Q1. The number of students in a class. Quantitative, discrete data.
The width of the rooms in the campus. Quantitative, continuous data.
Or, *any* other acceptable answers.

Q2. (a) $2^k = 30 \rightarrow k = \frac{\log(30)}{\log(2)} = 4.91 \approx 5\ classes$

(b) $i = \frac{57-24}{5} = 6.6 \approx 7\ \ (class\ width)$

(c)

Number of orders	Frequency	Relative frequency
24 - 30	6	0.20
31- 37	8	0.27
38 - 44	6	0.20
45 - 51	7	0.23
52 - 58	3	0.10

(d) $(0.2 + 0.23 + 0.1) \times 100\% = 53\%$

Q3. (a)

Stem	Leaf									
3	3	3	4	5	5	5	8	8		
4	0	0	1	1	5	6	6	6	7	7
5	1	3	4	6	6	8				
6	2	6	8							
7										
8	1	2	8							

Key: 3|3 = 33

The data is skewed to the right.

(b) $mean = \frac{1495}{30} = 49.83$, $median = 46$
Since $median < mean$, it suggests a right-skewed distribution. This coincide the findings in part (a).

(c) $Q_1 = 38,\ \ Q_3 = 56,\ \ IQR = 56 - 38 = 18,\ \ 1.5IQR = 27$
$Q_1 - 1.5IQR = 11$
$Q_3 + 1.5IQR = 83$

In this data, the observation 88 is well outside of the interval [11, 83]. This observation may be an outlier.

Q4. (a) $\sum mf = 379,\ \sum m^2 f = 6709$

$\bar{x} = \frac{379}{24} = 15.792$

$s^2 = \frac{1}{23}\left[6709 - \frac{379^2}{24}\right] = 31.4764$

$s = 5.61$

(b) $84\% \ \rightarrow\ 1 - \frac{1}{k^2} = 0.84 \ \rightarrow\ k = \pm 2.5$

$\bar{x} \pm 2.5s = 15.792 \pm 2.5(5.61) = 15.792 \pm 14.025$

According to the Chebyshev's theorem, at least 84% of the numbers computer sold for the past 24 weeks falls between 1.767 and 29.817 units.

Q5. There are 6 ways (6 outcomes of the green die) to get each 1, 2 or 3 on the red die. There are 2 ways to get a total of 11 from 2 dice, 5 and 6, or 6 and 5.

$P(win) = \frac{6+6+6+2}{36} = \frac{20}{36} = \frac{5}{9} = 0.5556$

Q6. Let D = assembled computer has a defective part in it

$P(X|D) = \frac{P(X \cap D)}{P(D)} = \frac{0.6(0.05)}{0.6(0.05)+0.4(0.1)} = 0.4286$

Q7. (a) $P(A \cap B) = \frac{1}{4} \times \frac{2}{5} = \frac{1}{10} = 0.1$

(b) $P(B) = \frac{3}{5} - \frac{2}{5} + \frac{1}{10} = \frac{3}{10} = 0.3$

(c) $P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{0.3}{0.7} = \frac{3}{7} = 0.4286$

(d) $P(A \cap B \cap C) = \frac{1}{2} \times \frac{1}{10} = \frac{1}{20} = 0.05$

$P(B \cap C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = 0.1$

$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.05}{0.1} = \frac{1}{2} = 0.5$

Q8. For components A and D to function properly,

$P(G) = P(A \cup D) = 1 - P(\bar{A} \cap \bar{D}) = 1 - 0.1(0.4) = 0.96$

For components B, C and E to function properly,

$P(H) = P(B \cup C \cup E) = 1 - P(\bar{B} \cap \bar{C} \cap \bar{E}) = 1 - 0.2(0.3)(0.5) = 0.97$

For the system to function properly,

$P(G \cap H) = 0.96 \times 0.97 = 0.9312$

Q9. (a) ${}^{16}P_{10} = 2.91 \times 10^{10}$

(b) $4! \times {}^{12}P_6 = 15966720$

(c) $n(ways) = n(2\ couples\ \&\ 1\ unmarried) + n(2\ couples\ \&\ 1\ married)$

$= {}^{3}C_2 \times {}^{4}C_1 + {}^{6}C_5 = 12 + 6 = 18$

Q10. (a)

<table>
<tr><td colspan="2" rowspan="2">$P_{X,Y}(x,y)$</td><td colspan="3">y</td><td rowspan="2">$P_X(x)$</td></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td rowspan="2">x</td><td>0</td><td>$\frac{1}{12}$</td><td>$\frac{2}{12}$</td><td>0</td><td>0.25</td></tr>
<tr><td>1</td><td>0</td><td>$\frac{4}{12}$</td><td>$\frac{5}{12}$</td><td>0.75</td></tr>
<tr><td colspan="2">$P_Y(y)$</td><td>$\frac{1}{12}$</td><td>$\frac{6}{12}$</td><td>$\frac{5}{12}$</td><td>1</td></tr>
</table>

(b) $P_{X,Y}(1,1) = 0,\ P_X(1)P_Y(1) = 0.75 \times \frac{1}{12} = 0.0625$

$Since\ P(x,y) \neq P(x)P(y)\ for\ at\ least\ one\ pair\ of\ (x,y),$

$\therefore\ random\ variables\ X\ and\ Y\ are\ not\ independent.$

Q11.

$$\begin{aligned} Var(3X-2Y) &= 9Var(X) + 4Var(Y) + 2(3)(-2)Cov(X,Y) \\ &= 9(4) + 4(16) - 12(2) = 76 \end{aligned}$$

Q12. (a) For independent X and Y, $Cov(X,Y) = 0, Var(X+Y) = Var(X) + Var(Y)$

$A = 125X$

$E(A) = 125E(X) = 125[(20)(0.5) + (-12)(0.5)] = 125(4) = 500$

$Var(A) = 125^2 Var(X) = 125^2[272 - 4^2] = 4000000$

$B = 100Y$

$E(B) = 100E(Y) = 100[(20)(0.5) + (-10)(0.5)] = (100)5 = 500$

$Var(B) = 100^2 Var(Y) = 100^2[250 - 5^2] = 2250000$

$C = 50X + 60Y$

$E(C) = 50E(X) + 60E(Y) = 50(4) + 60(5) = 500$

$Var(C) = 50^2 Var(X) + 60^2 Var(Y) = 50^2(256) + 60^2(225) = 1450000$

In terms of expected return, all three portfolios are equivalent. Portfolio C, where investment is split between two companies, has the lowest variance. Therefore, it is the least risky. It is better to invest in portfolio C.

(b) Only the volatility of portfolio C changes due to the correlation coefficient.

$Cov(X,Y) = -0.2 \times \sqrt{256} \times \sqrt{225} = -48$

$$\begin{aligned} Var(C) &= 50^2 Var(X) + 60^2 Var(Y) + 2(50)(60)Cov(X,Y) \\ &= 1450000 - 288000 = 1162000 \end{aligned}$$

Nevertheless, the diversified portfolio C is still optimal.