Physics 464: Problem Set 10

1. CTDL GVII problem 1.
2. CTDL GVII problem 2.
3. Probability density distributions in a one-electron atom.
   (a) Consider the angular distributions associated with the electron probability density of a one-electron atom in the f $(l = 3)$ state. For each possible value of m, draw three-dimensional, spherical-polar graphs.
   (b) For the f states, calculate the expectation values $\langle \hat{\mu}_l^2 \rangle$ and $\langle \hat{\mu}_m \rangle$ of the orbital magnetic moment operator
   $$\hat{\vec{\mu}}_l = -\frac{\mu_B}{\hbar} \hat{\vec{L}} \quad ,$$
   where $\mu_B = \frac{e\hbar}{2m_e c}$ is the Bohr magneton, in which you should express your answers. Note that $\hat{\vec{\mu}}_l^2 \sim \hat{\vec{L}}^2$ while $\hat{\mu}_m \sim L_z$, so the latter may give a different answer for each quantum number m.
   (c) Plot the radial probability density functions for the $n = 3$ states, plotting curves for $3s$, $3p$, and $3d$ radial distributions. (An s, p, d state is $l = 0, 1, 2$, respectively.) Calculate $\langle r \rangle$ for these states and indicate the value on the plot.
   (d) Draw the boundary surface plots that combine angular and radial probability distribution information for the $3p$ and $3d$ states for $m = 0$. Note the spherical surface at $r = \langle r \rangle$ and any nodal surfaces.
4. Dipole moments in a one-electron atom. The dipole moment operator for a one-electron atom is $\vec{d} = -e\vec{r}$, where $\vec{r}$ is the electron "position" vector.
   (a) Show that the expectation value of $\vec{d}$ vanishes for all stationary states (energy eigenstates) of a one-electron atom.
   (b) If a one-electron atom is probed by a time-dependent electromagnetic field, then transitions may occur. Suppose that the atom at time t is given in the state
   $$\Psi(\vec{r}, t) = \frac{1}{\sqrt{1 + \beta^2}} \left(\psi_{1s}(\vec{r}) e^{-iE_1 t/\hbar} + \beta \psi_{2p_0}(\vec{r}) e^{-iE_2 t/\hbar} \right) \quad ,$$
   where β is a real constant. ($2p_0$ is $n = 2$, $l = 1$, $m = 0$. E_n are the energy eigenstates in the absence of the external field.) Find the values of $\langle \hat{H} \rangle$, $\langle \hat{\vec{L}}^2 \rangle$, $\langle \hat{L}_z \rangle$ in this state. Calculate $\langle \vec{d} \rangle$ for this state and show that on average the atom behaves like an oscillating electric dipole. Calculate the frequency of oscillation.
   (c) Calculate $\langle \vec{d} \rangle$ when (i) the state ψ_{2p_0} is replaced by ψ_{1s} in Ψ above; (ii) the state ψ_{2p_0} is replaced with ψ_{3d_0} in Ψ above; (iv) What are the selection rules if the m values of the states in Ψ are different? What possible l and m values in Ψ are necessary so that $\langle \vec{d} \rangle \neq 0$