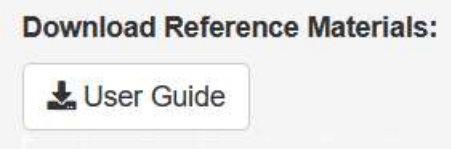
Prior Elicitation for Ordinal Models Application User Guide

Section 1: Initialize Prior Specification

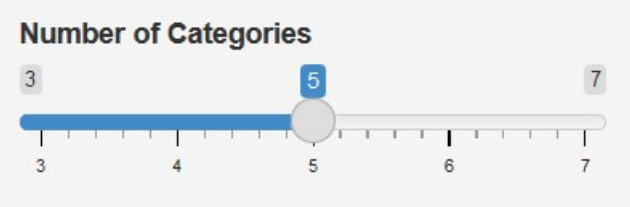
Section 1.1: Download Reference Materials



- You may also download this guide by clicking the ***User Guide*** button.

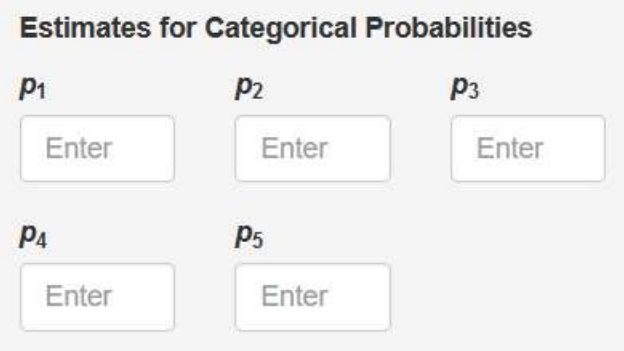
Section 1.2: Ordinal Model Specification

Section 1.2.1: Number of Categories



- Using the ***Number of Categories*** slider, you may select 3, 4, 5, 6, or 7 categories for the ordinal model. The rest of the sidebar panel will automatically adjust to accommodate the selected number of categories.

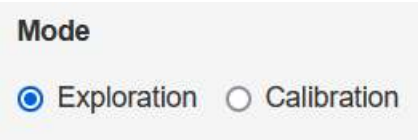
Section 1.2.2: Estimates for Categorical Probabilities



- Using these textboxes, you can specify initial estimates for the multinomial probability assigned to each category.

- Each probability should be strictly between 0 and 1. The specified probabilities should sum to 1 (to three decimal places). Please use decimals for the probabilities, not fractions.

Section 1.3: Select Mode



- The ***Mode*** radio button can be used to select the computational settings for the app. This will be later explained in more detail, but the marginal priors for the categorical probabilities $\boldsymbol{p}$ and the ordinal mean θ are estimated using simulation and kernel density estimation.
- "Exploration" mode can be used when exploring initial inputs for the ordinal prior (when we are not yet sure whether our initial inputs are adequate). In "Exploration" mode, it should take at most 5 to 10 seconds to update the plots on the ***Prior Specification*** tab. This mode can be used to quickly update the plots, but the marginal priors for $\boldsymbol{p}$ and θ are less precisely estimated.
- "Calibration" mode can be used when fine-tuning the inputs for the ordinal prior (when we know our current inputs are reasonable). In "Calibration" mode, it should take at most 15 to 20 seconds to update the plots on the ***Prior Visualization*** tab. This mode is slower, but the marginal priors for $\boldsymbol{p}$ and θ are more precisely estimated.

Section 1.4: Updating the *Prior Visualization* Tab

Section 1.4.1: The *Update* Button

- To have the changes from the sidebar panel reflected in the plot on the ***Prior Visualization*** tab, you must press the ***Update*** button. This is done to prevent the app from breaking if an invalid input is specified on the sidebar panel.

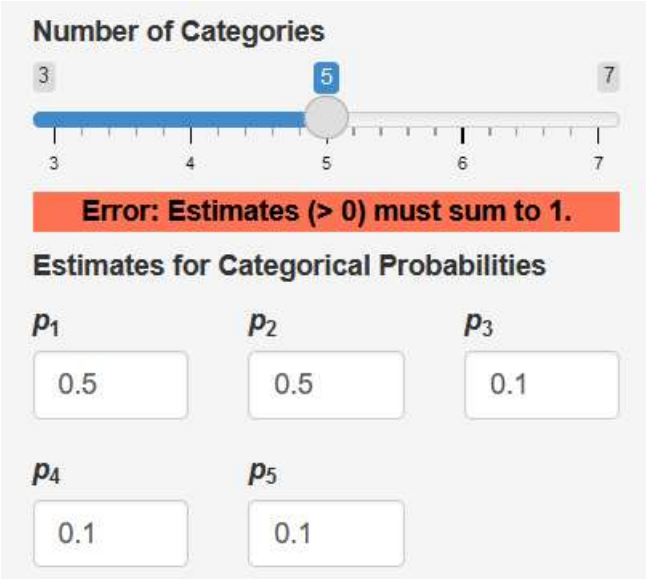
Section 1.4.2: Loading Message

- The following loading message appears when the app is initially loading or when the plots are repopulating after the ***Update*** button has been pressed. No updates to the plots should take more than one minute.

Loading plots. Please note this may take up to a minute. Thank you for your patience.

Section 1.4.3: Error Messages

- When an invalid input is specified, a red error message appears right above where the invalid input has been specified on the sidebar panel. The plot on the ***Prior Visualization*** tab is locked in place and appears as it did before the ***Update*** button was pressed.
- Some error messages will be discussed later in more detail. The following error message serves as an example. This is the error message that populates when the estimates for the categorical probabilities do not sum to 1.



Section 1.5: The *Reset* Button

- The ***Reset*** button in the upper right corner of the app can be used to reset the inputs on the sidebar panel and the plot on the ***Prior Visualization*** tab. The progress from your session will not be saved. We show how to save your progress before exiting or resetting the app later.

Section 2: Marginal Prior Specification

Section 2.1: Reparameterization

Section 2.1.1: Rationale

- We do not directly assign marginal prior distributions to the multinomial probabilities: $p_1, p_2, \ldots, p_w$, where w is the number of categories. This would present issues as we need to specify a joint prior for $\boldsymbol{p}$ such that its multinomial probabilities sum to 1. Therefore, we define new variables $Z_1, Z_2, \ldots, Z_w$ as in Elfadaly and Garthwaite (2017) such that

$$Z_1 = p_1, \quad Z_v = \frac{p_v}{1 - \sum_{t=1}^{v-1} p_t} \text{ for } v = 2, \ldots, w-1, \quad \text{and} \quad Z_w = 1.$$

- The inverse transformations are given by

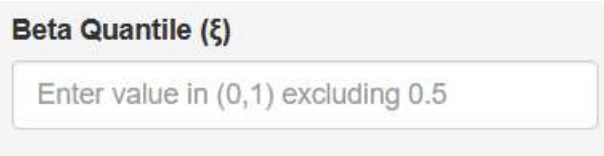
$$p_1 = Z_1 \quad \text{and} \quad p_v = Z_v \prod_{t=1}^{v-1} (1 - Z_t) \text{ for } v = 2, \ldots, w.$$

- By assigning beta distributions to $Z_1, Z_2, \ldots, Z_{w-1}$, we ensure that the corresponding multinomial probabilities sum to 1.

Section 2.1.2: Interpretation

- The $\boldsymbol{Z}$ variables have sensible interpretations. The variable Z_v represents the probability that an observation is assigned to category v given that it has not been assigned to categories 1, …, $v-1$.

Section 2.2: Choose Beta Quantile

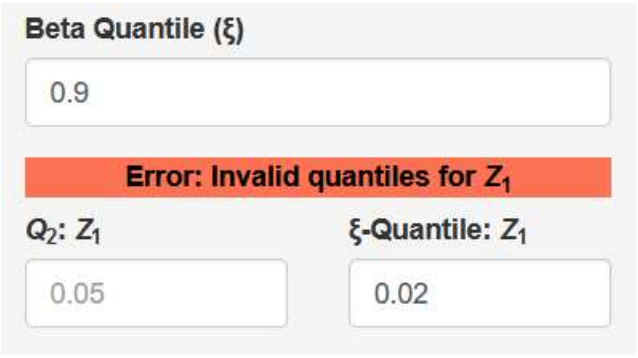


- Using the ***Beta Quantile (ξ)*** textbox, you can choose a quantile between 0 and 1 (exclusive) that is not the median (0.5). If an invalid quantile is entered into this textbox, an error message will appear when the ***Update*** button is pressed.
- As shown in **Section 2.3**, we will use this quantile and the median (Q_2) to specify marginal beta priors for $Z_1, Z_2, \ldots, Z_{w-1}$. We need to specify two distinct quantiles to uniquely define a beta distribution; this is why the chosen quantile cannot be the median.

Section 2.3: Specifying Marginal Beta Distributions

Section 2.3.1: Inputs

- Based the number of categories (w) selected, two textboxes will appear for each variable Z_v for $v = 1, \dots, w-1$. The first textbox is for the median (Q_2) of the beta distribution for Z_v, and the second textbox is for ξ-quantile of this distribution.
- Once the valid inputs have been entered into the ***Estimates for Categorical Probabilities*** textboxes and the ***Update*** button has been pressed, estimates for the medians of the beta distributions for $Z_1, \dots, Z_{w-1}$ are suggested in the $\boldsymbol{Q_2}$ textbox. These medians are suggested by applying the transformation in **Section 2.1.1** to the values from the ***Estimates for Categorical Probabilities*** textboxes.
- These estimates can be overwritten, but the suggested value will be used if nothing is entered into the $\boldsymbol{Q_2}$ textbox.
- If invalid quantiles are entered into the textboxes, an error message will appear the next time the ***Update*** button is pressed. The following error message serves as an example when invalid quantiles are chosen for Z_1.

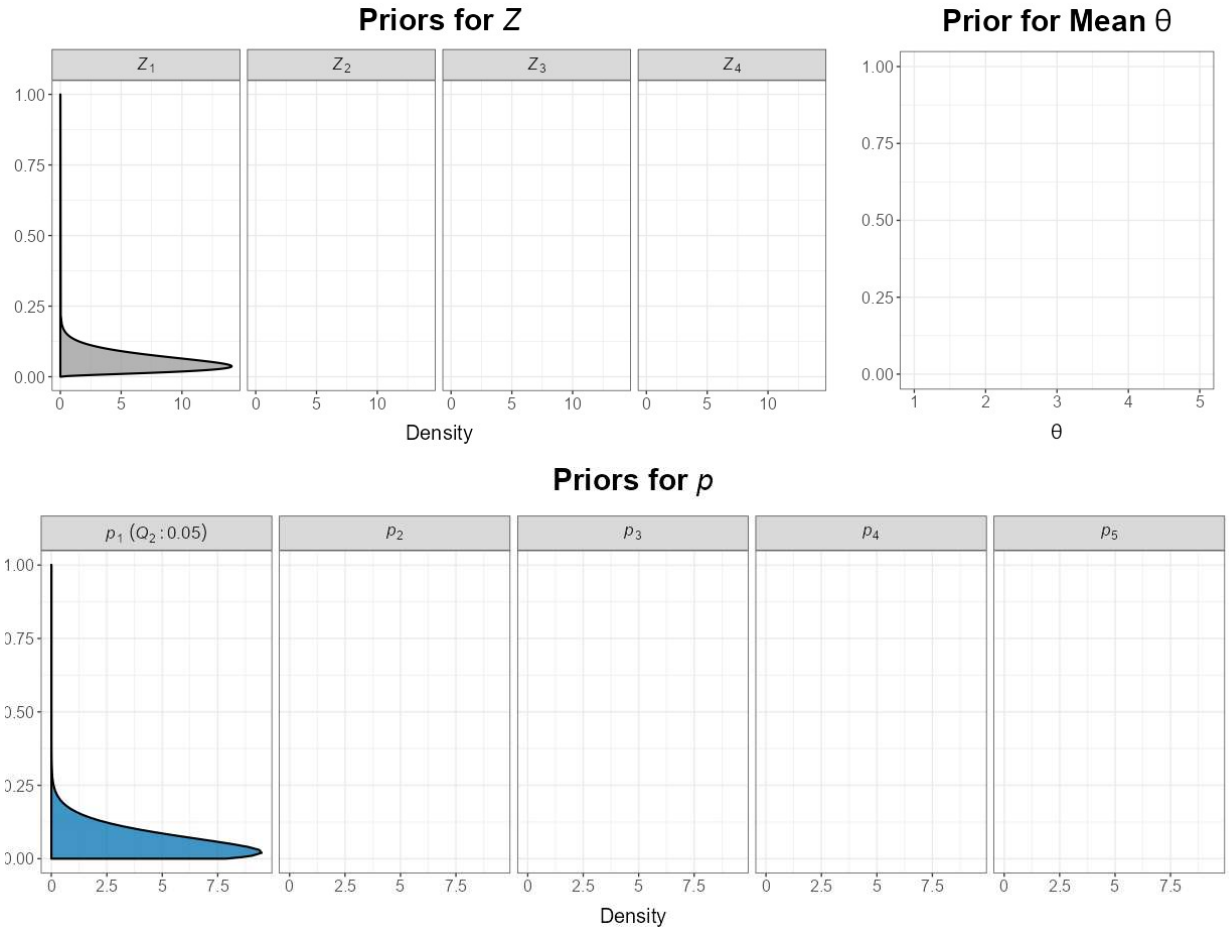


- A pair of quantiles may be deemed invalid for one of the following reasons. First, both quantiles must be numeric values between 0 and 1 (exclusive). If $\xi > 0.5$, then the ξ-quantile must be strictly greater than the median. This is why the example error message was returned. If $\xi < 0.5$, then the ξ-quantile must be strictly less than the median. Moreover, the textboxes must be filled out in descending order. That is, the textboxes for Z_1 cannot be empty if the textboxes for Z_2 have been filled in. This would return an error message for Z_1. The suggested value for

the median of Z_ν only counts as being filled in if there is a valid input in the ***ξ-Quantile*** textbox; otherwise, the ***Median*** textbox is treated as empty.

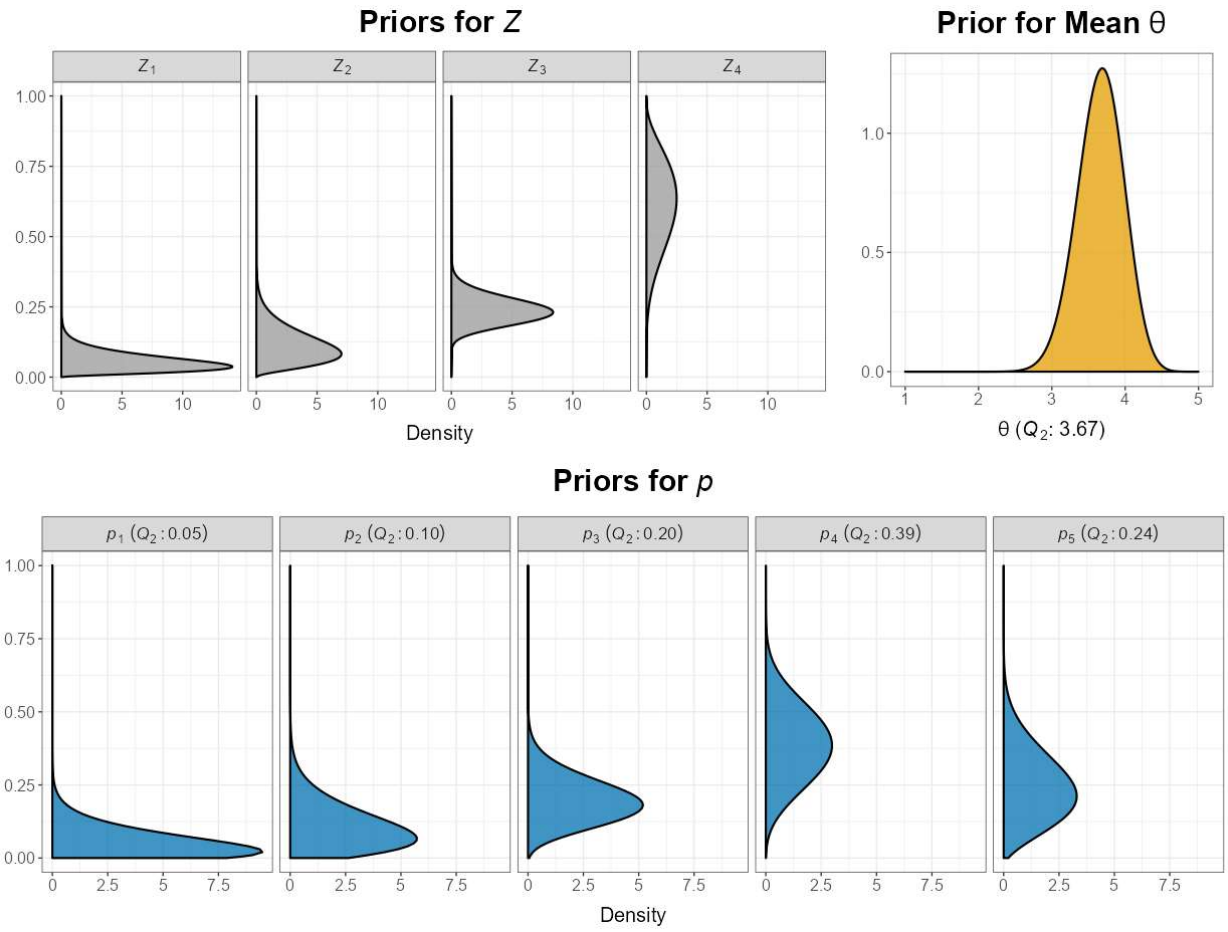
- We require that the textboxes be filled out in descending order due to the transformations in **Section 2.1.1**. For instance, p_1 is an expression of Z_1 and p_2 is an expression of Z_1 and Z_2. If the inputs for Z_2 have been filled in before the inputs for Z_1, then we cannot consider the marginal distribution of either p_1 or p_2!

Section 2.3.1: Incrementally Updating the Plots



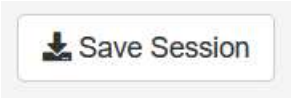
- The app is designed such that the plots on the ***Prior Visualization*** tab can be updated incrementally as the inputs in the ***Marginal Prior Specification*** section of the sidebar panel are chosen. The figure above shows an example plot where only the inputs for Z_1 have been filled in. You just need to press the ***Update*** button to have your changes from the sidebar panel reflected on the plot. Again, the plot will not update if there is an invalid input, and an appropriate error message will be displayed.

- We recommend first working with the inputs for Z_1, looking at the resulting beta distribution for Z_1 in the upper left plots and at the estimated marginal prior for p_1 in the bottom plots. For $v = 1$, these distributions are the same. According to the transformations in **Section 2.1.1**, the marginal prior for p_1 is not impacted by the distributions for $Z_v, v > 1$. Therefore, we suggest tweaking the inputs for Z_1 until satisfied with the plots for Z_1 and p_1. This process can be repeated with the inputs for Z_2; this time, the marginal priors for Z_2 and p_2 will be of interest. We can continue this process until the inputs for Z_{w-1} have been specified. At this point, all subplots in the main figure should be populated.
- For efficiency, the app is designed to detect the smallest value of $v = 1, \dots, w-1$ such that the inputs for Z_v have changed since the last time that the ***Update*** button was pressed. This has been done to promote prior specification for ordinal models as an incremental process. For example, let's suppose that the Z_2 inputs have changed since the last time the plots were updated but that the inputs for Z_1 have not changed. When the ***Update*** button is pressed, the app will detect this and update the plots for Z_2 and p_2 but not those for Z_1 and p_1.
- The following figure illustrates a completed plot (where all the marginal priors have been populated) when $w = 5$. The upper right plot shows the marginal prior for the ordinal mean θ, which has been estimated using simulation. The median of the prior for the ordinal mean, on a scale from 1 to w, is given in parentheses. The induced prior medians for $p_1, p_2, \dots, p_w$ are also denoted on their marginal plots.



Section 3: Saving Session Progress

Section 3.1: Save Session and Output



- Before exiting or refreshing the app, you can save the progress from your session using the ***Save Session*** button in the upper right corner of the ***Prior Visualization*** tab. Pressing this button will download a .zip file with the following four components.
    - 01_p_estimates.csv: this file contains the values from the ***Estimates for the Categorical Probabilities*** textboxes.
    - 02_Zquantile_inputs.csv: this file contains a table with the inputs from the $\boldsymbol{Q_2}$ and **ξ-Quantile** textboxes for $Z_1, \ldots, Z_{w-1}$, along with the value for ξ.

    - 03_marginal_beta_parameters.csv: this file contains a table with the shape and rate parameters for the marginal beta distributions for $Z_1, \ldots, Z_{w-1}$. These beta distributions are defined using the median and ξ-quantile inputs from the previous file.
    - 04_marginal_plots.pdf: this file contains the large plot from the ***Prior Visualization*** tab.
- The output in these files comes from the last time that the ***Update*** button was pressed without being stopped by an error message. That is, it comes from the last time that the plot on the ***Prior Visualization*** tab was successfully updated.
- You can save your progress when a session is partially complete. For instance, you may have only specified a marginal beta prior for Z_1. In that case, the beta parameters for Z_1 will be saved to the output files. The output files would contain ***NA*** values for the beta distribution parameters corresponding to $Z_2, \ldots, Z_{w-1}$.

References

Elfadaly, F.G. and P.H. Garthwaite (2017). Eliciting Dirichlet and Gaussian copula prior distributions for multinomial models. *Statistics and Computing 27* (2), 449-467.