ECE 230L, HW #7
Fall 2017

Solutions

Homework #7

MOSFETs – *100 points*

DUE @ Beginning of Class: Thursday, October 26

1) E-Book, problem 10.33 (8 points)

2) E-Book, problem 10.36, note that the MOSFET is biased in saturation region (8 points)

3) E-Book, problem 10.44 (10 points)

4) E-Book, problem 10.46 (16 points)

5) E-Book, problem 10.51 (6 points)

6) E-Book, problem 10.56 (14 points)

7) Use the *MOSFet* simulator on nanohub.org to obtain the I-V_g (@ V_d = 0.05 V and 1.0 V) and I-V_d (@ max V_d = 1.0 V and min V_g = 0.5 V, max V_g = 1.0 V with 3 curves) characteristics for a Si MOSFET with the following attributes (use default for everything else): (22 points)

Channel Length = 50 nm	Oxide Thickness = 2 nm
Junction Depth = 10 nm	Substrate Thickness = 10 nm

Provide plots of the subthreshold, transfer, and output curves, with each correctly labeled. Extract the following from the I-V curves, showing how the extraction was done on the actual plotted curves:

i. Subthreshold swing
ii. Threshold voltage
iii. ON-current @ V_{DD} = 1 V
iv. Transconductance
v. Calculate the mobility, assuming W = 1 μm and V_{DS} = 1 V

8) Now change the oxide thickness, T_{ox} to 10 nm in the *MOSFet* simulator and rerun (all other parameters the same as in problem #7). What is different about this new device's performance compared to the device in problem #7 (include I-V curves (subthreshold) and at least one extracted performance metric to support your observation)? (8 points)

9) Keeping T_{ox} = 10 nm, change the dielectric constant to 20 and rerun. Compare the subthreshold curves from this device with those from the device in problem #8. What changed? Why? (*hint:* consider what happens to C_{ox}) (8 points)

1) E-Book, problem 10.33 (8 points)

10.33 Consider an n-channel MOSFET with the following parameters: $k_n' = 0.18\ \mathrm{mA/V^2}$, $W/L = 8$, and $V_T = 0.4$ V. Determine the drain current I_D for (*a*) $V_{GS} = 0.8$ V, $V_{DS} = 0.2$ V; (*b*) $V_{GS} = 0.8$ V, $V_{DS} = 1.2$ V; (*c*) $V_{GS} = 0.8$ V, $V_{DS} = 2.5$ V; and (*d*) $V_{GS} = 1.2$ V, $V_{DS} = 2.5$ V.

a) Since $V_{GS} > V_T$ and V_{DS} is small:

$$I_D = \frac{k_n'}{2}\frac{W}{L}\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

$$= \frac{0.18\ \mathrm{mA}/\cancel{V^2}}{2}\cdot 8\left[2(0.8V - 0.4V)(0.2\cancel{V}) - (0.2\cancel{V})^2\right] = \boxed{0.0864\ \mathrm{mA}}$$ 2

b) $V_{DS} = 1.2V$ means device in saturation since: $V_{DS}(sat) = V_{GS} - V_T = 0.8V - 0.4V = 0.4V$

hence: $$I_D(sat) = \frac{k_n'}{2}\cdot\frac{W}{L}(V_{GS}-V_T)^2$$

$$= \frac{0.18\ \mathrm{mA/V^2}}{2}\cdot 8\left[0.4V\right]^2 = \boxed{0.115\ \mathrm{mA}}$$ 2

c) Same V_{GS} as part "b", but $V_{DS} = 2.5V$

Since device was already in saturation in part b), higher V_{DS} will not change I_D. Hence: $\boxed{I_D = 0.115\ \mathrm{mA}}$ 2

d) $V_{GS} = 1.2V$, $V_{DS} = 2.5V$

$$I_D(sat) = \frac{k_n'}{2}\cdot\frac{W}{L}(V_{GS}-V_T)^2$$

$$= \frac{0.18\ \mathrm{mA/V^2}}{2}\cdot 8(1.2V - 0.4V)^2 = \boxed{0.461\ \mathrm{mA}}$$ 2

2016

2) E-Book, problem 10.36, note that the MOSFET is biased in saturation region (8 points)

10.36 Consider a p-channel MOSFET with the following parameters: $k_p' = 0.12\ \text{mA/V}^2$ and $W/L = 20$. The drain current is 100 μA with applied voltages of $V_{SG} = 0$, $V_{BS} = 0$, and $V_{SD} = 1.0$ V. (*a*) Determine the V_T value. (*b*) Determine the drain current I_D for $V_{SG} = 0.4$ V, $V_{SB} = 0$, and $V_{SD} = 1.5$ V. (*c*) What is the value of I_D for $V_{SG} = 0.6$ V, $V_{SB} = 0$, and $V_{SD} = 0.15$ V?

a) With assumption of bias in saturation region:

$$I_D(sat) = \frac{K_p'}{2} \cdot \frac{W}{L}\left(V_{SG} + V_T\right)^2$$

$$0.1\ mA = \frac{0.12\ mA/V^2}{2} \cdot 20\left(0 + V_T\right)^2 \implies \boxed{V_T = 0.289\ V}$$ 2

b) $V_{SG} = 0.4V$, $V_{SB} = 0$, $V_{SD} = 1.5V$

Since $V_{SD} > V_{SD}(sat) = V_{SG} + V_T = 0.4V + 0.289V = 0.689V$ 2

↳ saturation region

$$I_D(sat) = \frac{K_p'}{2} \cdot \frac{W}{L}\left(V_{SG} + V_T\right)^2 = \frac{0.12\ mA/\cancel{V^2}}{2} \cdot 20\left(0.689\cancel{V}\right)^2 = \boxed{0.57\ mA}$$ 2

c) $V_{SG} = 0.6V$, $V_{SB} = 0$, $V_{SD} = 0.15V$

→ no longer in saturation:

$$I_D = \frac{K_p'}{2} \cdot \frac{W}{L}\left[2\left(V_{SG} + V_T\right)V_{SD} - V_{SD}^2\right]$$

$$= \frac{0.12\ mA/V^2}{2} \cdot 20\left[2\left(0.6V + 0.289V\right)\left(0.15V\right) - \left(0.15V\right)^2\right]$$

$$= \boxed{0.293\ mA}$$ 2

3) E-Book, problem 10.44 (10 points

10.44 The transconductance of an n-channel MOSFET is found to be $g_m = \partial I_D/\partial V_{GS} =$ 1.25 mA/V when measured at $V_{DS} = 50$ mV. The threshold voltage is $V_T = 0.3$ V. (*a*) Determine the conductance parameter K_n. (*b*) What is the current at $V_{GS} = 0.8$ V and $V_{DS} = 50$ mV? (*c*) Determine the current at $V_{GS} = 0.8$ V and $V_{DS} = 1.5$ V.

a) $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

since at small V_{DS}: $I_D = K_n\left[2(V_{GS} - V_T)(V_{DS}) - V_{DS}^2\right]$

$$g_m = \frac{d}{dV_{GS}}\left\{K_n\left[2(V_{GS}-V_T)(V_{DS}) - V_{DS}^2\right]\right\} = K_n(2V_{DS})$$

$$\Rightarrow 1.25 \text{ mA/V} = K_n\left(2\left(50\times10^{-3}\text{ V}\right)\right)$$

$$\boxed{K_n = 12.5 \text{ mA/V}^2}$$

4

b) $V_{GS} = 0.8$ V

$$I_D = K_n\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

$$= (12.5 \text{ mA}/\cancel{\text{V}^2})\left[2(0.8\text{ V} - 0.3\cancel{\text{V}})(0.05\cancel{\text{V}}) - (0.05\cancel{\text{V}})^2\right]$$

$$= \boxed{0.594 \text{ mA}}$$

4

c) $V_{DS} = 1.5$ V

→ since $V_{DS} > V_{DS}(sat) = V_{GS} - V_T = 0.8\text{ V} - 0.3\text{ V} = 0.5\text{ V}$

= saturation region:

$$I_D(sat) = K_n(V_{GS} - V_T)^2 = (12.5 \text{ mA/V}^2)(0.5\text{ V})^2 = \boxed{3.125 \text{ mA}}$$

2

4) E-Book, problem 10.46 **16 points**

10.46 One curve of an n-channel MOSFET is characterized by the following parameters: $I_D(\text{sat}) = 2 \times 10^{-4}$ A, $V_{DS}(\text{sat}) = 4$ V, and $V_T = 0.8$ V.

(*a*) What is the gate voltage?

(*b*) What is the value of the conduction parameter?

(*c*) If $V_G = 2$ V and $V_{DS} = 2$ V, determine I_D.

(*d*) If $V_G = 3$ V and $V_{DS} = 1$ V, determine I_D.

(*e*) For each of the conditions given in (*c*) and (*d*), sketch the inversion charge density and depletion region through the channel.

a) Since $V_{DS}(\text{sat}) = V_{GS} - V_T$

$4\text{V} = V_{GS} - 0.8\text{V} \Rightarrow \boxed{V_{GS} = 4.8\text{V}}$ 2

b) $I_D(\text{sat}) = K_n(V_{GS} - V_T)^2 \Rightarrow 2 \times 10^{-4}\text{A} = K_n(4.8\text{V} - 0.8\text{V})^2$

$\boxed{K_n = 12.5\ \mu\text{A}/\text{V}^2}$ 2

c) $V_G = 2\text{V}$, $V_{DS} = 2\text{V}$

since $V_{DS} > V_{DS}(\text{sat}) = V_{GS} - V_T = 2\text{V} - 0.8\text{V} = 1.2\text{V}$

→ saturation region

$I_D(\text{sat}) = (12.5 \times 10^{-6}\ \text{A}/\text{V}^2)(2\text{V} - 0.8\text{V})^2 = \boxed{18\ \mu\text{A}}$ 2

d) $V_G = 3\text{V}$, $V_{DS} = 1\text{V}$

Since $V_{DS} < V_{DS}(\text{sat}) = V_{GS} - V_T = 3\text{V} - 0.8\text{V} = 2.2\text{V}$ → linear region

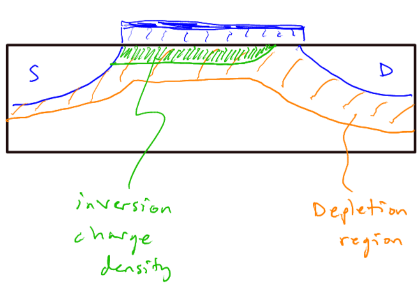
$I_D = K_n[2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$

$= (12.5 \times 10^{-6}\ \text{A}/\text{V}^2)[2(2.2\text{V})(1\text{V}) - (1\text{V})^2] = \boxed{42.5\ \mu\text{A}}$ 2

e)

For part (c)

V_{DS} is 0.8V beyond saturation ($V_{DS}(\text{sat})$)



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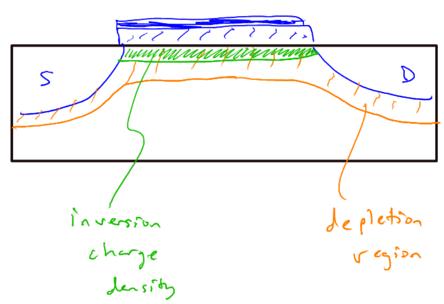
For part (d)

linear region with V_{DS} 1.2V < $V_{DS}(\text{sat})$



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✱ Sketches may vary but should show inversion layer significantly separated from drain here with a more expanded depletion region near the drain-side.

5) E-Book, problem 10.51 **6 points**

10.51 The substrate doping and body-effect coefficient of an n-channel MOSFET are $N_a = 10^{16}$ cm^{-3} and $\gamma = 0.12$ V$^{1/2}$, respectively. The threshold voltage is found to be $V_T = 0.5$ V when biased at $V_{SB} = 2.5$ V. What is the threshold voltage at $V_{SB} = 0$?

$$\Delta V_T = \gamma\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$ 2

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259\text{ V}) \ln\left(\frac{10^{16}\text{ cm}^{-3}}{1.5\times10^{10}\text{ cm}^{-3}}\right) = 0.3473\text{ V}$$

$$\Delta V_T = (0.12\text{ V}^{1/2})\left[\sqrt{2(0.3473\text{V}) + 2.5\text{V}} - \sqrt{2(0.3473\text{V})}\right] = 0.114\text{ V}$$ 2

Then: $V_T = V_{T0} + \Delta V_T$

$0.5\text{V} = V_{T0} + 0.114\text{ V}$

$\Rightarrow \boxed{V_{T0} = 0.386\text{ V}}$ 2

6) E-Book, problem 10.56 **14 points**

10.56 An n-channel MOSFET has the following parameters:

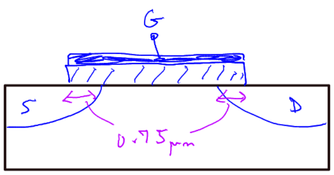
$\mu_n = 400\ \text{cm}^2/\text{V-s}$ $\quad t_{ox} = 500\ \text{Å}$
$L = 2\ \mu\text{m}$ $\quad W = 20\ \mu\text{m}$
$V_T = +0.75\ \text{V}$

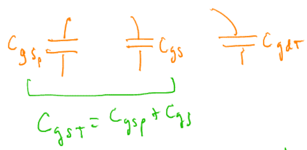
Assume the transistor is biased in the saturation region at $V_{GS} = 4$ V. (*a*) Calculate the ideal cutoff frequency. (*b*) Assume that the gate oxide overlaps both the source and drain contacts by 0.75 μm. If a load resistance of $R_L = 10\ \text{k}\Omega$ is connected to the output, calculate the cutoff frequency.

a) Ideal cut-off freq. with no overlap caps:

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\mu_n (V_{GS} - V_T)}{2\pi L^2} = \frac{(400\ \text{cm}^2/\text{V}\cdot\text{s})(4\text{V} - 0.75\text{V})}{2\pi (2\times 10^{-4}\ \text{cm})^2} = \boxed{5.17\ \text{GHz}}$$ **4**

b)





$$f_T = \frac{g_m}{2\pi (C_{gsT} + C_m)}$$

gate-drain overlap cap.

$$C_m = C_{gdT}(1 + g_m R_L)$$

$$C_{gdT} = C_{ox}(\text{overlap})\,W = \frac{\epsilon_{ox}\epsilon_0}{t_{ox}}(0.75\times 10^{-4}\ \text{cm})(20\times 10^{-4}\ \text{cm})$$

$$= \frac{(3.9)(8.85\times 10^{-14}\ \text{F/cm})}{500\times 10^{-8}\ \text{cm}}(0.75\times 10^{-4}\ \text{cm})(20\times 10^{-4}\ \text{cm}) = 1.035\times 10^{-14}\ \text{F}$$

$$g_m = \frac{W \mu_n C_{ox}}{L}(V_{GS} - V_T) = \frac{(20\times 10^{-4}\ \text{cm})(400\ \text{cm}^2/\text{V}\cdot\text{s})\overbrace{(3.9)(8.85\times 10^{-14}\ \text{F/cm})}^{C_{ox}}}{(2\times 10^{-4}\ \text{cm})(500\times 10^{-8}\ \text{cm})}(4\text{V} - 0.75\text{V})$$

$$= 0.8974\times 10^{-3}\ \text{S}$$ **2**

$$C_m = (1.035\times 10^{-14}\ \text{F})\left(1 + (0.8974\times 10^{-3}\ \text{S})(10\times 10^{3}\ \Omega)\right)$$

$$= 1.032\times 10^{-13}\ \text{F}$$ **2**

$$C_{gsT} = C_{gs} + C_{gsp} = C_{ox}(L\cdot W) + \underbrace{C_{ox}(\text{overlap})\,W}_{= C_{gdT} = 1.035\times 10^{-14}\ \text{F}}$$

$$= \frac{(3.9)(8.85\times 10^{-14}\ \text{F/cm})}{500\times 10^{-8}\ \text{cm}}(2\times 10^{-4}\ \text{cm})(20\times 10^{-4}\ \text{cm}) + 1.035\times 10^{-14}\ \text{F}$$

$$= 3.796\times 10^{-14}\ \text{F}$$ **2**

units: $\frac{A\ \cancel{V}}{\cancel{V}\ C} = \frac{\cancel{C}}{s\cdot \cancel{C}} = \text{Hz}$

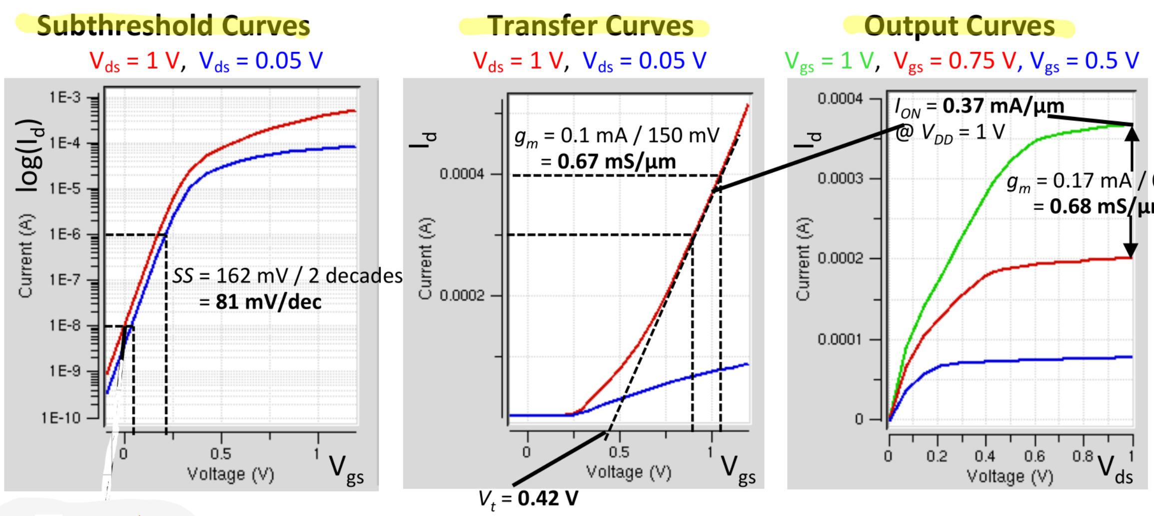
$$f_T = \frac{(0.8974\times 10^{-3}\ \text{S})}{2\pi(3.796\times 10^{-14}\ \text{F} + 1.032\times 10^{-13}\ \text{F})} = \boxed{1.01\ \text{GHz}}$$ **4**

7) Use the *MOSFet* simulator on nanohub.org to obtain the I-V_g (@ V_d = 0.05 V and 1.0 V) and I-V_d (@ max V_d = 1.0 V and min V_g = 0.5 V, max V_g = 1.0 V with 3 curves) characteristics for a Si MOSFET with the following attributes (use default for everything else): (**22 points**)

Channel Length = 50 nm Oxide Thickness = 2 nm
Junction Depth = 10 nm Substrate Thickness = 10 nm

Provide plots of the subthreshold, transfer, and output curves, with each correctly labeled. Extract the following from the I-V curves, showing how the extraction was done on the actual plotted curves:

i. Subthreshold swing
ii. Threshold voltage
iii. ON-current @ V_{DD} = 1 V
iv. Transconductance (show extraction in two ways)
v. Calculate the mobility, assuming $W = 1\mu m$, $V_{DS} = 1V$



2 pt. for each plot

i.	***SS* = 81 mV/dec**
ii.	**V_t = 0.42 V**
iii.	**I_{ON} = 0.37 mA/µm**
iv.	**g_m = 0.67 mS/µm**
v.	$\mu_n \approx 19.4\ cm^2/V\cdot s$

4 pts each, 2 for showing extraction on plot and 2 for right answer

4

NOTE: answers are approximate and will vary slightly but should be reasonably close

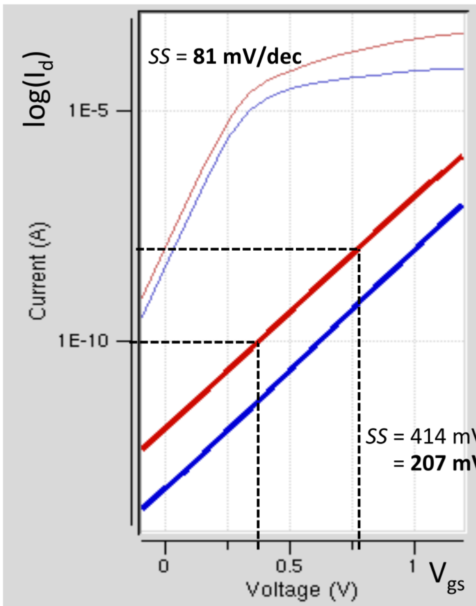
$$\mu_n = \frac{g_m L^2}{V_{ds} C_{ox}}$$

$$C_{ox} = \frac{\epsilon_{ox}\epsilon_0}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\ F/cm)}{2\times10^{-7}\ cm} = 1.726\times10^{-6}\ F/cm^2\ [L\cdot W]\ (50\times10^{-7}\ cm)(1\times10^{-4}\ cm)$$

$$= 8.629\times10^{-16}\ F$$

$$\mu_n = \frac{(0.67\times10^{-3}\ S)(50\times10^{-7}\ cm)^2}{(1V)(8.629\times10^{-16}\ F)} = 19.4\ cm^2/V\cdot s$$

8) Now change the oxide thickness, T_{ox} to 10 nm in the *MOSFet* simulator and rerun (all other parameters the same as in problem #7). What is different about this new device's performance compared to the device in problem #7 (include I-V curves (subthreshold) and at least one extracted performance metric to support your observation)? (**8 points**)



4 points for plots (can be on same axes or two diff plots)

2 pt. for extraction of metric

2 pt for explanation of some type

(just needs to be a logical comparison and explanation)

The subthreshold swing (*SS*) increases from 81 mV/dec to 207 mV/dec as the oxide thickness is increased from 2 nm to 10 nm. This device exhibits short channel effects due to the thick oxide and small channel length.

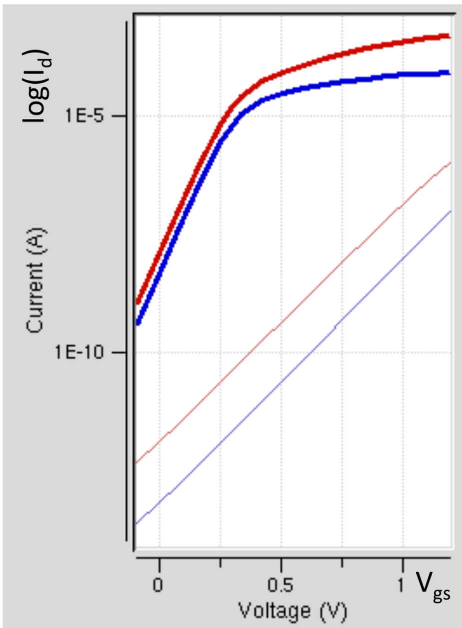
Another metric that can be compared is the threshold voltage, which is seen to shift dramatically (best observed in transfer curves).

9) Keeping T_{ox} = 10 nm, change the dielectric constant to 20 and rerun. Compare the subthreshold curves from this device with those from the device in problem #8. What changed? Why? (*hint:* consider what happens to C_{ox}) (**8 points**)

→ SS reduced with $\varepsilon_{ox} = 20$ owing to higher C_{ox}

6 pts for comparison plot(s)

2 pt for some observation like



t_{ox} = 10 nm, ε_{ox} = 20

t_{ox} = 10 nm, ε_{ox} = 3.9 (problem #8 device)

Equivalent oxide thickness (EOT) indicates how thick a silicon oxide would need to be to provide the same electrostatic effect as a high-k (higher dielectric constant) dielectric.

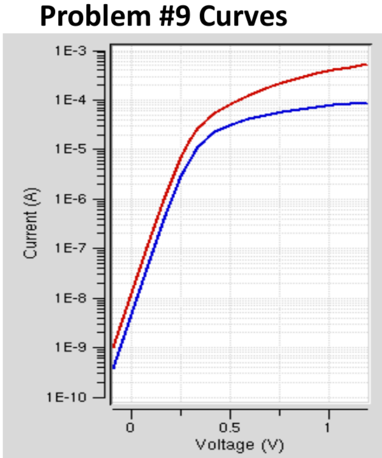
$$EOT = \frac{\varepsilon_{SiO_2} t_{ox}}{\varepsilon_{ox}} = \frac{3.9 t_{ox}}{\varepsilon_{ox}}$$

Side point explanation

In this case, EOT = 1.95 nm, which is why the curves look nearly identical to the device from problem #2 where the silicon oxide was 2 nm thick.

INTERESTING OBSERVATION (do these look the same??):

Problem #9 Curves



Problem #7 Curves