

# Homework 1 for STA 250 – Fall 2017

Due in class September 6, 2017 (Show your work!)

Note: This homework will be graded based only on participation and will count for 1% of the course grade. It is mainly for you (and me) to get a sense of whether you have the necessary background for this course.

1. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let  $p$  be the probability that the student knows the answer and  $1 - p$  the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability  $1/m$ , where  $m$  is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly?
2. A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 p.m., find the probability that the first to arrive has to wait longer than 10 mins.
3. Let  $Z$  be a **standard** normal random variable, and for a fixed  $x$ , set

$$X = \begin{cases} Z & \text{if } Z > x \\ 0 & \text{otherwise.} \end{cases}.$$

Show that  $E[X] = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ .

4. Let  $U$  be a **Uniform**(0, 1) random variable, and suppose that the conditional distribution of  $X$ , given that  $U = p$ , is **Binomial**( $n, p$ ). Find the probability mass function of  $X$ .
5. If  $X$  and  $Y$  are independent Poisson random variables with respective parameters  $\lambda_1$  and  $\lambda_2$ , what is the distribution of  $X + Y$ ?
6. If the distribution of  $X$  is **Uniform**(0, 1) and given  $X = x$ , the distribution of  $Y$  is **Uniform**(0,  $x$ ). Find  $E[Y]$  and  $\text{Var}[Y]$ .

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