<u>Homework #6</u>

pn Junction large-signal & MOS capacitors – *100 points*

DUE @ Beginning of Class: Thursday, October 19

1) E-Book, problem 1.42 (pg. 443), *hint:* voltage drop will be the same for each of the two top diodes in the circuit (5 points)

2) E-Book, problem 1.49 (pg. 444), assume an ideal diode (3 points)

3) E-Book, problem 10.7, given ϕ_{ms} = -0.9932 V (3 points)

4) E-Book, problem 10.9, see Example 10.2 for useful information (4 points)

5) E-Book, problem 10.12 (10 points)

6) E-Book, problem 10.14, requires some trial-and-error and Fig. 10.16 (8 points)

7) E-Book, problem 10.23, assume no charge in oxide and use Fig. 10.16 (16 points)

8) E-Book, problem 10.30 (16 points)

9) E-Book, problem 10.31, be sure to label your band diagrams (E_{Fm}, E_{Fi}, etc) (15 points)

10) The energy band diagram for an ideal MOS capacitor with t_{ox} = 0.2 μm operated at 300 K is given below. Note that the applied gate voltage causes band bending in the semiconductor such that $E_F = E_{Fi}$ at the Si-SiO_2 interface. Answer the following questions: (20 points)

a) Sketch the electrostatic potential inside the semiconductor as a function of position.

b) Roughly sketch the electric field inside the oxide and semiconductor as a function of position.

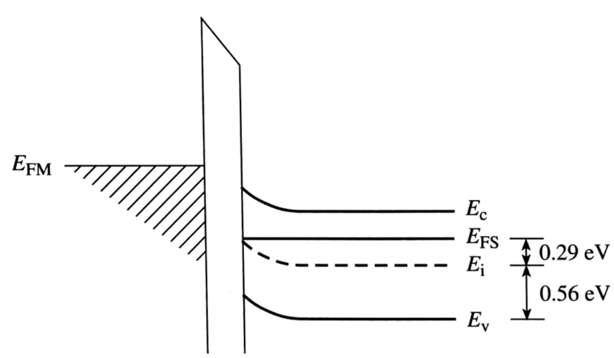
c) Do equilibrium conditions prevail inside the semiconductor?

d) What is the electron concentration at the Si-SiO_2 interface?

e) N_D = ?

f) ϕ_s = ?

g) What is the approximate applied V_G (indicate how you arrived at answer)?



1) E-Book, problem 1.42 (pg. 443), *hint:* voltage drop will be the same for each of the two top diodes in the circuit (5 points)

1.42 (a) The reverse-saturation current of each diode in the circuit shown in Figure P1.42 is $I_S = 6 \times 10^{-14}$ A. Determine the input voltage V_I required to produce an output voltage of $V_O = 0.635$ V. (b) Repeat part (a) if the 1 kΩ resistor is changed to $R = 500\ \Omega$.

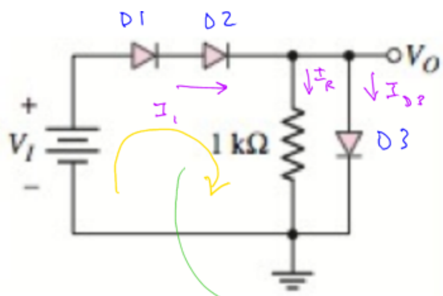


Figure P1.42

$V_I = V_{D1} + V_{D2} + V_R$

a) Since I_s is same for all diodes,

$V_{D1} = V_{D2}$

V_o in this case

$$I_{D3} = I_S \exp\left(\frac{V_a}{V_t}\right) = (6\times10^{-14}\,A)\exp\left(\frac{0.635\,V}{0.0259\,V}\right) = 2.67\,mA$$

$$I_R = \frac{0.635\,V}{1\times10^{3}\,\Omega} = 0.635\,mA$$

By KCL nodal analysis:

$$I_1 = I_R + I_{D3}$$

$$I_1 = 2.67\,mA + 0.635\,mA = 3.31\,mA$$

from $I_{D1} = I_S \exp\left(\frac{V_{D1}}{V_t}\right) \Rightarrow V_{D1} = V_t \ln\left(\frac{I_{D1}}{I_S}\right)$

$$V_{D1} = V_{D2} = (0.0259\,V)\ln\left(\frac{3.31\times10^{-3}\,A}{6\times10^{-14}\,A}\right) = 0.641\ V$$

By KVL loop:

$$V_I = V_{D1} + V_{D2} + V_R = 2(0.641\,V) + 0.635\,V = \boxed{1.92\ V}$$ 2

b) Repeat with $R = 500\,\Omega$

By the same approach as above:

$$\boxed{V_I = 1.93\ V}$$

~ same

2) E-Book, problem 1.49 (pg. 444) (3 points)

1.49 (a) In the circuit shown in Figure P1.49, find the diode voltage V_D and the supply voltage V such that the current is $I_D = 0.4$ mA. Assume the diode cut-in voltage is $V_\gamma = 0.7$ V. (b) Using the results of part (a), determine the power dissipated in the diode.

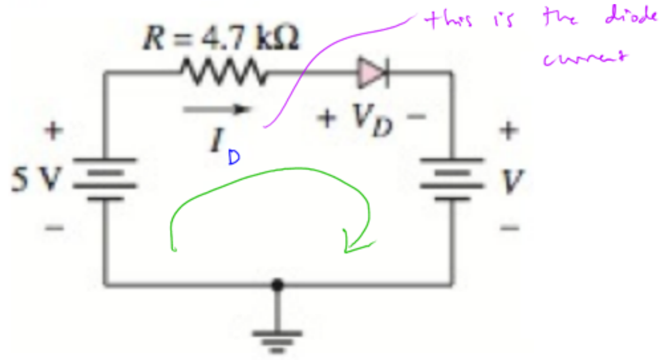


Figure P1.49

a) Since the diode is assumed ideal, V_D should just be $= V_\gamma = 0.7V$

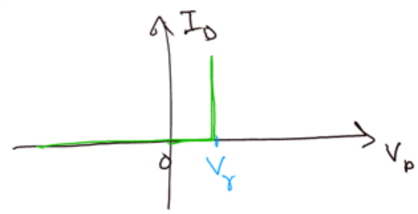
KVL gives:

$$5V = I_D R + V_D + V$$

$$5V = (0.4mA)(4.7k\Omega) + 0.7V + V$$

$$\Rightarrow \boxed{V = 2.42V}, \quad \boxed{V_D = 0.7V}$$

IDeal DIODE:



b) Dissipated power will be:

$$P = I_D \cdot V_D = (0.4mA)(0.7V) = \boxed{0.28 mW}$$

3) E-Book, problem 10.7 (3 points)

10.7 (*a*) Consider the MOS capacitor described in Problem 10.5. For an oxide thickness of $t_{ox} = 20$ nm $= 200$ Å and an oxide charge of $Q'_{ss} = 5 \times 10^{10}$ cm^{-2}, calculate the flat-band voltage. (*b*) Repeat part (*a*) for an oxide thickness of $t_{ox} = 8$ nm $= 80$ Å.

From 10.5: $N_a = 4\times10^{16}\ cm^{-3}$, Al $-$ SiO$_2$ $-$ Si

a) $V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$ 1

$C_{ox} = \frac{\epsilon_o \epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\ F/cm)}{20\times10^{-7}\ \cancel{cm}} = 1.726\times10^{-7}\ F/cm^2$

$V_{FB} = -0.9932\ V - \frac{5\times10^{10}\ \cancel{cm^{-2}}}{1.726\times10^{-7}\ F/\cancel{cm^2}}(1.6\times10^{-19}\ c) = \boxed{-1.04\ V}$ 1

units: $\frac{1}{F} = \frac{V}{c}\times c = V$

b) Using same approach:

$\boxed{V_{FB} = -1.012\ V}$ 1

4) E-Book, problem 10.9 (4 points)

10.9 Consider an aluminum gate–silicon dioxide–p-type silicon MOS structure with $t_{ox} = 450$ Å. The silicon doping is $N_a = 2 \times 10^{16}$ cm^{-3} and the flat-band voltage is $V_{FB} = -1.0$ V. Determine the fixed oxide charge Q'_{ss}.

$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_G}{2e} + \phi_{fp}\right)$

$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259\ V)\ln\left(\frac{2\times10^{16}\ cm^{-3}}{1.5\times10^{10}\ cm^{-3}}\right) = 0.365\ V$

$\phi_{ms} = 3.20\ V - \left(3.25\ V + \frac{1.12\ eV}{2e} + 0.365\ V\right) = -0.975\ V$ 1

for Al-Si, example 10.2 in E-book

$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \Rightarrow Q'_{ss} = (\phi_{ms} - V_{FB})\, C_{ox}$ 1

$C_{ox} = \frac{\epsilon_o \epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\ F/cm)}{45\times10^{-7}\ cm} = 7.67\times10^{-8}\ F/cm^2$ 1

$Q'_{ss} = [-0.975\ V - (-1\ V)](7.67\times10^{-8}\ F/cm^2) = 1.92\times10^{-9}\ c/cm^2$

OR 1

$\frac{Q'_{ss}}{e} = 1.2\times10^{10}\ cm^{-2}$

double the indicated points

5) E-Book, problem 10.12 (10 points)

10.12 A 400-Å oxide is grown on p-type silicon with $N_a = 5 \times 10^{15}$ cm^{-3}. The flat-band voltage is -0.9 V. Calculate the surface potential at the threshold inversion point as well as the threshold voltage assuming negligible oxide charge. Also find the maximum space charge width for this device.

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259\,\text{V}) \ln\left(\frac{5\times10^{15}\,\text{cm}^{-3}}{1.5\times10^{10}\,\text{cm}^{-3}}\right) = 0.329\,\text{V}$$

surface potential at threshold inversion:

$$\phi_s = 2\phi_{fp} = \boxed{0.659\,\text{V}} \quad 1$$

$$V_{FB} = \phi_{ms} - \frac{Q_{ss}'}{C_{ox}} = -0.9\,\text{V}$$

$$V_T = \frac{|Q_{SD}'(max)|}{C_{ox}} + V_{FB} + \phi_s$$

and: $|Q_{SD}'(max)| = e N_a x_{dT}$

$$x_{dT} = \left[\frac{4\epsilon_s\epsilon_0\phi_{fp}}{eN_a}\right]^{1/2} = \left[\frac{4(11.7)(8.85\times10^{-14}\,F/cm)(0.329\,V)}{(1.6\times10^{-19}\,c)(5\times10^{15}\,cm^{-3})}\right]^{1/2} = \boxed{0.413\times10^{-4}\,\text{cm}} \quad 1$$

$$|Q_{SD}'(max)| = (1.6\times10^{-19}\,c)(5\times10^{15}\,cm^{-3})(0.413\times10^{-4}\,cm) = 3.304\times10^{-8}\,c/cm^2$$

and: $$C_{ox} = \frac{\epsilon_{ox}\epsilon_0}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\,F/cm)}{40\times10^{-7}\,cm} = 8.63\times10^{-8}\,F/cm^2 \quad 1$$

$$V_T = \frac{3.304\times10^{-8}\,c/cm^2}{8.63\times10^{-8}\,F/cm^2} - 0.9\,V + 0.659\,V = \boxed{0.142\,\text{V}} \quad 2$$

double the indicated points

6) E-Book, problem 10.14 (8 points)

10.14 Consider a MOS device with the following parameters: p$^+$ polysilicon gate, n-type silicon substrate, $t_{ox} = 18$ nm $= 180$ Å, and $Q'_{ss} = 4 \times 10^{10}$ cm^{-2}. Determine the silicon doping concentration such that the threshold voltage is in the range $-0.35 \leq V_{TP} \leq -0.25$ V.

Based on the given info:

$$C_{ox} = \frac{\epsilon_{ox}\epsilon_0}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\,F/cm^2)}{18\times10^{-7}\,cm} = 1.92\times10^{-7}\,F/cm^{-3}$$ 1

unit conversion for Q'_{ss}:

$$Q'_{ss} = (1.6\times10^{-19}\,c)(4\times10^{10}\,cm^{-2}) = 6.4\times10^{-9}\,c/cm^2$$

For V_{TP}:

$$V_{TP} = -\frac{(|Q'_{SD}(max)| + Q'_{ss})}{C_{ox}} + \phi_{ms} - 2\phi_{fn}$$

Key eqn's with N_d dependence:

$$\phi_{fn} = V_t \ln\left(\frac{N_d}{n_i}\right) = (0.0259\,V)\ln\left(\frac{N_d}{1.5\times10^{10}\,cm^{-3}}\right)$$

$$|Q'_{SD}(max)| = eN_d x_{dT} = (1.6\times10^{-19}\,c)N_d\left[\frac{4(11.7)(8.85\times10^{-14}\,F/cm)\phi_{fn}}{(1.6\times10^{-19}\,c)N_d}\right]^{1/2}$$ 2

$$x_{dT} = \left[\frac{4\epsilon_r\epsilon_0\phi_{fn}}{eN_d}\right]^{1/2}$$

$$\phi_{ms} \simeq \text{Fig. 10.16}(N_d)$$

Use trial and error to find N_d

1

one solution: $\boxed{N_d = 5\times10^{16}\,cm^{-3}}$

yields

$V_{TP} = -0.303\,V$, which is within the given range

NOTE: actual answers for N_d will vary, but the approach to obtaining the answer should be as above.

double the indicated points

7) E-Book, problem 10.23 (16 points)

10.23 An ideal MOS capacitor with an n$^+$ polysilicon gate has a silicon dioxide thickness of t_{ox} = 12 nm = 120 Å on a p-type silicon substrate doped at $N_a = 10^{16}$ cm^{-3}. Determine the capacitance C_{ox}, C'_{FB}, C'_{min}, and $C'(\text{inv})$ at (*a*) f = 1 Hz and (*b*) f = 1 MHz. (*c*) Determine V_{FB} and V_T. (*d*) Sketch C'/C_{ox} versus V_G for parts (*a*) and (*b*).

a) For low-frequency, $f = 1\text{ Hz}$

$$C_{ox} = \frac{\epsilon_o \epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14}\text{ F/cm})}{12\times10^{-7}\text{ cm}} = \boxed{2.88\times10^{-7}\text{ F/cm}^2}$$ 1

$$C'_{FB} = \frac{\epsilon_{ox}\epsilon_o}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)\sqrt{\frac{V_t \epsilon_s \epsilon_o}{e N_a}}} = \frac{(3.9)(8.85\times10^{-14}\text{ F/cm})}{12\times10^{-7}\text{ cm} + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259\text{ V})(11.7)(8.85\times10^{-14}\text{ F/cm})}{(1.6\times10^{-19}\text{ C})(10^{16}\text{ cm}^{-3})}}} = \boxed{1.35\times10^{-7}\text{ F/cm}^2}$$ 1

$$C'_{min} = \frac{\epsilon_{ox}\epsilon_o}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)x_{dT}} = \frac{(3.9)(8.85\times10^{-14}\text{ F/cm})}{12\times10^{-7}\text{ cm} + \left(\frac{3.9}{11.7}\right)(3.0\times10^{-5}\text{ cm})} = \boxed{3.08\times10^{-8}\text{ F/cm}^2}$$ 1

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259\text{ V})\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.347\text{ V}$$

$$x_{dT} = \left[\frac{4\epsilon_s\epsilon_o\phi_{fp}}{eN_a}\right]^{1/2} = \left[\frac{4(11.7)(8.85\times10^{-14}\text{ F/cm})(0.347\text{ V})}{(1.6\times10^{-19}\text{ C})(10^{16}\text{ cm}^{-3})}\right]^{1/2} = 3.0\times10^{-5}\text{ cm}$$

at low frequency

$$C'(\text{inv}) = C_{ox} = \boxed{2.88\times10^{-7}\text{ F/cm}^2}$$ 1

b) $f = 1\text{ MHz}$

$\Rightarrow$ everything unchanged except $C'(\text{inv})$ which now $= C'_{min}$:

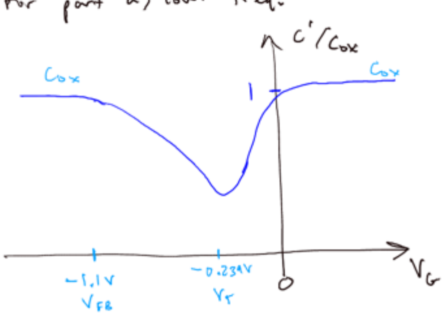
$$\boxed{\begin{array}{ll} C_{ox} = 2.88\times10^{-7}\text{ F/cm}^2 & C'_{min} = 3.08\times10^{-8}\text{ F/cm}^2 \\ C'_{FB} = 1.35\times10^{-7}\text{ F/cm}^2 & C'(\text{inv}) = C'_{min} = 3.08\times10^{-8}\text{ F/cm}^2 \end{array}}$$ 1

0.5

c) $V_{FB} = \phi_{ms} \simeq \boxed{-1.10\text{ V}}$ (from Fig 10.16) 0.5

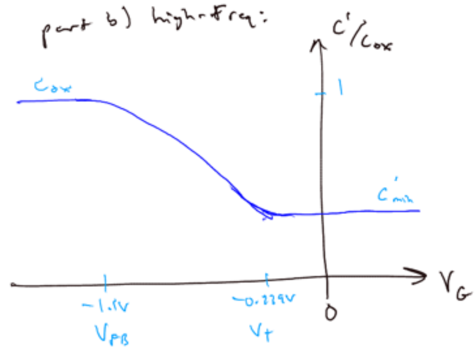
$$V_{TN} = \frac{\underbrace{|Q'_{SD}(\max)|}_{= eN_a x_{dT}}}{C_{ox}} + V_{FB} + 2\phi_{fp} = \frac{(1.6\times10^{-19}\text{ C})(10^{16}\text{ cm}^{-3})(3.0\times10^{-5}\text{ cm})}{2.88\times10^{-7}\text{ F/cm}^2} - 1.10\text{ V} + 2(0.347\text{ V}) = \boxed{-0.239\text{ V}}$$

d) For part a) low-freq:



1

part b) high-freq:



1

double the indicated points

8) E-Book, problem 10.30 (16 points)

10.30 The high-frequency C–V characteristic curve of a MOS capacitor is shown in Figure P10.30. The area of the device is 2×10^{-3} cm^2. The metal–semiconductor work function difference is $\phi_{ms} = -0.50$ V, the oxide is SiO_2, the semiconductor is silicon, and the semiconductor doping concentration is 2×10^{16} cm^{-3}. (*a*) Is the semiconductor n or p type? (*b*) What is the oxide thickness? (*c*) What is the equivalent trapped oxide charge density? (*d*) Determine the flat-band capacitance.

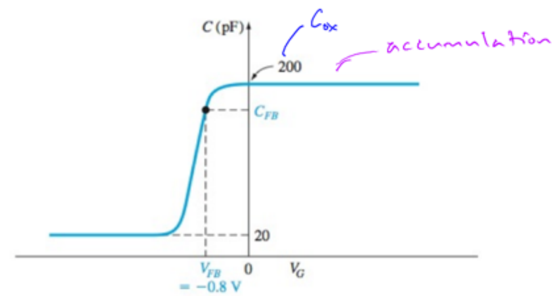


Figure P10.30 | Figure for Problem 10.30.

a) Semiconductor is n-type because accumulation region occurs at more positive V_G 1

b) $C'_{ox} = \frac{\epsilon_{ox}\epsilon_0}{t_{ox}} = \frac{C_{ox}}{A} = \frac{200 \times 10^{-12} F}{2\times10^{-3} cm^2} = 10^{-7} F/cm^2$ 1

$$t_{ox} = \frac{\epsilon_{ox}\epsilon_0}{10^{-7} F/cm^2} = \frac{(3.9)(8.85\times10^{-14} \cancel{F}/\cancel{cm})}{10^{-7} \cancel{F}/\cancel{cm^2}} = 3.45\times10^{-6} cm = \boxed{34.5 nm}$$ 2

c) $V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \Rightarrow -0.8V = -0.50V - \frac{Q'_{ss}}{10^{-7}F/cm^2}$

$\therefore Q'_{ss} = 3\times10^{-8} C/cm^2$ OR $1.88\times10^{11} cm^{-2}$ 2

either is acceptable

d)
$$C'_{FB} = \frac{\epsilon_{ox}\epsilon_0}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)\sqrt{V_t\left(\frac{\epsilon_s\epsilon_0}{eN_d}\right)}} = \frac{(3.9)(8.85\times10^{-14} F/cm)}{3.45\times10^{-6} cm + \left(\frac{3.9}{11.7}\right)\sqrt{(0.0259V)\left(\frac{(11.7)(8.85\times10^{-14}F/cm)}{(1.6\times10^{-19}C)(2\times10^{16} cm^{-3})}\right)}}$$

$= 7.82\times10^{-8} F/cm^2$

OR

$= 156 pF$

either is acceptable 2

9) E-Book, problem 10.31 (15 points)

10.31 Consider the high-frequency C–V plot shown in Figure P10.31. (*a*) Indicate which points correspond to flat-band, inversion, accumulation, threshold, and depletion modes. (*b*) Sketch the energy-band diagram in the semiconductor for each condition.

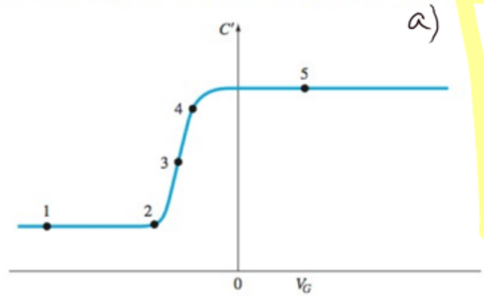


Figure P10.31 | Figure for Problem 10.31.

a)
1 – inversion
2 – threshold
3 – depletion
4 – flat-band
5 – accumulation

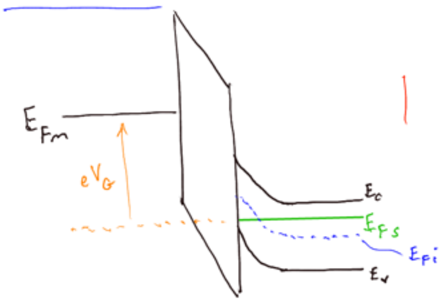
5

★ n-type semiconductor based on accumulation being at more positive V_G

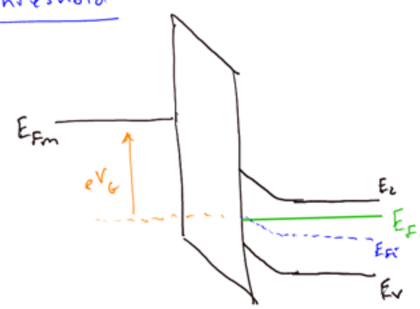
double the indicated points

b)

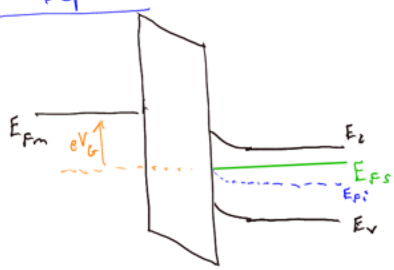
1 – inversion



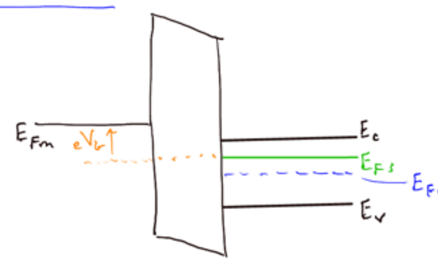
2 – Threshold



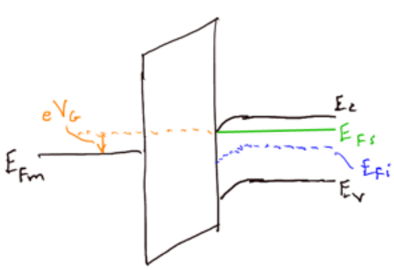
3 – Depletion



4 – Flat-Band



5 – Accumulation



double the indicated points

10) The energy band diagram for an ideal MOS capacitor with t_{ox} = 0.2 μm operated at 300 K is given below. Note that the applied gate voltage causes band bending in the semiconductor such that $E_F = E_{Fi}$ at the Si-SiO$_2$ interface. Answer the following questions: (20 points)

a) Sketch the electrostatic potential inside the semiconductor as a function of position.

b) Roughly sketch the electric field inside the oxide and semiconductor as a function of position.

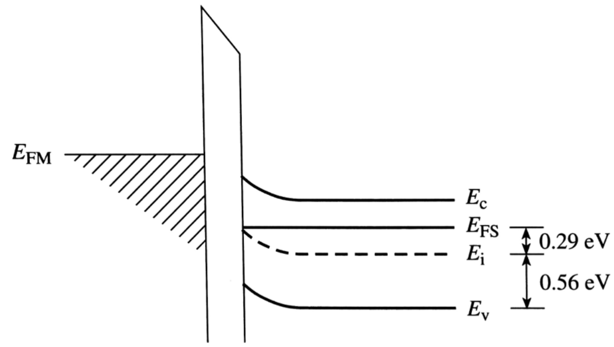
c) Do equilibrium conditions prevail inside the semiconductor?

d) What is the electron concentration at the Si-SiO$_2$ interface?

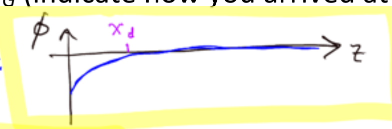
e) N_D = ?

f) ϕ_s = ?

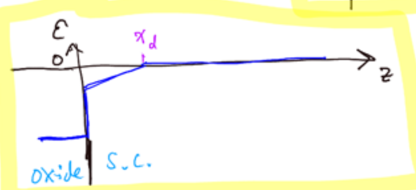
g) What is the approximate applied V_G (indicate how you arrived at answer)?



a) Potential will be the inverse of the bands: 1



b) $\mathcal{E}$-field is $\propto \frac{dE_c}{dz}$: 2



c) $\boxed{\text{Yes}}$, E_F is constant / flat in S.C. 1

d) Since $E_F = E_{Fi}$ at the Si-SiO$_2$ interface: $n = n_i \exp\left(\frac{\cancel{E_F - E_{Fi}}}{kT}\right) = n_i = \boxed{1.5\times10^{10}\ cm^{-3}}$ 1

e) $N_d = n_{bulk} = n_i \exp\left(\frac{E_{FS} - E_{Fi}}{kT}\right) = (1.5\times10^{10}\ cm^{-3}) \exp\left(\frac{0.29\ eV}{0.0259\ eV}\right) = \boxed{1.09\times10^{15}\ cm^{-3}}$ 1

f) $\phi_s = \frac{1}{e}\left[E_{Fi}]_{bulk} - E_{Fi}|_{surface}\right] = \boxed{-0.29\ V}$ 1

g) We know that at $V_G = 0$, $E_{Fm} = E_{FS}$. Hence the amount of applied V_G can be approximated by the difference between E_{Fm} and E_{FS}, as shown in the band diagram above, in purple. 1

$\therefore \boxed{V_G \simeq -0.8\ V}$ 2

answer will vary but should be negative and close to this (within 0.2 V) with an explanation