1 Conic Curves

Definition 1. *For a quadratic curve*

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0,$$

the discriminant is defined as $\Delta = B^2 - 4AC$.

Theorem 1. *The discriminant catagories the quadratic curve as:*

1. $\Delta < 0$*: Ellipse, point or null;*

2. *Delta* $= 0$*: Parabola, line, parallel line pair or null;*

3. *Delta* > 0*: Hyperbola or crossing lines.*

Theorem 2. *The equation of tangen line at* (x_0, y_0) *is*

$$Ax_0x + By_0y + C\frac{x_0y + y_0x}{2} + D\frac{x + x_0}{2} + E\frac{y + y_0}{2} + F = 0.$$

Theorem 3. *The equation of the chord crossing the two tangent points is*

$$Ax_0x + By_0y + C\frac{x_0y + y_0x}{2} + D\frac{x + x_0}{2} + E\frac{y + y_0}{2} + F = 0.$$

Theorem 4. *The equation of the intersection of the two tangent lines crossing the two intersection points of a chord is*

$$Ax_0x + By_0y + C\frac{x_0y + y_0x}{2} + D\frac{x + x_0}{2} + E\frac{y + y_0}{2} + F = 0.$$

2 General Geometry

Reflection 1. 为求解关于三角形之问题，可借助

1. 正弦定理获得正弦与边之关系；

2. 余弦定理获得余弦与相应边之关系；

3. 余弦定理将边的平方降次；

必要时，应组合上述反射。

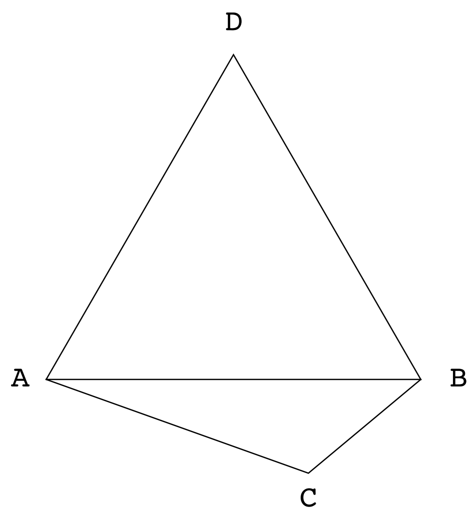

图 1:

Problem 1. *Given $AC = 4$,$BC = 2$, ΔABD is a equilateral triangle. Evaluate the maxium of $S_{\Delta ACD}$.*

Solution. With

$$a^2 = 20 - 16\cos C, \quad 4 = a^2 + 16 - 8a\cos A,$$

and

$$a\sin A = 2\sin C,$$

$$\begin{aligned} S = 2a\sin DAC &= 2a\left(\sin 60^\circ \cos A + \cos 60^\circ \sin A\right) \\ &= 2a\left(\frac{\sqrt{3}}{2}\frac{a^2+12}{8a} + \frac{1}{2}\frac{2\sin C}{a}\right) \\ &= \frac{\sqrt{3}}{2}\frac{1}{4}\left(a^2+12\right) + 2\sin C \\ &= \frac{\sqrt{3}}{2}\frac{1}{4}\left(32 - 16\cos C\right) + 2\sin C \le 4 + 4\sqrt{3}. \end{aligned}$$

□

3 Functions and Analysis

Reflection 2. *To compare two numbers involving a transcendental expression, one may construct a function whose variable substitute the transcen-*

dental and evaluate its minimum value. Where duplicated occurrence may give some clues to decide the position of the variable.

Problem 2. *Compare the following couples of numbers:*

1. $\log 3, \sqrt{3}\log 2$
2. $\log \pi, \sqrt{\frac{\pi}{e}}$
3. $2^{\sqrt{15}}, 15$
4. $3e\ln 2, 4\sqrt{2}$

Solution. With
$$f = \frac{\log x}{x},$$
which attains its maximum value at $x = e$ when $f(x) = 1/e$ and is monotone increasing for $x < e$, one may reduce the first comparison to
$$\frac{\log \sqrt{3}}{\sqrt{3}}, \frac{\log 2}{2},$$
whose relative sizes are obvious with the f.

The second one may amount to compare
$$\frac{\log \sqrt{\pi}}{\sqrt{\pi}}, \frac{\log e}{\sqrt{e}},$$
which is again obvious with the argument above. A similar argument applies to the rest cases. □