

Numeric types in Python:

```
>>> type(2)
<class 'int'>
```

Represents integers exactly

```
>>> type(1.5)
<class 'float'>
```

Represents real numbers approximately

```
>>> type(1+1j)
<class 'complex'>
```

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

This function represents a rational number

Constructor is a higher-order function

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```

list

list

list

Executing a for statement:

```
for <name> in <expression>:
    <suite>
```

1. Evaluate the header `<expression>`, which must yield an iterable value (a sequence)
2. For each element in that sequence, in order:
  - A. Bind `<name>` to that element in the current frame
  - B. Execute the `<suite>`

Unpacking in a for statement:

A sequence of fixed-length sequences

```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
```

A name for each element in a fixed-length sequence

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1
>>> same_count
2
```

..., -3, -2, -1, 0, 1, 2, 3, 4, ...

range(-2, 2)

Length: ending value – starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

List comprehensions:

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Short version: `[<map exp> for <name> in <iter exp>]`

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent
2. Create an empty *result list* that is the value of the expression
3. For each element in the iterable value of `<iter exp>`:
  - A. Bind `<name>` to that element in the new frame from step 1
  - B. If `<filter exp>` evaluates to a true value, then add the value of `<map exp>` to the result list

The result of calling `repr` on a value is what Python prints in an interactive session

The result of calling `str` on a value is what Python prints using the `print` function

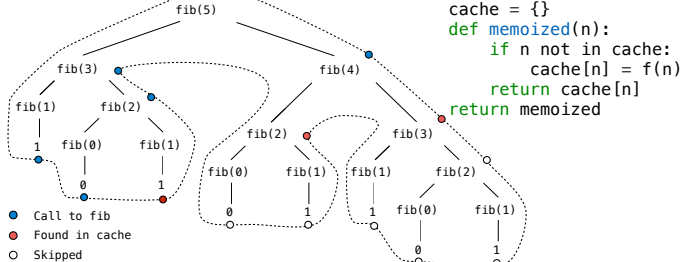
```
>>> 12e12
12000000000000.0
>>> print(today)
2014-10-13
>>> print(repr(12e12))
12000000000000.0
```

`str` and `repr` are both polymorphic; they apply to any object

`repr` invokes a zero-argument method `__repr__` on its argument

```
>>> today.__repr__()
'datetime.date(2014, 10, 13)'
>>> today.__str__()
'2014-10-13'
```

Memoization:



```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

**Type dispatching:** Look up a cross-type implementation of an operation based on the types of its arguments

**Type coercion:** Look up a function for converting one type to another, then apply a type-specific implementation.

$\Theta(b^n)$  Exponential growth. Recursive `fib` takes  $\Theta(\phi^n)$  steps, where  $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$ . Incrementing the problem scales  $R(n)$  by a factor

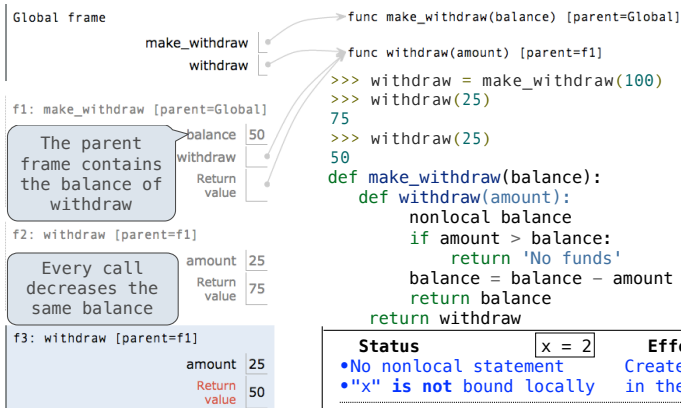
$\Theta(n^2)$  Quadratic growth. E.g., `overlap`. Incrementing  $n$  increases  $R(n)$  by the problem size  $n$

$\Theta(n)$  Linear growth. E.g., `factors` or `exp`

$\Theta(\log n)$  Logarithmic growth. E.g., `exp_fast`

$\Theta(1)$  Constant. The problem size doesn't matter

$R(n) = \Theta(f(n))$  means that there are positive constants  $k_1$  and  $k_2$  such that  $k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$  for all  $n$  larger than some  $m$



Strings as sequences:

```
>>> city = 'Berkeley'
>>> len(city)
8
>>> city[3]
'k'
>>> 'here' in "Where's Waldo?"
True
>>> 234 in [1, 2, 3, 4, 5]
False
>>> [2, 3, 4] in [1, 2, 3, 4]
False
```

List & dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
>>> a == b
False
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
>>> suits = ['coin', 'string', 'myriad']
>>> original_suits = suits
>>> suits.pop()
'myriad'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['heart', 'diamond']
>>> suits
['heart', 'diamond', 'spade', 'club']
>>> original_suits
['heart', 'diamond', 'spade', 'club']
```

Identity:

`<exp0> is <exp1>` evaluates to `True` if both `<exp0>` and `<exp1>` evaluate to the same object

Equality:

`<exp0> == <exp1>` evaluates to `True` if both `<exp0>` and `<exp1>` evaluate to equal values

Identical objects are always equal values

You can copy a list by calling the list constructor or slicing the list from the beginning to the end.

**Constants:** Constant terms do not affect the order of growth of a process

$\Theta(n)$   $\Theta(500 \cdot n)$   $\Theta(\frac{1}{500} \cdot n)$

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

$\Theta(\log_2 n)$   $\Theta(\log_{10} n)$   $\Theta(\ln n)$

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

```
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

Outer: length of a

Inner: length of b

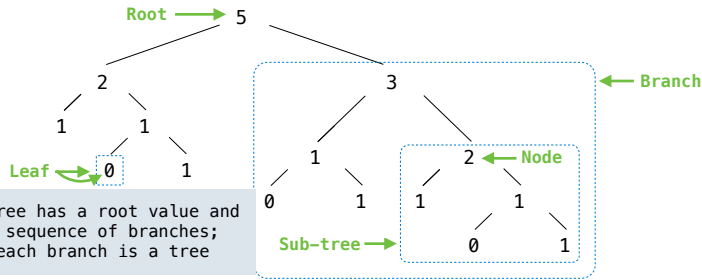
If  $a$  and  $b$  are both length  $n$ , then `overlap` takes  $\Theta(n^2)$  steps

**Lower-order terms:** The fastest-growing part of the computation dominates the total

$\Theta(n^2)$   $\Theta(n^2 + n)$   $\Theta(n^2 + 500 \cdot n + \log_2 n + 1000)$

Status	Effect
•No nonlocal statement	Create a new binding from name "x" to number 2 in the first frame of the current environment
•"x" is not bound locally	
•No nonlocal statement	Re-bind name "x" to object 2 in the first frame of the current environment
•"x" is bound locally	
•nonlocal x	
•"x" is bound in a non-local frame	Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound
•nonlocal x	
•"x" is not bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found
•nonlocal x	
•"x" is bound in a non-local frame	SyntaxError: name 'x' is parameter and nonlocal
•"x" also bound locally	

Tree data abstraction:



A tree has a root value and a sequence of branches; each branch is a tree

```
def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [root] + list(branches)

def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True

def is_leaf(tree):
    return not branches(tree)

def leaves(tree):
    """The leaf values in tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
    if is_leaf(tree):
        return [root(tree)]
    else:
        return sum([leaves(b) for b in branches(tree)], [])
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

tree(3, [tree(1), tree(2, [tree(1), tree(1)]), [3, [1], [2, [1], [1]]])

```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
    def is_leaf(self):
        return not self.branches

class BinaryTree(Tree):
    empty = Tree(None)
    empty.is_empty = True
    def __init__(self, entry, left=empty, right=empty):
        Tree.__init__(self, entry, (left, right))
        self.is_empty = False
    @property
    def left(self):
        return self.branches[0]
    @property
    def right(self):
        return self.branches[1]
    def is_leaf(self):
        return self.left.is_empty and self.right.is_empty
```

Built-in isinstance function: returns True if branch has a class that is or inherits from Tree

E: An empty tree

Bin = BinaryTree

t = Bin(3, Bin(1), Bin(7, Bin(5), Bin(9, Bin.empty, Bin(11))))

```
class Link:
    empty = ()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __getitem__(self, i):
        if i == 0:
            return self.first
        else:
            return self.rest[i-1]
    def __len__(self):
        return 1 + len(self.rest)

def extend_link(s, t):
    if s is Link.empty:
        return t
    else:
        return Link(s.first, extend_link(s.rest, t))

def map_link(f, s):
    if s is Link.empty:
        return s
    else:
        return Link(f(s.first), map_link(f, s.rest))
```

Some zero length sequence

Sequence abstraction special names:

- \_\_getitem\_\_ Element selection []
- \_\_len\_\_ Built-in len function

Yes, this call is recursive

>>> s = Link(3, Link(4))  
>>> extend\_link(s, s)  
Link(3, Link(4, Link(3, Link(4))))  
>>> square = lambda x: x \* x  
>>> map\_link(square, s)  
Link(9, Link(16))

Python object system:

Idea: All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

A new instance is created by calling a class

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

balance: 0 holder: 'Jim'

When a class is called:

1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

`__init__` is called a constructor

self should always be bound to an instance of the Account class or a subclass of Account

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

Function call: all arguments within parentheses

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Method invocation: One object before the dot and other arguments within parentheses

Call expression

Dot expression



&lt;expression&gt; . &lt;name&gt;

The &lt;expression&gt; can be any valid Python expression.

&lt;name&gt; must be a simple name.

Evaluates to the value of the attribute looked up by &lt;name&gt; in the object that is the value of the &lt;expression&gt;.

To evaluate a dot expression:

1. Evaluate the <expression> to the left of the dot, which yields the object of the dot expression
2. <name> is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
3. If not, <name> is looked up in the class, which yields a class attribute value
4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

Account class attributes	Instance attributes of jim_account	Instance attributes of tom_account
interest: 0.05 (withdraw, deposit, __init__)	balance: 0 holder: 'Jim' interest: 0.08	balance: 0 holder: 'Tom'

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
        or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```