**FE5222 ADP Project Two**

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# Introduction

Spread options have terminal payoffs based on the difference in prices between two underlying assets together with a strike. When the strike equals , the spread option is equivalent to an option to exchange one asset for another, where the Margrabe’s formula works as an explicit solution [6]. The Kirk’s approximation as published in 1995, is a valid formula when the strike is small but non-zero, where a special sigma is adopted into the generalized Black-Scholes option pricing [1,2,3].

In this project, we would investigate the spread option pricing via Kirk’s approximation, and employ the Monte Carlo (MC) simulation output as a benchmark. Results from both methods are compared and discussed in various scenarios.

# Materials and Methods

## Monte Carlo Simulation

## Kirk’s Approximation

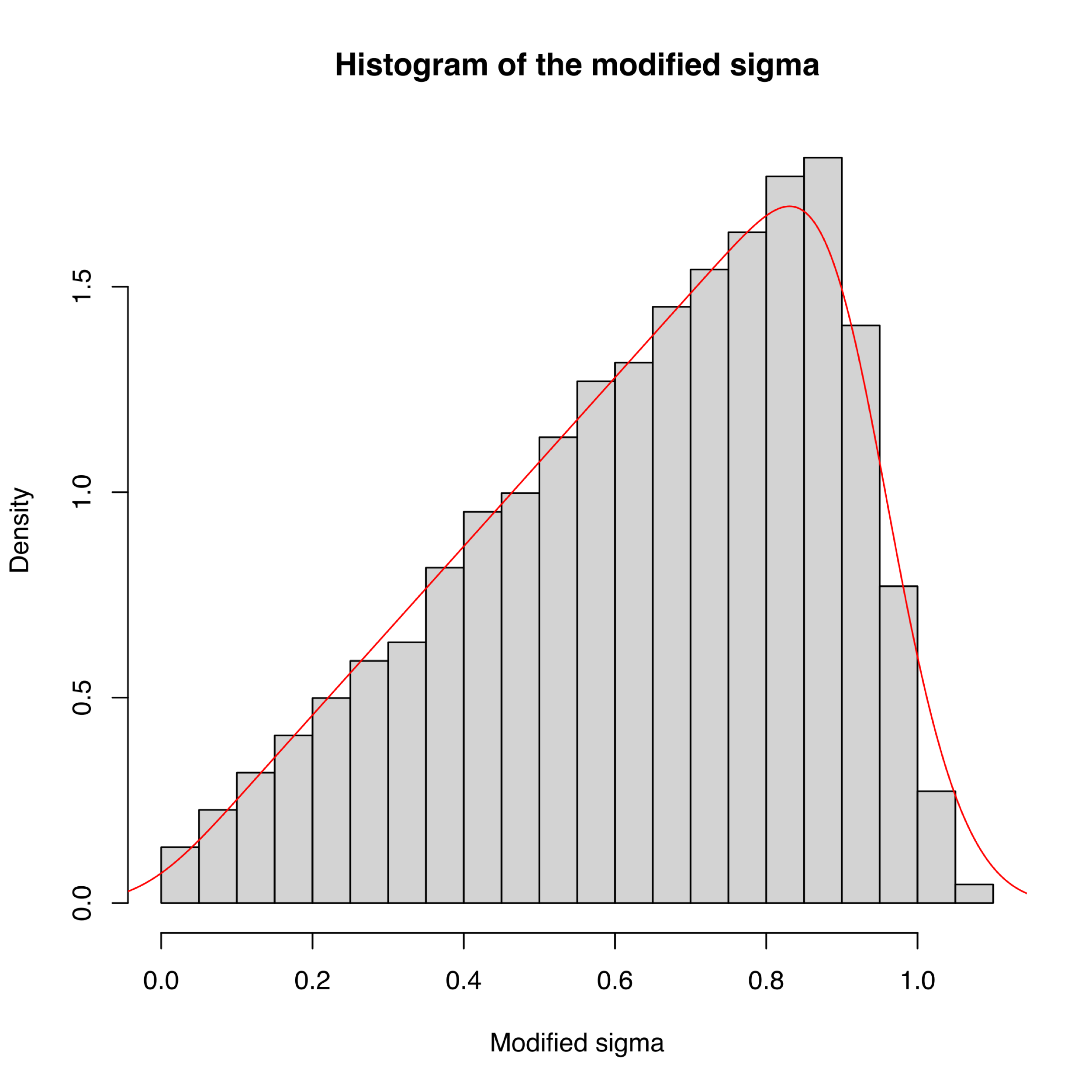
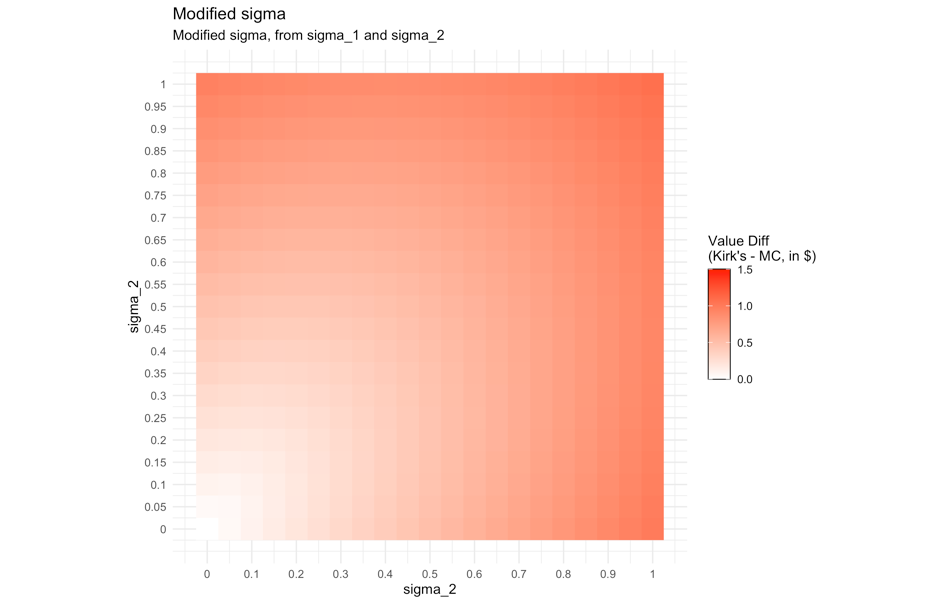


Figure 1. The modified , as calculated using and , ranges from 0.00 to 1.07

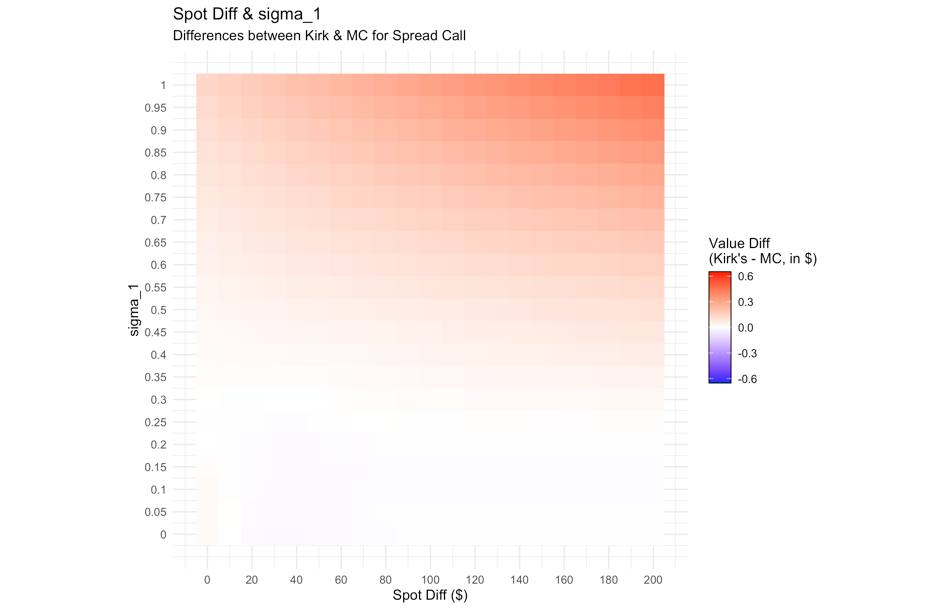
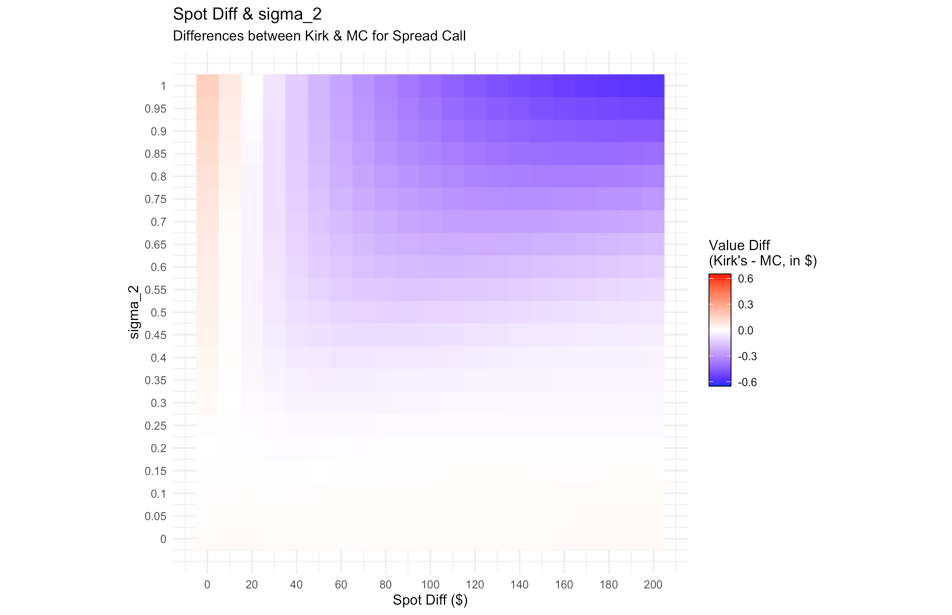
with a negative skewness of -0.51 and a negative excess kurtosis of -0.60.

## Comparison and Visualization

To compare the two investigated methods with respect to pricing parameters, we further performed pairwise pricing and visualized their differences (Kirk’s – MC’s) over two dimensional grids with red-white-blue color scales. To improve comparability, we kept the color scale bar centered around $0. Investigated pricing parameters with default values includes: Spot difference between the two stocks () = $110 - $100 = $10, Strike as in percentage of () = = 50%, volatility of the first stock () = 0.2, volatility of the second stock () = 0.2, instantaneous correlation () = 0.4, interest rate () = 0.08, and time to maturity () = 1. They are explored using equal spaced grids in corresponding plots but kept constant otherwise. To keep reproducibility and accelerate convergence, all Monte Carlo pricing schemes share the same initial random seed with 1,000,000 paths and antithetic variates. Visualizations are implemented in RStudio via packages ‘tidyverse’ and ‘ggplot2’ [4,5].

# Results and Discussion

## Grid comparison for pricing parameters

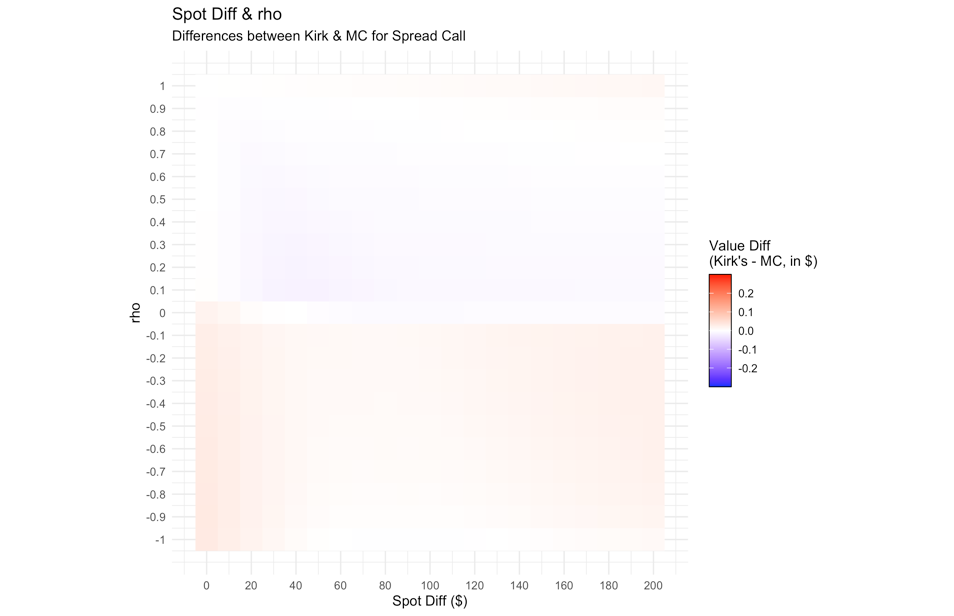
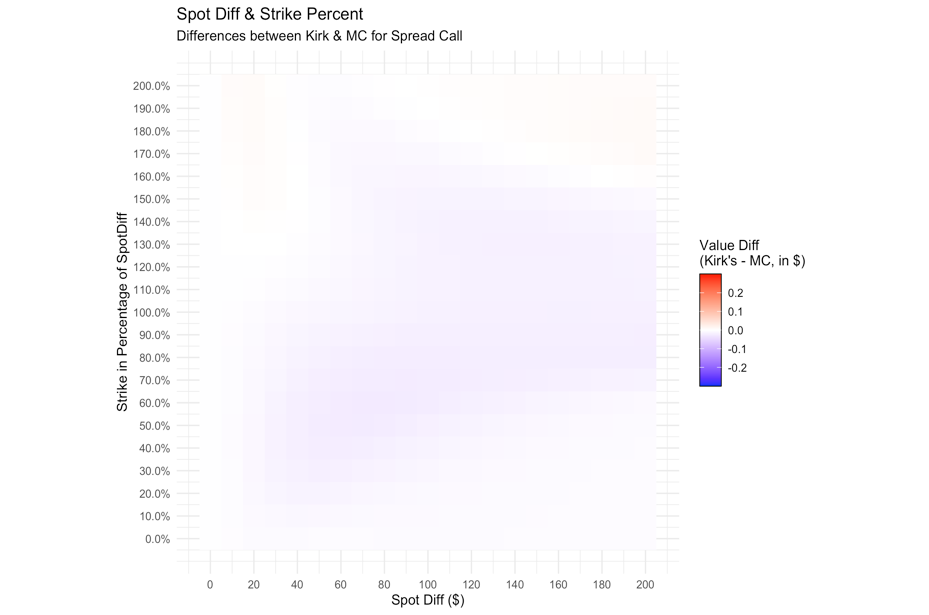
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(a) (b)

Figure 1. The differences between Kirk’s and MC among various  
(a) and , as well as (b) and .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.1(a). The value difference ranging from $-0.02 to $0.46 has its mean at $0.10 and median at $0.06 with a positive skewness of 1.10 and a positive excess kurtosis of 0.46. Generally, larger spot differences or larger would bring positive value differences.

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.1(b). The value difference ranging from $-0.57 to $0.16 has its mean at $-0.12 and median at $-0.05 with a negative skewness of -1.04 and a positive excess kurtosis of 0.10. At higher level, larger spot differences would bring negative value differences.

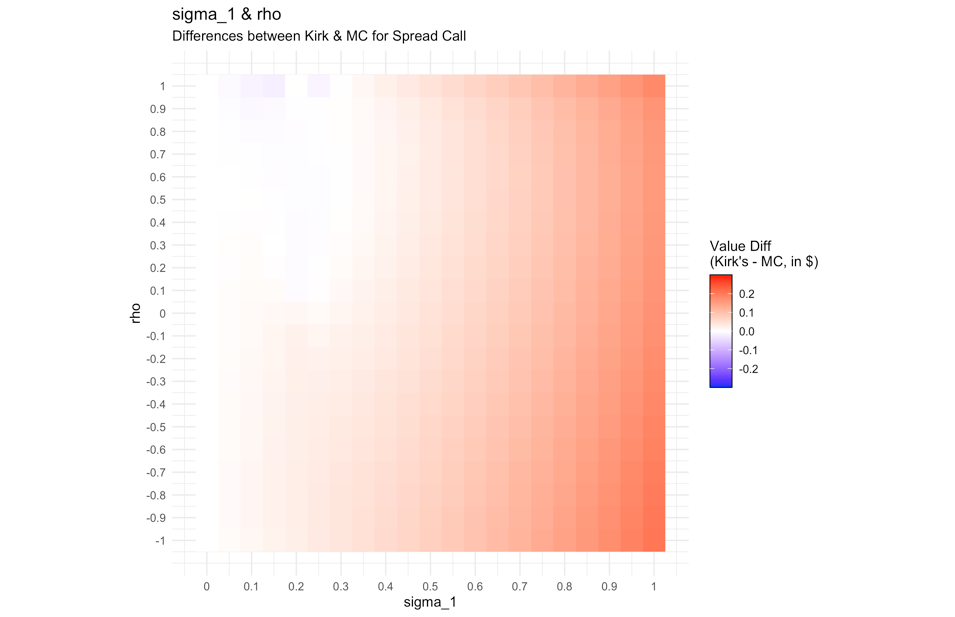
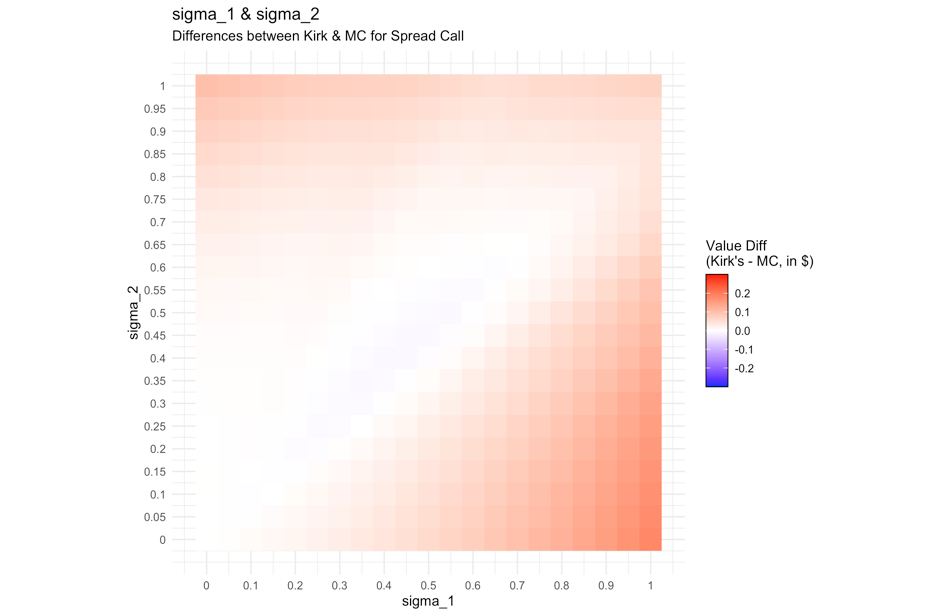
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(a) (b)

Figure 2. The differences between Kirk and MC among various  
(a) and , as well as (b) and Strike in percentage of .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.2(a). The value difference ranging from $-0.02 to $0.03 has its mean at $0.00 and median at $0.00 with a positive skewness of 0.40 and a negative excess kurtosis of -0.70. Generally, positive correlation would bring negative value differences, and vice versa.

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.2(b). The value difference ranging from $-0.03 to $0.01 has its mean at $-0.01 and median at $-0.01 with a positive skewness of 0.05 and a negative excess kurtosis of -0.92. The difference is more visible when the Strike is around the differences between two Spots.

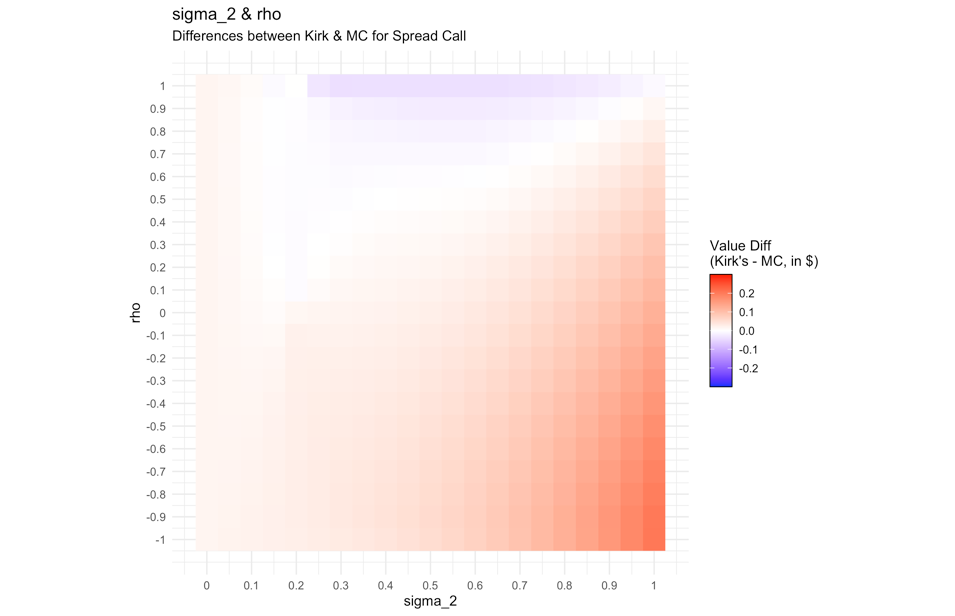
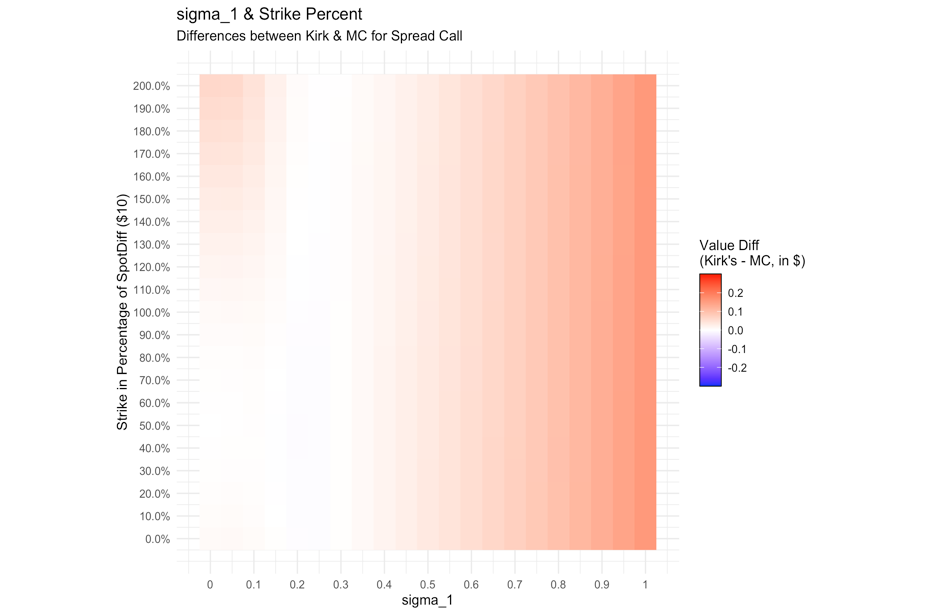
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(a) (b)

Figure 3. The differences between Kirk and MC among various   
(a) and , as well as (b) and .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.3(a). The value difference ranging from $-0.01 to $0.18 has its mean at $0.04 and median at $0.03 with a positive skewness of 1.24 and a positive excess kurtosis of 1.19. Both larger and larger would bring positive value differences. The difference is minimized when = .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.3(b). The value difference ranging from $-0.02 to $0.21 has its mean at $0.06 and median at $0.05 with a positive skewness of 0.60 and a negative excess kurtosis of -0.77.

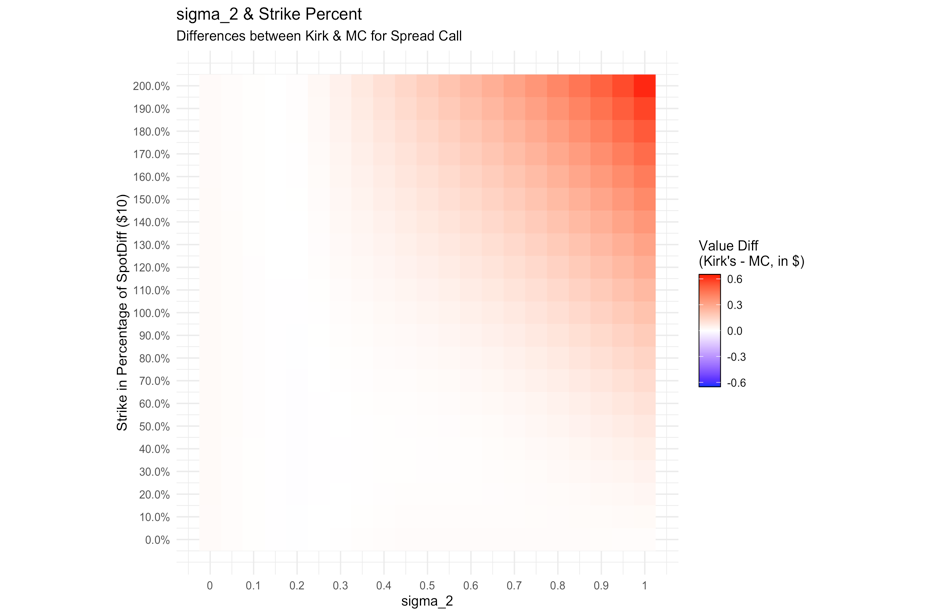
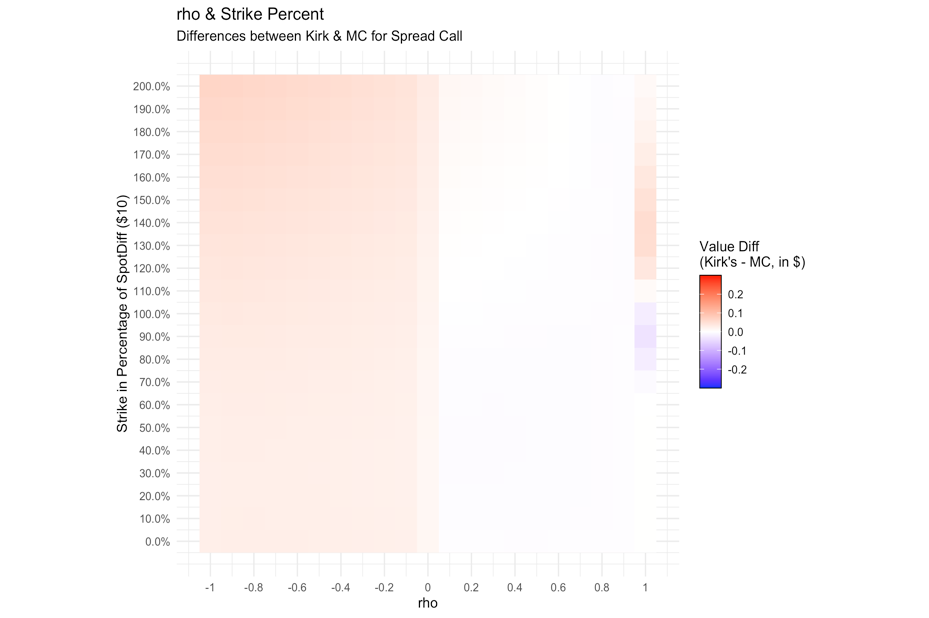
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(a) (b)

Figure 4. The differences between Kirk and MC among various   
(a) and Strike in percentage of , as well as (b) and .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.4(a). The value difference ranging from $0.00 to $0.16 has its mean at $0.05 and median at $0.04 with a positive skewness of 0.71 and a negative excess kurtosis of -0.74.

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.4(b). The value difference ranging from $-0.04 to $0.20 has its mean at $0.04 and median at $0.02 with a positive skewness of 1.23 and a positive excess kurtosis of 1.34. Positive would bring negative differences that may offset the positive differences brought by larger .

(a) (b)

Figure 5. The differences between Kirk and MC among various   
(a) and Strike in percentage of , as well as (b) and Strike in percentage of .

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.5(a). The value difference ranging from $0.00 to $0.62 has its mean at $0.08 and median at $0.02 with a positive skewness of 2.08 and a positive excess kurtosis of 4.17. Both larger and larger Strike would bring positive value differences.

The value differences between Kirk’s and MC’s among the inspected – pairs are plotted in Fig.5(b). The value difference ranging from $-0.04 to $0.06 has its mean at $0.02 and median at $0.02 with a positive skewness of 0.31 and a negative excess kurtosis of -0.99.

# Conclusion

In this project we investigated and implemented Monte Carlo together with Kirk’s approximation on spread call options pricing, then further compared their performances in various scenarios.

BBSR as a binomial-tree-based model showed efficient convergence within time steps and served as a stable benchmark. LSMC as a simulation-based model showed flexibility and good convergence using more than time steps and sample paths. The difference between LSMC and BBSR oscillates stronger when the Spot is around the Strike. With higher , the log-Normal distribution at each time step in the GBM paths would have higher mean and lead to stronger payoff asymmetry. In comparison, BBSR only involves sigma in the step. The oscillation shrinks with and gets stronger when the Spot is around the Strike. Higher could be bounding the discounted payoff and thus limiting the errors. Longer , given constant time steps , is involving wilder fluctuation per time step in the GBM paths and thus accumulates randomness. Higher Strike as allowing more paths to be ITM could be inviting more randomness and thus the asymmetry. Increments in involves more randomness than increments in . Increments in involves more randomness than decrements in .

# References

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2. Kirk, E., & Aron, J. (1995). Correlation in the energy markets. Managing energy price risk, 1, 71-78.
3. Li, M., Deng, S. J., & Zhoc, J. (2008). Closed-form approximations for spread option prices and Greeks. The Journal of Derivatives, 15(3), 58-80.
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6. Wilmott, P., 2007. Paul Wilmott introduces quantitative finance. John Wiley & Sons.

# Appendix:

1. Spread Call Monte Carlo Pricing and Kirk’s Approximation in Python Codes, by Gao Jichen
2. Comparison & Visualization in R Codes, by Cheng Tuoyuan