Quiz 3 for STA 250/MTH 342 – Fall 2017

Time: 25 mins. Closed book. Closed notes. Please show your work to get credits!

1. (10 pts) Let X be a discrete random variable that can take four values x_1, x_2, x_3,x_4 and x_5. Consider two hypotheses H_0 and H_1 which specify the distribution of X as follows.

X	H_0	H_1	LR	Rank
x_1	.2	.1	1/2	⑤
x_2	.1	.2	2	①
x_3	.3	.2	2/3	④
x_4	.2	.3	3/2	②
x_5	.2	.2	1	③

   (a) What is the most powerful level α test for H_0 versus H_1 at level $\alpha = .3$?

   (b) What is the Type II error rate β of this test?

(a) At level $\alpha = .3$, $R = \{x_2, x_4\}$. $\alpha = .1 + .2 = .3$

(b) $\beta = P(X \notin R \mid H_1) = P(X \in \{x_1, x_3, x_5\} \mid H_1)$

$= .1 + .2 + .2$

$= .5$

2. (10 pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. data from a Uniform distribution on the interval $[0, \theta]$.

   (a) Let $U = \hat{\theta}/\theta$ where $\hat{\theta} = \max\{X_1, X_2, \ldots, X_n\}$. Find the c.d.f of U and verify that this c.d.f does not depend on θ.

   (b) Suppose the c.d.f of U you find in part (a) is G, construct a 95% confidence interval for θ in terms of G.

(a) $U = \hat{\theta}/\theta$

$$P(U \le u) = P(X_1 \le \theta u, X_2 \le \theta u, \ldots, X_n \le \theta u)$$

$$= \prod_i P(X_i \le \theta u)$$

$$= \left(\frac{\theta u}{\theta}\right)^n$$

$$= u^n \quad \text{for any } 0 \le u \le 1.$$

which doesn't depend on θ.

(b) A 95% CI is given by

$$P\left(G^{-1}(.025) \le \hat{\theta}/\theta \le G^{-1}(.975)\right) = .95$$

Hence

$$P\left(\hat{\theta}/G^{-1}(.975) \le \theta \le \hat{\theta}/G^{-1}(.025)\right) = .95$$

The CI is $\left[\hat{\theta}/G^{-1}(.975), \hat{\theta}/G^{-1}(.025)\right]$.