Ideas in context

The Empire of Chance

How probability changed science and everyday life

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Zeno Swijtink
Theodore Porter
Lorraine Daston
John Beatty
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IDEAS IN CONTEXT

Edited by Wolf Lepenies, Richard Rorty, J. B. Schneewind and Quentin Skinner

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To the memory of Bill Coleman,
who helped sow the seeds for this book

CONTENTS

Acknowledgments

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Introduction

Fortuna, the fickle, wheel-toting goddess of chance, has never been a favorite of philosophy and the sciences. In that touchstone of medieval learning, Boethius' *Consolations of Philosophy*, sober Dame Philosophy warns that only when Fortuna "shows herself unstable and changeable, is she truthful," and preaches against the very existence of chance, conceived as "an event produced by random motion and without any sequence of causes." Dame Philosophy had illustrious allies. Despite the upheavals in science in the over two millennia separating the Athens of Aristotle from the Paris of Claude Bernard, they shared at least one article of faith: science was about causes, not chance.

Yet even as Bernard sought to banish chance and indeterminism from physiology, Fortuna already ruled a large and growing empire in the sciences. The laws of the realm were probability theory and statistics. By "taming chance," in Ian Hacking's evocative phrase, probability and statistics had reconciled Scientia to her arch-rival Fortuna. From its beginnings in the mid-seventeenth century, probability theory spread in the eighteenth century from gambling problems to jurisprudence, data analysis, inductive inference, and insurance, and from there to sociology, to physics, to biology, and to psychology in the nineteenth, and on to agronomy, polling, medical testing, baseball, and innumerable other practical (and not so practical) matters in the twentieth. But this triumphal march was emphatically not the simple accumulation of applications by a mathematical theory. Perhaps more than any other part of mathematics, probability theory has had a relationship of intimacy bordering upon identity with its applications. Indeed, there was arguably no "pure" theory of mathematical probability until 1933 (see 3.6), and until the early nineteenth century, the failure of an application threatened the theory itself (see 1.5). For much of its history, probability theory *was* its applications.

This means that probability theory was as much modified by its conquests as the disciplines it invaded. When, for example, probability became a tool for evaluating compilations of numbers about births, deaths, crimes, barometric fluctuations, dead letters, and other kinds of statistics, the very meaning of probability changed, from a degree of certainty in the mind to a ratio of events counted in the world (see 2.2). When the British polymath Francis Galton invented a way of measuring how much offspring peas deviated from their parent stock, he launched the analysis of correlations (see 2.5; 4.4). Factor analysis has its roots in educational psychology, analysis of variance in eugenics and agronomy, and so on.

It was in fact the rule for probabilistic ideas and techniques to originate in highly specific contexts, and to advance on the strength of vivid analogies. The normal or bell-shaped curve at first represented the probability of observational error in astronomy, then of nature's "errors" from *l'homme moyen* in sociology, then of anarchic individual gas molecules exhibiting orderly collective properties (see 2.5; 5.6). Eventually the normal curve came to represent the distribution of almost everything, from intelligence quotients to agricultural yields, and shed the particular interpretations derived from its early applications (see 8.1). But for almost a century such concrete analogies were the bridges over which it and other probabilistic notions passed from one discipline to another.

This book is about the applications of probability and statistics to science and life, where "application" is understood in this special sense: the mathematical tool shaped, but was also shaped by, its objects. The mathematical development of probability and statistics has been admirably treated in the work of such scholars as Isaac Todhunter, L. A. Maistrov, O. B. Sheynin, Stephen Stigler, and Ivo Schneider. Our primary concern, however, is not theirs. We analyze how probability and statistics transformed our ideas of nature, mind, and society. These transformations have been profound and wide-ranging, changing the structure of power as well as of knowledge. They have shaped modern bureaucracy as well as the modern sciences. The extraordinary range and significance of these transformations has rarely been appreciated, perhaps because the influence of probability on the various sciences has until now been studied only piecemeal. Also, the domain of probability and statistics was less often expanded by decisive conquest and revolution than by infiltration and alliance. In this book, we view these transformations synoptically, as a single historical movement – one whose influence on modern thought and life is second to no other area of scientific endeavor.

These encounters of probability and statistics with science have in no case been neutral – mere translations of extant ideas and methods into the language of mathematical probability. Even when probability has entered at the level of methods rather than of theories, the ultimate impact has transcended technique. When psychologists adopted inference statistics as a tool of the trade, they also came to view the same techniques as models of the mind (see 6.4). When biometricians warred with Mendelians over the proper approach to genetics, the view of biological inheritance implied by Pearsonian statistics was at issue (see 4.4). When physicians opposed the use of randomized clinical trials, they doubted not only the relevance of the results but also the ethics of the therapy (see 2.3; 4.2; 7.3). Whatever they touch, probability and statistics transform, and are themselves often transformed in the process. This book is a study in the interactive effects of quantification.

Not all important notions of chance, however, can be set to numbers – even in the sciences. Some of the most important influences of probabilistic ideas have involved qualitative ideas. Consider, for example, the extent to which discussions of chance permeate philosophy, raising issues of determinism, free will, causality, explanation, evidence, and inference. These strands are braided together with scientific themes in this book. The Belgian statistician Adolphe Quetelet and the British physicist James Clerk Maxwell grappled with the issue of free will in their work (see 2.6; 5.6); the probabilists Jakob Bernoulli and Laplace gave us our most lapidary statements of determinism (see 1.5; 8.3); chance as "absence of design" played a seminal role in evolutionary biology (see 4.3). This book documents how supple the bonds linking philosophical and probabilistic notions could be, depending on the context: statistics in nineteenth-century sociology, for example, was paired with the most inexorable brand of determinism, but in twentieth-century physics it implies the strictest indeterminism.

The empire of chance is too vast for us to map in its entirety in the compass of a single volume. We aim at a comprehensive, but not an exhaustive tour of its domains. We begin with two historical chapters that describe the origins and development of probability and statistics from the mid-seventeenth to the end of the nineteenth century. Here we introduce changing interpretations of the probability calculus, changing attitudes towards determinism, changing conceptions of averages and errors – all, again, in the context of changing applications. In each of the subsequent four chapters, we focus on one area of broad application: experimental methodology, biology, physics, and psychology. Our objective is not simply to list the points at which probability has entered these fields, but to examine the historical circumstances and conceptual consequences attendant to key applications. With chapter 7, we leave the sciences to assess the impact of probability and statistics on daily life, from weather reports to mammography. Again, we lay no claim to an exhaustive study of such applications – to catalogue them alone would require volumes. Rather, we have selected instances that show how deeply, if quietly, these applications have altered our values and assumptions about matters as diverse as legal fairness or human intelligence. Finally, we survey, from something like the victorious general's hilltop, the territory we have covered.

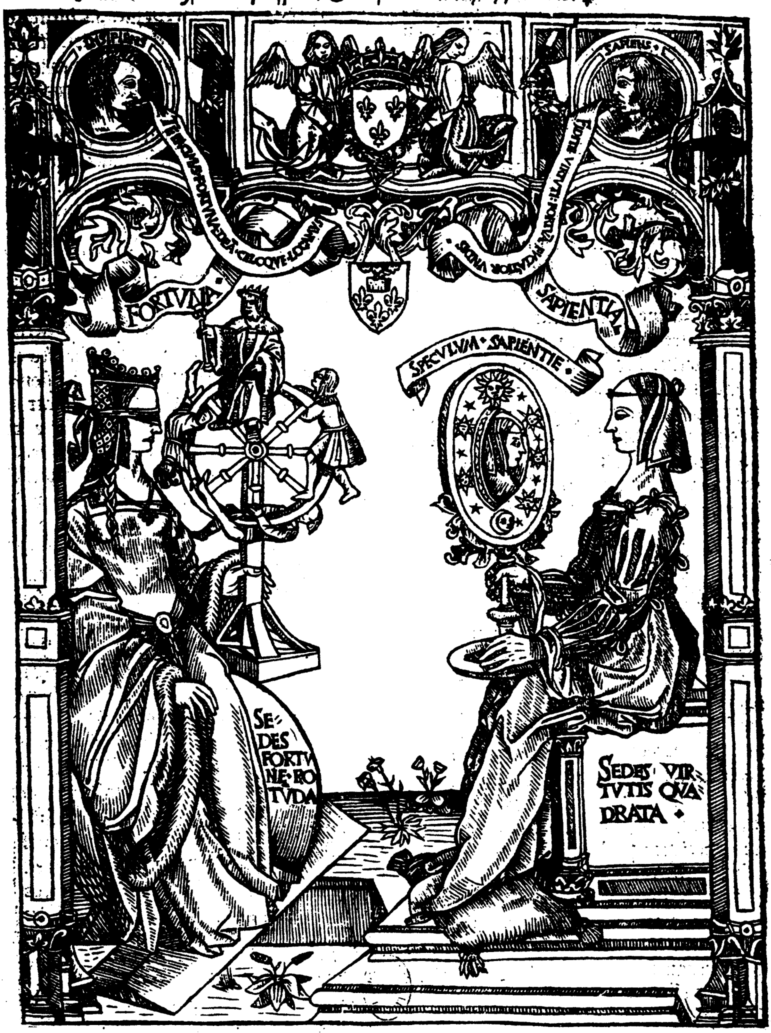
We envision a broad audience comprising both scholars from many fields and interested laymen, and have therefore kept technical material to a minimum. The handful of equations that do appear do not do double duty as explanations.

This is a book by several hands, but it is by no means an anthology. We tell a continuous story, with characters who appear and reappear, episodes that overlap and intersect, and common themes that repeat like a refrain. The plot line zigzags from one discipline to another – Fortuna did not honor these boundaries – and the reader will, for example, meet R. A. Fisher in the chapter on psychology as well as in those on experimental methodology and biology, and encounter debates over the implications of statistics for freedom of the will in physics as well as in sociology. A collaboration was essential because the scope and interdisciplinary nature of the topic required a range of knowledge that exceeds the competence of any single author. Some chapters were drafted by one of us, others by two or even three. But the entire manuscript was planned and then revised in light of criticism and discussion by all members of the group. That its contents reflect a diversity of interests will, we hope, be counted among its strengths rather than its weaknesses. We have not identified chapters by their original authors, but rather present the book as we conceived (and conceive) it: a collaborative work, with a narrative that stretches from beginning to end. Dutiful subjects of the empire of chance, we used a lottery to order our names on the title page.

As all scholars know, such a wholehearted collaboration requires special conditions. All six of us were members of the year-long research project on "The Probabilistic Revolution" at the Center for Interdisciplinary Studies in Bielefeld, Federal Republic of Germany. Essays written by participants in this project now fill two thick volumes (Krüger, Daston, and Heidelberger, eds., 1987; Krüger, Gigerenzer, and Morgan, eds., 1987) that demonstrate the importance of the topic across a wide range of fields, nations, and centuries. Our subset of six hoped to condense and connect the elements contained in these essays into a single narrative, one that viewed the growth of probabilistic and statistical ideas from a unified perspective. We also hoped to perpetuate the intense collegiality of the Bielefeld year with a project that would build upon our preliminary consensus, and one that would demand still greater collaborative efforts. In addition to our many discussions with our Bielefeld colleagues, we have drawn heavily on the collections mentioned above, as well as from recent books by Ian Hacking (1975; forthcoming), Donald MacKenzie (1981), Stephen Stigler (1986), Theodore Porter (1986), Gerd Gigerenzer and David Murray (1987), and Lorraine Daston (1988). We include also the results of much original research on crucial topics not addressed in the existing scholarly literature.

Fortuna's wheel governed not only the fates of men, but also her own. Few biographies contain as many ironic twists, turnabouts, and improbabilities as that of probability itself. Our story confounds expectation at many turns: we find physics borrowing from sociology; words that flip-flop meanings into their opposites; strange pairings of probability with determinism, or mechanism with chance. Philosophy scorned Fortuna as "changeable," and change, both subtle and dramatic, episodic and enduring, is the leitmotif of this study. Yet whereas the vicissitudes of fortune, as Gibbon says, "bury empires and cities in a common grave," we see no sign that Fortuna's empire will suffer a turn of the wheel.

Sensuyt le premier liure de francois petracque poete florentin/ des Remedes de fortune prospere. Translate de latin en francois.



Destre en la fleur de sa ieunesse. Et de lesperāce de viure longuement. Joye et Esperance. Chapitre premier.

Je suis en la fleur de mon aage/iay encores a viure lōguemēt. Raisō. Voicy la premiere veine desperance dhumaine nature Et qui a deceu et decevra dhōmes plusieurs milliers. Joye. Mon aage est en sa fleur. Raison. Cest vne ioye veine et briefue Ceste fleur fletrist et seiche pendant le tēps q̄ nous parlons Joye Mon aage est entiere. Raison. Qui doibt dire que laage soit entiere a laquelle il

A.i.

Fortuna (left) and Sapientia (right) are depicted here in traditional opposition. The slow breakdown of this opposition is the topic of this book. Source: Petrarch, *Remède de l'un et l'autre fortune prospère et adverse* (Paris, 1524); courtesy of the Bibliothèque Nationale, Paris.

God . . . has afforded us only the twilight of probability; suitable, I presume, to that state of mediocrity and probationership he has been pleased to place us in here. . .

John Locke (1690)

1

Classical probabilities, 1660–1840

1.1 INTRODUCTION

In July of 1654 Blaise Pascal wrote to Pierre Fermat about a gambling problem which came to be known as the Problem of Points: Two players are interrupted in the midst of a game of chance, with the score uneven at that point. How should the stake be divided? The ensuing correspondence between the two French mathematicians counts as the founding document in mathematical probability, even though it was not the first attempt to treat games of chance mathematically (Pascal, [1654] 1970, vol. 1, pp. 33–7; Cardano, [comp. *c.* 1525] 1966). Some years later, Pascal included among his *Pensées* an imaginary wager designed to convert sporting libertines: no matter how small we make the odds of God's existence, the pay-off is infinite; infinite bliss for the saved and infinite misery for the damned. Under such conditions, Pascal argued that rational self-interest dictates that we sacrifice our certain but merely finite worldly pleasures to the uncertain but infinite prospect of salvation (Pascal, [1669] 1962, pp. 187–90).

These two famous Pascal manuscripts, the one mathematical and the other philosophical, reveal the double root of the mathematical theory of probability. It emerged at the crux of two important intellectual movements of the seventeenth century: a new pragmatic rationality that abandoned traditional ideals of certainty; and a sustained and remarkably fruitful attempt to apply mathematics to new domains of experience. Neither would have been sufficient without the other. Philosophical notions about what happens only most of the time, and about the varying degrees of certainty connected with this unreliable experience date from antiquity, as do games of chance. But before *circa* 1650, no one attempted to quantify any of these senses of probability. Nor would the spirit of mathematical enterprise have alone sufficed, for quantification requires a subject matter, an interpretation to flesh out the mathematical formalism. This was particularly true for the calculus of probabilities, which until this century had no mathematical existence independent of its applications.

1.2 THE BEGINNINGS

The prehistory of mathematical probability has attracted considerable scholarly attention, perhaps because it seems so long overdue. Chance is our constant companion, and the mathematics of the earliest formulations of probability theory was elementary. Suggestive fragments of probabilistic thinking do turn up almost everywhere in the classical and medieval learned corpus: Around 85 B.C., Cicero connected that which usually happens with what is ordinarily believed in his rhetorical writings and called both *probabile* (Cicero, 1960, pp. 85–90). In a tenth-century manuscript, a monk enumerated all 36 possibilities for the toss of two dice (Kendall, 1956), and Talmudists reasoned probabilistically about inheritances and paternity (Rabinovitch, 1973). Yet none of these flowered into a mathematics of probability.

Several plausible hypotheses about why mathematical probability came about when it did also dissolve upon inspection. Maritime insurance expanded rapidly in Italy and the Low Countries during the commercial boom of the fifteenth and sixteenth centuries, but insurers did not collect statistics on shipwrecks, much less develop a mathematical basis for pricing premiums. It was the mathematicians who later – much later – influenced the insurers, not vice versa (Maistrov, [1964] 1974; Daston, 1987). Nor did any new recognition of chance inspire the mathematicians; on the contrary, the early probabilists from Pascal through Pierre Simon Laplace were determinists of the strictest persuasion (Kendall, 1956; Hacking, 1975). One might speculate that the mathematics of combinatorics was a precondition for the earliest versions of probability theory, but the two subjects appear to have developed in tandem, with probability theory often stimulating work in combinatorics rather than the reverse (Todhunter, 1865). The Renaissance doctrine of signatures linked the evidence of things with that of words in a way that parallels the objective and subjective senses of mathematical probabilities, but Cicero and the medieval rhetorical tradition that followed him had done so long before

(Hacking, 1975; Garber and Zabell, 1979). Similarly, the passion for gambling was hardly an invention of the seventeenth century, and so could not have been the catalyst that transformed qualitative probabilities into quantitative ones. It is, in short, easier to say where mathematical probability did *not* come from.

The very earliest writings on mathematical probability do supply some clues, however. If we return to the two Pascal musings, we discover that although they are recognizably part of what came to be called the calculus of probabilities, they are not cast in terms of probabilities. The fundamental concept was instead expectation, later defined as the product of the probability of an event e and its outcome value V:

$$P(e)V = E$$

So, for example, the expectation of someone holding one out of a thousand tickets for a fair lottery with a prize of $10,000 would be $10. As the definition implies, we now derive expectation from the probability, but for the early probabilists expectation was the prior and irreducible notion.

Expectation in turn was understood in terms of a fair exchange or contract. Pascal described his solution to the Problem of Points as rendering to each player what "in justice" belonged to him. In the first published treatise on mathematical probability, *De ratiociniis in ludo aleae* (1657), the Dutch mathematician and physicist Christiaan Huygens made expectation his departure point and defined it in terms of equity: equal expectations obtained in a fair game; that is, one that "worked to no one's disadvantage" (Huygens, [1657] 1920, p. 60). Since later probabilists would *define* a fair game as one in which the players possessed equal expectations, this definition of equal expectations in terms of a fair game strikes the modern reader as circular. But for the first generation of probabilists, notions of equity were intuitively clear enough to serve as the stuff of definitions and postulates.

These intuitions drew upon a category of legal agreement that had become increasingly important in sixteenth- and seventeenth-century commercial law, the aleatory contract. Jurists defined such agreements as the exchange of a present and certain value for a future, uncertain one – staking a gamble, purchasing an annuity, taking out an insurance policy, bidding on next year's wheat crop, or buying the next cast of a fisherman's net. Pascal's wager hinged upon a similar trade of the certain enjoyment of present vices for the uncertain joy of salvation. Aleatory contracts acquired prominence and a certain notoriety as the preferred way of exonerating merchants who made loans with interest from charges of usury

(Coumet, 1970). The element of risk, argued the canon lawyers, was the moral equivalent of labor, and therefore earned the merchant his interest as honestly as the sweat of his brow would have. Thus Jesuits successfully petitioned the Sacred Congregation for Propaganda in 1645 for a special dispensation for their Chinese converts, who were charging 30% interest on loans, on the condition "there is considered the equality and probability of the danger, and provided that there is kept a proportion between danger and what is received" (Noonan, 1957, p. 289).

It was this "proportion between danger and what is received," the element of equity fundamental to all contracts, that the mathematicians attempted to quantify in almost all of the early applications of probability theory. Both the problems they addressed – gambling stakes, annuity prices, future inheritances – and the terms in which they did so – using the concept of equal expectations – bear witness to the seminal influence of the law of aleatory contracts.

Pascal's wager is an example of how reasoning by expectations had become almost synonymous with a new brand of rationality by the mid-seventeenth century. His libertine interlocutor must be led back into the Christian fold by uncertain wagers rather than theological certainties. In the sixteenth century, Reformation controversies between Protestants and Catholics on the one hand, and the revival of the sceptical philosophy of Sextus Empiricus and his school on the other, combined to undermine the ideal of certain knowledge that had guided intellectual inquiry since Aristotle. In its place gradually emerged a more modest doctrine that accepted the inevitability of less than certain knowledge, but maintained nonetheless that it was still sufficient to guide the reasonable man in precept and in practice. Aristotle's dictum from the *Nicomachean Ethics* (1094b 24–25) was much quoted: "it is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits: it is evidently equally foolish to accept probable reasoning from a mathematician and to demand from a rhetorician demonstrative proofs."

The ultimate result of the Reformation and Counter-Reformation clashes over the fundamental principles of faith and their justification, and of the radical scepticism of Michel de Montaigne and other sixteenth-century thinkers was vastly to erode the domain of the demonstrative proof and to expand that of probable reasoning. Their immediate impact was more devastating, challenging all claims to any kind of knowledge whatsoever. Religious apologists who sought to undercut the other side's claims to legitimacy on the basis of either (ambiguous) revelation or (dubious) authority soon discovered that their destructive arguments were a double-edged sword. The revived pyrrhonism of the "libertins érudits" denied the reliability of even sense impressions and mathematical demonstrations; Descartes' *Meditations* began with a sceptical reverie of this extreme variety. Thus all of the traditional sources of certainty, religious and philosophical, came simultaneously under attack. Confronted with a choice between fideist dogmatism on the one hand and the most corrosive scepticism on the other, an increasing number of seventeenth-century writers attempted to carve out an intermediate position that abandoned all hope of certainty except in mathematics and perhaps metaphysics, and yet still insisted that men could attain probable knowledge. Or rather, they insisted that probable knowledge was indeed knowledge (Popkin, 1964; Shapiro, 1983).

In order to make their case for the respectability of the merely probable, these "mitigated sceptics" turned from rarified philosophical discourse to the conduct of daily life. The new criterion for rational belief was no longer a watertight demonstration, but rather that degree of conviction sufficient to impel a prudent man of affairs to action. For reasonable men that conviction in turn rested upon a combined reckoning of hazard and prospect of gain, i.e. upon expectation. Pascal's wager is about neither the bare probability of God's existence, nor the infinite bliss or misery that awaits saint or sinner, respectively. Rather, it is about the product of the two, significantly conceived in terms of a gamble, and the relationship between certain stake and uncertain pay-off, and thus a sterling example of the new rationality. Pascal's Port Royal colleagues Antoine Arnauld and Pierre Nicole made such mixed reasoning the *sine qua non* of rational judgment in their influential *Logique*, cautioning their readers that it is not enough to consider how good or bad an outcome is in itself, but also the likelihood that it will come to pass (Arnauld and Nicole, [1662] 1965, pp. 352–3). English and Dutch spokesmen for the new rationality of expectation preferred commercial analogies, but the idea was the same. John Wilkins, Anglican bishop and founding member of the Royal Society of London, argued in his *Of the Principles and Duties of Natural Religion* (1675) that just as merchants were willing to risk the perils of a long voyage in the name of profit, so "he that would act rationally, according to such Rules and Principles as all mankind do observe in the government of their Actions, must be persuaded to do the like" in matters of science and religion (Wilkins, [1675] 1699, p. 16). The emphasis upon action as the basis of belief, rather than the reverse, was key to the defense against scepticism, for as these writers were wont acidly to observe, even the most confirmed sceptic took his meals just as if the external world existed.

Expectation was thus central to the new rationality or "reasonableness," as it was sometimes called. The mitigated sceptics were less interested in equity than in rational belief, but they drew heavily upon the doctrine of aleatory contracts for examples to show that it was accepted practice and therefore reasonable to exchange a present, certain good – be it money to invest, a long-accepted scientific theory, or the indulgence of our lusts and passions – for a future, uncertain one – more money, a better theory, salvation. Mathematicians seeking to quantify the legal sense of expectations inevitably became involved in quantifying the new rationality as well. So began an alliance between mathematical probability theory and standards of rationality that stamped the classical interpretation as a "reasonable calculus"; as a mathematical codification of the intuitive principles underlying the belief and practice of reasonable men. The identification of classical probability theory with reasonableness was so strong that when the results of the one clashed with the other, it was the mathematicians who anxiously amended definitions and postulates to restore harmony, as we shall see below.

1.3 THE CLASSICAL INTERPRETATION

Thus the calculus of chance was in the first instance a calculus of expectations, and thereby an attempt to quantify the new, more modest doctrine of rationality that surfaces almost everywhere in seventeenth-century learned discourse. The first published works on the subject, from Huygens' little treatise of 1657 to Jakob Bernoulli's definitive *Ars conjectandi* of 1713, covered a range of topics that cohere only against this background. Aleatory contracts like gambling (Huygens, Pierre de Montmort, Jakob Bernoulli) and annuities (Johann De Witt, Halley, Nicholas Bernoulli), and later evidentiary problems like the evaluation of historical or courtroom testimony (John Craig, George Hooper, Nicholas and Jakob Bernoulli) constituted the domain of applications for the new theory. By the end of this period, probability had emerged as a distinct and primitive concept, although most of the applications continued to revolve around questions of expectation for some time thereafter.

Just what these probabilities measured was ambiguous from the outset, and remains a matter of controversy to this day. Originally the word "probability" had meant an opinion warranted by authority (Byrne, 1968); hence the Jesuit doctrine of probabilism, which casuists wielded to absolve almost every transgression on the grounds that one theologian or another had taken a mild view of the matter (Demain, 1935). However, the mitigated scepticism of the early seventeenth century modified even this qualitative sense of probability. The proponents of reasonableness spoke not of certainty but of certainties, ranging from the highest grade of "mathematical" certainty attained by demonstration, through the "physical" certainty of sensory evidence, down to the "moral" certainty based on testimony and conjecture. The precise descriptions of these levels varied slightly from author to author, but the notion of such an ordered scale, and the emphasis that most things admit only of moral certainty, remained a staple of the literature from Hugo Grotius' *De veritate religionis christianae* (1624) to John Locke's *Essay Concerning Human Understanding* (1690) and thereafter. When Bishop Joseph Butler claimed in 1736 that "probabilities are the very guide of life," he was by then repeating a cliché (Butler, 1736, p. iii).

In the context of these discussions, the very meaning of the word "probability" changed from its medieval sense of any opinion warranted by authority to a degree of assent proportioned to the evidence at hand, both of things and of testimony (Locke, [1690] 1959, IV. xv–xvi). These probabilities were qualitatively conceived, and owed much to the language and practice of legal evidence, as the numerous courtroom examples and analogies make clear (Daston, 1988, chapter 2). However, mathematicians like Gottfried Wilhelm Leibniz and Jakob Bernoulli seized upon the new "analysis of hazards" as a means of quantifying these degrees of certainty, and in so doing, converting the three ordered points into a full continuum, ranging from total disbelief or doubt to greatest certainty (J. Bernoulli, 1713, IV.i). Indeed, Leibniz described the fledgling calculus of probabilities as a mathematical translation of the legal reasoning that carefully proportioned degrees of assurance on the part of the judge to the kinds of evidence submitted (Leibniz, [comp. *c.* 1705] 1962, pp. 460–5). The fact that these legal probabilities were sometimes expressed in terms of fractions to create a kind of "arithmetic of proof" (for example, the testimony of a relative of the accused might count only ⅓ as much as that of an unimpeachable witness) may have made them seem mathematically tractable.

The mathematicians who set about trying to measure these probabilities in some non-arbitrary fashion came up with at least three methods: equal possibilities based on physical symmetry; observed frequencies of events; and degrees of subjective certainty or belief. (Other seventeenth-century meanings of "probability," such as the appearance of truth or the strength of analogy, were not successfully quantified.) The first was well suited to gambling devices like coins or dice but little else; the second depended on the collection of statistics and assumptions of long-term stability; and the third echoed the legal practice of proportioning degrees of certainty to evidence.

The various senses emerged from different contexts, and suggested different applications for the mathematical theory. Sets of equiprobable outcomes based on physical symmetry derived from gambling and were applied to gambling – very few other situations satisfy these conditions in an obvious way. Statistical frequencies originally came from mortality and natality data gathered by parishes and cities from the sixteenth century onwards. In 1662 the English tradesman John Graunt used the London bills of mortality to approximate a mortality table by assuming that roughly the same fraction of the population died each decade after the age of six (Graunt, [1662] 1975, pp. 29–30). (Since the bills of mortality registered only cause, not age at death, Graunt's table was based on informed guesswork about what diseases killed whom at what age, and the faith that mortality was regular.) Eighteenth-century authors collected more detailed demographic data and enlisted probability theory in order to compute the price of annuities, and later life insurance, and to argue for divine providence in human affairs.The epistemic sense of belief proportioned to evidence arose from legal theories about just how much and what kind of evidence was required to produce what degree of conviction in the mind of the judge, and inspired applications to the probabilities of testimony, both courtroom and historical, and of judgment.

Latter-day probabilists view these three answers to the question, "What do probabilities measure?" as quite distinct, and much ink has been spilt arguing their relative merits and compatibility (Nagel, 1955). In particular, a bold line is now drawn between the first two "objective" meanings of probability, which correspond to states of the world, and the third "subjective" sense, which corresponds to states of mind. Yet classical probabilists used "probability" to mean all three senses, shifting from one to another with an insouciance that bewilders their more nice-minded successors.

Why were classical probabilists able to conflate these different notions of probability so easily, and often very fruitfully? In part, because the objective and subjective senses were not then separated by the chasm that yawns between them in current philosophy. Legal theorists of the sixteenth and seventeenth centuries found it plausible to assume that conviction formed in the mind of the judge in proportion to the weight of the evidence presented, and Locke repeated the assumption in a more general context, invoking the qualitative probabilities of evidence: the rational mind assents to a claim "proportionably to the preponderancy of the greater grounds of probability on one side or the other" (Locke, [1690] 1959, vol. II, p. 366). At least two further elements were required to connect the objective and subjective senses of qualitative probabilities. First, precept had to be guaranteed in practice. It was not enough that the mind *should* apportion assent in strict relation to the evidence; it had to be shown that it actually did so. Second, the evidence had to be quantified.

The empiricist philosophy-*cum*-psychology of the late seventeenth and eighteenth centuries satisfied both desiderata. John Locke, David Hartley, and David Hume created and refined a theory of the association of ideas that made the mind a kind of counting machine that automatically tallied frequencies of past events and scaled degrees of belief in their recurrence accordingly. Hartley went so far as to provide a physiological mechanism for this mental record-keeping: each repeated sensation set up a cerebral vibration that etched an ever deeper groove in the brain, corresponding to an ever stronger belief that things would be as they had been. Hume notoriously rejected the rationality of such inferences to the future based on past experience, *pace* Locke and Hartley, but he retained the psychology that made them inevitable. Images of past experiences conjoin to heighten the vivacity of a mental impression, each repetition being "as a new stroke of the pencil, which bestows an additional vivacity on the colours." (Hume, [1739] 1975, p. 135). Since the mind irresistibly conferred belief in proportion to the vivacity of an idea, the more frequent the conjunction of events in past experience, the firmer the conviction that they would occur again. Locke and Hartley contended that this matching of belief to frequencies was rational (Hartley appealing explicitly to the calculus of probabilities; Locke, [1690] 1959, IV.xv; Hartley, 1749, vol. I, pp. 336–9). Hume replied that it was merely habitual, although his "Essay on Miracles" elevated belief based on unexceptioned past experience to at least a kind of reasonableness (Hume, [1758] 1955, chapter 10). All however concurred that the normal mind, when uncorrupted by upbringing or prejudice, irresistibly linked the subjective probabilities of belief with the objective probabilities of frequencies.

They also showed an increasing tendency to reduce all forms of evidence whatsoever to frequencies, in contrast to the legal doctrines that had originally been the prototype of degrees of belief proportioned to evidence. For the judge, the probative weight of eye-witness testimony that the accused had been seen fleeing the scene of the murder with unsheathed bloody sword derived from the quality of the evidence, not its quantity. It mattered not how many times in the past similar evidence had led to successful convictions. Locke remained very close to this legal tradition in his discussion of the kinds of evidence that create probabilities: number of witnesses, their skill and integrity, contradictory testimony, internal consistency, etc. He told the cautionary tale of the King of Siam, who dismissed the Dutch ambassador as a liar because his tales of ice-skating on frozen canals ran counter to the accumulated experience of generations of Siamese that water was always fluid. The King erred in trusting the mere quantity of his experience, without evaluating its breadth and variety. Yet Locke also made a place for "the frequency and constancy of experience" and for the number, as well as the credibility of testimonies (Locke, [1690] 1959, IV.xv). Later philosophical writings on probabilities narrowed the sense of evidence to the countable still further. Hume represents the endpoint of this evolution, in which evidence has become the sum of repeated, identical events. According to Hume, the mind not only counted; it was exquisitely sensitive to small differences in the totals: "When the chances or experiments on one side amount to ten thousand, and on the other to ten thousand and one, the judgment gives the preference to the latter, upon account of that superiority" (Hume, [1739] 1975, p. 141).

The guarantee that subjective belief was willy-nilly proportioned to objective frequencies and also, according to some authors, to physical symmetries allowed classical probabilists to slide from one sense of probability to another with little or no explicit justification. Only when associationist psychology shifted its emphasis to the illusion and distortions that prejudice and passion introduced into this mental reckoning of probabilities did the gap between subjective and objective probabilities become clear enough to demand a choice between the two. It was not so much the development and triumph of a thoroughgoing frequentist version of probability theory that marked the end of the classical interpretation, as the realization that a choice must be made between (at least) two distinct senses of probability. The range of problems to which the classical probabilists applied their theory shows that their understanding of probability embraced objective as well as subjective elements: statistical actuarial probabilities happily co-existed with epistemic probabilities of testimony in the work of Jakob Bernoulli or Laplace.

1.4 DETERMINISM

But the writings of these two towering figures in the history of mathematical probability also contained the manifestoes that, rightly or wrongly, led to the standard view of the classical interpretation as incorrigibly subjective. Both maintained that probabilities measure human ignorance, not genuine chance; that God (or Laplace's secularized super-intelligence) had no need of probabilities; that necessary causes, however hidden, governed all events. Therefore probabilities had to be states of mind rather than states of the world, the makeshift tools of intellects too feeble to penetrate immediately to the real nature of things. Theirs was an epistemological determinism that maintained that all events were in principle predictable, and that probabilities were therefore relative to our knowledge. Bernoulli remarked that backward peoples still gambled on eclipses that European astronomers could now predict; some day gambling on coins and dice would seem equally primitive when the science of mechanics was perfected (J. Bernoulli, 1713, IV.i).

The very mathematicians who had carved out a place for chance in the natural and moral sciences insisted to a man that chance, in Abraham De Moivre's words, "can neither be defined nor understood" (De Moivre, [1718] 1756, p. 253). They did concede that certain statistical rates varied from year to year and from place to place, but they were confident enough in the underlying regularity of phenomena like mortality to simplify and adjust the unruly data accordingly (Pearson, 1978, pp. 319–29). Variability, they believed, would prove just as illusory as chance when fully investigated.

In order to unknot the apparent paradox of the ardent determinism of the classical probabilists, we must look beyond probability theory to the panmathematical spirit of the period in which it emerged. Classical probability arose and flourished during a time of spectacular successes in fitting mathematics to whole new domains of experience, from rainbows to vibrating strings. Natural philosophers like Galileo assumed that if nature spoke the language of mathematics, this was because nature was fully determined, at least from God's viewpoint: the glue that connected causes and effects must be as strong as that which connected premises and conclusions in a mathematical argument. Determinism thus became a precondition for the mathematical description of nature.

At first glance, chance events therefore seemed the least likely candidates for mathematical treatment; even Pascal admitted that there was something paradoxical about a "géométrie du hasard" (Pascal, [1654] 1970, vol. I, part 2, p. 1034). The very earliest mathematical attempts to analyze gambling problems stumbled over just this problem. In his manuscript on the subject (composed *c.* 1525), the Italian physician, mathematician, and inveterate gambler Girolamo Cardano felt mathematically obliged to assert that each face of a die occurred once in every six rolls, although this flew in the face of his own experience at the gaming tables. He resolved the conflict with an appeal to the intervention of luck (he was a great believer in his own), which disrupted the necessary connection between the underlying probabilities and the actual events in favor of particular players (Cardano, [comp. *c.* 1525] 1966, pp. 264–5). He thus relinquished his claim to founding the mathematical theory of probability. Classical probability theory arrived when luck was banished; it required a climate of determinism so thorough as to embrace even variable events as expressions of stable underlying probabilities, at least in the long run. Determinism made a "geometry of chance" conceivable by anchoring variable events to constant probabilities, so that even fortuitous events met what were then the standards for applying mathematics to experience. Those standards were not compatible with older notions of chance as real, or with what we might call genuine randomness in the world.

"Chance" and "fortune" had been part of the philosophical vocabulary since Aristotle, meaning variously coincidence (meeting someone who owes you money on the way to the market), absence of purpose (often identified with necessity, as in the "blind necessity" of Epicurean atoms), or an ample endowment of the "external" goods of good health, wealth, beauty, and children (Sorabji, 1980). All of these meanings survived in ordinary usage, but only one played an important role in classical probability theory. This was the opposition of chance and purpose, particularly divine purpose, of which natural theologians and their probabilist allies like De Moivre made much. John Arbuthnot remarked upon the tiny probability that, year after year, male should exceed female births in a disproportion neatly arranged (or so Arbuthnot argued) to guarantee the future of the institution of monogamous marriage (Arbuthnot, 1710); Daniel Bernoulli pointed to the close alignment of the planets in the plane of the ecliptic as evidence for a single cause of the solar system (D. Bernoulli, 1752). Almost any symmetry or stability unlikely to have come about by "mere chance" – the intricate construction of the human eye; the regular mortality rate – became an argument "from design" for the existence of an intelligent and beneficent deity. The mathematical versions of the argument from design like Arbuthnot's were criticized by contemporaries like Nicholas Bernoulli and d'Alembert, who noted that all other irregular arrangements of planets or birth ratios were just as improbable. However, the natural theologians persisted until Darwin in seeing chance refuted everywhere by the traces of divine handiwork (see chapter 4). Indeed, such beliefs inspired many of the eighteenth-century statistical demographers, who, like the German pastor Johann Süssmilch, saw in rates of natality, marriage, and mortality, "a constant, general, complete, and beautiful order." (Süssmilch, [1741] 1775, vol. 1, p. 49). Only in the mid-nineteenth century, when the alleged statistical regularities were examined against the background of very different aims and controversies, was variability given its due and chance a new lease on life. But for the classical probabilists "chance" and "luck" that stood outside the causal order were superstitions. If we could see the world as it really was, penetrating to the "hidden springs and principles" of things, we would discover only necessary causes. Probabilities were merely provisional, a figment of human ignorance and therefore subjective.

The classical interpretation of mathematical probability was thus characterized in precept by determinism and therefore by a subjective slant, and in practice by a fluid sense of probability that conflated subjective belief and objective frequencies with the help of associationist psychology. It is however somewhat misleading to call this an "interpretation" of the mathematical theory, for to the classical probabilists the interpretation *was* the theory. The "doctrine of chances," or "art of conjecture," as probability theory was variously called in the eighteenth century, was a part of "mixed mathematics," a term deriving from Aristotle's explanation of how harmonics or optics mixed the forms of mathematics with the matter of sound and light (*Physics*, 193b22–194a15). In contrast to the more modern applied mathematics, mixed mathematics did not necessarily presuppose a prior and independent mathematical theory to be applied to various subject matters. Classical probability theory had no existence independent of its subject matter, *viz.* the beliefs and conduct of reasonable men. As Laplace put it in a famous passage, mathematical probability was in essence "only good sense reduced to a calculus" (Laplace, [1814] 1951, p. 6). Its status was less that of a mathematical theory with applications than that of a mathematical model of a certain set of phenomena, like the part of celestial mechanics that described lunar motion. As such, it was held up to empirical test. If astronomical theory failed to predict lunar perturbations, so much the worse for the theory. When the results of classical probability theory did not square with the intuitions of reasonable men, it was the mathematicians who returned to the drawing board.

1.5 REASONABLENESS

The protracted controversy over the St. Petersburg problem was just such a clash between reasonableness and the dictates of probability theory, and illustrates how seriously mathematicians took their task of modeling "good sense." The problem was first proposed by Nicholas Bernoulli in a letter to Pierre de Montmort, and published in the second edition of the latter's *Essai d'analyse sur les jeux de hasard* (1713). Pierre and Paul play a coin toss game with a fair coin. If the coin comes up heads on the first toss, Pierre agrees to pay Paul \$1; if heads does not turn up until the second toss, Paul receives \$2; if not until the third toss, \$4, and so on. Reckoned according to the standard method, Paul's expectation (and therefore the fair price of playing the game) would be:

$$E = (\tfrac{1}{2} \times \$1) + (\tfrac{1}{4} \times \$2) + (\tfrac{1}{8} \times \$4) + \ldots + [(\tfrac{1}{2})^{n} \times \$2^{n-1})] + \ldots$$

Since there is a small but finite chance that even a fair coin will produce an unbroken run of tails, and since the pay-offs increase in proportion to the decreasing probabilities of such an event, the expectation is infinite. However, as Nicholas Bernoulli and all subsequent commentators were quick to observe, no reasonable man would pay even a small sum to play the game. Although the mathematicians labeled this a paradox, it contained no contradiction between results derived from assumptions of equal validity. The calculation of expectation is straightforward, and there is nothing in the mathematical definition of expectation that precludes an infinite answer. Rather, it struck them as paradoxical that the results of the mathematical theory could be so at odds with the manifest dictates of good sense. Applied mathematicians in the modern sense might simply have questioned the suitability of the mathematical theory for this class of problems, but that route was not open to the mixed mathematicians of the eighteenth century. In their eyes the clash between mathematical results and good sense threatened the very validity of mathematical probability.

This is why the St. Petersburg problem, trivial in itself, became a *cause célèbre* among classical probabilists. In 1738 Nicholas' cousin Daniel Bernoulli published a resolution of the paradox in the annals of the Academy of St. Petersburg, the first of many such attempts. Daniel's memoir not only named the problem after the Academy; it also raised the fundamental issues of the definition of probabilistic expectation and its relationship to reasonableness that animated the subsequent controversy. In essence, he proposed a new notion of reasonableness, and a redefinition of expectation to match. Observing that the standard definition of expectation was like an impartial judge who ignores the individual characteristics of the risk-takers, Bernoulli argued that in situations like the St. Petersburg problem more than equity was at stake. Here the players acted more out of prudence than of fairness, and the definition of expectation had to be modified accordingly. In contrast to the "mathematical" expectation of equity Bernoulli proposed the "moral" expectation of prudence, defined as the product of the probability of the outcome and what later became known as its utility. That is, Bernoulli substituted values relative to individual preference for monetary values, with the understanding that the richer you are, the more it takes to make you happy. By making utility a logarithmic function of monetary wealth, he was able to derive reassuringly small expectations, depending on one's fortune, for the St. Petersburg game. He was also able to show that moral expectation harmonized with other widely accepted practices and beliefs: for example, sober men of affairs knew to avoid the gaming tables and to distribute their cargo among several ships, and moral (but not mathematical) expectation gave results that confirmed their wisdom (D. Bernoulli, [1738] 1954).

Significantly, Daniel Bernoulli's examples were drawn from the world of trade and commerce, in contrast to the legal examples that had dominated the earlier discussions of expectation. Nicholas Bernoulli, who was professor of both Roman and canon law at the University of Basel as well as an accomplished mathematician, objected that moral expectation failed "to evaluate the prospects of every participant in accord with equity and justice." His cousin Daniel replied that his new definition of expectation "harmonize[d] perfectly with experience." What was at issue between the Bernoulli cousins was not whether probabilistic expectation should model reasonableness, but rather wherein such reasonableness consisted. Nicholas sided with the older sense of equity derived from aleatory contracts; Daniel with the increasingly important sense of economic prudence, derived from commerce. The prototypical reasonable man was no longer an impartial judge but rather a canny merchant, and the mathematical theory of probability reflected that shift.

Daniel Bernoulli's solution to the St. Petersburg problem was by no means universally accepted by other classical probabilists. Some, like Jean d'Alembert, thought the problem lay not with the monetary values but with the probabilities – was it really physically possible for a fair coin to continuously turn up tails? (see Swijtink, 1986). Others, like Siméon-Denis Poisson, pointed out that the length of the game was limited by the wealth of the two players. Still others, like M. J. A. N. Condorcet, argued that mathematical expectation was indeed correct, but only when applied as an average to many repetitions of the game (Daston, 1980; Jorland, 1987). However, all agreed that the paradox was a real one, and struck at the very foundations of the mathematical theory. The fact that classical probabilists were willing to tinker with definitions and postulates as fundamental as expectation in order to realign their calculus with good sense attests to their commitment to the mixed mathematical goal of quantifying the reasonable.

Given this goal, the classical probabilists always ran the risk of superfluity. If their calculus yielded results that echoed what the enlightened had known all along – as preface after mathematical preface emphasized was the case – then all the elaborate machinery of equations and calculations did seem a belaboring of the obvious. The probabilists replied that, in Voltaire's words, common sense was not that common. Only a small elite of *hommes éclairés* could reason accurately enough by unaided intuition; the calculus of probabilities sought to codify these intuitions (which the probabilists believed to be actually subconscious calculations) for use by *hoi polloi* not so well endowed by nature. This mathematical model of good sense could be compared to spectacles. By applying the same optical principles responsible for normal eyesight it was possible to extend vision artificially; similarly, the calculus of probabilities formalized the good sense that came naturally to the fortunate few to help out the befuddled many. And several probabilists suggested that even *hommes éclairés* could sometimes benefit from mechanized reasoning when the issue at hand was extremely complicated or obscured by sophistry.

The ideal of a calculus of reasoning, a set of formal rules independent of content, exerted a certain fatal attraction for many seventeenth- and eighteenth-century thinkers. The probabilists' hope of turning the "art of conjecture" into such a calculus echoes the seventeenth-century fascination with method taken to an extreme. In the end, the methods of Bacon, Descartes, and a host of lesser lights always relied to some extent on judgment, and therefore were of limited use in truly perplexing situations. As Leibniz quipped apropros of Descartes' famous rules of method, one takes what one needs and does what one ought. Leibniz also best captured the allure of a formal calculus that eliminated personal discretion, and with it, strife. Observing that among the learned only mathematicians ever resolved their problems to everyone's satisfaction, he envisioned a kind of "universal characteristic" that would somehow assign numbers to fundamental ideas and invent arithmetic-like oper-