解角度