Tutorial 4 Continuous Distributions

1. The lifetime, in years, of some electronic component is a continuous random variable with the density

$$f(x)=\begin{cases}\dfrac{k}{x^4} & \text{for } x\geq 1\\ 0 & \text{for } x<1\end{cases}.$$

Find k, the cumulative distribution function, and the probability for the lifetime to exceed 2 years. [3, 1-1/x^3, 0.125]

2. The installation time, in hours, for a certain software module has a probability density function

$$f(x)=k(1-x^3) \text{ for } 0<x<1.$$

Find k and compute the probability that it takes less than $1/2$ hour to install this module. [4/3, 31/48]

3. Lifetime of a certain hardware is a continuous random variable with density

$$f(x)=\begin{cases}K-\dfrac{x}{50} & \text{for } 0<x<10 \text{ years}\\ 0 & \text{for all other } x\end{cases}$$

a. Find K. [0.2]
b. What is the probability of a failure within the first 5 years? [0.75]
c. What is the expectation of the lifetime? [10/3]

4. The time it takes a printer to print a job is an Exponential random variable with the expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be ready before 10:01? [0.875]

5. For some electronic component, the time until failure has Gamma distribution with parameters $\alpha=2$ and $\lambda=2$ (years^{-1}). Compute the probability that the component fails within the first 6 months. [0264]

6. Two computer specialists are completing work orders. The first specialist receives 60% of all orders. Each order takes her Exponential amount of time with parameter $\lambda_1=3\text{ hrs}^{-1}$. The second specialist receives the remaining 40% of orders. Each order takes him Exponential amount of time with parameter $\lambda_2=2\text{ hrs}^{-1}$.
A certain order was submitted 30 minutes ago, and it is still not ready. What is the probability that the first specialist is working on it? [0.4764]

7. The time X it takes to reboot a certain system has Gamma distribution with $E(X)=20$ min and $Std(X)=10$ min. [4, 0.2, 0.3528]
   a. Compute parameters of this distribution.
   b. What is the probability that it takes less than 15 minutes to reboot this system?

8. A certain system is based on two independent modules, A and B. A failure of any module causes a failure of the whole system. The lifetime of each module has a Gamma distribution, with parameters α and λ given in the table,

Component	α	λ (*years*$^{-1}$)
A	3	1
B	2	2

   a. What is the probability that the system works at least 2 years without a failure?
   b. Given that the system failed during the first 2 years, what is the probability that it failed due to the failure of component B (but not component A)? [0.062, 0.655]

9. Let Z be a Standard Normal random variable. Compute
   a. $P(Z<1.25)$ **b.** $P(Z\le 1.25)$ **c.** $P(Z>1.25)$
   d. $P(|Z|\le 1.25)$ **e.** $P(Z<6.0)$ **f.** $P(Z>6.0)$
   g. With probability 0.8, variable Z does not exceed what value?
   [0.8944, 0.8944, 0.1056, 0.7888, 1, 0, 0.8416]

10. Installation of some software package requires downloading 82 files. On the average, it takes 15 sec to download one file, with a variance of 16 sec^2. What is the probability that the software is installed in less than 20 minutes? [0.2033]

11. Seventy independent messages are sent from an electronic transmission centre. Messages are processed sequentially, one after another. Transmission time of each message is Exponential with parameter $\lambda=5$ min^{-1}. Find the probability that all 70 messages are transmitted in less than 12 minutes. Use the Central Limit Theorem. [0.1151]

12. A computer lab has two printers. Printer I handles 40% of all the jobs. Its printing time is Exponential with the mean of 2 minutes. Printer II handles the remaining 60% of jobs. Its printing time is Uniform between 0 minutes and 5 minutes. A job was printed in less than 1 minute. What is the probability that it was printed by Printer I? [0.5674]

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Q3. (c) A computer processes tasks in the order they are received. Each task takes an Exponential amount of time with the average of 2 minutes. Compute the probability that a package of 6 tasks is processed in less than 10 minutes.

(5 marks)

(d) Donald Electronics Sdn. Bhd. offers a "no hassle" returns policy. The number of items returned per day follows the normal distribution. The mean number of customer returns is 8 per day and the standard deviation is 2 per day.

(i) Compute the percent of the days are between 10 and 12 customers returning items. (4 marks)

(ii) Is there any chance of a day with no returns? Justify your answer.

(4 marks)

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Q2. (b) Suppose that the average number of machine breakdowns per working day during the production process is 3. By using normal approximation to Poisson distribution, find the probability of having less than 15 breakdowns over 10 working days. (7 marks)

(c) Random variable X is normally distributed with mean 52 and standard deviation 13.14. Correct to 2 decimal places, find

(i) the value of *m* if $P(X > m) = 0.25$, (4 marks)

(ii) the value of *n* if $P(X < k) = 0.20$. (4 marks)