**FE5222 ADP Project Two**

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# Introduction

Spread options have terminal payoffs based on the difference in prices between two underlying assets together with a strike. When the strike equals , the spread option is equivalent to an option to exchange one asset for another, where the Margrabe’s formula works as an explicit solution [6]. The Kirk’s approximation was published in 1995, as a valid formula when the strike is small but non-zero, where a special sigma is adopted into the generalized Black-Scholes option pricing [1,2,3].

In this project, we would investigate the spread option pricing via Kirk’s approximation, and employ the Monte Carlo simulation output as a benchmark. Results from both methods are compared and discussed in various scenarios.

# Materials and Methods

## Monte Carlo Simulation

## Kirk’s Approximation

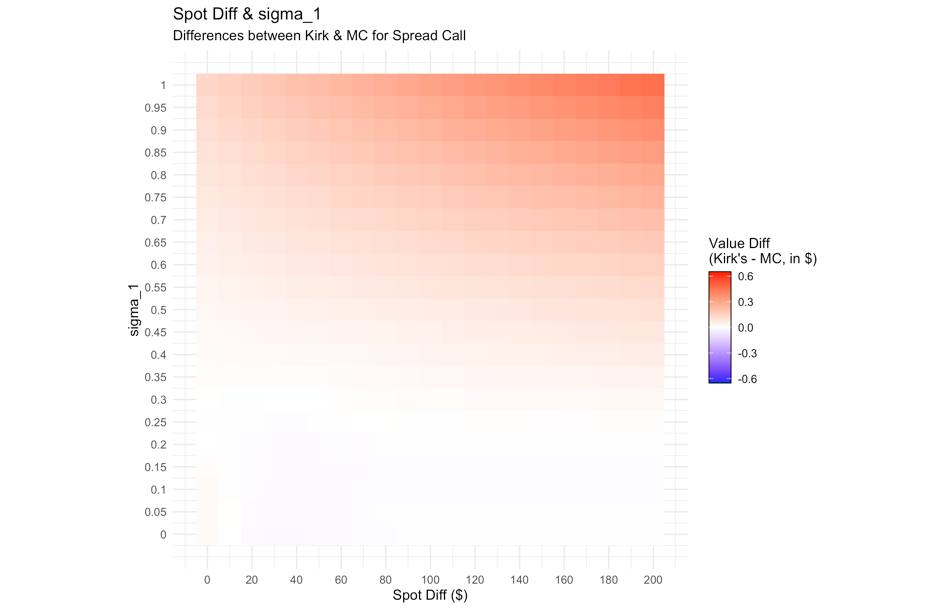
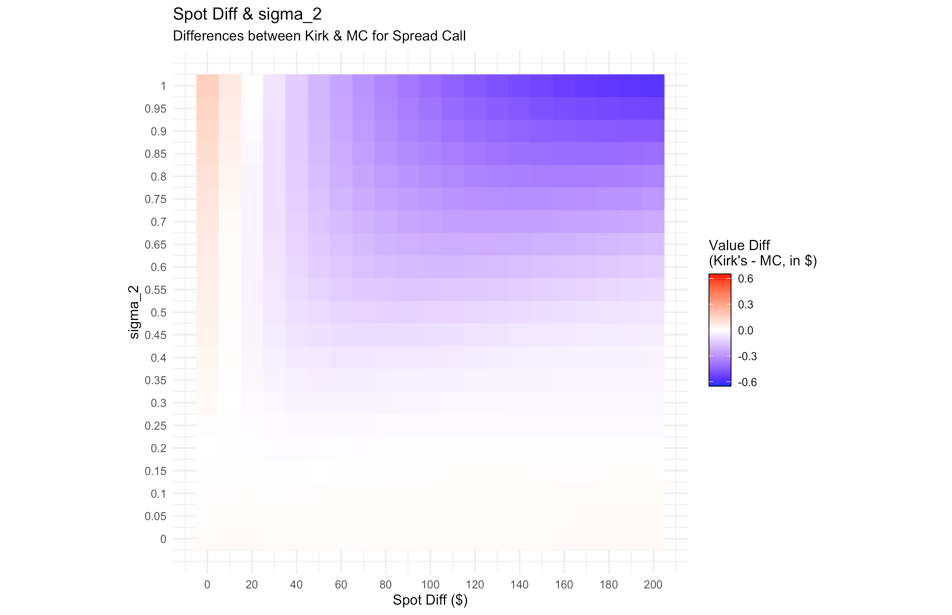
## Comparison and Visualization

To compare the two investigated methods with respect to **pricing parameters**, we further performed pairwise pricing and visualized their differences (Kirk’s - MC) over two dimensional grids with red-white-blue color scales. To improve comparability, we kept the color bar centered around $0. Investigated pricing parameters with default values includes: Spot = $100, Strike = $100, volatility () = 0.2, interest rate () = 0.06, and time to maturity () = 1. They are explored using equal spaced grids in corresponding plots but kept constant otherwise.

Investigated numerical parameters with default values includes: number of Monte Carlo paths () = 1000, number of time steps () = 252. We plotted the LSMC outputs at different and with the color bar centered around the BBSR benchmark value, then charted the BBSR outputs evolution at different . Visualizations were implemented in RStudio via packages ‘tidyverse’ and ‘ggplot2’ [4,5].

# Results and Discussion

## Grid comparison for pricing parameters

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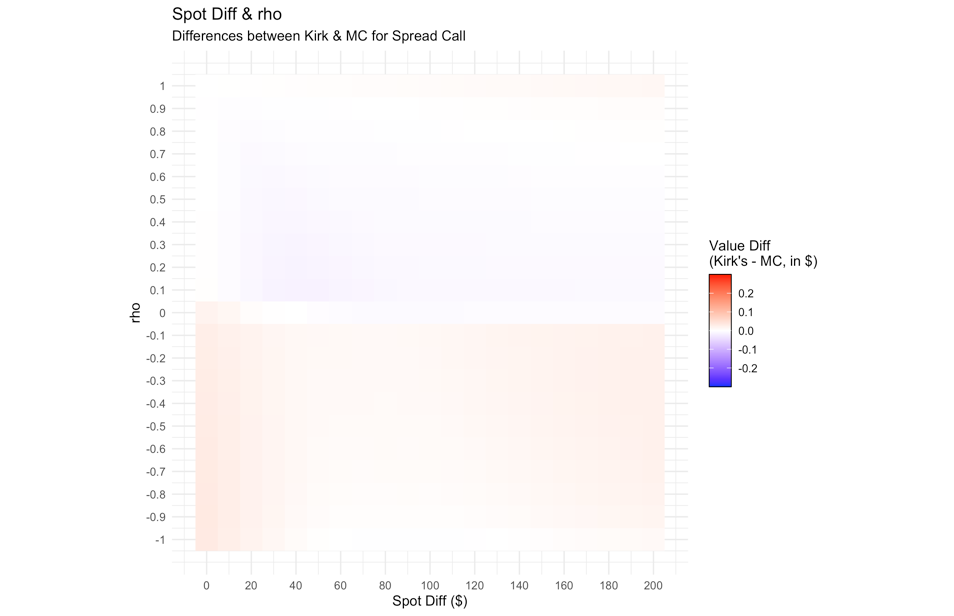
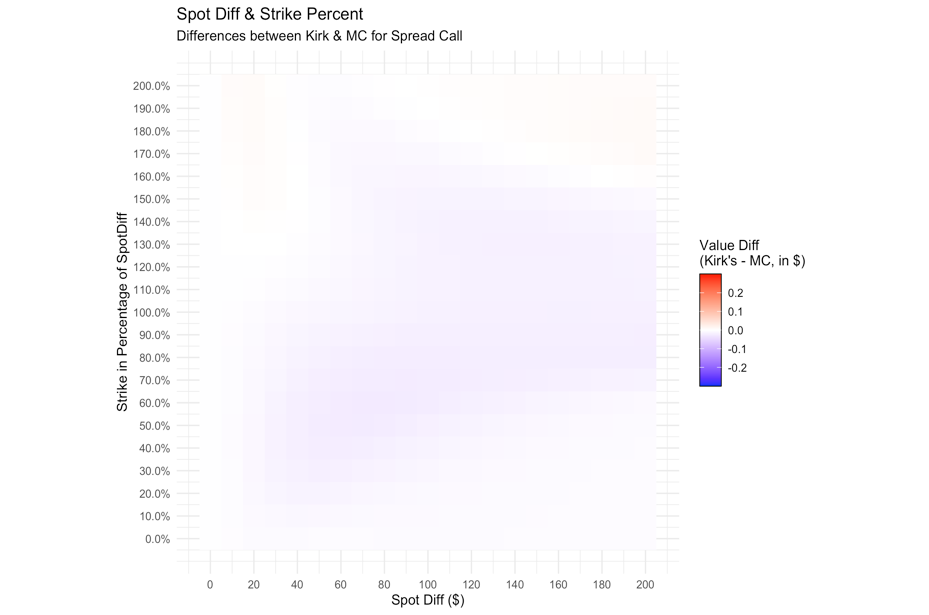
(a) (b)

Figure 1. The differences between Kirk and MC among various

(a) spot\_diff and , as well as (b) spot\_diff and volatility ().

From Fig.1(a), we observed that the LSMC outputs are deviated from BBSR less than $1 in absolute values within the inspected Spot-Strike pairs. The oscillation occurs mainly when the Spot is close to Strike and the differences are mainly positive. There might be more alternating between ITM and OTM if continue to hold for LSMC, in such scenarios upward bias due to the path used to calculate the cash flow also used for regression, more obvious (early exercise by look-ahead OTM); while the binomial trees are well arranged with approximately half leaves OTM and half leaves ITM at expiry.

From Fig.1(b), we observed that the LSMC outputs are deviated from BBSR less than 2$ in absolute values within the inspected Spot- pairs. The oscillation grows with and gets stronger when the Spot is around the Strike, with a slight skew towards higher spot. With higher , the log-Normal distribution at each time step in the GBM paths would have higher mean and lead to stronger payoff asymmetry. In comparison, BBSR only involves sigma in the step.

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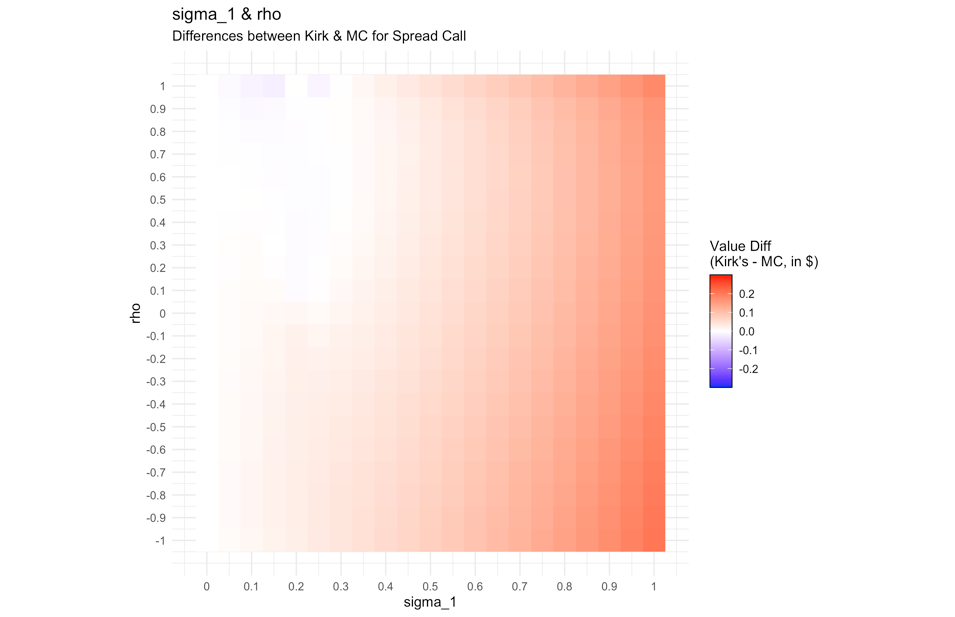
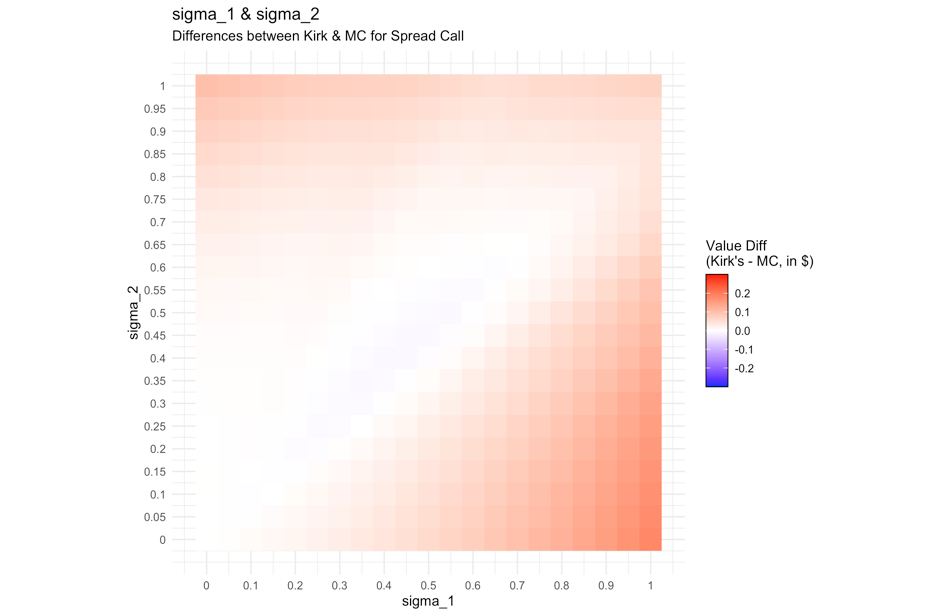
(a) (b)

Figure 2. The differences between Kirk and MC among various

(a) spot\_diff and , as well as (b) spot\_diff and strike percentage.

From Fig.2(a), we observed that the LSMC outputs are deviated from BBSR less than 1$ in absolute values within the inspected Spot-interest rate pairs. The oscillation shrinks with and gets stronger when the Spot is around the Strike. Higher could be bounding the discounted payoff and thus limiting the errors.

From Fig.2(b), we observed that the LSMC outputs are deviated from BBSR less than 1$ in absolute values within the inspected Spot-time to maturity pairs. The oscillation of the differences between two models grows with and gets stronger when the Spot is around the Strike. Longer , given constant time steps , is involving wilder fluctuation per time step in the GBM paths and thus accumulates randomness.

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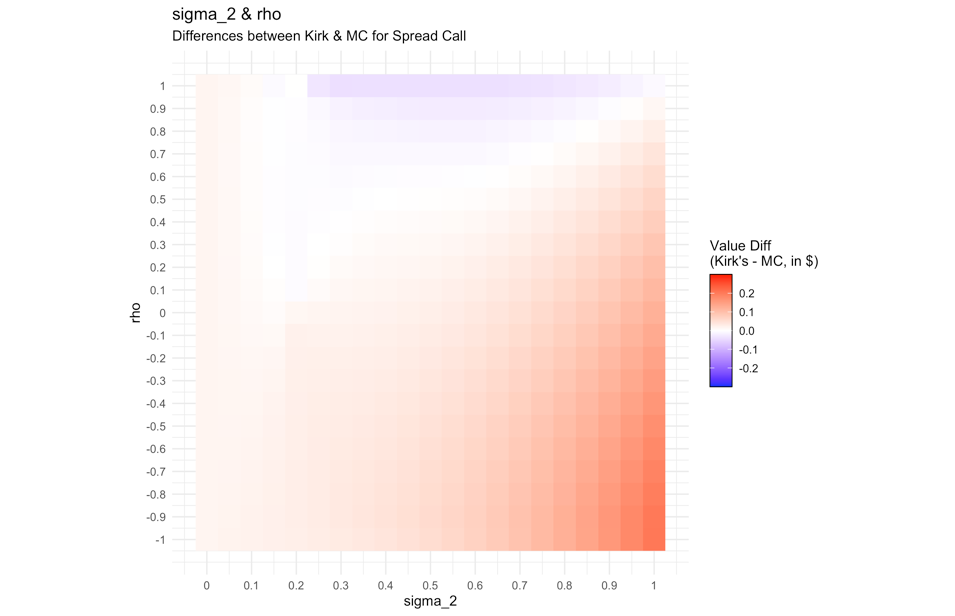
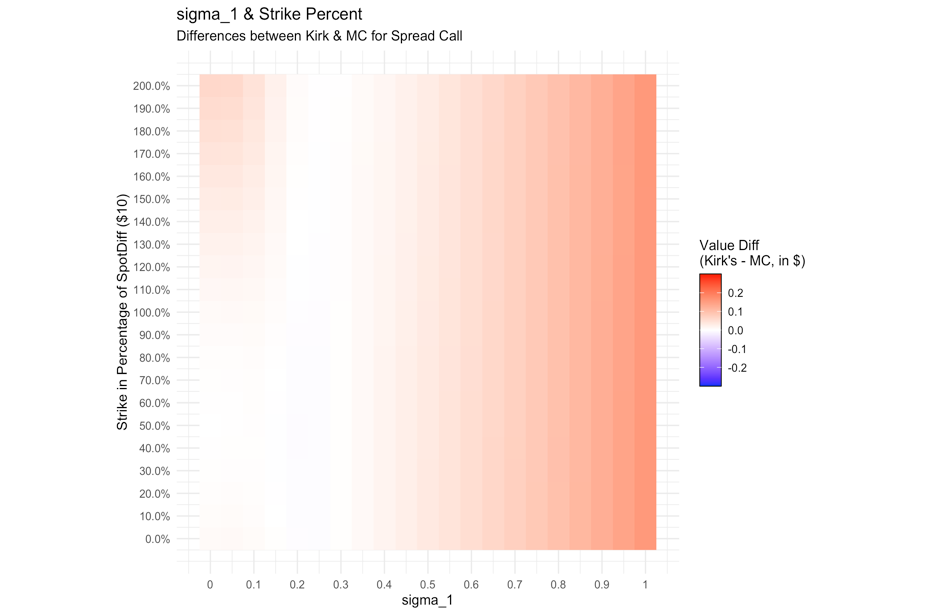
(a) (b)

Figure 3. The differences between Kirk and MC among various

(a) and , as well as (b) and .

From Fig.3(a), we observed that the LSMC outputs are deviated from BBSR less than 2$ in absolute values within the inspected Strike- pairs. The oscillation grows with sigma and gets stronger when the Strike is around the Spot, with a slight skew towards higher Strike. With higher , the log-Normal distribution at each time step in the GBM paths would have higher mean and lead to stronger payoff asymmetry. In comparison, BBSR only involves sigma in the step. Higher Strike as allowing more paths to be ITM could be inviting more randomness and thus the asymmetry.

From Fig.3(b), we observed that the LSMC outputs are deviated from BBSR less than 1$ in absolute values within the inspected Strike-interest rate pairs. The oscillation shrinks with and has a skew towards higher Strike. Higher could be bounding the discounted payoff and thus limiting the errors. Higher Strike as allowing more paths to be ITM could be inviting more randomness and thus the asymmetry.

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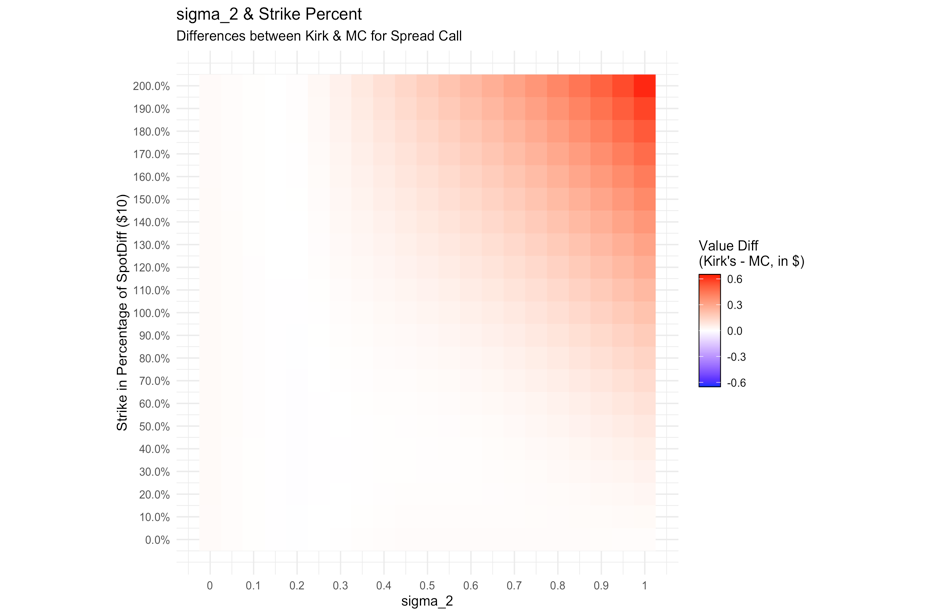
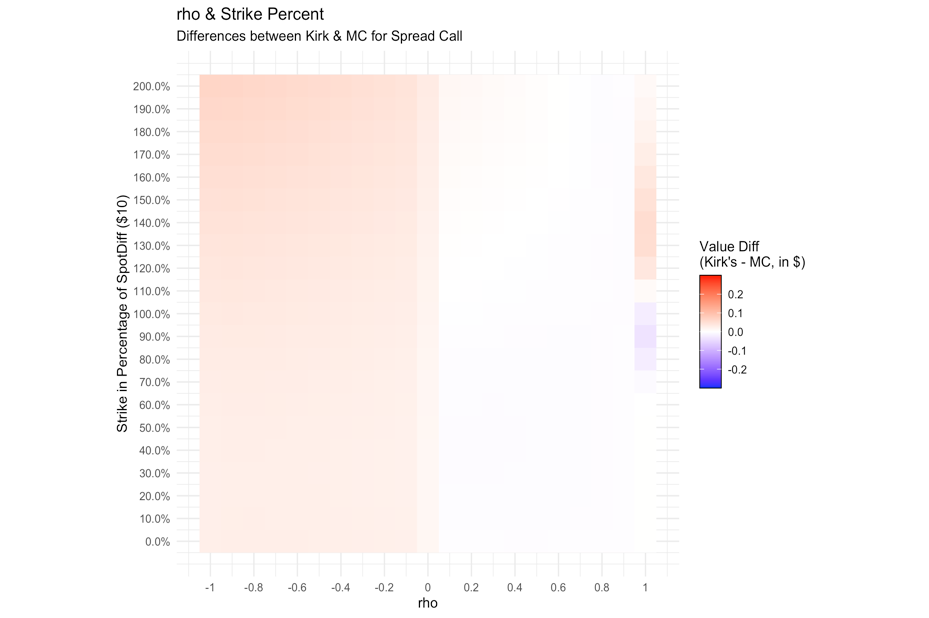
(a) (b)

Figure 4. The differences between Kirk and MC among various

1. and strike percentage, as well as (b) and .

From Fig.4(a), we observed that the LSMC outputs are deviated from BBSR less than 1$ in absolute values within the inspected Strike-time to maturity pairs. The oscillation grows with and has a slight skew towards higher Strike. Longer , given constant time steps , is involving wilder fluctuation per time step in the GBM paths and thus accumulates randomness. Higher Strike as allowing more paths to be ITM could be inviting more randomness and thus the asymmetry.

From Fig.4(b), we observed that the LSMC outputs are deviated from BBSR less than 2$ in absolute values within the inspected -interest rate pairs. The oscillation shrinks slightly with increasing but grows significantly with . Higher could be bounding the discounted payoff and thus limiting the errors. With higher , the log-Normal distribution at each time step in the GBM paths would have higher mean and lead to stronger payoff asymmetry. In comparison, BBSR only involves sigma at the step.

(a) (b)

Figure 5. The differences between Kirk and MC among various

(a) and strike percentage, as well as (b) and strike percentage.

From Fig.5(a), we observed that the LSMC outputs are deviated from BBSR less than 2$ in absolute values within the inspected -time to maturity pairs. The oscillation grows gradually with increasing and grows rapidly with increasing . Increments in involves more randomness than increments in .

From Fig.5(b), we observed that the LSMC outputs are deviated from BBSR less than 1$ in absolute values within the inspected interest rate-time to maturity pairs. The oscillation grows with longer but shrinks slowly with increasing . Increments in involves more randomness than decrements in .

# Conclusion and Future Research

In this project we investigated and implemented LSMC together with BBSR on American put options pricing, then further compared their performances in scenarios with various pricing parameters and numerical parameters. BBSR as a binomial-tree-based model showed efficient convergence within time steps and served as a stable benchmark. LSMC as a simulation-based model showed flexibility and good convergence using more than time steps and sample paths. The difference between LSMC and BBSR oscillates stronger when the Spot is around the Strike. With higher , the log-Normal distribution at each time step in the GBM paths would have higher mean and lead to stronger payoff asymmetry. In comparison, BBSR only involves sigma in the step. The oscillation shrinks with and gets stronger when the Spot is around the Strike. Higher could be bounding the discounted payoff and thus limiting the errors. Longer , given constant time steps , is involving wilder fluctuation per time step in the GBM paths and thus accumulates randomness. Higher Strike as allowing more paths to be ITM could be inviting more randomness and thus the asymmetry. Increments in involves more randomness than increments in . Increments in involves more randomness than decrements in .

For future research, we would like to experiment different basis functions in the regression, and apply LSMC on other path-dependent options. Their performances would be compared with other tree-based models.

# References

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2. Kirk, E., & Aron, J. (1995). Correlation in the energy markets. Managing energy price risk, 1, 71-78.
3. Li, M., Deng, S. J., & Zhoc, J. (2008). Closed-form approximations for spread option prices and Greeks. The Journal of Derivatives, 15(3), 58-80.
4. Wickham, H., 2016. ggplot2: elegant graphics for data analysis. springer.
5. Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L.D.A., François, R., Grolemund, G., Hayes, A., Henry, L., Hester, J. and Kuhn, M., 2019. Welcome to the Tidyverse. Journal of Open Source Software, 4(43), p.1686.
6. Wilmott, P., 2007. Paul Wilmott introduces quantitative finance. John Wiley & Sons.

# Appendix:

1. LSMC in Python Jupyter notebook, by Ho Ngok Chao
2. BBSR in Python Jupyter notebook, by Gao Jichen
3. Comparison & Visualization in R Codes, by Cheng Tuoyuan