Solutions

Homework #3

Thermal Equilibrium and Carrier Transport – *100 points*
DUE @ Beginning of Class: Thursday, September 21

1) Answer the following questions, showing your work where needed: (14 points)
   a) A Si wafer is uniformly doped *p*-type with $N_a = 10^{15}$ cm^{-3}. At T = 0 K, what are the equilibrium hole and electron concentrations?
   b) A semiconductor is doped with an impurity concentration N such that $N >> n_i$ and all the impurities are ionized. Also, $n_0 = N$ and $p_0 = n_i^2/N$. Is the impurity a donor or an acceptor? Explain.
   c) The electron concentration in a piece of Si maintained at 300 K under equilibrium conditions is 10^5 cm^{-3}. What is the hole concentration?
   d) For a silicon sample maintained at T = 300 K, the Fermi level is located 0.0259 eV above the intrinsic Fermi level. What are the hole and electron concentrations?
   e) In a nondegenerate germanium sample maintained under equilibrium conditions near room temperature, it is known that $n_i = 10^{13}$ cm^{-3}, $n_0 = 2p_0$, and $N_a = 0$. Determine n_0 and N_d.

2) For each of the following conditions, determine the position of E_{Fi}, compute $E_F - E_{Fi}$, and draw a carefully dimensioned energy band diagram (for instance, use graph paper) for the Si sample. Assume temperature independence for effective mass and use $m_p^* = 0.81m_0$ and $m_n^* = 1.18m_0$. NOTE: E_g (Si) = 1.08 eV at 450 K and 1.015 eV at 650 K. (20 points)
   a) T = 300 K, $N_a << N_d$, $N_d = 10^{15}$ cm^{-3}
   b) T = 300 K, $N_a = 10^{16}$ cm^{-3}, $N_d << N_a$
   c) T = 300 K, $N_a = 9 \times 10^{15}$ cm^{-3}, $N_d = 10^{16}$ cm^{-3}
   d) T = 450 K, $N_a = 0$, $N_d = 10^{14}$ cm^{-3}
   e) T = 650 K, $N_a = 0$, $N_d = 10^{14}$ cm^{-3}

3) E-Book, problem 4.17 (9 points)

4) E-Book, problem 4.26 (12 points)

5) E-Book, problem 4.34 (20 points)

6) E-Book, problem 4.47 (5 points)

7) E-Book, problem 4.55 (10 points)

8) E-Book, problem 4.62 (10 points)

1) Answer the following questions, showing your work where needed: (14 points)

a) A Si wafer is uniformly doped *p*-type with $N_a = 10^{15}$ cm^{-3}. At T = 0 K, what are the equilibrium hole and electron concentrations?

b) A semiconductor is doped with an impurity concentration *N* such that $N >> n_i$ and all the impurities are ionized. Also, $n_0 = N$ and $p_0 = n_i^2/N$. Is the impurity a donor or an acceptor? Explain.

c) The electron concentration in a piece of Si maintained at 300 K under equilibrium conditions is 10^5 cm^{-3}. What is the hole concentration?

d) For a silicon sample maintained at T = 300 K, the Fermi level is located 0.0259 eV above the intrinsic Fermi level. What are the hole and electron concentrations?

e) In a nondegenerate germanium sample maintained under equilibrium conditions near room temperature, it is known that $n_i = 10^{13}$ cm^{-3}, $n_0 = 2p_0$, and $N_a = 0$. Determine n_0 and N_d.

a) $N_a = 10^{15}\ cm^{-3}$, $T = 0\,K$

as $T \to 0$, $\boxed{n_0 \to 0 \text{ and } p_0 \to 0}$ 2

b) $N >> n_i$, $n_0 = N$, $p_0 = \frac{n_i^2}{N}$

Since $N >> n_i$, one would have:

$n_0 = N_d$, $p_0 = \frac{n_i^2}{N_d}$ if donor

$p_0 = N_a$, $n_0 = \frac{n_i^2}{N_a}$ if acceptor

We are given that $p_0 = \frac{n_i^2}{N}$, so it is $\boxed{\text{donor doping}}$. 2

c) $n_0 = 10^5\ cm^{-3}$, $T = 300\,K$

Since we are at 300K, the given $n_0 = 10^5\ cm^{-3}$ is clearly the minority carrier concentration. As always holds:

$n_0 p_0 = n_i^2 \rightarrow p_0 = \frac{n_i^2}{n_0} = \frac{(10^{10}\ cm^{-3})^2}{(10^5\ cm^{-3})} = \boxed{10^{15}\ cm^{-3}}$ 2

d) $T = 300K$, $E_F - E_{Fi} = 0.0259\ eV = kT$

$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] = (10^{10}\ cm^{-3})\, e^{1} = \boxed{2.72 \times 10^{10}\ cm^{-3}}$ 2

$p_0 = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = (10^{10}\ cm^{-3})\, e^{-1} = \boxed{3.68 \times 10^{9}\ cm^{-3}}$ 2

e) Ge $n_i = 10^{13}\ cm^{-3}$, $n_0 = 2p_0$, $N_a = 0$

Since $n_0 p_0 = \frac{n_0^2}{2} = n_i^2 \Rightarrow n_0 = \sqrt{2}\, n_i = \boxed{1.41 \times 10^{13}\ cm^{-3}}$ 2

Now, using charge neutrality: $p_0 - n_0 + N_d - N_a = n_0/2 - n + N_d = 0$

$\Rightarrow N_d = n_0/2 = \boxed{7.07 \times 10^{14}\ cm^{-3}}$ 2

2) For each of the following conditions, determine the position of E_{Fi}, compute $E_F - E_{Fi}$, and draw a carefully dimensioned energy band diagram (for instance, use graph paper) for the Si sample. Assume temperature independence for effective mass and use $m_p^* = 0.81m_0$ and $m_n^* = 1.18m_0$. NOTE: E_g (Si) = 1.08 eV at 450 K and 1.015 eV at 650 K. (20 points)

a) T = 300 K, $N_a << N_d$, $N_d = 10^{15}$ cm^{-3}
b) T = 300 K, $N_a = 10^{16}$ cm^{-3}, $N_d << N_a$
c) T = 300 K, $N_a = 9 \times 10^{15}$ cm^{-3}, $N_d = 10^{16}$ cm^{-3}
d) T = 450 K, $N_a = 0$, $N_d = 10^{14}$ cm^{-3}
e) T = 650 K, $N_a = 0$, $N_d = 10^{14}$ cm^{-3}

2 points/part below + 2 points per band diagram

First need to determine positioning of E_{Fi}. Since it is the Fermi level position for an intrinsic sample:

$$n_0 = p_0$$

$$N_c \exp\left[\frac{E_{Fi} - E_c}{kT}\right] = N_v \exp\left[\frac{E_v - E_{Fi}}{kT}\right]$$

Solving for E_{Fi}:

$$E_{Fi} = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)$$

Note that answers for Efi may vary by a bit if student's use Nv and Nc rather than the effective mass version of this expression. That is ok.

$$\frac{N_v}{N_c} = \frac{2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}}{2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2}} = \left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$\therefore E_{Fi} = \frac{E_c + E_v}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

for m_p^* and m_n^*, assume Temperature independence and DOS m^*:

$$m_p^* = 0.81 m_0, \quad m_n^* = 1.18 m_0 \Rightarrow \frac{m_p^*}{m_n^*} = 0.686$$

$$E_{Fi} = \frac{E_c + E_v}{2} - 0.283\,kT$$

a) $T = 300\text{K}$, $N_a << N_d$, $N_d = 10^{15}\text{cm}^{-3}$

$$E_{Fi} = \underbrace{\frac{E_c + E_v}{2}}_{=\text{ midgap} = E_{\text{midgap}}} - 0.283(0.026\text{eV}) = \boxed{7.4\text{meV below midgap}}$$

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = (0.026\text{eV})\ln\left(\frac{10^{15}}{10^{10}}\right) = \boxed{0.298\text{eV}}$$

b) $T = 300\text{K}$, $N_a = 10^{16}\text{cm}^{-3}$, $N_d << N_a$

$$E_{Fi} = \boxed{\text{midgap} - 7.4\text{meV}}$$

$$E_F - E_{Fi} = (0.026\text{eV})\ln\left(\frac{10^{16}}{10^{10}}\right) = \boxed{0.358\text{eV}}$$

c) $T = 300\text{K}$, $N_a = 9\times10^{15}\text{cm}^{-3}$, $N_d = 10^{16}\text{cm}^{-3}$

$$E_{Fi} = \boxed{\text{mid gap} - 7.4\text{ meV}}$$

$$E_F - E_{Fi} = kT\ln\left[\frac{N_d - N_a}{n_i}\right] = (0.026\text{ev})\ln\left[\frac{10^{16} - 9\times10^{15}}{10^{10}}\right] = \boxed{0.298\text{ eV}}$$

d) $T = 450\text{K}$, $N_a = 0$, $N_d = 10^{14}\text{cm}^{-3}$

$$E_{Fi} = \text{midgap} - 0.283\underbrace{(0.0388\text{eV})}_{kT\ @\ 450\text{K}} = \boxed{\text{midgap} - 0.011\text{ eV}}$$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_0}{n_i}\right)$$

n_i @ 450K = $5\times10^{13}\text{cm}^{-3}$

$$n_0 = \frac{N_d - \cancel{N_a}}{2} + \left[\left(\frac{N_d - \cancel{N_a}}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$= \frac{N_d}{2} + \sqrt{\frac{N_d^2}{4} + n_i^2} = 1.21\times10^{14}\text{cm}^{-3}$$

$$E_F - E_i = (0.0388\text{ev})\ln\left(\frac{1.21\times10^{14}}{5\times10^{13}}\right) = \boxed{0.034\text{eV}}$$

e) $T = 650\text{K}$, $N_a = 0$, $N_d = 10^{14}\text{cm}^{-3}$

$$E_{Fi} = \text{midgap} - 0.283\underbrace{(0.056\text{eV})}_{kT\ @\ 650\text{K}} = \boxed{\text{midgap} - 0.016\text{eV}}$$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_0}{n_i}\right)$$

n_i @ 650K ≅ 10^{16}cm^{-3}

$n_0 \simeq 10^{16}\text{cm}^{-3}$

$$\therefore E_F - E_i \cong \boxed{0}$$

3) E-Book, problem 4.17 (9 points)

4.17 Silicon at $T = 300$ K is doped with arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15}$ cm^{-3}. (*a*) Find $E_c - E_F$. (*b*) Determine $E_F - E_v$. (*c*) Calculate p_0. (*d*) Which carrier is the minority carrier? (*e*) Find $E_F - E_{Fi}$.

a) Find $E_c - E_F$

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$

$N_c = 2.8 \times 10^{19}$ cm^{-3} For Si @ 300 K

$$= (0.026\,eV) \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right) = \boxed{0.2148\,eV}$$

2

b) Determine $E_F - E_v$

$$E_F - E_v = E_g - (E_c - E_F) = 1.12\,eV - (0.2148\,eV) = \boxed{0.905\,eV}$$

2

c) Calculate p_0

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (1.04 \times 10^{19}\,cm^{-3}) \exp\left[\frac{-0.905\,eV}{0.026\,eV}\right] = \boxed{6.9 \times 10^3\,cm^{-3}}$$

1 2

d) minority carrier? $\boxed{\text{Holes}}$ because $p_0 n_0 = n_i^2$ and $p_0 << n_i$ 2

e) Find $E_F - E_{Fi}$

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = (0.026\,eV) \ln\left(\frac{7 \times 10^{15}}{1.5 \times 10^{10}}\right) = \boxed{0.338\,eV}$$

2

4) E-Book, problem 4.26 (12 points)

4.26 (*a*) Determine the values of n_0 and p_0 in GaAs at $T = 300$ K if $E_F - E_v = 0.25$ eV. (*b*) Assuming the value of p_0 in part (*a*) remains constant, determine the values of $E_F - E_v$ and n_0 at $T = 400$ K.

2 points/part

a) Determine n_o & p_o in GaAs at $T=300K$, $E_F - E_v = 0.25\,eV$

$$p_o = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (7\times10^{18}\,cm^{-3})\exp\left[\frac{-0.25\,eV}{0.026\,eV}\right] = \boxed{4.5\times10^{14}\,cm^{-3}}$$

$\hookrightarrow N_v = 7\times10^{18}\,cm^{-3}$ for GaAs at 300K

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.3\times10^{6}\,cm^{-3})^2}{4.5\times10^{14}\,cm^{-3}} = \boxed{1.17\times10^{-2}\,cm^{-3}}$$

b) If p_o is constant, determine $E_F - E_v$ and n_o at $T=400K$

kT @ $400K = 0.0345\,eV$

Since $N_v = 7\times10^{18}\,cm^{-3}$ for 300K, at 400K it will be:

Table 4.1 in text

$$N_v = (7\times10^{18}\,cm^{-3})\left(\frac{400K}{300K}\right)^{3/2} = 1.078\times10^{19}\,cm^{-3}$$

similarly for $N_c = 4.7\times10^{17}\,cm^{-3}$ at 300K:

$$N_c = (4.7\times10^{17}\,cm^{-3})\left(\frac{400K}{300K}\right)^{3/2} = 7.24\times10^{17}\,cm^{-3}$$

$$E_F - E_v = kT\ln\left(\frac{N_v}{p_o}\right) = (0.0345\,eV)\ln\left(\frac{1.078\times10^{19}\,cm^{-3}}{4.5\times10^{14}\,cm^{-3}}\right) = \boxed{0.348\,eV}$$

$$E_c - E_F = \underbrace{1.42\,eV}_{E_g \text{ of GaAs}} - 0.348\,eV = 1.07\,eV$$

$$n_o = N_c \exp\left[\frac{E_F - E_c}{kT}\right] = (7.24\times10^{17}\,cm^{-3})\exp\left[\frac{-1.07\,eV}{0.0345\,eV}\right]$$

$$= \boxed{2.4\times10^{4}\,cm^{-3}}$$

5) E-Book, problem 4.34 (20 points)

4.34 Determine the equilibrium electron and hole concentrations in silicon for the following conditions:

(a) $T = 300$ K, $N_d = 10^{15}$ cm^{-3}, $N_a = 4 \times 10^{15}$ cm^{-3}

(b) $T = 300$ K, $N_d = 3 \times 10^{16}$ cm^{-3}, $N_a = 0$

(c) $T = 300$ K, $N_d = N_a = 2 \times 10^{15}$ cm^{-3}

(d) $T = 375$ K, $N_d = 0$, $N_a = 4 \times 10^{15}$ cm^{-3}

(e) $T = 450$ K, $N_d = 10^{14}$ cm^{-3}, $N_a = 0$

Double the below point values

Determine n_o, p_o in Si for:

a) $T = 300K$, $N_d = 10^{15} cm^{-3}$, $N_a = 4\times 10^{15} cm^{-3}$

$$p_o \simeq N_a - N_d = 4\times10^{15} - 10^{15} = \boxed{3\times10^{15} cm^{-3}} = p_o \quad 1$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10} cm^{-3})^2}{3\times10^{15} cm^{-3}} = \boxed{7.5\times10^{4} cm^{-3}} = n_o \quad 1$$

b) $T = 300K$, $N_d = 3\times10^{16} cm^{-3}$, $N_a = 0$

$$n_o = N_d = \boxed{3\times10^{16} cm^{-3}} \quad 1$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10} cm^{-3})^2}{3\times10^{16} cm^{-3}} = \boxed{7.5\times10^{3} cm^{-3}} \quad 1$$

c) $T = 300K$, $N_d = N_a = 2\times10^{15} cm^{-3}$

with N_d and N_a =, the s.c. will be dominated by intrinsic carriers

$$n_o = p_o = n_i = \boxed{1.5\times10^{10} cm^{-3}} \quad 2$$

d) $T = 375K$, $N_d = 0$, $N_a = 4\times10^{15} cm^{-3}$

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right] = (2.8\times10^{19} cm^{-3})(1.04\times10^{19} cm^{-3})\left(\frac{375K}{300K}\right)^3 \exp\left[\frac{-1.12eV}{0.026eV}\left(\frac{300K}{375K}\right)\right]$$

temp change

$$\Rightarrow n_i = 7.33\times10^{11} cm^{-3}$$

$$p_o = N_a = \boxed{4\times10^{15} cm^{-3}} = p_o \quad 1$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(7.33\times10^{11} cm^{-3})^2}{4\times10^{15}} = \boxed{1.34\times10^{8} cm^{-3}} = n_o \quad 1$$

e) $T = 450K$, $N_d = 10^{14} cm^{-3}$, $N_a = 0$

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right] = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{450K}{300K}\right)^3 \exp\left[\frac{-1.12eV}{0.026eV}\left(\frac{300K}{450K}\right)\right]$$

$$n_i = 1.72\times10^{13} cm^{-3}$$

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d-N_a}{2}\right)^2 + n_i^2} = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + (1.72\times10^{13})^2} = \boxed{1.03\times10^{14} cm^{-3}} \quad 1$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.72\times10^{13})^2}{1.03\times10^{14}} = \boxed{2.88\times10^{12} cm^{-3}} \quad 1$$

6) E-Book, problem 4.47 (5 points)

4.47 In silicon at $T = 300$ K, it is found that $N_a = 7 \times 10^{15}$ cm^{-3} and $p_0 = 2 \times 10^4$ cm^{-3}. (*a*) Is the material n type or p type? (*b*) What are the majority and minority carrier concentrations? (*c*) What must be the concentration of donor impurities?

Si @ $T = 300K$, $N_a = 7 \times 10^{15}\ cm^{-3}$, $p_0 = 2 \times 10^{4}\ cm^{-3}$

a) $p_0 << n_i \Rightarrow \boxed{n\text{-type}}$ 1

b) majority carriers:

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10}\ cm^{-3})^2}{2 \times 10^{4}\ cm^{-3}} = \boxed{1.13 \times 10^{16}\ cm^{-3}}$$ 1

minority carrier:

$$p_0 = \boxed{2 \times 10^{4}\ cm^{-3}}$$ 1

c) what is N_d?

$$n_0 = N_d - N_a$$

$$\therefore N_d = \boxed{1.83 \times 10^{16}\ cm^{-3}}$$ 2

Double the below point values

7) E-Book, problem 4.55 (10 points)

4.55 (*a*) Silicon at $T = 300$ K is doped with donor impurity atoms at a concentration of $N_d = 6 \times 10^{15}$ cm^{-3}. (*i*) Determine $E_c - E_F$. (*ii*) Calculate the concentration of additional donor impurity atoms that must be added to move the Fermi energy level a distance kT closer to the conduction band edge. (*b*) Repeat part (*a*) for GaAs if the original donor impurity concentration is $N_d = 1 \times 10^{15}$ cm^{-3}.

a) Si, $T = 300\,K$, $N_d = 6 \times 10^{15}\,cm^{-3}$

i) $E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right) = (0.026\,eV) \ln\left(\frac{2.8 \times 10^{19}\,cm^{-3}}{6 \times 10^{15}\,cm^{-3}}\right) = \boxed{0.219\,eV}$ 1

ii) $E_c - E_F = 0.219\,eV - 0.026\,eV = 0.193\,eV$ 1

$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = (2.8 \times 10^{19}) \exp\left[\frac{-0.193\,eV}{0.026\,eV}\right] = 1.63 \times 10^{16}\,cm^{-3}$

additional dopants: $1.63 \times 10^{16} - 6 \times 10^{15} = \boxed{1.03 \times 10^{16}\,cm^{-3}}$ 1

b) GaAs, $T = 300\,K$, $N_d = 10^{15}\,cm^{-3}$

i) $E_c - E_F = (0.026\,eV) \ln\left(\frac{4.7 \times 10^{17}\,cm^{-3}}{10^{15}\,cm^{-3}}\right) = \boxed{0.159\,eV}$ 1

ii) $E_c - E_F = 0.159\,eV - 0.026\,eV = 0.133\,eV$

$N_d = (4.7 \times 10^{17}) \exp\left[\frac{-0.133\,eV}{0.026\,eV}\right] = 2.72 \times 10^{15}\,cm^{-3}$

additional dopants: $2.72 \times 10^{15}\,cm^{-3} - 10^{15}\,cm^{-3} = \boxed{1.72 \times 10^{15}\,cm^{-3}}$ 1

Double the below point values

8) E-Book, problem 4.62 (10 points)

4.62 Silicon atoms, at a concentration of 7×10^{15} cm^{-3}, are added to gallium arsenide. Assume that the silicon atoms act as fully ionized dopant atoms and that 5 percent of the concentration added replace gallium atoms and 95 percent replace arsenic atoms. Let $T = 300$ K. (*a*) Determine the donor and acceptor concentrations. (*b*) Is the material n type or p type? (*c*) Calculate the electron and hole concentrations. (*d*) Determine the position of the Fermi level with respect to E_{Fi}.

Si atoms $< 7 \times 10^{15}$ cm^{-3} added to GaAs

$\underbrace{\text{5\% replace Ga}}_{\text{Donor}}$, $\underbrace{\text{95\% replace As}}_{\text{acceptor}}$

a)

$N_d = (0.05)(7 \times 10^{15}\,\text{cm}^{-3}) = \boxed{3.5 \times 10^{14}\,\text{cm}^{-3}}$

$N_a = (0.95)(7 \times 10^{15}\,\text{cm}^{-3}) = \boxed{6.65 \times 10^{15}\,\text{cm}^{-3}}$ 1

b) $N_a > N_d \Rightarrow \boxed{p\text{-type}}$ 1

c) $p_o = N_a - N_d = 6.65 \times 10^{15} - 3.5 \times 10^{14} = \boxed{6.3 \times 10^{15}\,\text{cm}^{-3}}$ 1

$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{6.3 \times 10^{15}} = \boxed{5.14 \times 10^{-4}\,\text{cm}^{-3}}$ 1

1

d) $E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right) = (0.026\,\text{eV}) \ln\left(\frac{6.3 \times 10^{15}}{1.8 \times 10^6}\right) = \boxed{0.57\,\text{eV}}$