Homework 8 for STA 250/MTH 342 – Fall 2017

Due at the beginning of class on November 8, 2017

1. D&S (4th Ed.) Exercise 9.2.12 (page 558) (The book uses $\alpha(\delta)$ and $\beta(\delta)$ to denote the Type I error rate and Type II error rate of a test δ.)

2. Let X be a discrete random variable that can take four values x_1, x_2, x_3 and x_4. Consider two hypotheses H_0 and H_1 which specify the distribution of X as follows.

X	H_0	H_1
x_1	.2	.1
x_2	.3	.3
x_3	.3	.1
x_4	.2	.5

   (a) Compare the likelihood ratio (LR) for each possible value of X and order the x_i according to the LR.

   (b) What is the LR test (in terms of the rejection region) for testing H_0 versus H_1 at level $\alpha = .2$? What is the test at level $\alpha = 0.5$?

3. A coin is thrown independently 10 times to test the null hypothesis that the probability of head $p = 0.5$ versus the alternative that $p \neq 0.5$. The test rejects if either 0 or 10 heads are observed.

   (a) Is the null hypothesis simple or composite? How about the alternative?

   (b) What is the level (or size) of the test?

   (d) If in fact the probability of heads is $p = .4$, what is the power of the test for the alternative $p = 0.4$? What about for $p = .1$?

   (e) Write the power of as a function of the probability of heads, $\pi(p)$.

4. True or false (give reasons):

   (a) The size of a test is equal to the probability that the null hypothesis is true.

   (b) The power of a test is the probability that the alternative hypothesis is true.

   (c) If the level of a test is decreased, the type 2 error is expected to decrease.

   (d) A type I error occurs when the data falls outside the rejection region of the test.

(e) The type II error of a test is determined by the null distribution of the test.

5. Suppose that we wish to test the null hypothesis, H_0 that the proportion p of ledger sheets with errors is equal to .2 versus the alternative, H_a, that the proportion is smaller than .2, by using the following test procedure. Two ledger sheets are selected at random. If both are error-free, we reject H_0. If one or more contains an error, we look at two more sheet. If they are both error-free, we reject H_0. In all other cases we accept H_0.

   (a) In the context of this problem, what is a type I error?

   (b) What is the type I error rate α of this test?

   (c) What is the type II error for the simple alternative $p = 0.3$?

   (d) Calculate the power $\pi(p)$ as a function of p.

6. Let $X_1, X_2, \ldots, X_n$ be i.i.d. data from a $\mathrm{N}(0, \sigma^2)$ distribution. Find the rejection region for the LR test at level $\alpha = .10$ of

$$H_0 : \sigma^2 = 1 \quad \text{vs} \quad H_1 : \sigma^2 = 2.$$