Solutions

Homework #5

pn Junction Operation and Small Signal Model – *100 points*

DUE @ Beginning of Class: Thursday, October 12

****Make sure to use the "Note" on page 251.**

1) E-Book, problem 8.3 (12 points)

2) E-Book, problem 8.8, you can use the ideal diode equation (10 points)

3) E-Book, problem 8.10, you can approximate the current as $I = I_s e^{\frac{V_a}{V_t}}$ (16 points)

4) E-Book, problem 8.15, you can use the ideal diode equation (20 points)

5) E-Book, problem 8.28 (12 points)

6) E-Book, problem 8.37 (8 points)

7) E-Book, problem 8.42, (16 points)

8) E-Book, problem 1.38 (pg. 443) (6 points)

(double all labeled point values)

1) E-Book, problem 8.3 (12 points)

8.3 The doping concentrations in a GaAs pn junction are $N_d = 10^{16}\ \text{cm}^{-3}$ and $N_a = 4 \times 10^{16}\ \text{cm}^{-3}$. Find the minority carrier concentrations at the edges of the space charge region for (*a*) $V_a = 0.90$ V, (*b*) $V_a = 1.10$ V, and (*c*) $V_a = -0.95$ V.

Assuming complete ionization:

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.8 \times 10^6\ \text{cm}^{-3})^2}{4 \times 10^{16}\ \text{cm}^{-3}} = 8.1 \times 10^{-5}\ \text{cm}^{-3}$$

Table 4.2 in book, $n_i = 1.8 \times 10^6\ \text{cm}^{-3}$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.8 \times 10^6\ \text{cm}^{-3})^2}{10^{16}\ \text{cm}^{-3}} = 3.24 \times 10^{-4}\ \text{cm}^{-3}$$

a) $V_a = 0.9\text{V}$

$$p_n(x_n) = p_{no} \exp\left(\frac{V_a}{V_t}\right) = (3.24 \times 10^{-4}\ \text{cm}^{-3}) \exp\left(\frac{0.9\text{V}}{0.026\text{V}}\right) = \boxed{4.0 \times 10^{11}\ \text{cm}^{-3}}$$

$V_t = \frac{kT}{e} = 0.026\text{V}$ for R.T.

$$n_p(-x_p) = n_{po} \exp\left(\frac{V_a}{V_t}\right) = (8.1 \times 10^{-5}\ \text{cm}^{-3}) \exp\left(\frac{0.9\text{V}}{0.026\text{V}}\right) = \boxed{10^{11}\ \text{cm}^{-3}}$$

b) $V_a = 1.10\text{V}$

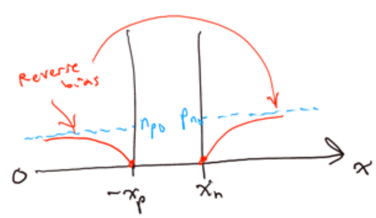
Similarly as to part a):

$$\boxed{\begin{aligned} p_n(x_n) &= 9.03 \times 10^{14}\ \text{cm}^{-3} \\ n_p(-x_p) &= 2.26 \times 10^{14}\ \text{cm}^{-3} \end{aligned}}$$

c) $V_a = -0.95\text{ V}$

With a reverse-bias, minority carrier conc. at space charge edges will be ~ 0:

$$\boxed{\begin{aligned} p_n(x_n) &= 0 \\ n_p(-x_p) &\simeq 0 \end{aligned}}$$



(double all labeled point values)

2) E-Book, problem 8.8, you can use the ideal diode equation (10 points)

8.8 A one-sided p^+n silicon diode has doping concentrations of $N_a = 5 \times 10^{17}$ cm^{-3} and $N_d = 8 \times 10^{15}$ cm^{-3}. The minority carrier lifetimes are $\tau_{n0} = 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 2 \times 10^{-4}$ cm^2. Calculate the (*a*) reverse-biased saturation current, and (*b*) the forward-bias current at (*i*) $V_a = 0.45$ V, (*ii*) $V_a = 0.55$ V, and (*iii*) $V_a = 0.65$ V.

a) $$J_s = \frac{e D_n n_{po}}{L_n} + \frac{e D_p p_{no}}{L_p} = e n_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}} \right]$$

$$p_{no} = \frac{n_i^2}{N_d} \qquad L_p = \sqrt{D_p \tau_p}$$

$$n_{po} = \frac{n_i^2}{N_a} \qquad L_n = \sqrt{D_n \tau_n}$$

$$J_s = (1.6\times10^{-19}\,c)(1.5\times10^{10}\,cm^{-3})^2 \left[\frac{1}{5\times10^{17}\,cm^{-3}}\sqrt{\frac{25\,cm^2/s}{10^{-7}\,s}} + \frac{1}{8\times10^{15}\,cm^{-3}}\sqrt{\frac{10\,cm^2/s}{8\times10^{-8}\,s}} \right]$$

units: $\frac{C}{\cancel{cm^{6}}} \cdot \frac{\cancel{cm^{3}}\,cm}{s} = A/cm^2$

$$= 5.15\times10^{-11}\,A/cm^2$$

$$I_s = J_s A = (5.15\times10^{-11}\,A/cm^2)(2\times10^{-4}\,cm^2) = \boxed{1.03\times10^{-14}\,A}$$

b) $$I = I_s\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

i. $V_a = 0.45V$

$$I = (1.03\times10^{-14}\,A)\left[\exp\left(\frac{0.45V}{0.026V}\right) - 1\right] = \boxed{3.38\times10^{-7}\,A}$$

ii. $V_a = 0.55V$

$$\boxed{I = 1.6\times10^{-5}\,A}$$

iii. $V_a = 0.65V$

$$\boxed{I = 7.42\times10^{-4}\,A}$$

☆ Note: Answers will be slightly different but still ok if $V_t = 0.0259$ V is used instead.

(double all labeled point values)

3) E-Book, problem 8.10, you can approximate the current as $I = I_s e^{\frac{V_a}{V_t}}$ (16 points)

8.10 Fill in the missing data in the following table.

Case	V_a (V)	I(mA)	I_s (mA)	J_s (mA/cm²)	A(cm²)
1	0.65	0.50			2×10^{-4}
2	0.70		2×10^{-12}		1×10^{-3}
3		0.80		1×10^{-7}	1×10^{-4}
4	0.72	1.20		2×10^{-8}	

Case 1

$$I = I_s \exp\left(\frac{V_a}{V_t}\right) \Rightarrow 0.50\times10^{-3}\,A = I_s \exp\left(\frac{0.65V}{0.0259V}\right)$$

$$I_s = 6.31\times10^{-15}\,A = \boxed{6.31\times10^{-12}\,mA}$$

$$J_s = \frac{I_s}{A} = 3.15\times10^{-11}\,A/cm^2 = \boxed{3.15\times10^{-8}\,mA/cm^2}$$

Case 2

$$I = I_s \exp\left(\frac{V_a}{V_t}\right) = (2\times10^{-12}\,mA)\exp\left(\frac{0.70V}{0.0259V}\right) = \boxed{1.09\,mA}$$

$$J_s = \frac{I_s}{A} = \frac{1.09\,mA}{1\times10^{-3}\,cm^2} = \boxed{2\times10^{-9}\,mA/cm^2}$$

Case 3

$$I = A J_s \exp\left(\frac{V_a}{V_t}\right) \Rightarrow V_a = V_t \ln\left[\frac{I}{A J_s}\right] = (0.0259V)\ln\left[\frac{0.80\,mA}{(10^{-4}\,cm^2)(10^{-7}\,mA/cm^2)}\right]$$

$$= \boxed{0.65V}$$

$$I_s = A J_s = (10^{-4}\,cm^2)(10^{-7}\,mA/cm^2) = \boxed{10^{-11}\,mA}$$

Case 4

$$I_s = \frac{I}{\exp(V_a/V_t)} = \frac{1.20\,mA}{\exp\left(\frac{0.72V}{0.0259V}\right)} = \boxed{1.01\times10^{-12}\,mA}$$

$$A = \frac{I_s}{J_s} = \frac{1.01\times10^{-12}\,mA}{2\times10^{-8}\,mA/cm^2} = \boxed{5.07\times10^{-5}\,cm^2}$$

(double all labeled point values)

4) E-Book, problem 8.15, you can use the ideal diode equation (20 points)

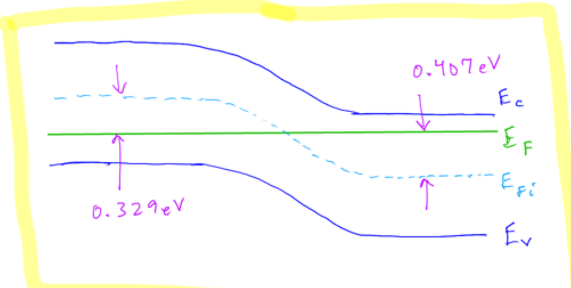
8.15 A silicon pn junction with a cross-sectional area of 10^{-4} cm^2 has the following properties at $T = 300$ K:

n region	p region
$N_d = 10^{17}$ cm^{-3}	$N_a = 5 \times 10^{15}$ cm^{-3}
$\tau_{p0} = 10^{-7}$ s	$\tau_{n0} = 10^{-6}$ s
$\mu_n = 850$ cm^2/V-s	$\mu_n = 1250$ cm^2/V-s
$\mu_p = 320$ cm^2/V-s	$\mu_p = 420$ cm^2/V-s

(*a*) Sketch the thermal equilibrium energy-band diagram of the pn junction, including the values of the Fermi level with respect to the intrinsic level on each side of the junction. (*b*) Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5 V. (*c*) Determine the ratio of hole current to total current at the space charge edge x_n.

a) p-side: $E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = 0.0259\,eV \ln\left(\frac{5\times10^{15}\,cm^{-3}}{1.5\times10^{10}\,cm^{-3}}\right) = 0.329\,eV$ 1

n-side: $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = 0.0259\,eV \ln\left(\frac{10^{17}\,cm^{-3}}{1.5\times10^{10}\,cm^{-3}}\right) = 0.407\,eV$ 1



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b) From the given info we can find: $D_n = \mu_n \frac{kT}{e} = (1250\,cm^2/V\cdot s)(0.0259\,V) = 32.4\,cm^2/s$

$D_p = \mu_p \frac{kT}{e} = (320\,cm^2/V\cdot s)(0.0259\,V) = 8.29\,cm^2/s$

see problem 2 solution

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right] = (1.6\times10^{-19}\,C)(1.5\times10^{10}\,cm^{-3})^2\left[\frac{1}{5\times10^{15}}\sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}}\right]$$

$$= 4.43\times10^{-11}\,A/cm^2$$

$$I_s = A J_s = (10^{-4}\,cm^2)(4.43\times10^{-11}\,A/cm^2) = \boxed{4.43\times10^{-15}\,A}$$ 1

$$I = I_s \exp\left(\frac{V_0}{V_t}\right) = (4.43\times10^{-15}\,A)\exp\left(\frac{0.5\,V}{0.0259\,V}\right) = \boxed{1.07\times10^{-6}\,A}$$ 2

c) Hole current: $I_p = en_i^2 A\left(\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right)\exp\left(\frac{V_0}{V_t}\right) = (3.28\times10^{-16}\,A)\exp\left(\frac{V_0}{V_t}\right)$

Then: $$\frac{I_p}{I} = \frac{3.28\times10^{-16}\,A\,\cancel{\exp(V_0/V_t)}}{4.43\times10^{-15}\,A\,\cancel{\exp(V_0/V_t)}} = \boxed{0.074}$$ 2

(double all labeled point values)

5) E-Book, problem 8.28 (12 points)

8.28 Consider a silicon pn junction diode with an applied reverse-biased voltage of V_R = 5V. The doping concentrations are $N_a = N_d = 4 \times 10^{16}$ cm^{-3} and the cross-sectional area is $A = 10^{-4}$ cm^2. Assume minority carrier lifetimes of $\tau_0 = \tau_{n0} = \tau_{p0} = 10^{-7}$ s. Calculate the (*a*) ideal reverse-saturation current, (*b*) reverse-biased generation current, and (*c*) the ratio of the generation current to ideal saturation current.

a) $I_s = A e n_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}} \right]$

see problem 2

$$= (10^{-4}\,cm^2)(1.6\times10^{-19}\,C)(1.5\times10^{10}\,cm^{-3})^2 \left[\frac{1}{4\times10^{16}\,cm^{-3}}\sqrt{\frac{25\,cm^2/s}{10^{-7}\,s}} + \frac{1}{4\times10^{16}\,cm^{-3}}\sqrt{\frac{10\,cm^2/s}{10^{-7}\,s}} \right]$$

$$= \boxed{2.32\times10^{-15}\,A}$$ 2

b) $I_{gen} = \frac{A e n_i W}{2\tau_0}$

to find W, need V_{bi}: $V_{bi} = \frac{kT}{e}\ln\left[\frac{N_a N_d}{n_i^2}\right] = (0.0259\,V)\ln\left[\frac{(4\times10^{16}\,cm^{-3})^2}{(1.5\times10^{10}\,cm^{-3})^2}\right] = 0.77\,V$

then: $W = \left[\frac{2\epsilon_s\epsilon_0(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$

$$= \left[\frac{2(11.7)(8.85\times10^{-14}\,F/cm)(0.77V+5V)}{(1.6\times10^{-19}\,C)}\left(\frac{8\times10^{16}\,cm^{-3}}{(4\times10^{16}\,cm^{-3})^2}\right)\right]^{1/2} = 6.11\times10^{-5}\,cm$$

$$I_{gen} = \frac{(10^{-4}\,\cancel{cm^2})(1.6\times10^{-19}\,C)(1.5\times10^{10}\,\cancel{cm^{-3}})(6.11\times10^{-5}\,\cancel{cm})}{2(10^{-7}\,s)} = \boxed{7.33\times10^{-11}\,A}$$ 3

c)

$$\frac{I_{gen}}{I_s} = \frac{7.33\times10^{-11}\,A}{2.32\times10^{-15}\,A} = \boxed{3.2\times10^{4}}$$ 1

(double all labeled point values)

6) E-Book, problem 8.37 (8 points)

8.37 (*a*) Calculate the small-signal diffusion capacitance and diffusion resistance of a silicon pn junction diode biased at $I_{DQ} = 1.2$ mA. Assume the minority carrier lifetimes are 0.5 μs in both the n and p regions. (*b*) Repeat part (*a*) for the case when the diode is biased at $I_{DQ} = 0.12$ mA.

a) $$r_d = \frac{V_t}{I_{DQ}} = \frac{0.0259\,V}{1.2\times10^{-3}\,A} = \boxed{21.6\,\Omega}$$

units: $\frac{A\cdot s}{V} = \frac{C}{V} = F$

$$C_d = \frac{1}{2V_t}\left(I_{po}\tau_{po} + I_{no}\tau_{no}\right) = \frac{1}{2V_t}\tau_o\left(I_{po}+I_{no}\right) = \frac{I_{DQ}\,\tau_o}{2V_t} = \frac{(1.2\times10^{-3}A)(0.5\times10^{-6}s)}{2(0.0259\,V)}$$

$$= \boxed{1.16\times10^{-8}\,F}$$

$I_{DQ} = I_{po} + I_{no}$

$\tau_{no} = \tau_{po} = \tau_o$

b) $I_{DQ} = 0.12$ mA

$$r_d = \frac{0.0259\,V}{0.12\times10^{-3}\,A} = \boxed{216\,\Omega}$$

$$C_d = \frac{(0.12\times10^{-3}A)(0.5\times10^{-6}s)}{2(0.0259\,V)} = \boxed{1.16\times10^{-9}\,F}$$

(double all labeled point values)

7) E-Book, problem 8.42, (16 points)

8.42 A one-sided p^+n silicon diode has doping concentrations of $N_a = 4 \times 10^{17}$ cm^{-3} and $N_d = 8 \times 10^{15}$ cm^{-3}. The diode cross-sectional area is $A = 5 \times 10^{-4}$ cm^2. (*a*) The maximum diffusion capacitance is to be limited to 1 nF. Determine (*i*) the maximum current through the diode, (*ii*) the maximum forward-bias voltage, and (*iii*) the diffusion resistance. (*b*) Repeat part (*a*) if the maximum diffusion capacitance is limited to 0.25 nF.

a) $C_{d_{max}} = 1\,nF$

i. Since $N_a >> N_d \Rightarrow I_{po} >> I_{no}$

$$C_d = \frac{1}{2V_t}\left(I_{po}\tau_{po} + \cancel{I_{no}\tau_{no}}\right) = \frac{I_{po}\tau_{po}}{2V_t}$$ |

$$\hookrightarrow I_{po} = \frac{2C_d V_t}{\tau_{po}} = \frac{2(1\times10^{-9}F)(0.0259V)}{10^{-7}\,s} = 5.18\times10^{-4}A = \boxed{0.518\,mA}$$ |

table value in book

ii. $$I_{po} = Aen_i^2\left[\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]\exp\left(\frac{V_a}{V_t}\right)$$ |

$$0.518\times10^{-3}A = (5\times10^{-4}cm^2)(1.6\times10^{-19}c)(1.5\times10^{10}cm^{-3})^2\left[\frac{1}{8\times10^{15}cm^{-3}}\sqrt{\frac{10\,cm^2/s}{10^{-7}s}}\right]\exp\left(\frac{V_a}{V_t}\right)$$

$$\Rightarrow V_a = (0.0259V)\ln\left(\frac{0.518\times10^{-3}A}{2.25\times10^{-14}A}\right) = \boxed{0.618\,V}$$ |

iii. $$r_d = \frac{V_t}{I_D} = \frac{0.0259V}{0.518\times10^{-3}A} = \boxed{50\,\Omega}$$ |

b) Using the same approach as above, values are:

i. $\boxed{I_{po} = 0.13\,mA}$ |

ii. $\boxed{V_a = 0.58\,V}$ |

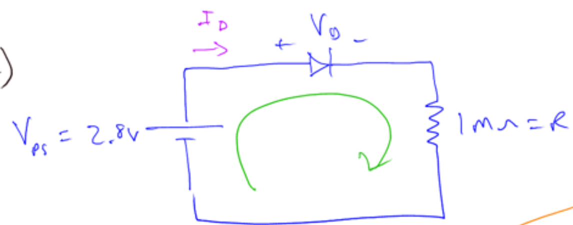
iii. $\boxed{r_d = 200\,\Omega}$ |

(double all labeled point values)

8) E-Book, problem 1.38 (pg. 443) (6 points)

1.38 A pn junction diode is in series with a 1 MΩ resistor and a 2.8 V power supply. The reverse-saturation current of the diode is $I_S = 5 \times 10^{-11}$ A. (a) Determine the diode current and voltage if the diode is forward biased. (b) Repeat part (a) if the diode is reverse biased.

a)



KVL gives:

$$V_{PS} = V_D + I_D R$$

$$2.8\text{ V} = V_D + I_D (10^6\,\Omega)$$

Ideal Diode eqn:

$$I_D = \frac{2.8V - V_D}{10^6\,\Omega} \quad \text{AND} \quad I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (5 \times 10^{-11}\text{A}) \exp\left(\frac{V_a}{0.0259V}\right)$$

By Iteration (trial and error) to satisfy both equations:

$$\boxed{V_D = 0.282\text{ V}, \quad I_D = 2.52 \times 10^{-6}\text{ A}}$$

b) In R.B., $I_D = -I_S = \boxed{-5 \times 10^{-11}\text{ A}}$

$$\boxed{V_D = -2.8\text{ V}}$$