Objective:

- Learning and Implementing Segment Tree: A type of binary tree.

Segment Tree   *[Taken from: http://www.geeksforgeeks.org/segment-tree-set-1-sum-of-given-range/]*

Let us consider the following problem to understand Segment Trees.

We have an array arr[0 . . . n-1]. We should be able to

1. Find the sum of elements from index l to r where 0 <= l <= r <= n-1

2. Change value of a specified element of the array arr[i] = x where 0 <= i <= n-1.

A **simple solution** is to run a loop from l to r and calculate sum of elements in given range. To update a value, simply do arr[i] = x. The first operation takes $O(n)$ time and second operation takes $O(1)$ time.

Another solution is to create another array and store sum from start to i at the ith index in this array. Sum of a given range can now be calculated in $O(1)$ time, but update operation takes $O(n)$ time now. This works well if the number of query operations is large and very few updates.

What if the number of query and updates are equal? **Can we perform both the operations in $O(\log n)$ time once given the array?** We can use a *Segment Tree* to do both operations in $O(Logn)$ time.

Representation of Segment trees

1. Leaf Nodes are the elements of the input array.

2. Each *internal node* represents some merging of the leaf nodes. The merging may be different for different problems. For this problem, merging is sum of leaves under a node.

An array representation of tree is used to represent Segment Trees. For each node at index i, the left child is at index 2*i+1, right child at 2*i+2 and the parent is at $\left\lfloor (i-1)/2 \right\rfloor$.
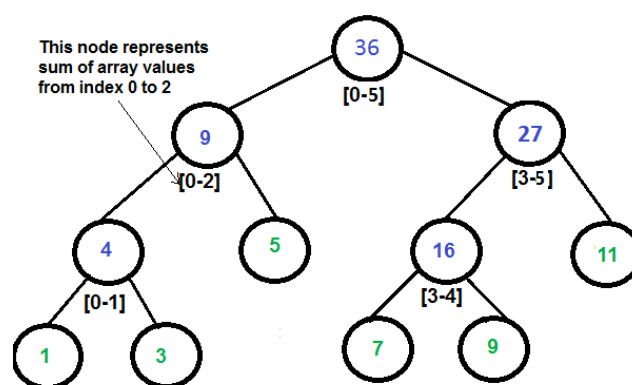
Segment Tree for input array {1, 3, 5, 7, 9, 11}

Construction of Segment Tree from given array We start with a segment arr[0 . . . n-1]. and every time we divide the current segment into two halves(if it has not yet become a segment of length 1), and then call the same procedure on both halves, and for each such segment we store the sum in corresponding node. All levels of the constructed segment tree will be completely filled except the last level. Also, the tree will be a Full Binary Tree because we always divide segments in two halves at every level. Since the constructed tree is always full binary tree with n leaves, there will be n-1 internal nodes.

So total number of nodes will be 2*n – 1. Height of the segment tree will be $\lceil \log_2 n \rceil$. Since the tree is represented using array and relation between parent and child indexes must be maintained, size of memory allocated for segment tree will be $2 * 2^{\lceil \log_2 n \rceil} - 1$.

Query for Sum of given range Once the tree is constructed, how to get the sum using the constructed segment tree. Following is algorithm to get the sum of elements.

```
int getSum(node, l, r)
{
    if range of node is within l and r
        return value in node
    else if range of node is completely outside l and r
        return 0
    else
     return getSum(node's left child, l, r) +
          getSum(node's right child, l, r)
}
```

Update a value Like tree construction and query operations, update can also be done recursively. We are given an index, which needs to be updated. Let *diff* be the value to be added. We start from root of the segment tree, and add *diff* to all nodes, which have given index in their range. If a node doesn't have given index in its range, we don't make any changes to that node.

Implementation: Following is implementation of segment tree. The program implements construction of segment tree for any given array. It also implements query and update operations.

```c
// Program to show segment tree operations like construction, query and update
#include <stdio.h>
#include <math.h>

// A utility function to get the middle index from corner indexes.
int getMid(int s, int e) {  return s + (e -s)/2;  }

// A recursive function that constructs Segment Tree for array[ss..se].
// si is index of current node in segment tree st
int constructSTUtil(int arr[], int ss, int se, int *st, int si)
{
    // If there is one element in array, store it in current node of
    // segment tree and return
    if (ss == se)
    {
        st[si] = arr[ss];
        return arr[ss];
    }

    // If there are more than one elements, then recur for left and
    // right subtrees and store the sum of values in this node
    int mid = getMid(ss, se);
    st[si] =  constructSTUtil(arr, ss, mid, st, si*2+1) +
            constructSTUtil(arr, mid+1, se, st, si*2+2);
    return st[si];
}

/* Function to construct segment tree from given array. This function
   allocates memory for segment tree and calls constructSTUtil() to
   fill the allocated memory */
int * constructST(int arr[], int n)
{
```

```cpp
    // Allocate memory for segment tree
    int x = (int)(ceil(log2(n))); //Height of segment tree
    int max_size = 2*(int)pow(2, x) - 1; //Maximum size of segment tree
    int * st = new int[max_size];

    // Fill the allocated memory st
    constructSTUtil(arr, 0, n-1, st, 0);

    // Return the constructed segment tree
    return st;
}

// Driver program to test above functions
int main()
{
    int arr[] = {1, 3, 5, 7, 9, 11};
    int n = sizeof(arr)/sizeof(arr[0]);

    // Build segment tree from given array
    int *st = constructST(arr, n);

    // Print sum of values in array from index 1 to 3
    printf("Sum of values in given range = %d\n", getSum(st, n, 1, 3));

    // Update: set arr[1] = 10 and update corresponding segment
    // tree nodes
    updateValue(arr, st, n, 1, 10);

    // Find sum after the value is updated
    printf("Updated sum of values in given range = %d\n",
                                        getSum(st, n, 1, 3));

    return 0;
}
```

Time Complexity: Time Complexity for tree construction is O(n). There are total 2n-1 nodes, and value of every node is calculated only once in tree construction.

Time complexity to query is O(Logn). To query a sum, we process at most four nodes at every level and number of levels is O(Logn).

The time complexity of update is also O(Logn). To update a leaf value, we process one node at every level and number of levels is O(Logn).

Your Task:

I hope you comprehend the concept of segment trees and its construction process.

- The next step for you is to implement 'getSum' and 'updateValue' function described in driver function

- The next step will be to implement the above concept in Object Oriented way by designing an ADT named 'SegmentTree' with appropriate data members and member functions.