Homework 11 for STA 250/MTH 342 – Fall 2017

Due at noon on Monday December 11, 2017

1. D&S (4th Ed.) Exercise 10.1.8 (page 633)
2. D&S (4th Ed.) Exercise 10.3.4 (page 645)
3. D&S (4th Ed.) Exercise 10.4.5 (page 652)
4. Going back to Problems 3 and 4 in HW9. Find expressions for the p-values of the corresponding generalized likelihood ratio test. (Express it in the cdf of the corresponding sampling distribution.)
5. In class we have seen that the two-sample t-test for the comparison of the means of two normal sampes of size m and n with common unknown variance rejects when the absolute value of the t-statistic

$$T = \frac{\bar{X} - \bar{Y}}{s_{pooled} \cdot \sqrt{\frac{1}{m} + \frac{1}{n}}}$$

is larger than some constant C, where

$$s_{pooled}^2 = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}.$$

   (a) What is the sampling distribution of T when $\mu_X = \mu_Y$?

   (b) More generally, whether $\mu_X = \mu_Y$ or not, find the sampling distribution of

$$\frac{\bar{X} - \bar{Y} - \mu_X + \mu_Y}{s_{pooled} \cdot \sqrt{\frac{1}{m} + \frac{1}{n}}}.$$

   (c) Based on (b), find a $(1-\alpha) \times 100\%$ confidence interval for the difference between the two means $\mu_X - \mu_Y$.

   (d) Show that the level α two-sample t-test for

$$H_0 : \mu_X = \mu_Y \quad \text{vs} \quad H_1 : \mu_X \neq \mu_Y$$

   rejects if and only if the CI you found in (c) does not contain 0.

Month	Number of Deaths	Days/Month
Jan.	1668	31
Feb.	1407	28
Mar.	1370	31
Apr.	1309	30
May	1341	31
June	1338	30
July	1406	31
Aug.	1446	31
Sept.	1332	30
Oct.	1363	31
Nov.	1410	30
Dec.	1526	31

6. The following table gives the number of deaths due to accidental falls for each month during 1970. Is there any evidence for a departure from the uniformity in the rate over time? Is there a seasonal pattern to this death rate? If so, describe its pattern and speculate as to the causes. (Hints: (i) In formulating the null hypothesis, you need to take into account the number of days each month has. (ii) To investigate whether there is a seasonal trend, compute the deviation $X_i - m_i$, and see if there is a pattern.)

7. Nylon bars were tested for brittleness (Bennett and Franklin 1954). Each of 280 bars was molded under similar conditions and was tested in five places. Assuming that each bar has uniform composition, the number of breaks on a given bar should be binomially distributed with five trials and an unknown probability p of failure. If the bars are all of the same uniform strength, p should be the same for all of them; if they are of different strengths, p should vary from bar to bar. Thus, the null hypothesis is that the p's are all equal. The following table summarizes the outcome of the experiment. Under the given assumption, the data in

Breaks/Bar	Frequency
0	157
1	69
2	35
3	17
4	1
5	1

the table consist of 280 observations of independent Binomial random variables. Carry out a χ^2 test and compute the corresponding p-value. Note that you may need to pool some of the blocks together. Provide the reasoning behind the pooling.