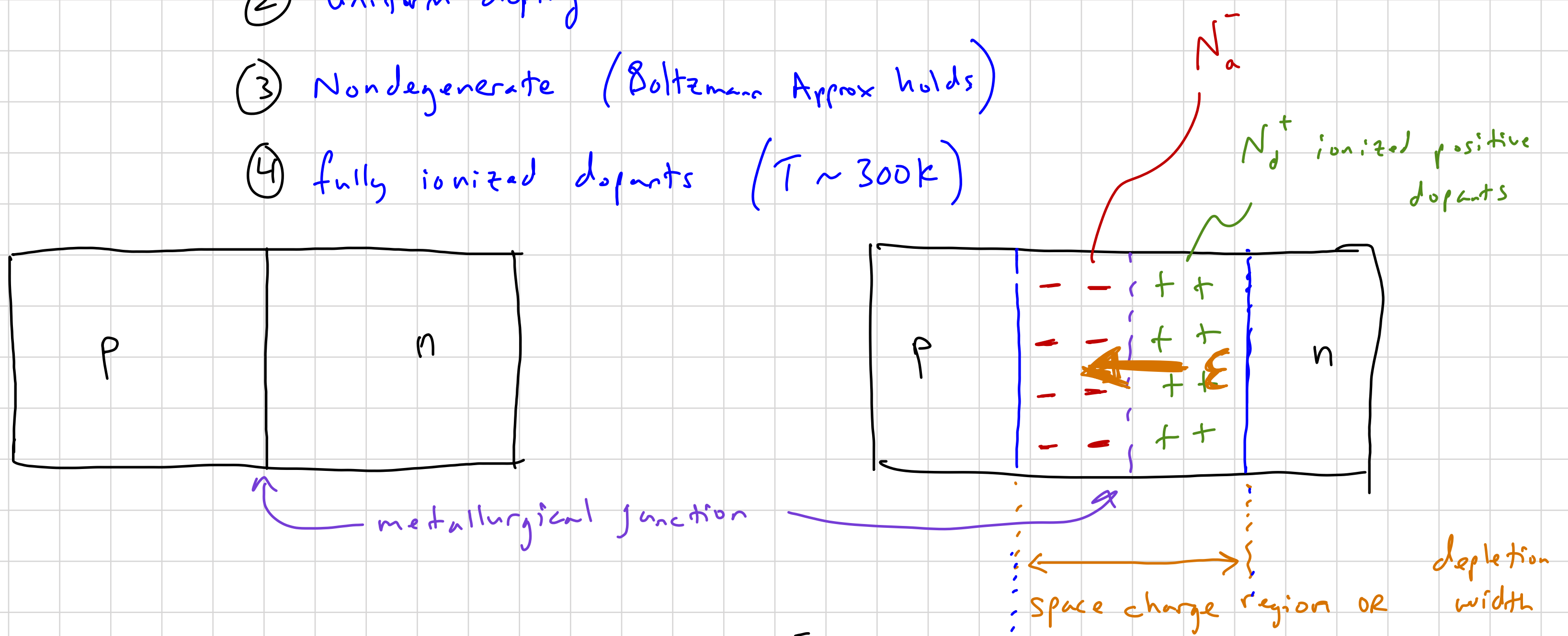
Lecture # 8

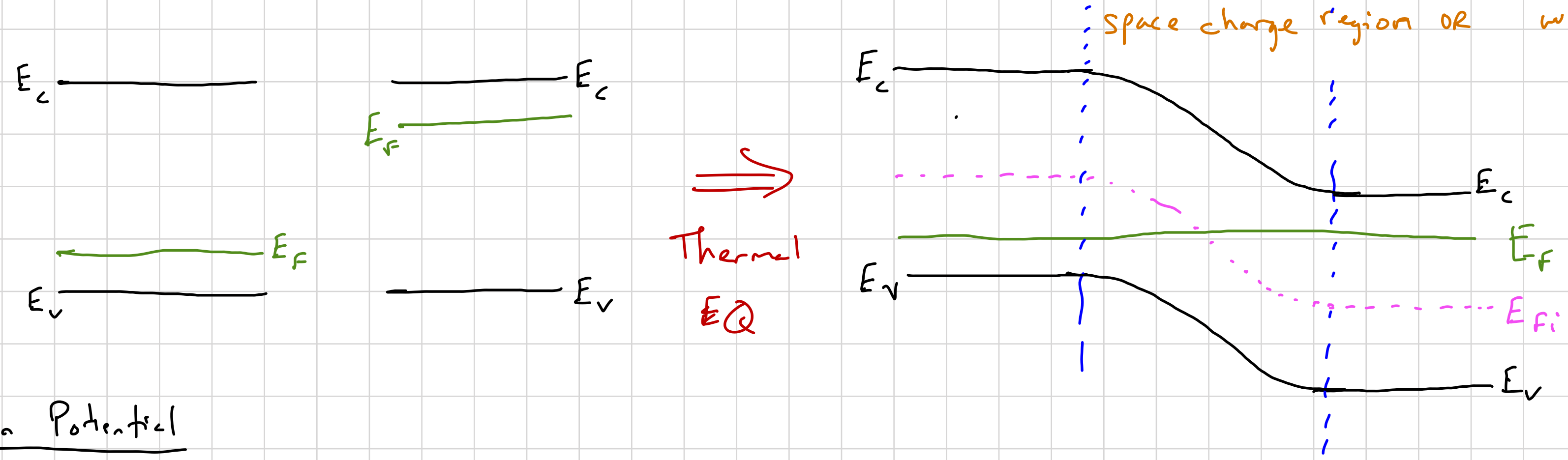
# pn Junction Diodes

Conceptually, a pn junction is formed when a p-type S.C. region is brought into contact with an n-type region with an abrupt junction.

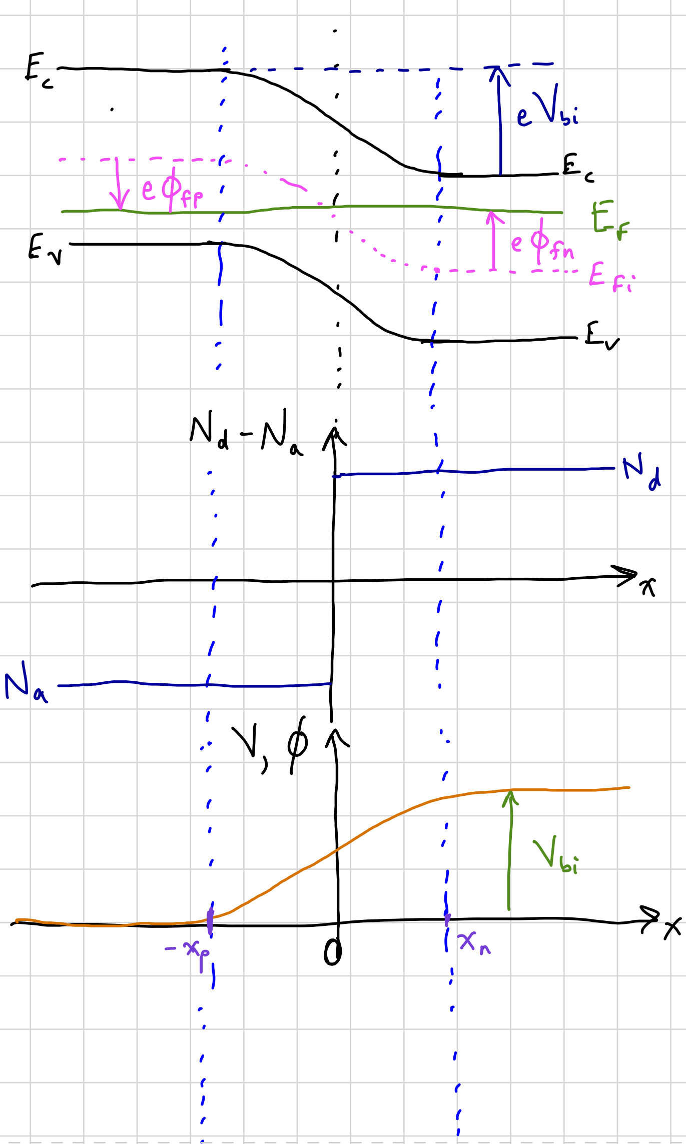
We will make the following assumptions in this analysis:

1. Abrupt junctions (step junctions)
2. Uniform doping
3. Nondegenerate (Boltzmann Approx holds)
4. fully ionized dopants ($T \sim 300K$)




## Built-in Potential




$$V_{bi} = |\phi_{fn}| + |\phi_{fp}|$$

Electric field: $\mathcal{E} = -\dfrac{dV}{dx}$

Integrating across depletion region:

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}\, dx = \int_{\phi(-x_p)}^{\phi(x_n)} d\phi = \phi(x_n) - \phi(-x_p)$$

Since EQ (no applied bias):

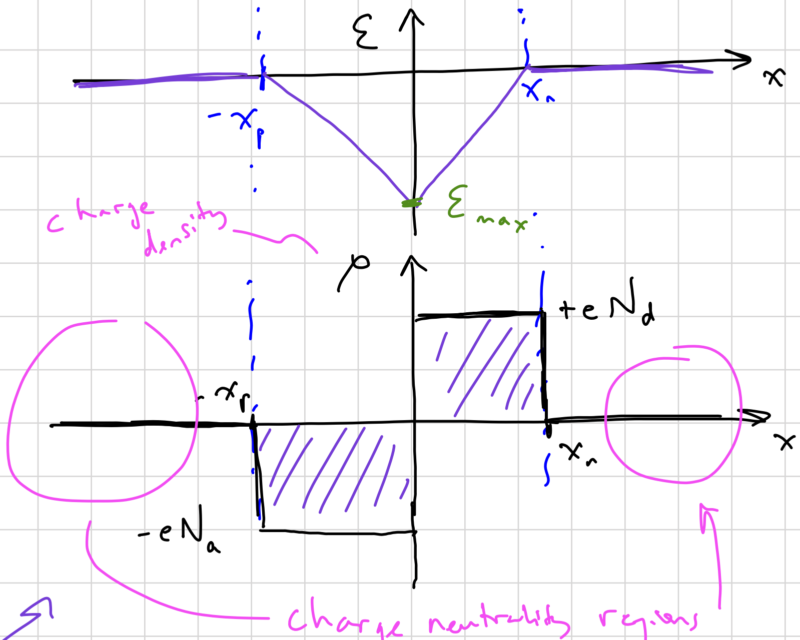
$$J_n = e\mu_n n \mathcal{E} + e D_n \frac{dn}{dx} = 0$$

solve for $\mathcal{E}$ and use Einstein relation: $\dfrac{D_n}{\mu_n} = \dfrac{kT}{e}$

$$\mathcal{E} = -\frac{D_n}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{e}\frac{dn/dx}{n}$$

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}\, dx = \frac{kT}{e}\int_{n(-x_p)}^{n(x_n)} \frac{1}{n}\, dn = \frac{kT}{e}\ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

Step junction, so: $n(x_n) = N_d$, $n(-x_p) = \dfrac{n_i^2}{N_a}$




$$\boxed{V_{bi} = \underbrace{\frac{kT}{e}}_{V_t \text{ (thermal voltage)}} \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

★NOTE: $N_a$ and $N_d$ from now on indicate doping on p-type or n-type side, respectively

## Electric field

poisson eqn: $\frac{d\varepsilon}{dx} = \frac{\rho}{\epsilon_0 \epsilon_s}$

charge density

permittivity of free space

book just gives $\epsilon_s$ but it is: $\epsilon_s \epsilon_0$

↑ relative dielectric constant

$$\varepsilon = \int \frac{\rho(x)}{\epsilon_s \epsilon_0} dx$$

apply charge density conditions and get:

$$\boxed{\varepsilon = -\frac{eN_a}{\epsilon_s \epsilon_0}(x + x_p) \qquad -x_p \le x \le 0}$$

$$\boxed{\varepsilon = -\frac{eN_d}{\epsilon_s \epsilon_0}(x_n - x) \qquad 0 \le x \le x_n}$$

Importantly at $x = 0$: $N_a x_p = N_d x_n$

★ charges/$cm^2$ is = on both sides!!

-potential in the junction: $\phi(x) = -\int \varepsilon(x)\, dx$

$$\Rightarrow \boxed{\phi(x) = \frac{eN_a}{2\epsilon_s \epsilon_0}(x + x_p)^2 \qquad -x_p \le x \le 0}$$

$$\boxed{\phi(x) = \frac{eN_d}{2\epsilon_s \epsilon_0}\left(x_n x - \frac{x^2}{2}\right) + \frac{eN_a}{2\epsilon_s \epsilon_0} x_p^2 \qquad 0 \le x \le x_n}$$

Use this to develop another expression for $V_{bi}$:

$$\boxed{V_{bi} = \left|\phi(x = x_n)\right| = \frac{e}{2\epsilon_s \epsilon_0}\left(N_d x_n^2 + N_a x_p^2\right)}$$

## Space charge width (W) — depletion width

→ distance from $-x_p$ to $x_n$, since: $x_p = \frac{N_d}{N_a} x_n$

$$x_n = \left[\frac{2\epsilon_s \epsilon_0}{e} \frac{N_a}{N_d(N_a + N_d)} V_{bi}\right]^{1/2}$$

$$x_p = \left[\frac{2\epsilon_s \epsilon_0}{e} \frac{N_d}{N_a(N_a + N_d)} V_{bi}\right]^{1/2}$$

$$\Rightarrow W = \left[\frac{2\epsilon_s \epsilon_0}{e} \left(\frac{N_a + N_d}{N_a N_d}\right) V_{bi}\right]^{1/2}$$