UNIVERSITI TUNKU ABDUL RAHMAN

ACADEMIC YEAR 2016/2017

SEPTEMBER EXAMINATION

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

TUESDAY, 13 SEPTEMBER 2016 TIME : 2.00 PM – 4.00 PM (2 HOURS)

BACHELOR OF COMPUTER SCIENCE (HONS)
BACHELOR OF INFORMATION SYSTEMS (HONS)
INFORMATION SYSTEMS ENGINEERING
BACHELOR OF INFORMATION SYSTEMS (HONS)
BUSINESS INFORMATION SYSTEMS
BACHELOR OF INFORMATION TECHNOLOGY (HONS)
COMMUNICATIONS AND NETWORKING

Instructions to Candidates :

This question paper consists of TWO (2) questions in Section A and THREE (3) questions in Section B.

Answer **ALL** questions in **Section A** and **ONLY TWO (2)** questions in **Section B**.

Each question carries 25 marks.

Should a candidate answer more than TWO (2) questions in section B, marks will only be awarded for the FIRST TWO (2) questions in that section in the order the candidate submits the answers.

Answer questions only in the answer booklet provided.

UCCD1143 PROBABILITY AND STATISTICS FOR COMPUTING

SECTION A (Answer ALL Questions)

Q1. [Combinatorics]

(a) How many **permutations** are there for arranging r objects from n **distinguishable** objects **without** repetition? (1 mark)

(b) How many **permutations** are there for arranging r objects from n **distinguishable** objects **with** the allowance of repetition? (1 mark)

(c) How many **permutations** are there for arranging r objects from n **indistinguishable** objects **without** repetition? (1 mark)

(d) How many **permutations** are there for arranging r objects from n **indistinguishable** objects **with** the allowance of repetition? (1 mark)

(e) How many **combinations** are there for choosing r objects from n **distinguishable** objects **without** repetition? (1 mark)

(f) How many **combinations** are there for choosing r objects from n **distinguishable** objects **with** the allowance of repetition? (1 mark)

(g) How many **combinations** are there for choosing r objects from n **indistinguishable** objects **without** repetition? (1 mark)

(h) How many **combinations** are there for choosing r objects from n **indistinguishable** objects **with** the allowance of repetition? (1 mark)

(i) How many ways are there for n persons to sit in a row? (1 mark)

(j) How many ways are there for n persons to sit in a round table? (1 mark)

(k) How many ways are there for $n \geq 3$ stones to be placed in a row, while three of them are indistinguishable? (1 mark)

(l) How many ways are there for $n \geq 4$ stones to be placed in a round table, while three of them are indistinguishable? (2 marks)

Q1. (Continued)

(m) There are two distinguishable rows where the first row has r seats and the second row has n-r seats, and $0 < r < n$. How many ways are there for n persons to sit in the two rows? (2 marks)

(n) There are two distinguishable round tables where the first one has r seats and the second one has n-r seats, where $0 < r < n$. How many ways are there for n persons to sit in the two tables? (3 marks)

(o) How many different words can be constructed using GGGTTKKKK (9 letters)? (2 marks)

(p) A password has 8 characters, and 7 of them are chosen from '0' to '9' and 'A' to 'Z' (36 choices), while one of them is from 'A' to 'Z' (26 choices). A student argued that the number of passwords is $(8)(26)(36^7)$ because there are 8 ways to choose the position which allows only 'A' to 'Z', and of course 26 choices for this position, and each of the remaining 7 positions has 36 choices. Explain why the student is wrong. (5 marks)

[Total : 25 marks]

Q2. [Simple Probability, Geometric Distribution, Binomial Distribution]
A fair dice has uniform probability, p, for all faces 1, 2, ..., and 6.

(a) What is the probability of getting a "4" in a single throw? (2 marks)

(b) What is the probability of not getting a "4" in a single throw? (2 marks)

(c) Throwing the dice twice, what is the probability of not getting a "4" in the first throw, and getting a "4" in the second throw? (2 marks)

(d) Throwing the dice for n times, what is the probability of not getting a "4" in the first n-1 throws, and getting a "4" in the last throw? (2 marks)

(e) Throwing the dice repeatedly, what is the probability that "4" will never occur? (2 marks)

(f) The expected number of throws for the first "4" to occur is 6. Directly from the definition of expectation, without simplification, give the infinite series that leads to the expected value 6. (3 marks)

(g) Recall that the expected number of throws for the first "4" to occur is 6. Suppose that a student cannot get a "4" in his first 3 throws. What is the expected number of times that he still needs to throw the dice before he can see the first "4"? (2 marks)

(h) Throwing the dice for 10 times, what is the probability of getting exactly two "4"s and one "5" (while the remaining throws cannot give "4" and "5") ? (4 marks)

(i) Prove that the probability of getting exactly three "4"s from n throws is less than that of getting exactly three "4"s from n+1 throws, for all $3 \leq n < 17$. (6 marks)

[Total : 25 marks]

SECTION B (Answer Any Two Questions)

Q3. [Conditional Probability and Event Independency and Exclusion]

(a) Let A and B be events in a sample space. Suppose P(A|B)=0.7, P(A|$\overline{B}$)=0.1, and P(B)=0.6.

(i) Find P($\overline{B}$). (2 marks)

(ii) Find P($\overline{A}$|B). (2 marks)

(iii) Use P(X) = P(X|Y)P(Y) + P(X|$\overline{Y}$)P($\overline{Y}$), for any events X and Y, to find P(A). (2 marks)

(iv) Are A and B independent? Explain. (3 marks)

(v) Find P(B|A).
[Hint: use the above results.] (3 marks)

(b) Prove that for any events X and Y in a sample space, with both P(X), P(Y) >0, if X and Y are mutually exclusive, then X and Y are NOT independent. (4 marks)

(c) Let Ω be a sample space having outcomes 1, 2,..., 10, and each has uniform probability 1/10. Let event X be {1, 2, 3, 4, 5}.
Construct a non-empty event Y in Ω, such that X and Y are NEITHER mutually exclusive NOR independent. (4 marks)

(d) Prove that A and B are independent if and only if $\overline{A}$ and $\overline{B}$ are independent. (5 marks)

[Total : 25 marks]

Q4. [Continuous Random Variables]

At 1:00pm, Romeo and Juliet start their lunch. Romeo's finishing time is uniformly distributed between 1:10pm and 1:30pm. For Juliet, the time required to finish her lunch follows an exponential distribution with density function $f(t) = \lambda e^{-\lambda t}$, and the probability that she can finish her lunch in t minutes is $\int_0^t f(x)dx$. Students may leave their answer in terms of e (natural number), whenever applicable.

(a) What is the probability density function of Romeo's finishing time? (2 marks)

(b) What is the expectation of Romeo's finishing time? (2 marks)

(c) What is the variance of Romeo's finishing time?
[Hint: (length of interval)^2 / 12.] (2 marks)

(d) What is the expectation of the time required by Juliet, in terms of λ? (2 marks)

(e) What is the variance of the time required by Juliet, in terms of λ? (2 marks)

(f) What is the probability that the time required by Juliet is more than its expected value (the answer of (d)) ? (2 marks)

(g) What is the probability, in terms of λ, that Romeo finishes his lunch earlier than Juliet? (5 marks)

(h) What is the probability, in terms of λ, that Romeo finishes his lunch later than Juliet? (1 mark)

(i) What is the probability that their finishing time is exactly the same? (i.e. the difference between their finishing time tends to 0.) (1 mark)

(j) What is the probability, in terms of λ, that the time gap between their finishing time is less than one minute? (5 marks)

(k) Suppose Romeo and Juliet's expected finishing time are the same. What is the value of λ? (1 mark)

[Total : 25 marks]

Q5. [Normal Distribution]

(a) Let $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$. Answer the following questions.

[Hint: no need to use integration.]

(i) What is the value of $\int_{-\infty}^{\infty} f(x)dx$? (1 mark)

(ii) What is the value of $\int_{-\infty}^{\infty} xf(x)dx$? (1 mark)

(iii) What is the value of $\int_{-\infty}^{\infty} x^2 f(x)dx$? (3 marks)

(b) In a road of speed limit 100 km/hr (km per hour), any vehicle with speed (even slightly) over the limit will certainly be caught by radar. Students A and B both drive with mean speed 90 km/hr, and standard deviation 10km/hr. Suppose that their speed follows normal distribution.

(i) What is the probability that student A's vehicle is caught? (3 marks)

(ii) Student A can change his mean speed but not the standard deviation due to his driving habit. In order to reduce the probability to 0.05, what is his new mean speed? (3 marks)

(iii) Student B can change the standard deviation of his speed but not the mean due to his driving habit. In order to reduce the probability to 0.05, what is the new standard deviation of his speed? (3 marks)

(c) Answer the following questions for Z-test.

(i) Given a sample of size n=144 with sample mean $\overline{X}$ = 21, and sample variance s^2=25, compute the interval for the population mean μ for 95% confidence. State the assumptions you make. (3 marks)

(ii) Construct a scenario of right-tail test such that a null hypothesis can be rejected with 95% confidence level, but cannot be rejected when the confidence level is increased to 99%. (4 marks)

(iii) Construct a scenario such that with 95% confident level a null hypothesis can be rejected in right-tail test, but cannot be rejected in two-sided test. (4 marks)

[Total : 25 marks]

Appendix

1. (i) (1-α)100% confidence interval for θ:

$$[\hat{\theta} - z_{\alpha/2}Std(\hat{\theta}),\ \hat{\theta} + z_{\alpha/2}Std(\hat{\theta})].$$

(ii) Z-test, $Z = \frac{\hat{\theta}-\theta}{\sigma/\sqrt{n}}$.

2. Standard Normal Distribution (The right side)

$$\Phi(z) = P\{Z \le z\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-x^2/2}\,dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986