Tutorial 3   Discrete Random Variables and Their Distributions

1. A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3.
    a.   Compute the probability mass function (*pmf*) of X, the number of corrupted files.
    b.   Draw a graph of its cumulative distribution function (*cdf*).

2. Every day, the number of network blackouts has a distribution (*pmf*)

x	0	1	2
$P(x)$	0.7	0.2	0.1

    A small internet trading company estimates that each network blackout results in a $500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

3. There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let X be the number of errors in these three blocks. Compute $E(X)$ and $Var(X)$.

4. Every day, the number of traffic accidents has the probability mass function

x	0	1	2	more than 2
$P(x)$	0.6	0.2	0.2	0

    independently of other days. What is the probability that there are more accidents on Friday than on Thursday?

5. Two random variables, X and Y, have the joint distribution $P(x, y)$,

$P(x, y)$		x	
		0	1
y	0	0.5	0.2
	1	0.2	0.1

    Are X and Y independent? Explain.

6. Let X and Y be the number of hardware failures in two computer labs in a given month. The joint distribution of X and Y is given in the table below.

$P(x, y)$		x		
		0	1	2
y	0	0.52	0.20	0.04
	1	0.14	0.02	0.01
	2	0.06	0.01	0

    a.   Compute the probability of at least one hardware failure.
    b.   From the given distribution, are X and Y independent? Why or why not?

7.   Shares of company A are sold at $10 per share. Shares of company B are sold at $50 per share. According to a market analyst, 1 share of each company can either gain 1, with probability 0.5, or lose $1, with probability 0.5, independently of the other company. Which of the following portfolios has the lowest risk:
   a.   100 shares of A
   b.   50 shares of A + 10 shares of B
   c.   40 shares of A + 12 shares of B

8.   Shares of company A cost $10 per share and give a profit of X%. Independently of A, shares of company B cost 50 per share and give a profit of Y%. Deciding how to invest $1,000, Michael chooses between 3 portfolios:
   a.   100 shares of A,
   b.   50 shares of A and 10 shares of B,
   c.   20 shares of B.

   The distribution of X is given by probabilities:
   $$P(X=-3)=0.3, \ P(X=0)=0.2, \ P(X=3)=0.5.$$
   The distribution of Y is given by probabilities:
   $$P(Y=-3)=0.4, \ P(Y=3)=0.6.$$
   Compute expectations and variances of the total dollar profit generated by portfolios (a), (b), and (c). What is the least risky portfolio? What is the most risky portfolio?

9.   A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other.
   a.   Find the probability of exactly 3 defective computers in a shipment of twenty.
   b.   Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

10.   A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the probability that it entered at least 10 computers.

11.   Every day, a lecture may be cancelled due to inclement weather with probability 0.05. Class cancellations on different days are independent.
   a.   There are 15 classes left this semester. Compute the probability that at least 4 of them get cancelled.
   b.   Compute the probability that the tenth class this semester is the third class that gets cancelled.

12.   Messages arrive at an electronic message centre at random times, with an average of 9 messages per hour.
   a.   What is the probability of receiving at least five messages during the next hour?
   b.   What is the probability of receiving exactly five messages during the next hour?

13.   On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.
   a.   Compute the probability that less than 10 computers crashed.
   b.   Compute the probability that exactly 10 computers crashed.
   You may want to use a suitable approximation.

14.   Network breakdowns are unexpected rare events that occur every 3 weeks, on the average. Compute the probability of more than 4 breakdowns during a 21-week period.

UECM1633 Probability and Statistics for Computing

Answers to selected exercises.
1.   (a) P(0)=0.42, P(1)=0.46, P(2)=0.12
2.   E(Y) = 200 dollars, Var(Y) = 110, 000 squared dollars
3.   E(X) = 0.6, Var(X) = 0.24
4.   0.28
5.   Dependent.
6.   (a) 0.48   (b) Dependent.
7.   Third portfolio.
8.   (a) E(Profit)=6, Var(Profit)=684.   (b) E(Profit)=6, Var(Profit)=387.
        (c) E(Profit)=6, Var(Profit)=864. The least risky portfolio is (b);
           the most risky portfolio is (c).
9.   (a) 0.0596   (b) 0.9860
10.   0.2447
11.   (a) 0.0055   (b) 0.00314
12.   (a) 0.945   (b) 0.061
13.   (a) 0.968   (b) 0.018
14.   0.827