

## Problem Set #1

Quiz, 5 questions

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1.

Given an adjacency-list representation of a directed graph, where each vertex maintains an array of its outgoing edges (but *not* its incoming edges), how long does it take, in the worst case, to compute the in-degree of a given vertex? As usual, we use  $n$  and  $m$  to denote the number of vertices and edges, respectively, of the given graph. Also, let  $k$  denote the maximum in-degree of a vertex. (Recall that the in-degree of a vertex is the number of edges that enter it.)

- ☐  $\theta(k)$
  - ☐  $\theta(n)$
  - ☐ Cannot determine from the given information
  - ☐  $\theta(m)$
-

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2.

Consider the following problem: given an undirected graph  $G$  with  $n$  vertices and  $m$  edges, and two vertices  $s$  and  $t$ , does there exist at least one  $s$ - $t$  path?

If  $G$  is given in its adjacency list representation, then the above problem can be solved in  $O(m + n)$  time, using BFS or DFS. (Make sure you see why this is true.)

Suppose instead that  $G$  is given in its adjacency \*matrix\* representation. What running time is required, in the worst case, to solve the computational problem stated above? (Assume that  $G$  has no parallel edges.)

☐  $\theta(m + n \log n)$ ☐  $\theta(n * m)$ ☐  $\theta(m + n)$ ☐  $\theta(n^2)$

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3.

This problem explores the relationship between two definitions about graph distances. In this problem, we consider only graphs that are undirected and connected. The *diameter* of a graph is the maximum, over all choices of vertices  $s$  and  $t$ , of the shortest-path distance between  $s$  and  $t$ . (Recall the shortest-path distance between  $s$  and  $t$  is the fewest number of edges in an  $s$ - $t$  path.)

Next, for a vertex  $s$ , let  $l(s)$  denote the maximum, over all vertices  $t$ , of the shortest-path distance between  $s$  and  $t$ . The *radius* of a graph is the minimum of  $l(s)$  over all choices of the vertex  $s$ .

Which of the following inequalities always hold (i.e., in every undirected connected graph) for the radius  $r$  and the diameter  $d$ ? [Select all that apply.]

☐

$r \geq d$

☐

$r \geq d/2$

☐

$r \leq d/2$

☐

$r \leq d$ 

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4.

Consider our algorithm for computing a topological ordering that is based on depth-first search (i.e., NOT the "straightforward solution"). Suppose we run this algorithm on a graph  $G$  that is NOT directed acyclic. Obviously it won't compute a topological order (since none exist). Does it compute an ordering that minimizes the number of edges that go backward?

For example, consider the four-node graph with the six directed edges  $(s, v), (s, w), (v, w), (v, t), (w, t), (t, s)$ . Suppose the vertices are ordered  $s, v, w, t$ . Then there is one backwards arc, the  $(t, s)$  arc. No ordering of the vertices has zero backwards arcs, and some have more than one.

- ☐ Always
  - ☐ If and only if the graph is a directed cycle
  - ☐ Sometimes yes, sometimes no
  - ☐ Never
- 

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point

5.

On adding one extra edge to a directed graph  $G$ , the number of strongly connected components...?

- ☐ ...will definitely not change (no matter what  $G$  is)
  - ☐ ...could remain the same (for some graphs  $G$ )
  - ☐ ...never decreases by more than 1 (no matter what  $G$  is)
  - ☐ ...never decreases (no matter what  $G$  is)
-

☐ I, **David Bai**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

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