UNIVERSITI TUNKU ABDUL RAHMAN

ACADEMIC YEAR 2016/2017

APRIL EXAMINATION

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

MONDAY, 8 MAY 2017 TIME : 9.00 AM – 11.00 AM (2 HOURS)

BACHELOR OF COMPUTER SCIENCE (HONS)
BACHELOR OF INFORMATION SYSTEMS (HONS)
INFORMATION SYSTEMS ENGINEERING

Instructions to Candidates :

This question paper consists of TWO (2) questions in Section A and THREE (3) questions in Section B.

Answer **ALL** questions in **Section A** and **ONLY TWO (2)** questions in **Section B**.

Each question carries 25 marks.

Should a candidate answer more than TWO (2) questions in section B, marks will only be awarded for the FIRST TWO (2) questions in that section in the order the candidate submits the answers.

Answer questions only in the answer booklet provided.

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

SECTION A (Answer ALL Questions)

Q1. [Combinatorics]

(a) How many **permutations** are there for arranging r objects from n **distinguishable** objects **with** the allowance of repetition? (1 mark)

(b) How many **permutations** are there for arranging r objects from n **distinguishable** objects **without** repetition? (1 mark)

(c) How many **permutations** are there for arranging r objects from n **indistinguishable** objects **with** the allowance of repetition? (1 mark)

(d) How many **permutations** are there for arranging r objects from n **indistinguishable** objects **without** repetition? (1 mark)

(e) How many **combinations** are there for choosing r objects from n **distinguishable** objects **with** the allowance of repetition? (1 mark)

(f) How many **combinations** are there for choosing r objects from n **distinguishable** objects **without** repetition? (1 mark)

(g) How many **combinations** are there for choosing r objects from n **indistinguishable** objects **with** the allowance of repetition? (1 mark)

(h) How many **combinations** are there for choosing r objects from n **indistinguishable** objects **without** repetition? (1 mark)

(i) How many ways are there for n persons to sit in a row? (1 mark)

(j) How many ways are there for n persons to sit in a round table? (1 mark)

(k) How many ways are there for $n \geq 100$ stones to be placed in a row, while three of them are indistinguishable? (1 mark)

(l) How many ways are there for $n \geq 100$ stones to be placed in a round table, while three of them are indistinguishable? (2 marks)

Q1. (Continued)

(m) There are two distinguishable rows where the first row has m seats and the second row has n seats. How many ways are there for m+n persons to sit in the two rows? (2 marks)

(n) There are two distinguishable round tables where the first one has m seats and the second one has n seats. How many ways are there for m+n persons to sit in the two tables? (3 marks)

(o) How many different words can be constructed using GGHHTQKKK (9 letters)? (2 marks)

(p) A password has 8 characters, and 7 of them are chosen from '0' to '9' and 'A' to 'Z' (36 choices), while one of them is from '0' to '9' (10 choices). A student argued that the number of passwords is $(8)(10)(36^7)$ because there are 8 ways to choose the position which allows only '0' to '9', and of course 10 choices for this position, and each of the remaining 7 positions has 36 choices. Explain why the student is wrong. (5 marks)

[Total : 25 marks]

Q2. [Simple Probability, Geometric Distribution, Binomial Distribution, Z-test]

(a) Consider tossing an **unfair** coin, with probability ¼ of getting a head.

(i) What is the probability of **not** getting a head in a single throw? (2 marks)

(ii) Tossing the coin for n times, what is the probability of getting n-1 tails and a head? (2 marks)

(iii) Tossing the coin for n times, what is the expected number of heads? (2 marks)

(iv) Tossing the coin for n times, what is the variance of the number of heads? (2 marks)

(v) Tossing the coin for n times, what is the probability of **not** getting a head in the first n-1 tosses, and then, getting a head in the last toss? (2 marks)

(vi) Tossing the coin repeatedly, what is the probability that head will never occur? (2 marks)

(vii) What is the expected number of tosses for the first head to occur? (2 marks)

(viii) Using the identity, $\sum_{i=2}^{\infty}(i)(i-1)x^{i-2} = \frac{2}{(1-x)^3}$, or otherwise, find the expected number of tosses for the **second** head to occur. (4 marks)

(b) Answer the following questions for Z-test.

(i) Given a sample of size n=121 with sample mean $\overline{X}$ = 20, and sample variance s^2=49, compute the interval for the population mean μ for 95% confidence. State the assumptions you make. (3 marks)

(ii) Discuss when confidence level 99% is better than confidence level 95% in testing a hypothesis? (2 marks)

(iii) Discuss when both-sided test is better than one-sided (left- or right-) test in testing a hypothesis with the same confidence level? (2 marks)

[Total : 25 marks]

SECTION B (Answer Any Two Questions)

Q3. [Conditional Probability, Event Independency and Exclusion]

(a) Suppose that events A and B are independent. Answer the following questions in terms of A and B.

(i) What is P(A|B) ? (2 marks)

(ii) What is P(B|A) ? (2 marks)

(iii) What is $P(\overline{A}|B)$? (2 marks)

(iv) What is $P(A|\overline{B})$? (2 marks)

(v) What is $P(A \cap B \mid A)$? (2 marks)

(b) Suppose that events A and B are mutually exclusive. Answer the following questions in terms of A and B.

(i) What is P(A|B) ? (2 marks)

(ii) What is P(B|A) ? (2 marks)

(iii) What is $P(\overline{A}|B)$? (2 marks)

(iv) What is $P(A|\overline{B})$? (2 marks)

(v) What is $P(A \cap B \mid A)$? (2 marks)

(c) Prove that A and B are independent if and only if $\overline{A}$ and B are independent. [Note, it is B, NOT $\overline{B}$, in the second phrase.] (5 marks)

[Total : 25 marks]

Q4. [Uniform Distribution]

Tom and Jerry are friends. At 6:00pm, they start their dinner together. Tom's finishing time is uniformly distributed between 6:20pm and 6:30pm. For Jerry, his finishing time is uniformly distributed between 6:15pm and 6:45pm. After both of them finish their dinner, they will leave the table together. The time unit is measured in minutes. No need to do integration.

(a) What is the probability density function of Tom's finishing time? (2 marks)

(b) What is the expectation of Tom's finishing time? (2 marks)

(c) What is the variance of Tom's finishing time? (2 marks)

(d) What is the probability density function of Jerry's finishing time? (2 marks)

(e) What is the expectation of Jerry's finishing time? (2 marks)

(f) What is the variance of Jerry's finishing time? (2 marks)

(g) What is the probability that Tom waits for Jerry? (6 marks)

(h) What is the expected waiting time regardless of who finishes the dinner earlier? (7 marks)

[Total : 25 marks]

Q5. [A **challenging** question about Exponential, Normal, and Uniform, Distributions]
Company A's annual profit follows a distribution with density function $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right)$, for $x > 0$. Note, $\mu=0$ and $\int_0^\infty f(x)dx=0.5$. The company can lose money annually, and the *negative* profit follows a uniform distribution with density function $f(x)=\frac{1}{k}$, for $x_0 < x < 0$, where $k > 0$ and x_0 is a certain negative number and $-x_0$ is the maximum annually loss. Suppose that f is continuous at $x = 0$. The probability that the company earns at most $t > 0$ RM is $\int_0^t f(x)dx+0.5$. The probability that the company earns at most $t < 0$ RM (*i.e.*, loses at least $-t > 0$ RM) is $\int_{x_0}^t f(x)dx$.

The annual profit of Company B falls into an exponential distribution with density function $g(t)=\lambda e^{-\lambda t}$, and the probability that the company earns at most $t > 0$ RM is $\int_0^t g(x)dx$.

The profit and loss in this question are measured annually. Answer the following questions in terms of e (the natural number), π, λ, and σ, whenever applicable. No need to do integration.

(a) What is the probability that Company A records a profit?
What is the probability that Company A records a loss? (2 marks)

(b) What is the value of k?
[Hint: f is continuous at $x = 0$.] (3 marks)

(c) Using the result of (b), what is the maximum loss for Company A?
[Hint: f is uniform for $x_0 < x < 0$.] (1 mark)

(d) Assume that the company records a loss. Using the result of (c), what is the expected loss? (2 marks)

(e) Using the result in (d), and the identity $\int_0^\infty xf(x)dx=\frac{\sigma}{\sqrt{2\pi}}$, what is the expected profit earned by Company A? (2 marks)

(f) What is the probability that Company A records a profit at most 2σ? (4 marks)

Q5. (Continued)

(g) What is the probability that Company B records a loss? (2 marks)

(h) What is the expected profit earned by Company B? (2 marks)

(i) What is the probability that Company B records a profit at most 2σ? (3 marks)

(j) By comparing (e) and (h), how do we determine which company has higher ability of profiting? (2 marks)

(k) By comparing (f) and (i), how do we determine which company has higher ability of profiting? (2 marks)

[Total : 25 marks]

Appendix

Standard Normal Distribution (The right side)

$$\Phi(z) = P\{Z \le z\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2}\, dx$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986