UNIVERSITI TUNKU ABDUL RAHMAN

ACADEMIC YEAR 2017/2018

DECEMBER EXAMINATION

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

FRIDAY, 15 DECEMBER 2017 TIME : 9.00 AM – 11.00 AM (2 HOURS)

BACHELOR OF COMPUTER SCIENCE (HONS)
BACHELOR OF INFORMATION SYSTEMS (HONS)
INFORMATION SYSTEMS ENGINEERING
BACHELOR OF INFORMATION SYSTEMS (HONS)
BUSINESS INFORMATION SYSTEMS
BACHELOR OF INFORMATION TECHNOLOGY (HONS)
COMPUTER ENGINEERING

Instructions to Candidates :

This question paper consists of **FIVE (5)** questions.

Answer **any FOUR (4)** questions only. Each question carries 25 marks.

Should a candidate answer more than **FOUR (4)** questions, marks will only be awarded for the **FIRST FOUR (4)** questions in the order the candidate submits the answers.

Candidates are allowed to use a calculator.

Answer questions only in the answer booklet provided.

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

Q1. (a) How many ways are there to seat 4 people in a row? (2 marks)

(b) In how many ways can the letters of the word 'APPLE' be arranged? (3 marks)

(c) A fair dodecahedral dice has uniform probability, p, for all faces 1, 2,..., and 12.

(i) What is the probability of getting '5' in a single throw? (2 marks)

(ii) What is the probability of getting '5' in first throw and getting '4' in second throw? (3 marks)

(iii) What is the probability that '7' does not appear in the first n throws? Give your answer in terms of n. (3 marks)

(d) You toss a fair coin three times:

(i) What is the probability of three tails, TTT? (3 marks)

(ii) What is the probability that you observe exactly one head? (4 marks)

(iii) Given that you have observed at least one head, what is the probability that you observe at least two heads? (5 marks)

[Total : 25 marks]

Q2. (a) The discrete random variable X has probability mass function

$$f(x) = \begin{cases} kx, & if\ x = 1,2,3,4,5 \\ 0, & otherwise \end{cases}$$

where k is a positive constant.

(i) Find the value of k. (3 marks)

(ii) Calculate the mean of X. (4 marks)

(iii) Calculate the variance of X. (5 marks)

Q2. (Continued)

(b) A random variable has a Poisson distribution with positive mean and such that $P(X = 2) = 3\,P(X = 4)$.

Compute the value of $P(X = 6)$. (6 marks)

(c) There is a large number of color light bulbs which are continually being switched on and off. Individual lights fail at random times, and each day the display is inspected and any failed lights are replaced. The number of lights that failed in any one-day period has a Poisson distribution with mean 2.2.

Calculate the probability that out of any seven days, there are at least two days, in each of which at least four light bulbs need to be replaced. (7 marks)

[Total : 25 marks]

Q3. (a) The continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0, \\ \dfrac{5}{4} - x, & 0 \le x < 1, \\ \dfrac{1}{4x^2}, & x \ge 1. \end{cases}$$

(i) Find the cumulative distribution function of X. (6 marks)

(ii) Find the probability that an observed value of X is greater than three. (3 marks)

(b) Mr. Naruto arrives at a bus stop at 10:00 a.m., knowing that the bus will arrive at some time uniformly distributed between 10:00 a.m. and 10:30 a.m.

(i) What is the probability that he has to wait longer than 10 minutes? (3 marks)

(ii) If at 10:15 a.m. the bus has not yet arrived, what is the probability that he has to wait at least an additional 10 minutes? (5 marks)

Q3. (Continued)

(c) A train is scheduled to arrive at the Kampar Station at 8.15 a.m., but the actual time of arrival is normally distributed about a mean of 8.18 a.m. with a standard deviation of 3.7 minutes.

(i) Find the probability that the train arrives before 8.22 a.m. (4 marks)

(ii) Find the probability that the train is later than 8.15 a.m. (4 marks)

[Total : 25 marks]

Q4. (a) The following measurements were recorded for the temperature, in Celsius, of a certain area in Kampar.

25.6	30.2	32.1	35.8	27.9
28.4	27.6	29.6	36.2	33.3
29.7	34.5	28.8	27.6	31.1
29.2	32.9	34.5	27.6	31.4

Assume that the measurement is a simple random sample.

(i) Calculate the sample mean for these data. (3 marks)

(ii) Calculate the sample median. (3 marks)

(iii) Plot the data using a dot plot. (3 marks)

(iv) Compute the 10% trimmed mean for the above data set? (2 marks)

(b) The weights of a random sample of 100 college students show a mean of 80.5 kg and a standard deviation of 8.0 kg.

$n = 100, \quad \bar{x} = 80.5, \quad \sigma = 8.0, \quad$ and $\quad z_{0.005} = 2.575$

(i) Construct a 99% confidence interval for the mean weight of all college students. (3 marks)

(ii) What can we assert with 99% confidence about the possible size of our error if we estimate the mean weight of all college students to be 80.5 kg? (2 marks)

Q4. (Continued)

(c) A manufacturer has produced a new polyester nylon string, which the company claims has a mean breaking strength of 18 kg with a standard deviation of 0.8 kg. To test the hypothesis that $\mu = 18$ kg against the alternative that $\mu < 18$ kg, a random sample of 50 lines will be tested. The critical region is defined to be $\bar{x} < 17.9$.

(i) Find the probability of committing a type I error when H_0 is true. (3 marks)

(ii) Evaluate β for the alternatives $\mu = 17.8$ and $\mu = 17.9$ kg. (6 marks)

[Total : 25 marks]

Q5. (a) An electronic manufacturer believes that the proportion of orders for raw material arriving late is $p = 0.7$. If a random sample of 10 orders shows that 4 or fewer arrived late, the hypothesis that $p = 0.7$ should be rejected in favor of the alternative $p < 0.6$

(i) Use binomial distribution, find the probability of committing a type I error if the true proportion is $p = 0.7$. (3 marks)

(ii) Use binomial distribution, find the probability of committing a type II error for the alternatives $p = 0.4$ and $p = 0.5$. (6 marks)

(iii) Assume that 50 orders are selected and the critical region is defined to be $x \leq 30$, where x is the number of orders in the sample that arrived late. Use normal approximation, find the probability of committing a type I error if the true proportion is $p = 0.7$ and the probability of committing a type II error for the alternatives $p = 0.5$. (8 marks)

(b) Assume that the prior distribution for the proportion p of snacks from a vending machine that overflows is

p	0.05	0.10	0.15
$n(p)$	0.4	0.5	0.1

If 3 of the next 10 drinks from this machine overflow, find

(i) the posterior distribution for the proportion p, and (6 marks)

(ii) the Bayes estimate of p. (2 marks)

[Total : 25 marks]

Appendix

1. Standard Normal Probabilities Table

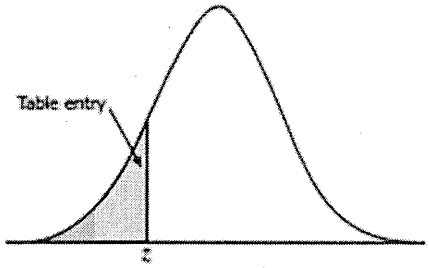

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

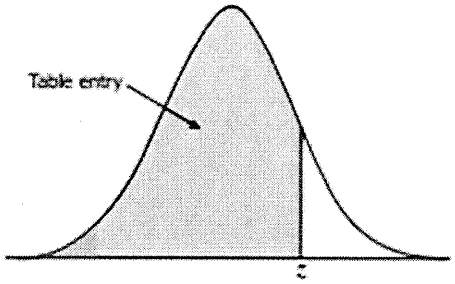

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998