Homework 9 for STA 250/MTH 342 – Fall 2017

Due at the beginning of class on November 20, 2017

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. data from a Poisson(θ) distribution.

   (a) Find the likelihood ratio for testing

   $$H_0 : \theta = \theta_0 \quad \text{vs} \quad H_1 : \theta = \theta_1$$

   where θ_1 is greater than θ_0.

   (b) Use the fact that the sum of independent Poisson random variables follows a Poisson distribution to explain how to determine a rejection region for a test at level α.

   (c) Is this test uniformly most powerful (UMP) for testing

   $$H_0 : \theta = \theta_0 \quad \text{vs} \quad H_1 : \theta > \theta_0?$$

   Provide your reasoning.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. data from a distribution with the following pdf

   $$\begin{aligned} f(x|\theta) &= \frac{1}{\theta} m x^{m-1} e^{-x^m/\theta} \quad \text{for } x > 0, \\ &= 0 \quad \text{otherwise,} \end{aligned}$$

   where $m > 0$ is a known constant.

   (a) Find the UMP level α test for $H_0 : \theta = \theta_0$ vs $H_1 : \theta > \theta_0$.

   (b) If $n = 20$, $\theta_0 = 100$ and we want to have a level $\alpha = .05$, what is the rejection region of the test, and what is its power function?

   (c) Again, if $\theta_0 = 100$, $\alpha = .05$, to have a test with Type II error rate β no larger than .05, how big does the sample size n have to be, for the alternative $\theta = \theta_1 = 105$?

3. Let $X_i \sim$ **Binomial**(n_i, p_i) for $i = 1, 2, \ldots, m$, be independent. Derive the (generalized) likelihood ratio test for the hypothesis

   $$H_0 : p_1 = p_2 = \ldots = p_m \quad \text{vs} \quad H_1 : \text{otherwise.}$$

   (The test statistic you get for this test may be very complicated and you don't have to simplify it. As a result, you will not be able to find the cutoff constant C to make the test level α. We will introduce some techniques for you to find the cutoff later in the course.)

4. Suppose that $X_1, X_2, \ldots, X_{n_1}$, $Y_1, Y_2, \ldots, Y_{n_2}$, and $W_1, W_2, \ldots, W_{n_3}$ are independent random variables from normal distributions with respective unknown means μ_X, μ_Y, and μ_W and unknown variances σ_X^2, σ_Y^2, and σ_W^2. Find the (generalized) likelihood ratio test for

$$H_0 : \sigma_X^2 = \sigma_Y^2 = \sigma_W^2 \quad \text{vs} \quad H_1 : \text{otherwise.}$$

   (Again, you don't need to find the corresponding constant C that makes this test level α.)

5. D&S (4th Ed.) Exercise 9.3.17 (page 567)

6. D&S (4th Ed.) Exercise 9.5.4 (page 585)