Tutorial 5 Statistical Inference

1. In order to ensure efficient usage of a server, it is necessary to estimate the mean number of concurrent users. According to records, the average number of concurrent users at 100 randomly selected times is 37.7, with a standard deviation $\sigma = 9.2$.
   a. Construct a 90% confidence interval for the expectation of the number of concurrent users. [36.19, 39.21]
   b. At the 1% significance level, do these data provide significant evidence that the mean number of concurrent users is greater than 35? [yes]

2. Salaries of entry-level computer engineers have Normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries (in $1000s):
   30, 50, 70
   a. Construct a 90% confidence interval for the average salary of an entry-level computer engineer. [16.28, 83.72]
   b. Does this sample provide a significant evidence, at a 10% level of significance, that the average salary of all entry-level computer engineers is different from $80,000? Explain. [no]

3. We have to accept or reject a large shipment of items. For quality control purposes, we collect a sample of 200 items and find 24 defective items in it.
   a. Construct a 96% confidence interval for the proportion of defective items in the whole shipment. [0.073, 0.167]
   b. The manufacturer claims that at most one in 10 items in the shipment is defective. At the 4% level of significance, do we have sufficient evidence to disprove this claim? Do we have it at the 15% level? [no, no]

   Now having looked at the collected sample, we consider an alternative supplier. A sample of 150 items produced by the new supplier contains 13 defective items. Is there significant evidence, at a 5% level of significance, that the quality of items produced by the new supplier is higher than the quality of items produced by the previous supplier? What is the p-value? [no, 0.1587]

4. A company opened a new movie theater. Before setting the price of a movie ticket, the company wants to find the average price charged by other movie theaters. A random sample of 100 movie theaters taken by the company showed that the mean price of a movie ticket is RM6.75 with a standard deviation of RM0.80.
   a. What is the point estimate of the mean price of a movie tickets for all theaters? What is the margin of error associated with this estimate? [6.75, 0.1568]
   b. Make a 95% confidence interval for the population mean, μ. [6.59, 6.91]

5. The standard deviation for a population is $\sigma = 7.14$. A random sample selected from this population gave a mean equal to 55.63.
   a. Make a 95% confidence interval for μ assuming $n = 196$. [54.63, 56.63]
   b. Construct a 95% confidence interval for μ assuming $n = 100$. [54.23, 57.03]
   c. Determine a 95% confidence interval for μ assuming $n = 49$. [53.63, 57.63]
   d. Does the width of the confidence intervals constructed in (a) to (c) increase as the sample size decreases? Explain why.

6. Let μ be the mean hourly wages of carpenters. A random sample of 30 carpenters yielded a 95% confidence interval for μ of RM11.45 to RM14.35.
   a. Find the value of $\bar{X}$ for this sample. [12.90]
   b. Find the 99% confidence interval for μ based on this sample. [10.99, 14.81]

7. A mail-order company promises its customers that the products ordered will be mailed within 72 hours after an order is placed. Recently, in a sample of 50 orders, it is found that 42 of them were mailed within 72 hours of the placement of the orders.
   a. Construct a 98% confidence interval of the percentage of all orders that are mailed within 72 hours of their placement. [71.94%, 96.06%]
   b. Suppose the confidence interval obtained in (a) is too wide. How can the width of this interval be reduced? Discuss all possible alternatives. Which of these alternatives is the best in the practical sense?
   c. If a new survey it to be done, what is the estimate of the sample size that would limit the maximum error to within 0.02 of the population proportion for a 99% confidence interval? [2230]

8. A random sample of 20 managers was taken, and they were asked whether or not they usually take work home. The responses of these managers are given below.

Yes	Yes	No	No	No	Yes	No	No	No	No
No	Yes	Yes	No	Yes	Yes	No	No	No	Yes

   Make a 99% confidence interval for the percentage of all managers who take work home. [11.79%, 68.21%]

9. A company that produces detergents wants to estimate the mean amount of detergent in 64-ounce jugs at a 99% confidence level. The company knows that the standard deviation of the amounts of detergent in such jugs is 0.2 ounces. How large a sample should the company take so that the estimate is within 0.04 ounces of the population mean? [166]

10. Tony's Pizza guarantees all pizzas will be delivered within 30 minutes of the placement of orders. An agent wants to estimate the proportion of all pizzas delivered within 30 minutes by Tony's Pizza. What is the most conservative estimate of the sample size that would limit the maximum error to within 0.02 of the population proportion for a 99% confidence interval? [4147]

11. The average running time of a certain rechargeable battery is known to be 8.5 hours. A change in the production method has been proposed, and a sample of 60 batteries produced by the new method has a mean running time of 8.62 hours and a standard deviation of 0.55 hours.
   a. Show that for the null hypothesis, $H_0 : \mu = 8.5$, the test statistic has the value $z = 1.69$.
   b. If the level of significance is 0.05 and if the alternative hypothesis is $H_1 : \mu \neq 8.5$, what conclusion do you reach? [do not reject]
   c. An engineer examines the data and makes the inference that the change in production was made for the purpose of improving the battery running time. He claims that the alternative hypothesis must be $H_1 : \mu > 8.5$. What conclusion does he reach? [reject]
   d. A second engineer notes that the production method change might have been made for reasons such as reducing cost or increasing the number of times that the battery can be recharged. She claims that the alternative hypothesis must be $H_1 : \mu < 8.5$. What conclusion does she reach? [do not reject]

12. The health of the bear population in National Park is monitored by periodic measurements taken from anesthetized bears. A sample of 54 bears has a mean weight of 182.9 lb. Assuming that σ is known to be 121.8 lb, use a 0.10 significance level to test the claim that the population mean of all such bear weights is less than 200 lb. Use the *p*-value approach for this test. [do not reject]

13. Chubs Baby Food Company wishes to compare the weight gain of infants using its brand versus its competitor's. A sample of 40 babies using the Chubs products revealed a mean weight gain of 7.6 pounds in the first three months after birth. For the Chubs brand, the population standard deviation of the sample is 2.3 pounds. A sample of 55 babies using the competitor's brand revealed a mean increase in weight of 8.1 pounds. The population standard deviation is 2.9 pounds. At the 0.05 significance level, can we conclude that babies using the Chubs brand gained less weight? Compute the p-value and interpret it. [do not reject]

14. Music streaming services are the most popular way to listen to music. Data gathered over the last 12 months show Apple Music was used by an average of 1.65 million households with a sample standard deviation of 0.56 million family units. Over the same 12 months Spotify was used by an average of 2.2 million families with a sample standard deviation of 0.30 million. Assume the population standard deviations are not the same. Using a significance level of 0.05, test the hypothesis of no difference in the mean number of households picking either service. [reject]

15. The owner of Bun 'N' Run Hamburgers wishes to compare the sales per day at two locations. The mean number sold for 10 randomly selected days at the Northside site was 83.55, and the standard deviation was 10.50. For a random sample of 12 days at the Southside location, the mean number sold was 78.80 and the standard deviation was 14.25. Assume equal population standard deviations. At the 0.05 significance level, is there a difference in the mean number of hamburgers sold at the two locations? What is the p-value? [reject]

16. As part of a recent survey among dual-wage-earner couples, an industrial psychologist found that 990 men out of the 1500 surveyed believed the division of household duties was fair. A sample of 1600 women found 970 believed the division of household duties was fair. At the 0.01 significant level, it is reasonable to conclude that the proportion of men who believe the division of household duties is fair is larger? What is the p-value? [reject]