Ph 464 Problem Set 1

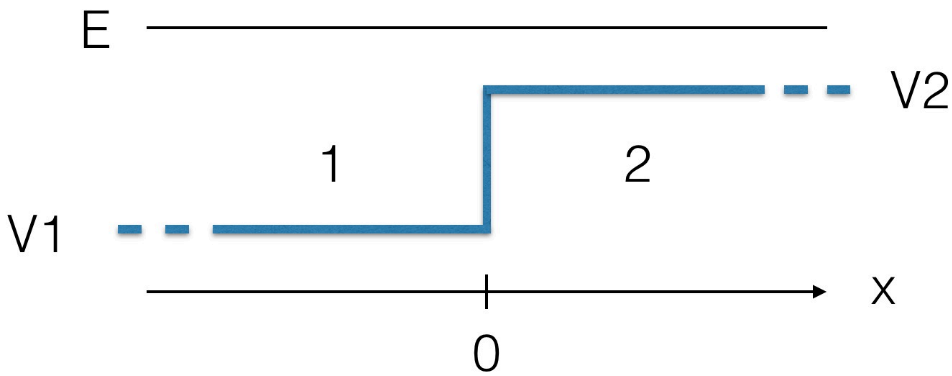
due Monday 4 Sep 2017 5pm

20% penalty for 24 hours late without permission, 40% penalty for 48 hours late without permission, no acceptance past 48 hours without permission.

1. C-TDL Chapter I, Problem 1.

2. Single-Step Potential

   (a) Find the general solution to the time-independent Schroedinger Equation (TISE) for the potential for

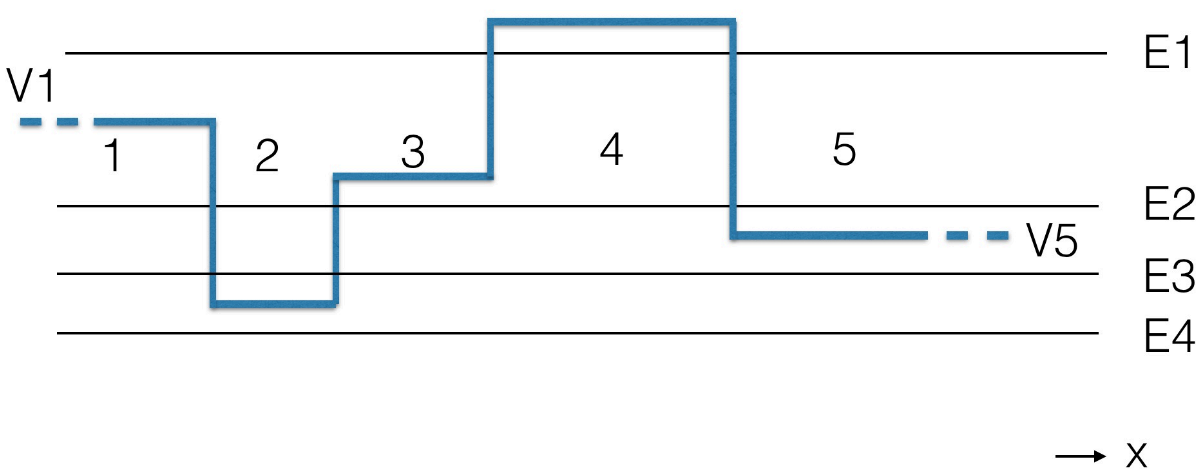


   $E > V_2$. How many arbitrary constants appear in the solution, for a given E, V_1, and V_2? What is the phase change for a particle incident from the right and reflected? Explain.

   (b) Recall that if we pick a particular set of values for the arbitrary constants in (a), we are dealing with <u>one function</u> (not two). The function has a different form in region I and region II. Also recall that if we have a set of functions $f_1(x), ..., f_n(x)$, the functions are <u>linearly dependent</u> if we can find a set of coefficients $\lambda_1, ..., \lambda_n$ such that $\lambda_1 f_1(x) + ... + \lambda_n f_n(x) = 0$. How many linearly <u>in</u>dependent solutions are there to the problem in part (a)?

   (c) What is the number of linearly independent solutions for $V_1 < E < V_2$? For $E < V_1 < V_2$? Explain.

3. Linearly Independent Solution of the TISE (think about the number of arbitrary constants, and boundary conditions). Consider a general square well potential with four steps:



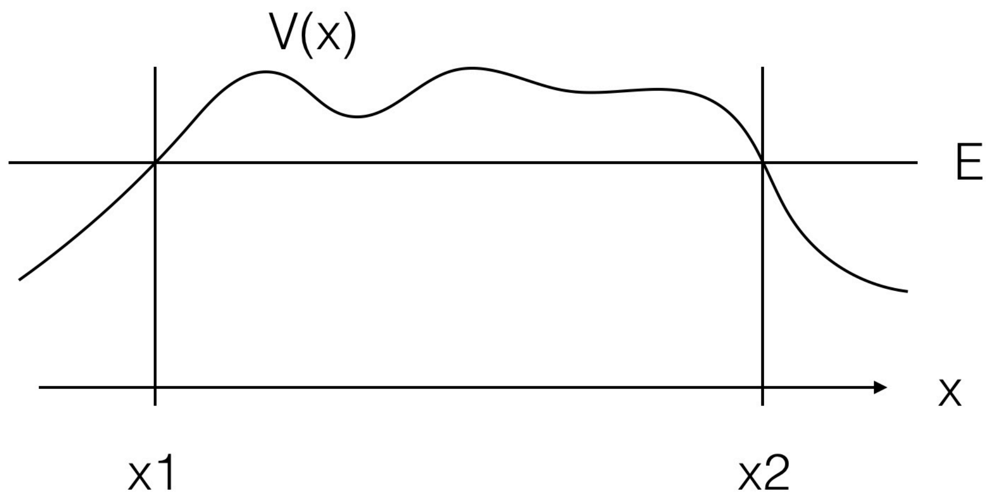
(a) Explain clearly how many linearly independent solutions of the TISE there are for $E = E_1$, for $E = E_2$, for $E = E_3$, and for $E = E_4$.

(b) Does the number of linearly independent solutions depend on whether V_4 is greater than E_1 or not?

(c) For $E = E_3$, consider the equations given by the boundary conditions for the arbitrary coefficents which enter into the solutions for $\Psi(x)$ in regions 1 through 5. What must happen to the determinant of the coefficients if there is a non-trivial solution? If such a solution exists, what is special about E_3 (think about region 2)?

4. Barrier Penetration for a General Potential (see C-TDL pp. 72-74)

(a) Consider a general potential $V(x)$ which acts as a "thick" barrier: Show that the penetration probablility,



for the case $\alpha(x)(x_2 - x_1) >> 1$, is given roughly by

$$T \simeq e^{-2\int_{x_2}^{x_1} \alpha(x)dx} \tag{1}$$

where $\alpha(x) = \sqrt{2m(V(x) - E)}/\hbar$ and where factors like $\frac{16E(V(x)-E)}{V(x)}$ have order of magnitude 1.

(b) A car weighing 1000kg runs out of gas just as it is approaching a hill, as shown. Its kinetic energy is not enough to carry it over the top. You can approximate the part of the hill that sticks up above the maximum classical height by $H(x) = x(l - x)/L$, with l=100m, L=250m. Estimate the probablility that the car will get across by quantum mechanical barrier penetration.

5. C-LDL Chapter I, Problem 2.