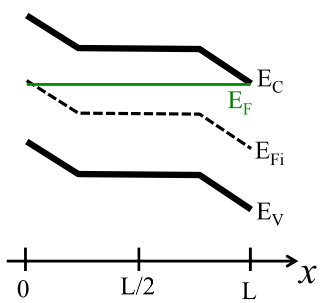
Solutions

Homework #4

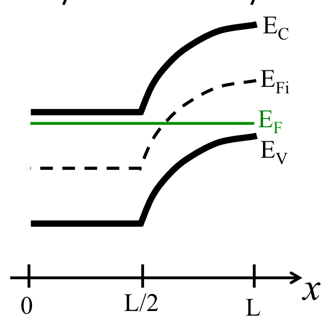
Carrier Transport and pn Junctions – *100 points*
DUE @ Beginning of Class: Tuesday, October 3

1) Answer the following questions, for each of the band diagrams given below (a-c): (24 points)

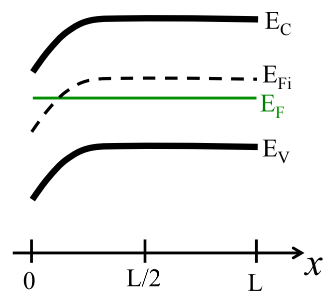
i) Do equilibrium conditions prevail? How do you know?
ii) Sketch the electrostatic potential (ϕ) inside the semiconductor as a function of x.
iii) Sketch the electric field (E) inside the semiconductor as a function of x.
iv) Roughly sketch n and p versus x.
v) Roughly sketch the electron drift-current density and the electron diffusion-current density as a function of x, on the same plot. Be sure to graph the proper polarity of the current densities and clearly identify the two current components. Give a brief explanation for how you arrived at your sketch.



a)



b)



c)

2) E-Book, problem 5.5 (4 points)

3) E-Book, problem 5.9 (9 points)

4) E-Book, problem 5.23 (16 points)

5) E-Book, problem 5.31 (6 points)

6) E-Book, problem 5.36 (6 points)

7) E-Book, problem 7.2 (9 points)

8) E-Book, problem 7.6 (10 points)

9) E-Book, problem 7.29, assume $V_R >> V_{bi}$ (12 points)

10) E-Book, problem 7.36 – asking to find N_a (4 points)

1) Answer the following questions, for each of the band diagrams given below (a-c): (24 points)

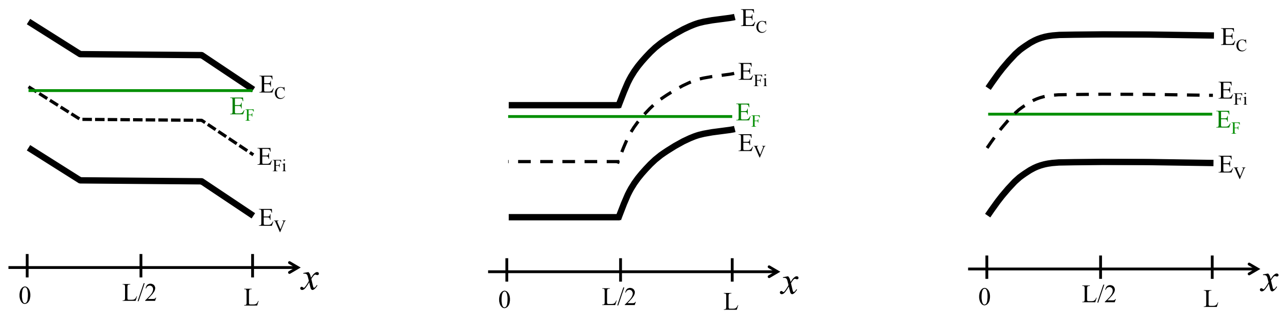
i) Do equilibrium conditions prevail? How do you know?

ii) Sketch the electrostatic potential (ϕ) inside the semiconductor as a function of *x*.

iii) Sketch the electric field (*E*) inside the semiconductor as a function of *x*.

iv) Roughly sketch *n* and *p* versus *x*.

v) Roughly sketch the electron drift-current density and the electron diffusion-current density as a function of *x*, on the same plot. Be sure to graph the proper polarity of the current densities and clearly identify the two current components. Give a brief explanation for how you arrived at your sketch.

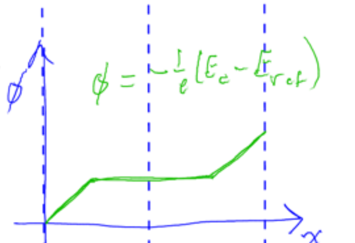


a) b) c)

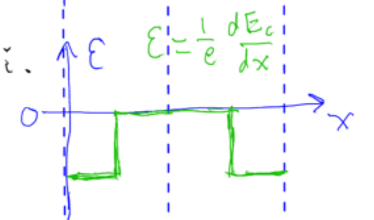
a)

1 i. Yes, E_F is constant.

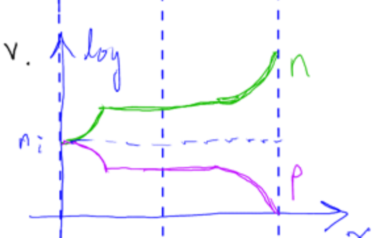
1 ii.



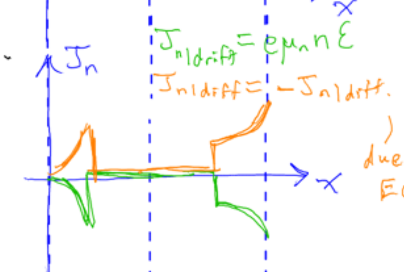
2 iii.



2 iv.



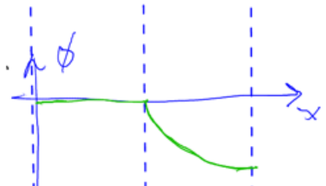
2 v.



b)

1 i. Yes, E_F is constant.

1 ii.



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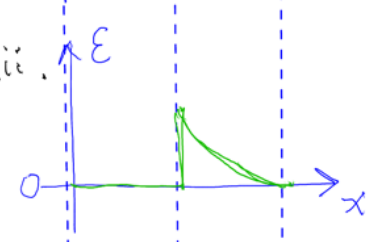

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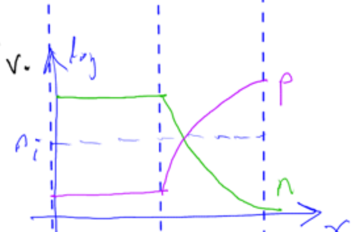
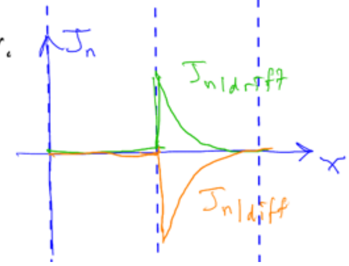


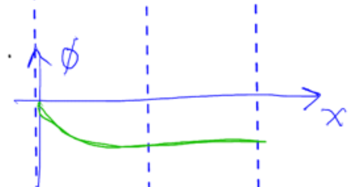
2 v.



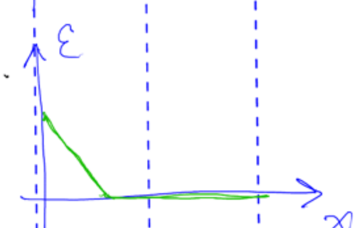
c)

1 i. Yes, E_F is constant.

1 ii.



2 iii.



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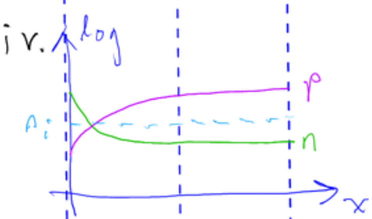
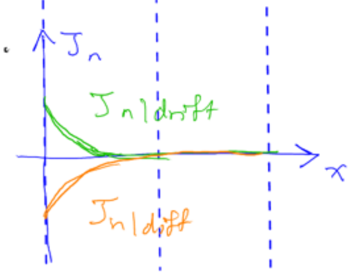


2 v.



2) E-Book, problem 5.5 (4 points)

5.5 A silicon sample is 2.5 cm long and has a cross-sectional area of 0.1 cm^2. The silicon is n type with a donor impurity concentration of $N_d = 2 \times 10^{15}$ cm^{-3}. The resistance of the sample is measured and found to be 70 Ω. What is the electron mobility?

Si, $L = 2.5\,cm$, $A = 0.1\,cm^2$, $N_d = 2\times10^{15}\,cm^{-3}$, $R = 70\,\Omega$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{e\mu_n N_d A} \quad \textbf{2}$$

↳ n-type, so: $\sigma = e\mu_n N_d$

$$\Downarrow$$

$$\mu_n = \frac{L}{eN_d RA} = \frac{2.5\,cm}{(1.6\times10^{-19}\,C)(2\times10^{15}\,cm^{-3})(70\,\Omega)(0.1\,cm^2)}$$

$$= \boxed{1116\ cm^2/V\cdot s} \quad \textbf{2}$$

units: $\frac{cm^2 A}{C\,\cancel{cm^{-3}}\,V\,cm^2} = \frac{cm^2\cancel{A}}{\cancel{A}\cdot s\cdot V} = \frac{cm^2}{V\cdot s}$

3) E-Book, problem 5.9 (9 points)

5.9 (*a*) A GaAs semiconductor resistor is doped with donor impurities at a concentration of $N_d = 2 \times 10^{15}$ cm^{-3} and has a cross-sectional area of 5×10^{-5} cm^2. A current of $I = 25$ mA is induced in the resistor with an applied bias of 5 V. Determine the length of the resistor. (*b*) Using the results of part (*a*), calculate the drift velocity of the electrons. (*c*) If the bias applied to the resistor in part (*a*) increases to 20 V, determine the resulting current if the electrons are traveling at their saturation velocity of 5×10^6 cm/s.

a)

From Fig. 5.3, $\mu_n \simeq 8000\ \text{cm}^2/\text{V}\cdot\text{s}$ at $N_d = 2\times10^{15}\ \text{cm}^{-3}$

$$R = \frac{V}{I} = \frac{5\text{V}}{0.025\text{A}} = 200\ \Omega = \frac{L}{(e\mu_n N_d)A}$$

$$\Rightarrow L = e\mu_n N_d RA = (1.6\times10^{-19}\text{C})(8000\ \text{cm}^2/\text{V}\cdot\text{s})(2\times10^{15}\ \text{cm}^{-3})(200\ \Omega)(5\times10^{-5}\ \text{cm}^2)$$

units $= \frac{\text{C}\ \text{cm}^2\ \cancel{\text{V}}\ \text{cm}^2}{\text{cm}^3\ \text{A}\cdot\cancel{\text{V}}\cdot\text{s}} = \frac{\text{cm}\cdot\cancel{\text{A}}}{\cancel{\text{A}}} = \text{cm}$ $= \boxed{0.026\ \text{cm}}$ **3**

b) $$J = \frac{I}{A} = en_0 v_{dn} \Rightarrow v_{dn} = \frac{I}{en_0 A} = \frac{0.025\text{A}}{(1.6\times10^{-19}\text{C})(2\times10^{15}\ \text{cm}^{-3})(5\times10^{-5}\ \text{cm}^2)} = \boxed{1.56\times10^6\ \text{cm/s}}$$

units: use $A = C/s$

3

c) $$I = en_0 v_{sat} A = (1.6\times10^{-19}\text{C})(2\times10^{15}\ \cancel{\text{cm}^{-3}})(5\times10^6\ \cancel{\text{cm}}/\text{s})(5\times10^{-5}\ \cancel{\text{cm}^2}) = \boxed{80\ \text{mA}}$$ **3**

4) E-Book, problem 5.23 (16 points)

5.23 Consider three samples of silicon at $T = 300$ K. The n-type sample is doped with arsenic atoms to a concentration of $N_d = 5 \times 10^{16}$ cm^{-3}. The p-type sample is doped with boron atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. The compensated sample is doped with both the donors and acceptors described in the n-type and p-type samples. (*a*) Find the equilibrium electron and hole concentrations in each sample, (*b*) determine the majority carrier mobility in each sample, (*c*) calculate the conductivity of each sample, (*d*) and determine the electric field required in each sample to induce a drift current density of $J = 120$ A/cm^2.

1 each

a) Find n_o & p_o in each sample

n-type: $n_o = N_d = \boxed{5\times10^{16}\ cm^{-3}}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10}\ cm^{-3})^2}{5\times10^{16}\ cm^{-3}} = \boxed{4.5\times10^{3}\ cm^{-3}}$$

p-type: $p_o = N_a = \boxed{2\times10^{16}\ cm^{-3}}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10}\ cm^{-3})^2}{2\times10^{16}\ cm^{-3}} = \boxed{1.13\times10^{4}\ cm^{-3}}$$

compensated: $n_o = N_d - N_a = 5\times10^{16}\ cm^{-3} - 2\times10^{16}\ cm^{-3} = \boxed{3\times10^{16}\ cm^{-3}}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10}\ cm^{-3})^2}{3\times10^{16}\ cm^{-3}} = \boxed{7.5\times10^{3}\ cm^{-3}}$$

b) majority carrier mobility in each

using Fig. 5.3: n-type: $\mu_n \cong 1100\ cm^2/V\cdot s$

p-type: $\mu_p \cong 400\ cm^2/V\cdot s$

compensated: $\mu_n \cong 1000\ cm^2/V\cdot s$

c) conductivity:

n-type: $\sigma = e\mu_n n_o = (1.6\times10^{-19}\ C)(1100\ cm^2/V\cdot s)(5\times10^{16}\ cm^{-3}) = \boxed{8.8\ S/cm}$

p-type: $\sigma = e\mu_p p_o = (1.6\times10^{-19}\ C)(400\ cm^2/V\cdot s)(2\times10^{16}\ cm^{-3}) = \boxed{1.28\ S/cm}$

compensated: $\sigma = e\mu_n n_o = (1.6\times10^{-19}\ C)(1000\ cm^2/V\cdot s)(3\times10^{16}\ cm^{-3}) = \boxed{4.8\ S/cm}$

d) E for $J = 120\ A/cm^2$

n-type: $J = \sigma E \Rightarrow E = \frac{J}{\sigma} = \frac{120\ A/cm^2}{8.8\ S/cm} = \boxed{13.6\ V/cm}$

p-type: $E = \frac{J}{\sigma} = \frac{120\ A/cm^2}{1.28\ S/cm} = \boxed{93.75\ V/cm}$

compensated: $E = \frac{J}{\sigma} = \frac{120\ A/cm^2}{4.8\ S/cm} = \boxed{25\ V/cm}$

5) E-Book, problem 5.31 (6 points)

5.31 The electron diffusion current density in a semiconductor is a constant and is given by $J_n = -2$ A/cm². The electron concentration at $x = 0$ is $n(0) = 10^{15}$ cm^{-3}. (*a*) Calculate the electron concentration at $x = 20\ \mu$m if the material is silicon with $D_n = 30$ cm²/s. (*b*) Repeat part (*a*) if the material is GaAs with $D_n = 230$ cm²/s.

a) $x = 20\ \mu m$, $n = ?$

units: $\frac{C \cdot cm^2 \cdot}{s \cdot cm^4} = A/cm^2$

$$J_{n|diff} = e D_n \frac{dn}{dx} = e D_n \frac{\Delta n}{\Delta x}$$

$$-2\ A/cm^2 = (1.602\times10^{-19}\ C)(30\ cm^2/s)\left[\frac{10^{15}\ cm^{-3} - n(x_1)}{0 - 20\times10^{-4}\ cm}\right]$$

$$\Rightarrow n(x_1) = \boxed{1.67\times10^{14}\ cm^{-3}}$$ **3**

b) GaAs : $D_n = 230\ cm^2/s$

Recompute same as but with $230\ cm^2/s = D_n$ gives:

$$n(x_1) = \boxed{8.91\times10^{14}\ cm^{-3}}$$ **3**

6) E-Book, problem 5.36 (6 points)

5.36 The total current in a semiconductor is constant and equal to $J = -10$ A/cm^2. The total current is composed of a hole drift current and electron diffusion current. Assume that the hole concentration is a constant and equal to 10^{16} cm^{-3} and assume that the electron concentration is given by $n(x) = 2 \times 10^{15} e^{-x/L}$ cm^{-3} where $L = 15\ \mu$m. The electron diffusion coefficient is $D_n = 27$ cm^2/s and the hole mobility is $\mu_p = 420$ cm^2/V-s. Calculate (*a*) the electron diffusion current density for $x > 0$, (*b*) the hole drift current density for $x > 0$, and (*c*) the required electric field for $x > 0$.

a) $J_{n|diff}$ for $x > 0$

$$J_{n|diff} = e D_n \frac{dn}{dx} = e D_n \frac{d}{dx}\left[2\times10^{15} e^{-x/L}\ cm^{-3}\right] = \frac{-e D_n (2\times10^{15}) e^{-x/L}}{L}$$

$$= \frac{-(1.602\times10^{-19} C)(27\ cm^2/s)(2\times10^{15}) e^{-x/L}\ cm^{-3}}{15\times10^{-4}\ cm}$$

$$= \boxed{-5.76 e^{-x/L}\ A/cm^2}$$

2

b) $J_{p|drift}$ for $x > 0$

$$J_{p|drift} = J_{total} - J_{n|diff} = -10\ A/cm^2 - (-5.76 e^{-x/L}\ A/cm^2)$$

$$= \boxed{\left[5.76 e^{-x/L} - 10\right] A/cm^2}$$

2

c) $\mathcal{E}$ for $x > 0$

$$J_{p|drift} = \sigma \mathcal{E} = e \mu_p p \mathcal{E} = 5.76 e^{-x/L} - 10$$

$$(1.602\times10^{-19} C)(420\ cm^2/V\cdot s)(10^{16}\ cm^{-3})\mathcal{E} = \left[5.76 e^{-x/L} - 10\right] A/cm^2$$

$$\Rightarrow \boxed{\mathcal{E} = \left[8.57 e^{-x/L} - 14.88\right] V/cm}$$

2

7) E-Book, problem 7.2 (9 points)

7.2 Calculate the built-in potential barrier, V_{bi}, for Si, Ge, and GaAs pn junctions if they each have the following dopant concentrations at $T = 300$ K: $n_i = 1.8\times10^{6}\ cm^{-3}$ $n_i = 2.4\times10^{13}\ cm^{-3}$ $n_i = 1.5\times10^{10}\ cm^{-3}$

(a) $N_d = 10^{14}\ \text{cm}^{-3}$ $N_a = 10^{17}\ \text{cm}^{-3}$

(b) $N_d = 5\times10^{16}$ $N_a = 5\times10^{16}$

(c) $N_d = 10^{17}$ $N_a = 10^{17}$

$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.026\,V)\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

a) $$V_{bi}^{Si} = (0.026V)\ln\left(\frac{(10^{17}\,cm^{-3})(10^{14}\,cm^{-3})}{(1.5\times10^{10}\,cm^{-3})^2}\right) = 0.635\ V$$

$V_{bi}^{Ge} = 0.253\ V$ 3

$V_{bi}^{GaAs} = 1.10\ V$

b) Same expression for V_{bi} with new doping:

$V_{bi}^{Si} = 0.778\ V$

$V_{bi}^{Ge} = 0.396\ V$ 3

$V_{bi}^{GaAs} = 1.25\ V$

c) $V_{bi}^{Si} = 0.814\ V$

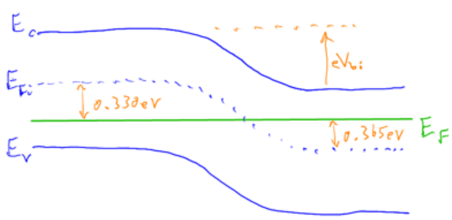
$V_{bi}^{Ge} = 0.432\ V$ 3

$V_{bi}^{GaAs} = 1.28\ V$

8) E-Book, problem 7.6 (10 points)

7.6 A silicon pn junction in thermal equilibrium at $T = 300$ K is doped such that $E_F - E_{Fi} = 0.365$ eV in the n region and $E_{Fi} - E_F = 0.330$ eV in the p region. (*a*) Sketch the energy-band diagram for the pn junction. (*b*) Find the impurity doping concentration in each region. (*c*) Determine V_{bi}.

a)



3

$E_g = 1.12eV$

$E_c - E_{Fi} \simeq 0.56eV \simeq E_{Fi} - E_v$

$\therefore E_c - E_F = 0.56eV - 0.365eV = 0.195eV$

$\therefore E_F - E_v = 0.56eV - 0.33eV = 0.23eV$

b) $N_d = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$

$= (1.5\times10^{10}\text{cm}^{-3})\exp\left[\frac{0.365eV}{0.026eV}\right] = \boxed{1.98\times10^{16}\text{ cm}^{-3}}$ n-region **2**

$N_a = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$

$= (1.5\times10^{10}\text{cm}^{-3})\exp\left[\frac{0.330eV}{0.026eV}\right] = \boxed{5.12\times10^{15}\text{ cm}^{-3}}$ p-region **2**

c) $V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)$

$= (0.026V)\ln\left(\frac{(5.12\times10^{15}\text{cm}^{-3})(1.98\times10^{16}\text{cm}^{-3})}{(1.5\times10^{10}\text{cm}^{-3})^2}\right)$

$= \boxed{0.695V}$ **3**

9) E-Book, problem 7.29, assume $V_R >> V_{bi}$ (12 points)

7.29 Consider a silicon pn junction with the doping profile shown in Figure P7.29. $T = 300$ K. (*a*) Calculate the applied reverse-biased voltage required so that the space charge region extends entirely through the p region. (*b*) Determine the space charge width into the n$^+$ region with the reverse-biased voltage calculated in part (*a*). (*c*) Calculate the peak electric field for this applied voltage.

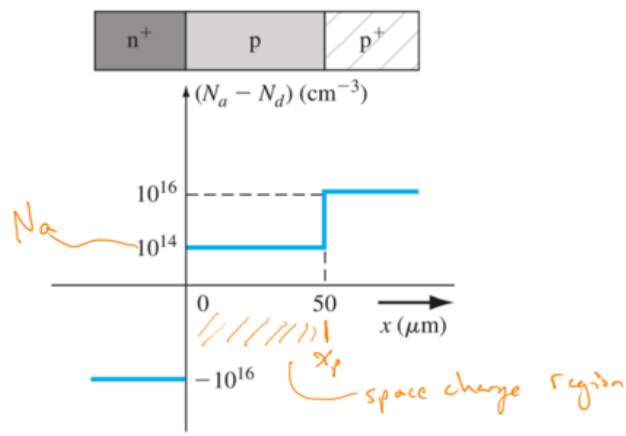


Figure P7.29 | Figure for Problem 7.29.

a) Want $x_p = 50 \times 10^{-4}$ cm

Assume a one-sided junction ($n^+ p$)

and $V_R >> V_{bi}$:

$$x_p \simeq \left[\frac{2\epsilon_s \epsilon_0}{e N_a} V_R\right]^{1/2}$$

2

$$(50 \times 10^{-4}\text{ cm}) \doteq \left[\frac{2(11.7)(8.85\times10^{-14}\text{ F/cm})}{(1.602\times10^{-19}\text{ C})(10^{14}\text{ cm}^{-3})} V_R\right]^{1/2}$$

$$\Rightarrow \boxed{V_R = 193\text{ V}}$$

2

b) Since $x_p N_a = x_n N_d$

$$\Rightarrow x_n = \frac{x_p N_a}{N_d} = \frac{(50\times10^{-4}\text{ cm})(10^{14}\,\cancel{\text{cm}^{-3}})}{10^{16}\,\cancel{\text{cm}^{-3}}} = \boxed{5\times10^{-5}\text{ cm}}$$

4

c) $$|\mathcal{E}_{max}| \simeq \frac{2V_R}{W} = \frac{2V_R}{x_n + x_p}$$

$$= \frac{2(193\text{V})}{50\times10^{-4}\text{ cm} + 5\times10^{-5}\text{ cm}} = \boxed{7.7\times10^{4}\text{ V/cm}}$$

4

10) E-Book, problem 7.36 – asking to find N_a (4 points)

7.36 Design an abrupt silicon n⁺p junction diode that has a reverse breakdown voltage of 80 V.

$V_B = 80\,V$

2 $$N_a = \frac{\epsilon_s \epsilon_0 \mathcal{E}_{crit}^2}{2eV_B} = \frac{(11.7)(8.85\times10^{-14}\,\cancel{F}/cm)(4\times10^5\,\cancel{V}/cm)^2}{2(1.602\times10^{-19}\,\cancel{C})(80\,\cancel{V})}$$

cm^{-3}

$$= \boxed{6.47\times10^{15}\ cm^{-3}}$$ **2**

from

$$V_B = \frac{\epsilon_s \epsilon_0 \mathcal{E}_{crit}^2}{2eN_B}$$