Equation Sheet

Please tear off this page and keep it with you

Equations and constants that *may* be helpful:

$$h = \frac{na}{A}, k = \frac{na}{B}, l = \frac{na}{C} \qquad E = \frac{\hbar^2 k^2}{2m^*} \qquad v = \frac{1}{\hbar}\frac{dE}{dk} \qquad \boldsymbol{m}^* = \hbar^2 \left(\frac{\boldsymbol{d^2E}}{\boldsymbol{dk^2}}\right)^{-1}$$

$$g_c(E) = \frac{4\pi}{h^3}\left(2m_n^*\right)^{3/2}\sqrt{E - E_C} \qquad g_v(E) = \frac{4\pi}{h^3}\left(2m_p^*\right)^{3/2}\sqrt{E_V - E} \qquad f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$n_0 = N_C e^{\left(\frac{-(E_C - E_F)}{kT}\right)} \qquad p_0 = N_V e^{\left(\frac{-(E_F - E_V)}{kT}\right)} \qquad N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \qquad N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_0 = n_i e^{\left(\frac{(E_F - E_{Fi})}{kT}\right)} \qquad p_0 = n_i e^{\left(\frac{(E_{Fi} - E_F)}{kT}\right)} \qquad n_i^2 = N_C N_V e^{\left(\frac{-E_g}{kT}\right)} = n_0 p_0$$

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$E_{Fi} = \frac{E_C + E_V}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) \qquad N_{C,V}(T) = N_{C,V\to 300K}\left(\frac{T}{300K}\right)^{3/2}$$

$$J_{drift} = \sigma E \qquad \sigma = e(\mu_n n + \mu_p p) = \frac{1}{\rho} \qquad \mu = \frac{e\tau_c}{m_c^*} \qquad J_{diff} = eD_n\frac{dn}{dx} - eD_p\frac{dp}{dx}$$

$$J = \frac{I}{A} \qquad V = IR \qquad \rho = \frac{RA}{L} \qquad \frac{D}{\mu} = \frac{kT}{e} \qquad \phi = -\frac{1}{e}\left(E_C - E_{ref}\right) \qquad E = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

$$k = 8.62x10^{-5}\,eV/K = 1.38x10^{-23}\,J/K \qquad h = 4.14x10^{-15}\,eV\cdot s = 6.63x10^{-34}\,J\cdot s \qquad \hbar = \frac{h}{2\pi}$$

$$q = 1.602x10^{-19}\,C$$

Some potentially useful information for Si at T = 300 K: N_c = 2.8x10^{19} cm^{-3}, N_v = 1.04x10^{19} cm^{-3}, n_i = 1.5x10^{10} cm^{-3}, m_n^* = 1.08m_0, m_p^* = 0.56m_0, E_g = 1.12 eV

Exam 1

Semiconductor Materials

Name: ___________________________________ **Net ID: ______________**

- **5 questions.**
- **20 points per question.**
- **Please write neatly (legibly). When given, write final answers in the provided boxes.**
- **Partial credit will be given IF you:**
  - **Show your work whenever it is possible.**

Good luck!

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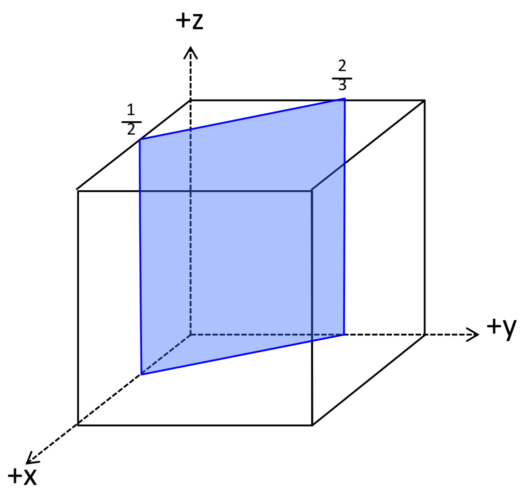
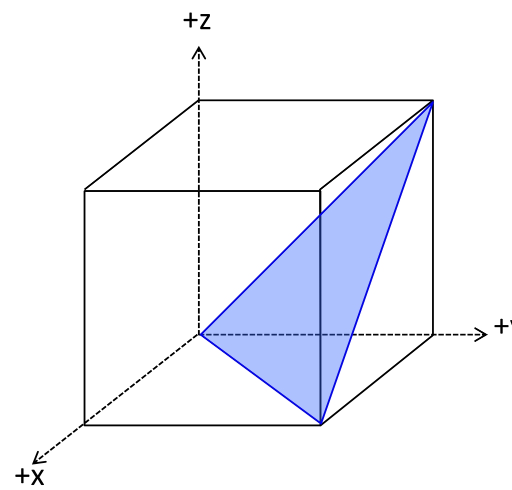
$$k = 8.62x10^{-5} eV/K = 1.38x10^{-23} J/K \qquad h = 4.14x10^{-15} eV \cdot s = 6.63x10^{-34} J \cdot s \qquad \hbar = \frac{h}{2\pi}$$

$$q = 1.602x10^{-19} C$$

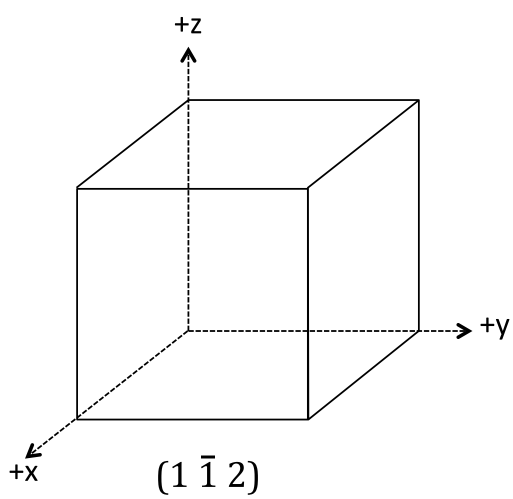
1) Crystal Structure of Solids

a. For each of the planes in the cubic unit cells below, determine the Miller indices in the most simplified, integer form. Write the answers on the given lines.

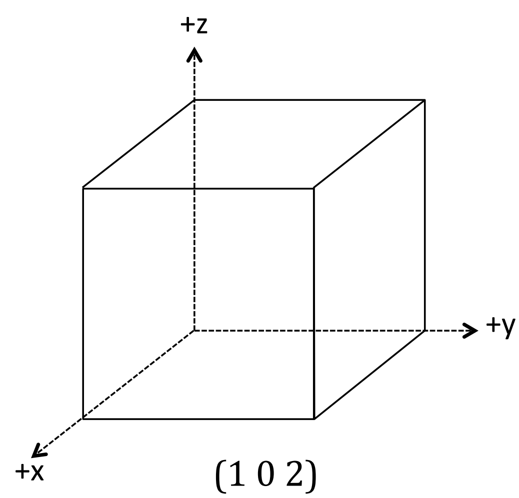




b. Using the two given unit cells below, draw the indicated plane (it's ok if the plane has to go outside of the unit cell area).



$(1\ \bar{1}\ 2)$



$(1\ 0\ 2)$

c. Calculate the surface density of atoms in the (100) plane of an FCC crystal with $a = \sqrt{2}$ Å.

=

2) Quantum Theory of Solids

a. Consider a 2D material that has a parabolic dispersion relation (E-*k*): $E = \frac{\hbar^2 k^2}{2m^*}$

i) Derive the density of states expression **per unit length** for this 2D material. HINT: One path is to consider there is 1 state every 2π/L x 2π/L area in *k*-space where L is crystal length in each dimension.

$g_{2D} =$

ii) What would this expression be if the dispersion relation were: $E = \sqrt{\hbar k}$

$g_{2D} =$

b. Using sketches, describe how an E-*k* diagram is related to an energy band diagram for a semiconductor. Be sure to label the conduction band, valence band, E_g, E_C, and E_V.

3) Thermal Equilibrium

Some potentially useful information for Si at T = 300 K: $N_c = 2.8\text{x}10^{19}$ cm^{-3}, $N_v = 1.04\text{x}10^{19}$ cm^{-3}, $n_i = 1.5\text{x}10^{10}$ cm^{-3}, $m_n^* = 1.08m_0$, $m_p^* = 0.56m_0$, $E_g = 1.12$ eV

a. Assuming E_g is independent of T, for each of the following cases in Si determine the electron and hole concentrations, $\boldsymbol{n_0}$ and $\boldsymbol{p_0}$:, and indicate if the material is n-type, p-type, or neither.

i. T = 300 K, $N_a = 10^{12}$ cm^{-3}, $N_d = 10^{13}$ cm^{-3}

n_0 =
p_0 =
type =

ii. T = 300 K, $N_a = 10^{14}$ cm^{-3}, $N_d = 10^{18}$ cm^{-3}

n_0 =
p_0 =
type =

iii. T = 900 K, $N_a = 10^{12}$ cm^{-3}, $N_d = 10^{14}$ cm^{-3}

n_0 =
p_0 =
type =

b. If the conduction band in Si were perfectly symmetric to the valence band:

i. What would be the position of the intrinsic Fermi level (E_{Fi}) relative to the maximum of the valence band, $E_{Fi} - E_V$? Show how you arrived at your answer.

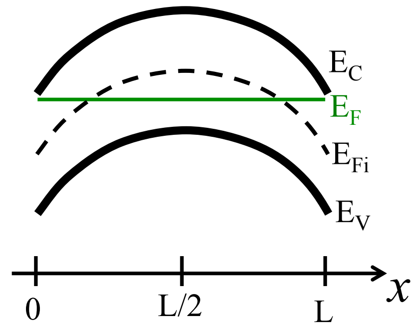
$E_{Fi} - E_V$ =

ii. How close would the Fermi level (E_F) come to the valence band maximum (E_V) if the Si were at 300 K and doped with $N_a = 10^{14}$ cm^{-3}, $N_d = 10^{18}$ cm^{-3}?

$E_F - E_V$ =

4) Band Diagrams

Answer the following questions for the band diagram given to the right:



a. Do equilibrium conditions prevail? How do you know?

b. Sketch the electrostatic potential (ϕ) inside the semiconductor as a function of *x*. Create your axes in the space to the right (below the band diagram)→

b.

c. Sketch the electric field (*E*) inside the semiconductor as a function of *x*.

c.

d. Roughly sketch *n*, *p*, and n_i versus *x*.

d.

e. Roughly sketch the electron drift-current density and the electron diffusion-current density as a function of *x*, on the same plot. Be sure to graph the proper polarity of the current densities and clearly identify the two current components.

e.

5) Carrier Transport—Drift and Diffusion

a. You are an engineer tasked with designing an n-type Si resistor with a resistivity (ρ) of 3.5 kΩ•cm that will give a resistance of 1 kΩ when measured. The following potentially useful parameters are given:

$T = 300$ K, $A = 10^{-5}$ cm^2, $\mu_n = 1000$ cm^2/Vs, $\mu_p = 800$ cm^2/Vs, $D_n = 25$ cm^2/s, $D_p = 10$ cm^2/s

i. How long (L) will your resistor need to be in order to meet these requirements?

L =

ii. What doping density will you use for N_a and N_d?

N_a =
N_d =

iii. What current density (J) will flow if the doping is uniform and a voltage of 2 V is applied across the resistor?

J =

b. Because your boss is crazy, he is insistent that the resistor you built have an unrealistically steep doping gradient such that $dn/dx = 10^{18}$ cm^{-4}. Assuming that the resultant diffusion current contributes to the total current flowing in the resistor, **what percentage** of the total current density will now come from diffusion current?

% =