| Lecture #6 | Semiconductors in EQ (cont.)

→ once doped, s.c. is now "extrinsic"
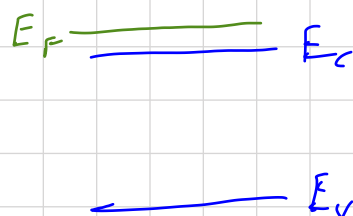
→ Ionization energy is the E to ionize the dopants
    ~ for si, this is ~kT

→ It turns out that the product $n_o p_o$ in an extrinsic S.C. is: $n_o p_o = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$

∴ $n_o p_o = n_i^2$ ✗ considered a material parameter for a given T   ★ Same as for intrinsic!
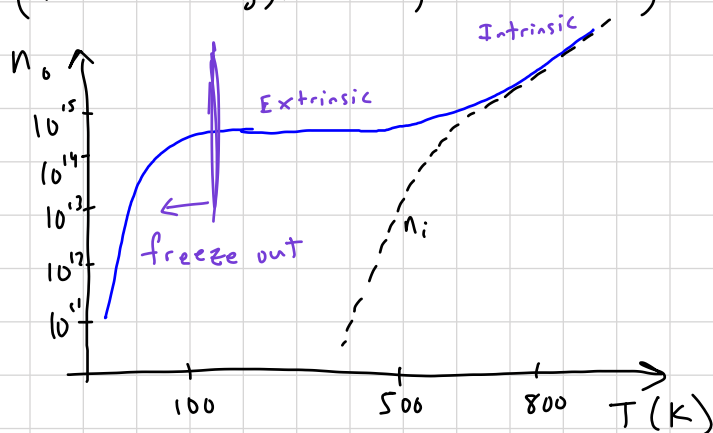
→ If E_F is within E_c or E_v ≡ degenerate S.C.

E_F ———————— E_c

——————— E_v

n-type degenerately doped S.C., Boltzmann Approx. does not apply!

→ Temperature (thermal energy) is key to ionizing dopants:

→ E_F position will depend on temperature and doping concentration, N_d / N_a

donor dopant conc. ↑   acceptor dopant conc. ↗

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right) = kT \ln\left(\frac{N_c}{N_d}\right)$$

→ Thermal EQ
→ nondegenerate
→ n-type

└ when $N_d \gg n_i$

Charge Neutrality — in thermal EQ, S.C. is electrically neutral ≡ net charge density = 0

$$n_o + N_a^- = p_o + N_d^+$$

↖ ionized acceptors   ↪ ionized donors

assume complete ionization of all dopants ⟿ $n_o p_o = n_i^2$

$$n_o + N_a = \frac{n_i^2}{n_o} + N_d$$

$$n_o^2 - (N_d - N_a) n_o - n_i^2 = 0$$

↓

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

→ Thermal EQ

→ full ionization

$$p_0 = \left(\frac{N_a - N_d}{2}\right) + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

general case solutions

Special Cases

① Intrinsic S.C. $\left(N_a = 0, N_d = 0\right)$

$$n_0 = n_i \quad , \quad p_0 = n_i$$

② Doped, nondegenerate S.C. with either: $N_d - N_a \simeq N_d \gg n_i$ OR $N_a - N_d \simeq N_a \gg n_i$

→ most typical case found in devices

$$n_0 \simeq N_d \qquad\qquad p_0 \simeq N_a$$

OR

$$p_0 = \frac{n_i^2}{N_d} \qquad\qquad n_0 \simeq \frac{n_i^2}{N_a}$$

③ Doped S.C. with $n_i \gg |N_d - N_a|$ — "lightly doped"

→ at reasonable temperatures, n_i will exceed N_d or N_a

$$n_0 = p_0 = n_i$$

④ Compensated S.C. where N_a and N_d are comparable and nonzero $\left(N_a - N_d \simeq 0\right)$

→ use full expressions for n_0, p_0

⑤ S.C. at very high temp, $n_i \gg |N_a - N_d|$

$$n_0 \simeq p_0 \simeq n_i$$

Carrier Transport — Drift

Review of some basics electrical behaviors:

Bar of s.c. material

Area = A

voltage (V) , current (A), $Amp = \frac{C}{s}$

Ohm's Law: $V = I R$ — resistance (Ω)

Current density: $J = \frac{I}{A} \left(A/cm^2 \right) = \sigma \mathcal{E}$ — electric field
— conductivity

electric field: $\mathcal{E} = \frac{V}{L} \left(V/cm \right)$

resistivity: $\rho = \frac{RA}{L}$ $(\Omega\text{-}cm)$ or $R = \frac{\rho L}{A}$

conductivity: $\sigma = \frac{1}{\rho}$ $\left(S/cm \right)$

★ Direction of current flow:
- h^+ go with $\mathcal{E}$-field, hence J_p goes with $\mathcal{E}$
- e^- go against $\mathcal{E}$-field, but J_n still goes with $\mathcal{E}$

it's a convention!

Drift current — charged carrier motion due to an applied $\mathcal{E}$-field

→ Last time we deduced how to determine # of e^-s (n_o) and h^+ (p_o) and now need to see how they move when an $\mathcal{E}$-field is applied

negative because e^- are neg. carriers

e^- current

average drift velocity

h^+ current

$$J_n|_{drift} = -e\, n_o\, v_{dn}$$

$$J_p|_{drift} = e\, p_o\, v_{dp}$$

elementary charge (q)

Why "average" drift velocity?
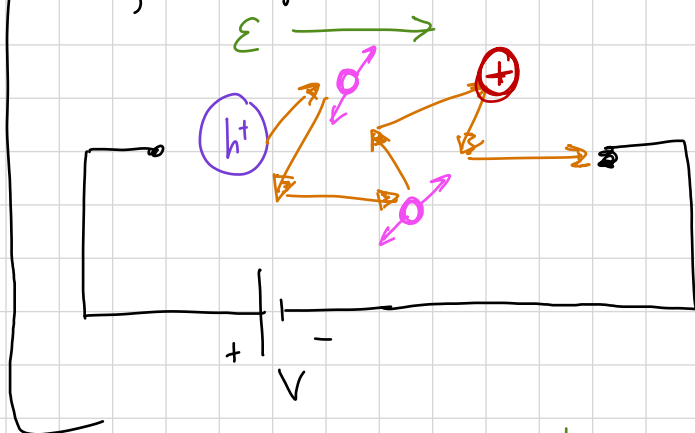
Side note

★ Carriers will scatter as they are accelerated by applied $\mathcal{E}$-field:

Lattice scattering: Thermal energy $(T > 0K)$ causes lattice atoms to vibrate, disrupting potential profile and scattering carriers.

Impurity scattering: ionized impurities disrupt periodic potential, causing scattering

e^- mobility

$$v_{dn} = -\mu_n \mathcal{E}$$

$$v_{dp} = \mu_p \mathcal{E}$$
— hole mobility

mobility defines how well a carrier will move through a crystal at a given T and $N_{a,d}$

units: $cm^2/V\cdot s$

$$J_n|_{drift} = -en_o(-\mu_n \mathcal{E}) = e\mu_n n_o \mathcal{E} \qquad J_p|_{drift} = e\mu_p p_o \mathcal{E}$$

$$\bigstar \quad \boxed{J_{drift} = e(\mu_n n_o + \mu_p p_o)\mathcal{E}} \qquad \text{Total drift current density}$$

Mobility

→ one of the most quoted properties of a S.C.

Looking back at impact of $\mathcal{E}$-field on carriers:

$$F = m^*_{n,p} \frac{dv}{dt} = e\mathcal{E}$$

assume m^* and $\mathcal{E}$ are constant

$$v = \frac{e\mathcal{E}t}{m^*_{n,p}}$$

Note on carrier motion: most is thermal (from $T > 0K$) and small $\mathcal{E}$-field just adds some directionality

→ mean time between scattering events/collisions

$$v = \left(\frac{e\tau_{cp}}{m^*_p}\right)\mathcal{E}$$

$\bigstar$ use a statistical model to determine impact of collision events and their frequency:

$$v = \mu_p \mathcal{E} \rightarrow \boxed{\mu_p = \frac{e\tau_{cp}}{m^*_p}} \quad \text{and} \quad \boxed{\mu_n = \frac{e\tau_{cn}}{m^*_n}}$$

→ two scattering events degrade μ:

Think about them as resistance added in path of current flow: $\left(\begin{array}{c}\text{just an}\\ \text{analogy}\end{array}\right)$

R_L R_I ← impurity scattering
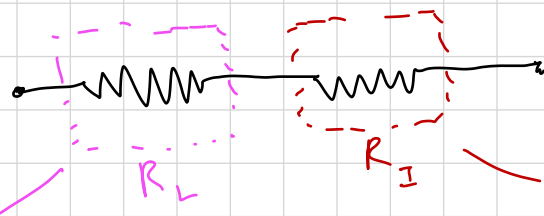
lattice scattering

→ No doping dependence

→ Decreases with ↓ T

→ Increases with ↓ T

→ Increases with ↑ N_a or ↑ N_d

"total" mobility : $\quad \dfrac{1}{\mu} = \dfrac{1}{\mu_L} + \dfrac{1}{\mu_I}$

Conductivity σ

$$J_{drift} = e(\mu_n n + \mu_p p)\mathcal{E} = \sigma \mathcal{E} \qquad \bigstar$$

$$\boxed{\sigma = e(\mu_n n_o + \mu_p p_o)} \quad S/cm$$

Resistivity ρ

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n_o + \mu_p p_o)} \quad \Omega\text{-cm}$$

Unique case (but happens frequently):

$$N_a \gg n_i \quad, \quad N_d = 0$$

$$\rightarrow \sigma \cong e \mu_p p_o = e \mu_p N_a$$

assuming complete ionization

velocity saturation

at low-to-moderate $\mathcal{E}$-fields, v_{drift} is linearly $\propto \mathcal{E}$

✷ $\mathcal{E}$-field does not substantially impact e^- energy but at higher $\mathcal{E}$-field it does lead to added scattering effects

For holes: $\quad v_d = \dfrac{\mu \mathcal{E}}{\left[1 + \left(\dfrac{\mu \mathcal{E}}{v_{sat}} \right) \right]} = \begin{cases} \mu \mathcal{E} \quad \dots \quad \mathcal{E} \rightarrow 0 \; (\text{small } \mathcal{E}) \\[2ex] v_{sat} \quad \dots \quad \mathcal{E} \rightarrow \infty \; (\text{large } \mathcal{E}) \end{cases}$