

C level

$$\begin{aligned}
 1. \quad & N + \frac{N}{2} + \frac{N}{4} + \dots + 2 + 1 \\
 &= N + \frac{N}{2} + \frac{N}{4} + \dots + 16 + 8 + 4 + 2 + 1 \\
 &= N + \frac{N}{2} + \frac{N}{4} + \dots + 16 + 8 + 4 + 2 + 1 + 1 - 1 \\
 &= N + \frac{N}{2} + \frac{N}{4} + \dots + 16 + 8 + 4 + 2 + 2 - 1 \\
 &= N + \frac{N}{2} + \frac{N}{4} + \dots + 16 + 8 + 4 + 4 - 1 \\
 &\dots \\
 &= N + \frac{N}{2} + \frac{N}{4} + \frac{N}{4} - 1 \\
 &= 2N - 1 \\
 &O(2N - 1) = O(N)
 \end{aligned}$$

2. Every element in values only be calculated once.

$$O(N)$$

$$\begin{aligned}
 3. \quad & O(\log_2(N)) \quad O(\log_3(N)) \\
 &= O\left(\frac{\lg(N)}{\lg 2}\right) = O\left(\frac{\lg(N)}{\lg 3}\right)
 \end{aligned}$$

$$(\log_x y = \frac{\log_z y}{\log_z x})$$

B level

1. For each $\text{fib}(n)$, runtime is runtime of $\text{fib}(n-1)$ plus runtime of $\text{fib}(n-2)$

runtime of $\text{fib}(0), \text{fib}(1) = 1$, $\text{fib}(2) = 2$, $\text{fib}(3) = 3$, $\text{fib}(4) = 5$.

i:	0	1	2	3	4	5
runtime:	1	1	2	3	5	8
fib(i):	0	1	1	2	3	5

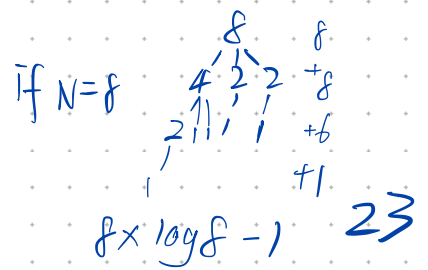
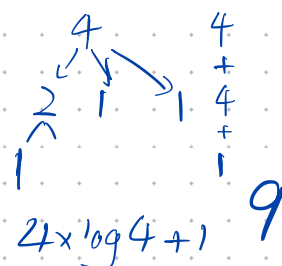
\therefore runtime $\text{fib}(n+1)$ for each:

$$\begin{aligned}
 \text{runtime of print-fib} &= \text{runtime of } \sum_{i=0}^{n-1} \text{fib}(i) \\
 &= \sum_{i=1}^n \text{fib}(i) \\
 &= \text{fib}(1) + \text{fib}(2) + \text{fib}(3) + \dots + \text{fib}(n) \\
 &= 1 + 1 + 2 + 3 + \dots + \text{fib}(n) \\
 &= \text{fib}(2) + \text{fib}(3) + \dots + \text{fib}(n) \\
 &= 2 \text{fib}(n)
 \end{aligned}$$

$$2. O(N \cdot \text{fib}(N+1))$$

$$3. O(n^3)$$

4. $n=1$, count 1
 $n=2$, count 2 = 3 if $n=4$,



if $n=16$: $\frac{16}{8} \cdot 4 = 16 + 2^3 + 4 + 4$
 $16 \times \log 16 - 7 = 57$
 $16 \times 4 - 7$

$4 \times 2 + 1$

$8 \times 3 - 1$

$\therefore O(N \cdot \log N)$

Intuition: function will print N and split itself. The splitted part is same size N with original one. The generations that it can split is $\log_2 N$. Every generation print N , totally be $(N \cdot \log N)$.

5. Problem 8

a) $\oplus(N^6)$ $\oplus(2^N)$ $\oplus(N)$

b) T T T T

c) NEI NEI FALSE TRUE NEI

d) $\oplus(N)$ e) N, N^2

$\oplus 2^N$

f) N, N^3

$\oplus \log N$

$\oplus N$

time to compare in a linked list
 length of List is N , each compare cost N time. So one compare cost N^2 .
 $1^2 + 2^2 + 3^2 + 4^2 + \dots + N^2 = N^3$

6. Problem 4

a) $i+=1$
 $i*=2$
 $i*=N$

c) Is this MITO Museum?

d) $b_1 \approx 1$ $b_2 \approx 3$

b) $\Theta(N^2 \cdot \log N)$
 $\mathcal{O}(N)$