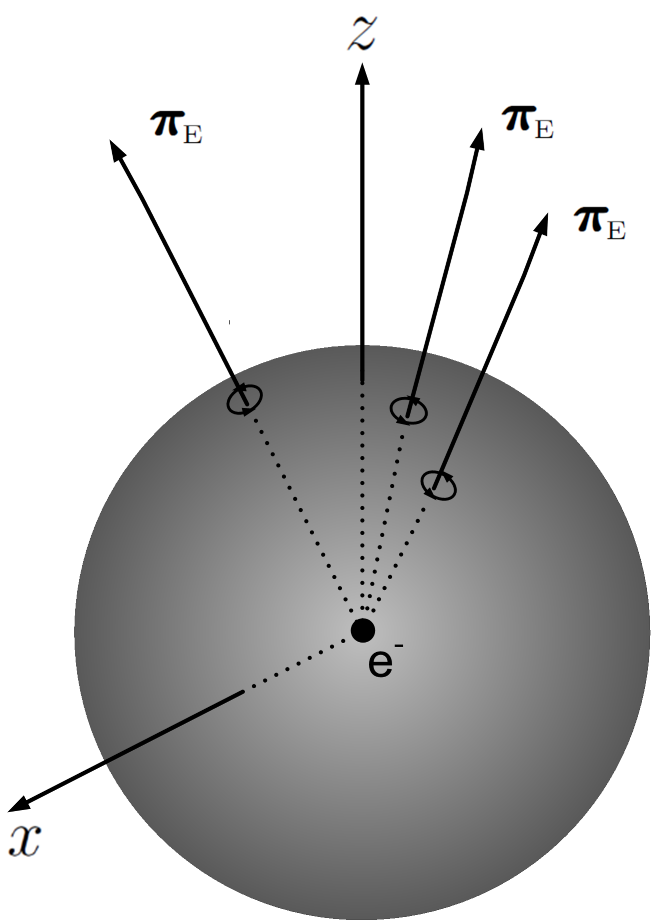
Inertial Vacuum Power

Dr. Christopher Bradshaw Hayes

May 29, 2023



Abstract

The principle of causality requires the introduction of electromagnetic flux into the velocity fields of the classical electron. The inherent violation of energy conservation relies heavily on the use of vacuum gauge potentials in multiple scenarios for the calculation of inertial vacuum power.

Links to youtube videos:
https://youtu.be/PHEeNRdzCX4
https://youtu.be/gk8QoH8dPXE

Vacuum
Gauge
Electrodynamics

Contents

List of Figures

1 Electromagnetic Flux for a Classical Electron

1.1 Momentum in Electromagnetic Fields

A classical electron is created at the origin of a coordinate system and moving along a worldline with four-velocity β^ν as illustrated in Figure 1. Strict enforcement of causality requires the spatial extent of the surrounding fields to be constrained by a causality sphere. This function, given by

$$\vartheta = \vartheta(t_r/\gamma) \tag{1.1}$$

is a *one* propagating in all directions at light speed from where the particle was created, and taking an argument which can be shown to be proportional to the retarded time. Not shown by the diagram are particle accelerations and (perhaps)

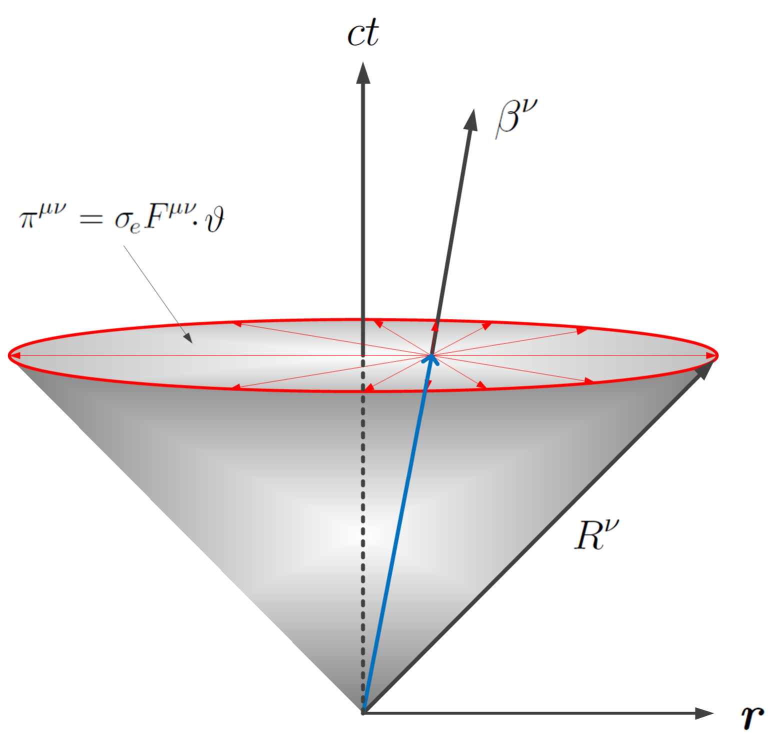


Figure 1: *Spacetime diagram illustrating the causal reach of the fields for an inertial electron created at the origin of coordinates.*

the simultaneous appearance of the anti-particle which would share the same causality sphere. These topics are important but introduce unnecessary complications to the ideas presented here.

The imposition of causality represents a large rift from the conventional theory and elicits discussion of its consequences. First, it will be necessary to re-interpret the electromagnetic fields surrounding the particle as carriers of momentum and energy.

For the specific case of inertial motion, this can be accomplished by multiplying the velocity electric and magnetic field vectors by an elementary charge density:

$$\boldsymbol{\pi}_{\mathrm{E}} = \sigma_e \mathbf{E}_v \qquad\qquad \boldsymbol{\pi}_{\mathrm{B}} = \sigma_e \mathbf{B}_v \tag{1.2}$$

This means electrons and positrons will have identical fields $\boldsymbol{\pi}_{\mathrm{E}}$ and $\boldsymbol{\pi}_{\mathrm{B}}$ in the new theory, and both will satisfy the same set of re-defined Maxwell-Lorentz equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\pi}_{\mathrm{E}} = 4\pi f_o \qquad\qquad \boldsymbol{\nabla} \times \boldsymbol{\pi}_{\mathrm{B}} = \frac{4\pi}{c}\boldsymbol{f} + \frac{1}{c}\frac{\partial \boldsymbol{\pi}_{\mathrm{E}}}{\partial t} \tag{1.3a}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\pi}_{\mathrm{E}} = -\frac{1}{c}\frac{\partial \boldsymbol{\pi}_{\mathrm{B}}}{\partial t} \qquad\qquad \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_{\mathrm{B}} = 0 \tag{1.3b}$$

The sources here are still those of a point particle but now re-interpreted as force densities. Discrete symmetry operations can be performed[1] showing C- and P- invariance, but violations of T-symmetry occur in the two source equations. However, this is not in conflict with the CPT-theorem as the question of how fast a Coulomb field radiates has nothing to do with an electromagnetic force it might exert.

Radiating fields still allow for momentum and energy attributed to Poynting's theorem, but require a slightly new formulation of the Poynting vector inclusive of a fundamental pressure constant[2]

$$\mathbf{S} = \frac{c}{\mu_e}\boldsymbol{\pi}_{\mathrm{E}} \times \boldsymbol{\pi}_{\mathrm{B}} \tag{1.5}$$

To be clear, causality requires a rest frame classical electron to possess an electric radiation field $\boldsymbol{\pi}_{\mathrm{E}}$. In a moving frame, both $\boldsymbol{\pi}_{\mathrm{E}}$ and $\boldsymbol{\pi}_{\mathrm{B}}$ together comprise an electromagnetic radiation field, but momentum $\mathbf{S}$ attributed to the Poynting vector lacks the r^{-2} character and does not qualify as radiation. During particle accelerations, the Poynting vector generalizes to

$$\mathbf{S} = \frac{c}{\mu_e}\left[(\boldsymbol{\pi}_{\mathrm{E}} + \boldsymbol{\pi}_{\mathrm{E}a}) \times (\boldsymbol{\pi}_{\mathrm{B}} + \boldsymbol{\pi}_{\mathrm{B}a})\right] \tag{1.6}$$

Expanding term-by-term, the first three still do not qualify as radiation fields, but the last term does qualify. In summary then, for the most general motions of the electron, causality requires two forms of radiation—one associated only with the velocity fields and *not* related to the Poynting vector, and another associated only with the acceleration fields and determined specifically *by* the Poynting vector.

[1] See Appendix for symmetry operations performed on momentum flux fields.

[2] The pressure constant given by

$$\mu_e = 4\pi\sigma_e^2 \tag{1.4}$$

is fundamental to classical electron theory as it represents the self-induced outward pressure attributed to the particles' own surface charge density.

It is not difficult to include radiating velocity fields into the general scheme of theoretical physics since their presence amounts to only a slight modification of the 1905 postulates. The postulate of relativity remains the same but the constancy of light postulate must be altered to read:

> ***–Postulate 2: Electromagnetic radiation fields propagate at speed c independent of the motion of the source.***

This is mainly just a generalization of the Einstein postulate to include all possible forms of radiation, but it's also a unifying statement which renders the descriptive title, '*static field*' as an unsubstantiated figment of the imagination.

The essential feature arising from the enforcement of causality is therefore the idea that velocity fields are emitted by a charged particle resulting in a flagrant violation of energy conservation. While this is cause for concern, there is a real possbility that radiating fields might generate a cosmology adequately explaining the presence of dark energy in the universe. Before this can be accomplished however, an overwhelming abundance of questions regarding properties of radiated energy must be answered. For example, do radiating Coulomb fields exert forces, does the field carry spin, and can it be absorbed by other particles? These are difficult questions that are addressed on limited basis in the rest of this investigation, and in the Appendix.

1.2 Radiated Flux from Vacuum Gauge Potentials

Vacuum Gauge Potentials in the Rest Frame: The problem of a classical particle with propagating fields can be addressed in a flat spacetime with a metric defined by the signature $(+---)$. Working in the rest frame only, begin with a disturbance at the origin of a coordinate system in the form of a radial oscillator:

$$\boldsymbol{r}_e(\tau) \equiv r_e\,\hat{\boldsymbol{r}} \cdot \mathrm{e}^{i\omega_e \tau} \tag{1.7}$$

Referring to r_e as the classical radius of the electron, this oscillator may be viewed as the spatial boundary of a propagating vacuum field represented by

$$\boldsymbol{\mathcal{U}}(r,\tau) = \frac{r_e^2}{r}\hat{\boldsymbol{r}} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \tag{1.8}$$

The action of $\boldsymbol{\mathcal{U}}$ is a radiator of longitudinal spherical waves produced by the oscillator which travel at light speed but—like transverse electromagnetic waves—require no medium for their propagation. An immediate link to classical electromagnetism follows by simply multiplying both sides of the vacuum field by the constant $4\pi\sigma_e$. The result is the vacuum gauge magnetic vector potential:

$$\boldsymbol{A}(r,\tau) = \frac{e}{r}\hat{\boldsymbol{r}} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \tag{1.9}$$

This field can be used along with its magnitude to generate a set of rest frame vacuum gauge potentials $A^\nu = (A, \boldsymbol{A})$ pointing along the light cone and satisfying the null condition $A^\nu A_\nu = 0$. Using the elementary area $a_e = 4\pi r_e^2$, rest frame energy flux is given by

$$\langle \mathbf{S} \rangle_{rest} = \frac{1}{2a_e} A^* \boldsymbol{A} c = \left[\tfrac{1}{2}\boldsymbol{\pi}_{\mathrm{E}} c\right]_{rest} \tag{1.10}$$

Vacuum Gauge Potentials Relative to a Moving Frame: The Lorentz transformation of A^ν can be performed with various levels of rigor and generally requires separate transformations for the four-vector and the oscillator. As the field itself is moving at light speed, the goal will be the derivation of the moving frame potentials in terms of the distance $\boldsymbol{R}$ from the retarded position of the source to the present time field point. To accomplish this write the light cone condition

$$R^\nu = x^\nu - w^\nu \tag{1.11}$$

where w^ν is the spacetime position of the particle at the retarded time. The formula for the moving frame vacuum gauge potentials may be written

$$\boxed{A^\nu(r,t) = \frac{eR^\nu}{\rho^2} \cdot \mathrm{e}^{i\omega_e z}} \tag{1.12}$$

The uncomplicated form of this result is visibly attractive, and derives from the invariant nature of the expanding light sphere. In the moving frame the argument z of the exponential is given by

$$z \equiv \frac{1}{\gamma}(t_r + \tau_e) = \frac{1}{\gamma}(t - R/c + \tau_e) \tag{1.13}$$

The presence of the constant $\tau_e \equiv r_e/c$ is useful for ensuring a simple complex oscillator at the classical radius but introduces difficulties for the transformation which must be carefully resolved. This problem is adequately addressed in the Appendix. Also of interest is the overall factor $1/\gamma$ in the argument z which can be absorbed into a moving frame frequency defined by $\omega_e' = \omega_e/\gamma$.

In the moving frame, the radius of the electron becomes self-evident by decomposing the velocity potentials into separate angular and radial functions defined relative to the instantaneous retarded position of the particle

$$A^\nu(r,t) = 4\pi\, \sigma_e(\theta,\phi) \cdot \mathrm{u}^\nu(R) \cdot \mathrm{e}^{i\omega_e z} \tag{1.14}$$

and having individual components given by

$$\sigma_e(\theta,\phi) \equiv \frac{\sigma_e}{\gamma^2(1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})^2} \qquad\qquad \mathrm{u}^\nu(R) \equiv \frac{{r_e}^2}{R^2}\, R^\nu \cdot \mathrm{e}^{i\omega_e z} \tag{1.15}$$

The function $\sigma_e(\theta,\phi)$ serves as a Doppler charge density, which can be readily integrated over solid angle to produce the charge of the electron. The function $\mathrm{u}^\nu(R)$ may be referred to as a vacuum dilatation function. While it is not a four-vector, the dependency of $\mathrm{u}^\nu(R)$ on the retarded vector R^ν implies an equal time radius $R = r_e$ in the moving frame, and leads to the oscillating null radius four-vector:

$$\mathrm{u}^\nu(r_e) = r_e\, n^\nu \cdot \mathrm{e}^{i\omega_e \tau} \tag{1.16}$$

In other words, the theory of radiating fields guarantees that the classical particle—synonymous with the oscillator $\mathrm{u}^\nu(r_e)$—will maintain a spherical profile of radius r_e in any reference frame. This is absolutely necessary for the covariant generation of spherical vacuum waves.

With an understanding of moving frame vacuum gauge potentials, a covariant formula for electromagnetic momentum flux can be determined by writing the potentials as the sum of timelike and spacelike components:

$$A^\nu = A_t^\nu + A_s^\nu \qquad \begin{cases} A_t^\nu & = \dfrac{e}{\rho}\beta^\nu \cdot \mathrm{e}^{i\omega_e z} \\ \\ A_s^\nu & = \dfrac{e}{\rho} U^\nu \cdot \cdot \mathrm{e}^{i\omega_e z} \end{cases} \tag{1.17}$$

In terms of the potentials, the momentum flux tensor (proportional to the field strength tensor) takes the form

$$\pi^{\mu\nu} = \frac{1}{a_e}[A_s^{*\mu},\, A_t^\nu] \tag{1.18}$$

An important covariant flux integral over a timelike surface is

$$dE^\nu = -\oint_{R=r_e} \left[\tfrac{1}{2}\pi^{\mu\nu}c\right] U_\mu R^2 d\Omega d\tau = -P_{in}\beta^\nu d\tau \tag{1.19}$$

Dividing both sides by $d\tau$ determines the four-vector radiation rate per unit proper time comparable to a similar formula for the particle's acceleration fields. The radiation rate includes a minus sign while the magnitude of the radiated power is

$$P_{in} = \frac{e^2 c}{2r_e^2} = \frac{mc^2}{\tau_e} \tag{1.20}$$

This value of this power is extremely large—on the order of ten billion Watts.

2 Calculations of Inertial Vacuum Power

In this section, several methods for calculating inertial vacuum power are explored providing additional perspective and serving to broaden the overall scope of the radiation-based theory.

2.1 Vacuum Power from Vacuum Gauge Potentials

As already shown in the rest frame, vacuum gauge potentials offer and independent calculation of P_{in} without any knowledge of the electromagnetic fields. Using the potentials in (1.12) one can write the radiated flux as

$$\langle \mathbf{S}_v \rangle = \frac{1}{2a_e} A^* \boldsymbol{A} c \tag{2.1}$$

Neither the scalar nor vector potential in this formula have any invariant status, but the vacuum gauge potentials can be transformed to any frame leaving the mathematical structure of (1.12) as an invariant quantity. Radiated power follows exactly as in Jackson (Second Edition), section 14.3, ecxept that orifice in the vacuum possesses an inward unit surface area as in Figure 2. This requires the inclusion of a minus sign in the power integral as the outward momentum flux points opposite to the direction of the surface element:

$$\frac{dP_{in}}{d\Omega} = -R^2 \langle \mathbf{S}_v \rangle \cdot \hat{\boldsymbol{n}}(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{n}}) \tag{2.2}$$

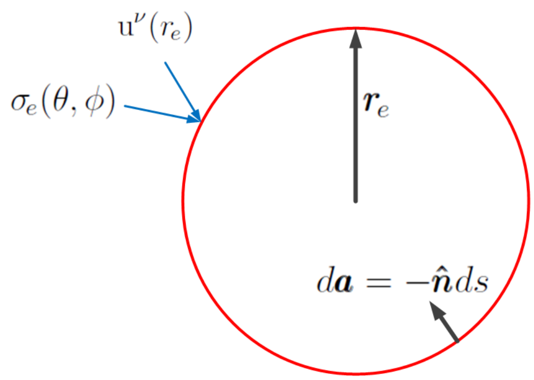


Figure 2: *Diagram illustrating the boundary of the vacuum orifice and an element of surface area at the boundary.*

Particle Accelerations: The previous calculation can be extended to include particle accelerations. In this case the symmetric stress tensor of the electromagnetic theory inherits a radiation term, falling off like ρ^{-2}, which can be written directly in terms of vacuum gauge potentials:

$$\Theta_3^{\mu\nu} = -\frac{1}{4\pi c^4}(\xi^2 + a^\lambda a_\lambda) A^\mu A^\nu \tag{2.3}$$

With the introduction of a complex phase, energy flux associated with the time-space components of this tensor may be added to the velocity flux

$$\langle \mathbf{S}_v \rangle + \langle \mathbf{S}_a \rangle = \frac{1}{a_e}\left[\frac{1}{2} - \frac{\tau_e^2}{c^2}(\xi^2 + a^\lambda a_\lambda)\right] A^* \boldsymbol{A} c \tag{2.4}$$

Inserting this new flux into (2.2) and integrating over solid angle then generates a power formula which is simply the sum of the inertial power and the Liénard generalization to the Larmor power formula

$$P_{total} = -\frac{mc^2}{\tau_e}\left[1 - \frac{4}{3}\frac{\tau_e^2}{c^2}\, a^\nu a_\nu\right] \tag{2.5}$$

The quantity $P_{total}\beta^\nu$ gives the four-vector radiation rate per unit proper time and both velocity power and acceleration power appear with a minus sign.

2.2 Vacuum Power from Compression Waves

Another interpretation of radiated vacuum power can be realized from a time derivative of equation (1.7) which determines the velocity of the oscillator[3]:

$$\boldsymbol{v}(\tau) = \Re e\left[c\,\hat{\boldsymbol{r}} \cdot \mathrm{e}^{i\omega_e \tau}\right] \tag{2.6}$$

But $\boldsymbol{v}(\tau)$ can be used to determine a velocity field from the radial boundary condition at r_e similar to equation (1.8). Defining a scalar pressure field by $cp = \mu_e|\boldsymbol{v}|$ determines the set:

$$p(r,\tau) = \frac{\mu_e r_e}{r} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \qquad\qquad \boldsymbol{v}(r,\tau) = \frac{c r_e}{r}\hat{\boldsymbol{r}} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \tag{2.7}$$

In the rest frame, these fields can be linked to a linearized Euler equation and source continuity equation, while showing properties similar to sound waves emitted by a spherically vibrating source. For relative motion, equations (2.7) can be awarded the status as covariant four-vectors in the form of a timelike pressure field and a spacelike velocity field:

$$p^\nu = \frac{\mu_e r_e}{\rho}\beta^\nu \cdot \mathrm{e}^{i\omega_e z} \qquad\qquad v^\nu = \frac{c r_e}{\rho}\, U^\nu \cdot \mathrm{e}^{i\omega_e z} \tag{2.8}$$

In terms of these fields the electromagnetic flux tensor is written

$$\pi^{\mu\nu} = \frac{1}{c}[v^{*\mu},\, p^\nu] \tag{2.9}$$

Pressure and velocity four-vectors may also be combined to form a related null pressure field

$$c\mathcal{P}^\nu \equiv cp^\nu + \mu_e v^\nu \equiv [c\mathcal{P}, \mu_e \boldsymbol{\mathcal{V}}] \quad \text{where} \quad c\mathcal{P} = \mu_e|\boldsymbol{\mathcal{V}}| \tag{2.10}$$

[3] Calculations in this section closely follow the theory of sound waves in chapter 9 (section 49) Fetter and Walecka, Theoretical Mechanics of Particles and Continua, 1980.

Naturally, this four-vector is proportional to the vacuum gauge velocity potentials and features moving frame scalar and vector components given by

$$\mathcal{P} = \mu_e r_e \frac{R}{\rho^2} \cdot e^{i\omega_e z} \qquad\qquad \boldsymbol{\mathcal{V}} = c r_e \frac{\boldsymbol{R}}{\rho^2} \cdot e^{i\omega_e z} \tag{2.11}$$

The field $\mathcal{P}$ radiates spherical compression waves similar to acoustic phenomena. A formula for radiated vacuum power is still equation (2.2) but radiated energy flux averaged over time will be written

$$\langle \mathbf{S}_v \rangle = \frac{1}{2}\mathcal{P}^* \boldsymbol{\mathcal{V}} \tag{2.12}$$

This result and others like it are testament to the ubiquitous nature of the vacuum fields radiated by the classical electron. With two sets of defined pressure fields, one can ask what exactly is being compressed. This question can be answered by asserting the existence of a medium sitting on top of Minkowski space in which the classical electron appears as an excited state.

Power Formula as Radiated Quanta: Either of (2.12) or (2.1) can be inserted into the differential power formula of (2.2), but this formula also has an interpretation in terms particle emissions. This is most easily illustrated by considering motion along the z-axis and writing

$$\frac{dP_{in}}{d\Omega} = \frac{dN(\theta)}{d\Omega} \cdot \varepsilon(\theta) \tag{2.13}$$

Here $dN/d\Omega$ is the particle emission rate into solid angle, and $\varepsilon(\theta)$ is the energy radiated as a function of polar angle. Using the classical radius formula $mc^2 = e^2/2r_e$, and considering the invariant Dirac frequency $\omega_D = mc^2/\hbar$, it is possible to deduce

$$\frac{dN(\theta)}{d\Omega} = \frac{1}{4\pi}\frac{\omega_D}{\gamma^3(1-\beta\cos\theta)^2} \qquad\qquad \varepsilon(\theta) = \frac{\hbar\omega_e}{\gamma(1-\beta\cos\theta)} \tag{2.14}$$

The functional form of $dN/d\Omega$ indicates that it derives largely from a Lorentz transformation of solid angle, while integrating shows that the number of particles radiated per unit time is altered by time dilation $N_{tot} = \omega_D/\gamma$.

2.3 Vacuum Power from Source Current

Calculating momentum flux velocity fields is the primary purpose of vacuum gauge potentials, but derivatives can still be applied to equation (1.12) to determine the field strength tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = F_M^{\mu\nu} \cdot \mathrm{e}^{i\omega_e z} \tag{2.15}$$

The subsscript M on the right indicates the conventional Maxwell fields which are modulated by the complex exponential. Now determine the four-current density from a divergence operation

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}\left[J_e^\nu \cdot \mathrm{e}^{i\omega_e z} + J_N^\nu\right] \tag{2.16}$$

Here J_e^ν is the conventional point four-current density associated with the classical particle which is now a point oscillator. Equally important however, is the additional null source-current given by

$$J_{\scriptscriptstyle N}^\nu = -\frac{i\omega_e e}{4\pi\rho^3} R^\nu \cdot \mathrm{e}^{i\omega_e z} \tag{2.17}$$

It is easy to show that $\partial_\nu J_{\scriptscriptstyle N}^\nu = 0$ even though it doesn't appear to be a conserved current. It's properties are most easily addressed in the rest frame where the charge density and the electric current density take the form radiating spherical waves out of phase with the electric field and determined by:

$$\varrho_{\scriptscriptstyle N} = -\frac{ie\omega_e}{4\pi c r^2} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \qquad\qquad \mathbf{J}_{\scriptscriptstyle N} = -\frac{ie\omega_e}{4\pi r^2} \cdot \mathrm{e}^{i\omega_e(\tau - r/c + \tau_e)} \hat{\boldsymbol{r}} \tag{2.18}$$

As the three-vector current radiates isotropically through the vacuum boundary, it cannot be associated with any magnetic field. This can by proved by inserting $\mathbf{J}_{\scriptscriptstyle N}$ into the Biot-Savart law to determine a magnetic field of zero. In addition, Ampère's law with the Maxwell correction in the rest frame is

$$\mathbf{0} = \boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}_{\scriptscriptstyle N} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{2.19}$$

Without a magnetic field, $\mathbf{J}_{\scriptscriptstyle N}$ follows from a time derivative of the electric field vector. For reference, the front page diagram shows the magnetic flux lines generated by the current density, which curl around the electric flux field and symmetrically cancel themselves out.

In an arbitrary moving frame, vacuum power can be calculated by forming two invariant timelike and spacelike integrals. Here we are forced to identify the frequency $\omega_e = 1/\tau_e$ and write vacuum power as:

$$P_{in} = \frac{1}{2} V^* I \quad \text{where} \quad \begin{cases} V(\tau) = \dfrac{1}{c}\displaystyle\oint_{r_e} J_{\scriptscriptstyle N}^\nu \beta_\nu R^2 d\Omega = -\dfrac{ie}{r_e} \cdot \mathrm{e}^{i\omega_e \tau} \\[3ex] I(\tau) = \displaystyle\oint_{r_e} J_{\scriptscriptstyle N}^\nu U_\nu R^2 d\Omega = \dfrac{iec}{r_e} \cdot \mathrm{e}^{i\omega_e \tau} \end{cases} \tag{2.20}$$

The integrals represent an oscillating voltage and current source at the electron radius which combine to produce the power formula with an inclusive minus sign. In addition, the calculation promotes the idea that the oscillating radius vector behaves as a spacetime aperture featuring its own radiation resistance–similar to antenna's in classical radiation theory. Using MKS units, this radiation resistance is given by

$$\mathcal{R} = \frac{1}{4\pi}\sqrt{\frac{\mu_o}{\epsilon_o}} \tag{2.21}$$

3 Summary: Implications of the Radiation-Based Theory

The insertion of momentum and energy into the velocity fields of the classical electron has led to a theory of vacuum gauge potentials with some very impressive results. Unfortunately, with comsological implications notwithstanding, these results might appear to be in serious conflict with classical electrodynamics. Specifically, the velocity fields in equation (2.15) have been reduced to high frequency travelling waves, and their ability to exert forces has been completely compromised. But this cannot be an issue for the electron which mediates the electromagnetic force with transverse-polarized photons derived from particle acceleration fields. Furthermore, velocity fields of equation (2.15) can still be used to construction of the velocity portion of the symmetric stress tensor which now appears as

$$\Theta_{\mu\nu} \equiv \frac{1}{4\pi}\left[F^*_{\mu\lambda}F^{\lambda}{}_{\nu} + \frac{1}{4}\mathrm{g}_{\mu\nu}F^*_{\alpha\lambda}F^{\alpha\lambda}\right] \tag{3.1}$$

Overall, it seems very plausible to construct a radiation-based theory for the classical particle. Its success however, will depend on the ability to re-construct the macroscopic theory of "static" fields as a limit involving the transmission of large numbers of photons. Success also requires an explanation for the unsettling value of inertial vacuum power at 1.74×10^{10} Watts. We have no answer for this except to propose that ground state atoms do not radiate, and that a very large absorption cross-section may exist for vacuum quanta.

A Derivatives of the Null Vector

The covariant derivative of R^ν is

$$\partial^\mu R^\nu = \mathrm{g}^{\mu\nu} - \frac{R^\mu \beta^\nu}{\rho} \tag{A.1}$$

The trace of the resulting matrix gives the 4-divergence $\partial_\nu R^\nu = 3$. In terms of individual components–and with the inclusion of a sign–a useful construction is:

$$-\partial^\mu R^\nu = \begin{bmatrix} -\dfrac{\partial R}{\partial ct} & -\dfrac{\partial \boldsymbol{R}}{\partial ct} \\ \boldsymbol{\nabla} R & \boldsymbol{\nabla} \boldsymbol{R} \end{bmatrix} \tag{A.2}$$

where individual components are given by

$$\frac{\partial R}{\partial ct} = 1 - \frac{\gamma R}{\rho} \qquad\qquad \frac{\partial \boldsymbol{R}}{\partial ct} = \frac{-\gamma R \boldsymbol{\beta}}{\rho} \tag{A.3}$$

$$\boldsymbol{\nabla} R = \frac{\gamma \boldsymbol{R}}{\rho} \qquad\qquad \boldsymbol{\nabla} \boldsymbol{R} = \mathbf{1} + \frac{\gamma \boldsymbol{R}\boldsymbol{\beta}}{\rho} \tag{A.4}$$

The determinant of (A.2) can be written $\det[\partial^\mu R^\nu] = 0$. The divergence and curl of $\boldsymbol{R}$ may be written

$$\boldsymbol{\nabla} \cdot \boldsymbol{R} = 3 + \frac{\gamma \boldsymbol{R} \cdot \boldsymbol{\beta}}{\rho} = Tr[\boldsymbol{\nabla} \boldsymbol{R}] \tag{A.5}$$

$$\boldsymbol{\nabla} \times \boldsymbol{R} = \frac{\gamma}{\rho} \boldsymbol{R} \times \boldsymbol{\beta} \tag{A.6}$$

Let $\boldsymbol{w}(ct_r)$ be the retarded position of a charged particle at time ct_r. The light cone condition is defined by

$$\boldsymbol{R} \equiv \boldsymbol{r} - \boldsymbol{w}(ct_r) \qquad\qquad R \equiv ct - ct_r \tag{A.7}$$

Derivatives of the retarded time with respect to present time coordinates are

$$\frac{\partial ct_r}{\partial ct} = \frac{\gamma R}{\rho} \tag{A.8}$$

$$\boldsymbol{\nabla} ct_r = \frac{-\gamma \boldsymbol{R}}{\rho} \tag{A.9}$$

B Lorentz Transformation of the Causality Sphere

An essential problem for a theory of radiating electromagnetic fields is the development of an appropriate transformation law for the causality step which necessarily includes the particle radius as a phase. In the rest frame the causality step is

$$\vartheta = \vartheta(c\tau - s + r_e) \tag{B.1}$$

A graphical depiction of the this function is provided in figure 3 showing a temporal expansion over a time interval $c\tau + r_e$. Unfortunately, the presence of r_e in the argu-

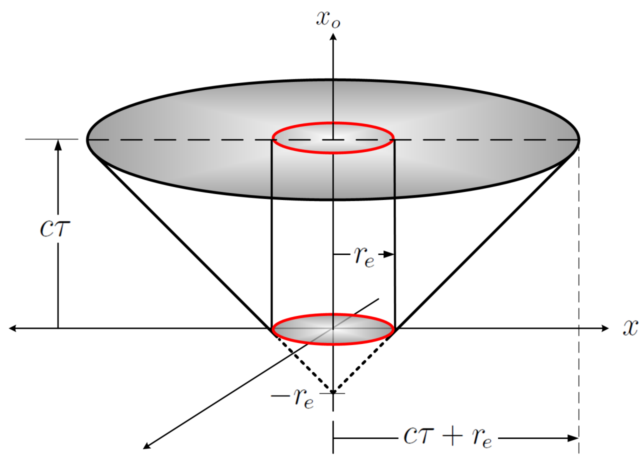


Figure 3: *Spacetime diagram indicating the expansion of the radial step $\vartheta(c\tau - s + r_e)$ in the rest frame. The inclusion of the phase r_e defines two regions inside and outside the particle radius.*

ment leads to difficulties when transforming to a moving frame. To understand why, insert the general homogeneous Lorentz transformation

$$c\tau = \gamma\,(ct - \boldsymbol{\beta}\cdot\boldsymbol{r}) \tag{B.2a}$$

$$\boldsymbol{s} = \boldsymbol{r} + \frac{\gamma - 1}{\beta^2}(\boldsymbol{\beta}\cdot\boldsymbol{r})\boldsymbol{\beta} - \gamma ct\boldsymbol{\beta} \tag{B.2b}$$

showing that the step transforms as

$$L[\vartheta(c\tau - s + r_e)] = \vartheta(\gamma ct - \gamma\boldsymbol{\beta}\cdot\boldsymbol{r} - \rho + r_e) \tag{B.3}$$

The problem is to determine the collection of points which represents the boundary of the expanding step in the moving frame. Letting $\vartheta = \vartheta(Z)$, then the vanishing of Z implies the covariant condition

$$(x^\nu + r_e\beta^\nu)\cdot(x_\nu + r_e\beta_\nu) = 0 \tag{B.4}$$

This is the equation of a sphere with a radius $ct+\gamma r_e$ which expands about the point $-\gamma\boldsymbol{\beta}r_e$ so the moving frame step function is

$$L[\vartheta] = \vartheta(ct - \|\boldsymbol{r} - \gamma\boldsymbol{\beta}r_e\| + \gamma r_e) \tag{B.5}$$

In short, the homogeneous transformation is not a useful theoretical tool for keeping the mathematics inside the argument of the step simple.

Based on the previous result it is reasonable to enquire how the Lorentz transformation might be tailored to shift the origin of coordinates so that the causality step still expands from the spatial origin in the moving frame. This can be accomplished by defining proper frame and moving frame time coordinates as

$$c\hat{\tau} \equiv c\tau + r_e \qquad\qquad c\hat{t} \equiv ct + r_e \tag{B.6}$$

Replacing the time coordinates in (B.2) with the hatted coordinates renders a new set of equations

$$c\hat{\tau} = \gamma\left(c\hat{t} - \boldsymbol{\beta}\cdot\boldsymbol{r}\right) \tag{B.7a}$$

$$\boldsymbol{s} = \boldsymbol{r} + \frac{\gamma-1}{\beta^2}(\boldsymbol{\beta}\cdot\boldsymbol{r})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}c\hat{t} \tag{B.7b}$$

This new set can be tested immediately to show that

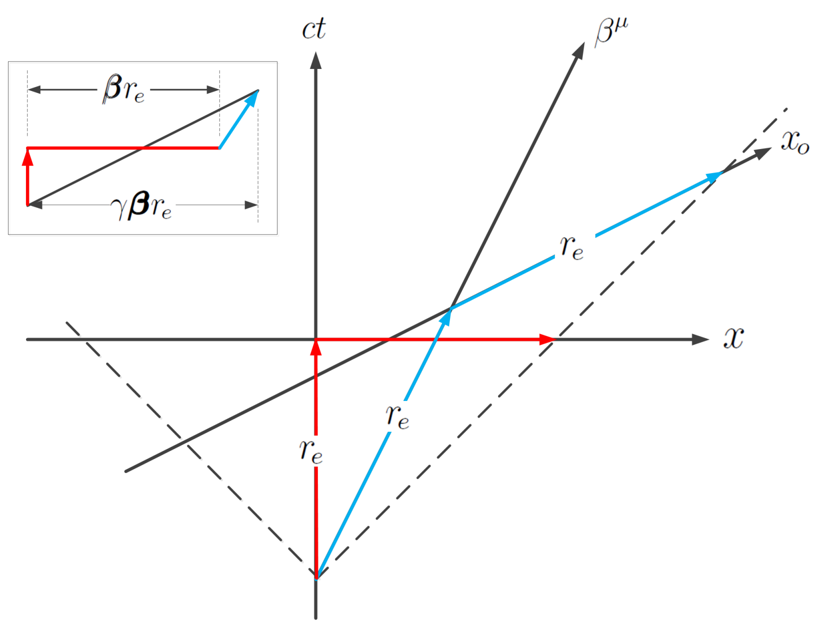


Figure 4: *Spacetime diagram based on equations (B.7). The transformation produces timelike and spacelike radius vectors (shown in red and blue) which are the same in both frames.*

$$L[\vartheta(c\tau - s + r_e)] = \vartheta(ct - r + r_e) \tag{B.8}$$

It is also important to consider equations (B.7) in terms of the retarded time ct_r and its associated vector $\boldsymbol{R}$. Relative to the moving frame, the position of the electron as a function of time is given by

$$w^\nu(ct_r) = (ct_r, \boldsymbol{\beta}c\hat{t}_r) \tag{B.9}$$

and the retardation condition is $R^\nu = x^\nu - w^\nu$. This substitution derives the following relations

$$c\tau + r_e = \rho + \frac{1}{\gamma}c\hat{t}_r \tag{B.10}$$

$$\boldsymbol{s} = \boldsymbol{R} + \frac{\gamma - 1}{\beta^2}(\boldsymbol{\beta} \cdot \boldsymbol{R})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}R \tag{B.11}$$

Extraneous terms involving the radius drop out of the second equation which is important because it preserves the definition of the covariant scalar $\rho \equiv R^\nu\beta_\nu$. In terms of the new coordinates the causality sphere can be written

$$\vartheta = \vartheta[\gamma^{-1}(c\hat{t}_r)] \tag{B.12}$$

This is a sensible result which vanishes for retarded times less then r_e. Finally, equations (B.7) can be re-arranged to look something like a Poincaré transformation except the coordinate shifts are functions of particle velocity

$$\alpha_\mu \equiv [\gamma - 1, -\gamma\boldsymbol{\beta}]r_e \tag{B.13}$$

C Discrete Symmetries

Classical electromagnetism is known to be invariant under charge, parity, and time reversal operations. This is still true for the field strength tensor of equation (2.15). These fields—generated by changes to the source current—are high frequency outgoing travelling waves which are incoming waves upon application of the T-operator. For the classical particle however, some changes to discrete symmetries are necessary to address the problems of infinite self-energy and particle stability which are not relevant to the macroscopic theory. These changes imply the flux fields satisfying equations (1.3). In this case the radiation-based theory determines the following:

- C-Symmetry: The square of the charge appears in all equations. This seems to be equivalent to the statement that the theory is invariant under the operation of charge conjugation.

- P-Symmetry: $\boldsymbol{\pi}_{\mathrm{E}}$ remains odd under a parity operation since field momentum appears to move opposite to the original coordinate system. But $\boldsymbol{\pi}_{\mathrm{B}}$ must still remain an axial vector owing to the relation

$$\boldsymbol{\pi}_{\mathrm{B}} = \boldsymbol{\beta} \times \boldsymbol{\pi}_{\mathrm{E}} \tag{C.1}$$

  Operations on equations (1.3) show that the momentum flux formulation remains invariant under parity.

- T-Symmetry: In the vacuum theory, electric flux fields radiate field energy. Clearly $\mathrm{T}[\boldsymbol{\pi}_{\mathrm{E}}] = -\boldsymbol{\pi}_{\mathrm{E}}$. Since the T- operation also changes the sign of $\boldsymbol{\beta}$ this requires $\mathrm{T}[\boldsymbol{\pi}_{\mathrm{B}}] = \boldsymbol{\pi}_{\mathrm{B}}$. Operations on equations (1.3) show that both source equations violate time reversal invariance.