Homework 6 for STA 250/MTH 342 – Fall 2017

Due at the beginning of class on October 18th, 2017

1. D&S (4th Ed.) Exercise 8.7.6 (page 513)
2. D&S (4th Ed.) Exercise 8.2.6 (page 472)
3. D&S (4th Ed.) Exercise 8.3.6 (page 479)
4. Suppose X and Y are independent random variables, X has Gamma$(1, 2)$ distribution and Y has Gamma$(3, 2)$ distribution.
   (a) Find the pdf of $Z = X + Y$.
   (b) Find $E(Z)$ in two ways: directly using the density for Z you just found and using the fact that the expectation of a sum is the sum of the expectations.
   (c) Now if U is another random variable independent of X whose pdf is
   $$f_U(u) = \begin{cases} 2u & \text{for } 0 \leq u \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$
   Find the pdf of $W = U + X$.
   (d) Find Var(W).
5. Normal approximation for the Poisson distribution.
   (a) For λ large, argue that a Poisson(λ) random variable should follow approximately a normal distribution. Find the mean and variance of this approximate normal distribution. (For this problem you can just consider the case when λ is an integer, although the result holds more generally for other values of λ as well.)
   (b) Use the result to part (a), find the approximate value of $P(X \leq 90)$ for $\lambda = 100$.
6. Suppose the data are n i.i.d. observations $X_1, X_2, \ldots, X_n$, from a Gamma(α, β) distribution where α is known and β unknown.
   (a) What is the MLE $\hat{\beta}$ for β?
   (b) What is the Fisher's information for β from a single observation?
   (c) Find the (approximate) sampling distribution of $\hat{\beta}$.
   (d) For this part only, suppose $n = 40$, $\alpha = 5$, and $\beta = 2$. What (approximately) is the probability $P(|\hat{\beta} - 2| < 0.1)$?

(e) What is the MLE for $\theta = \beta^2$, $\hat{\theta}$?

(f) Find the Fisher's information for θ from a single observation. Hint: Exercise 8.8.6.

(g) Find the (approximate) sampling distribution for the MLE of θ.