UNIVERSITI TUNKU ABDUL RAHMAN

ACADEMIC YEAR 2017/2018

SEPTEMBER EXAMINATION

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

SUNDAY, 3 SEPTEMBER 2017 TIME : 2.00 PM – 4.00 PM (2 HOURS)

BACHELOR OF COMPUTER SCIENCE (HONS)
BACHELOR OF INFORMATION SYSTEMS (HONS)
INFORMATION SYSTEMS ENGINEERING
BACHELOR OF INFORMATION SYSTEMS (HONS)
BUSINESS INFORMATION SYSTEMS
BACHELOR OF INFORMATION TECHNOLOGY (HONS)
COMPUTER ENGINEERING
BACHELOR OF INFORMATION TECHNOLOGY (HONS)
COMMUNICATIONS AND NETWORKING

Instruction to Candidates :

This question paper consists of FIVE (5) questions.

Answer **any FOUR (4)** questions only. Each question carries 25 marks.

Should a candidate answer more than FOUR (4) questions, marks will only be awarded for the FIRST FOUR (4) questions in the order the candidate submits the answers.

Candidates are allowed to use a calculator.

Answer questions only in the answer booklet provided.

Q1. (a) How many ways are there to seat 15 people in a row? (2 marks)

(b) How many 3 digit numbers can you make using the digits 1, 2 and 3 without repetitions? (3 marks)

(c) In how many ways can the letters of the word 'COOPERATIVE' be arranged? (4 marks)

(d) How many integer solutions are there for the equation

$$a_1 + a_2 + a_3 + a_4 + a_5 = 15$$

(i) where $a_i \geq 0$ for each a_i? (2 marks)

(ii) where $a_i > 0$ for each a_i? (2 marks)

(iii) where $a_i \geq 2$ for each a_i? (3 marks)

(e) How many ways can 10 teachers and 4 students be seated

(i) around a round table? (2 marks)

(ii) in a row if no two students can sit next to each other? (3 marks)

(iii) around a round table if no two students are allowed to sit next to each other? (4 marks)

[Total : 25 marks]

Q2. (a) Let X be the number of cups of coffee you drink per day. Suppose X is a number from $\{0, 1, 2, 3\}$ with equal probability. (Hence you drink at most 3 cups of coffee per day.) Let Y be the number of cups of coffee you drink before dinner each day. (Hence Y cannot be larger than X.) Suppose Y is a number from 0 to X with equal probability.

(i) Give all the values for $P(X, Y)$, the joint probability distribution. (4 marks)

(ii) Find $P(X)$ for $0 \leq X \leq 3$, and $P(Y)$ for $0 \leq Y \leq 3$. (2 marks)

(iii) Show that X and Y are not independent. (3 marks)

(iv) Find the expected value for X and Y respectively. (2 marks)

(v) Give all the values for $P(X \mid Y)$ for $0 \leq X \leq 3$ and $0 \leq Y \leq 3$. (4 marks)

(b) A guy/girl who knows you smiled at you. You know that he/she smiles at 99% of the people he/she knows and likes. You also know that he/she smiles at the people he/she knows 50% of the time, and that he/she likes 1% of all the people he/she knows.

(i) Find the probability that he/she likes you. (4 marks)

(ii) How will each of the following conditions affect the chances that he/she likes you?

a) If he/she smiles more at the people he/she likes,
b) If he/she smiles more at people,
c) If he/she likes more people he/she knows. (6 marks)

[Total : 25 marks]

Q3. (a) Let X be a continuous random variable with the probability density function:

$$f(x)=\begin{cases} c-x^2 & 1<x<2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the value of c. (5 marks)

(ii) Find the expected value of X. (4 marks)

(iii) Find the variance of X. (4 marks)

(b) Alice goes to lunch at time x, which has the following probability density function:

$$f(x)=\begin{cases} \frac{10}{3}-x^2 & 1\le x\le 2 \\ 0 & \text{otherwise.} \end{cases}$$

On the other hand, Bob first comes into the office at some time y between 1:00pm and 2:00pm, with uniform probability.

(i) Derive $P(x, y)$, the joint density of x and y. (4 marks)

(ii) Find the probability that Bob returns after Alice goes out for lunch.
(8 marks)
[Total : 25 marks]

Q4. (a) The per-day number of downloads N of your app on Playstore follows a Poisson distribution with rate λ=100 (per day).

(i) Give the expected value and variance for N. (3 marks)

(ii) Find $P\{N < 100\}$, the probability for the total number of downloads in a day to be less than 100. ($P\{N > 100\}$=0.4734 at λ=100.) (5 marks)

(iii) State whether $P\{N < 100\}$ is higher or lower for each of your other apps which have respectively,

a. λ=99 (per day),
b. λ=5 (per hour),
c. λ=0.05 (per minute). (6 marks)

(b) A company claims that its smartphone boots up within 30 seconds. You tried booting the device 10 times and obtained the times: 31.2, 33.8, 29.2, 30.6, 31.6, 30.2, 31.1, 32.4, 32.5 and 33.1.

(i) Find the average and standard deviation of the times you obtained. (5 marks)

(ii) Assuming that σ = 3.0, do we have sufficient reason to reject the company's claim at p=0.05? (Use $z_{0.05}$=1.645 and $z_{0.025}$=1.96.) (6 marks)

[Total : 25 marks]

Q5. You have a coin that, when flipped, ends up head with probability p and ends up tail with probability $1-p$. You flipped the coin 3 times, and obtained 1 head and 2 tails.

(a) Find the maximum likelihood estimate of p. (8 marks)

(b) Show that $P\{1 \text{ head out of } 3 \mid p\} \propto p(1-p)^2$. (2 marks)

(c) Show that $P\{\, p \mid 1 \text{ head out of } 3\} \propto \text{Beta}(p;\ \alpha + 1, \beta + 2)$, using the Beta distribution as a conjugate prior. (6 marks)

(d) Assuming that $\alpha = 1$ and $\beta = 1$ in Q5.(c), estimate p by integrating p over [0,1] under the distribution $\text{Beta}(p;\ 2,\ 3)$. (9 marks)

[Total : 25 marks]

Appendix

1. Algebra:
   (i) $\ln x^y = y \ln x$
   (ii) $b^3-a^3=(b-a)(b^2+ab+a^2)$
   (iii) For $ax^2+bx+c=0$, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

2. Calculus:
   (i) $\dfrac{d}{dx}\ln x=\dfrac{1}{x}$
   (ii) $\displaystyle\int_a^b x^n dx=\frac{1}{n+1}x^{n+1}\Big|_a^b$

3. Statistics:
   (i) $(1+\alpha)100\%$ confidence interval for θ:
   $$[\hat{\theta}-z_{\alpha/2}Std(\hat{\theta}),\ \hat{\theta}+z_{\alpha/2}Std(\hat{\theta})].$$
   (ii) Z-test, $Z=\frac{\hat{\theta}-\theta}{\sigma/\sqrt{n}}$.
   (iii) Probability mass function for a Poisson distribution,
   $$P\{N=n\}=e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$
   (iv) Probability density function for a Gamma distribution
   $$\text{Gamma}(x\mid\alpha,\beta)=\begin{cases}\dfrac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda\beta} & \text{if } x\ge 0\\ 0 & \text{otherwise}\end{cases}$$
   where $\Gamma(x)$ is the Gamma function (the exact form of $\Gamma(x)$ is not needed in this examination).
   (iv) Probability density function for a Beta distribution,
   $$\text{Beta}(p;\ \alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-\text{p})^{\beta-1},$$
   where $\Gamma(x)$ is the Gamma function (the exact form of $\Gamma(x)$ is not needed in this examination).