Comparing COVID-19 Exposure and Propagation with Quantum Walk Modeling

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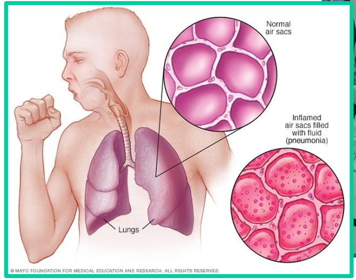
2019 Novel COVID-19 Coronavirus

Symptoms

- Fever
- Difficulty breathing
- Pneumonia
- **Lung damage/failure**

Socioeconomic Effects

- **Worldwide pandemic**
- Nationwide lockdown orders
- Supply chain disruption
- Medical politicization



Types of COVID-19 Transmission

Aerosol

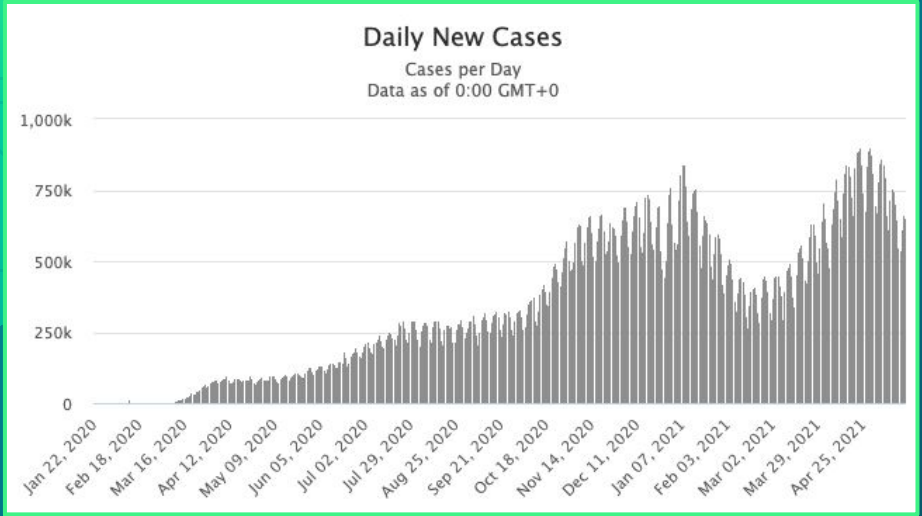
- Infected individuals emit viral particles when coughing, sneezing, or talking, exposing others in personal space

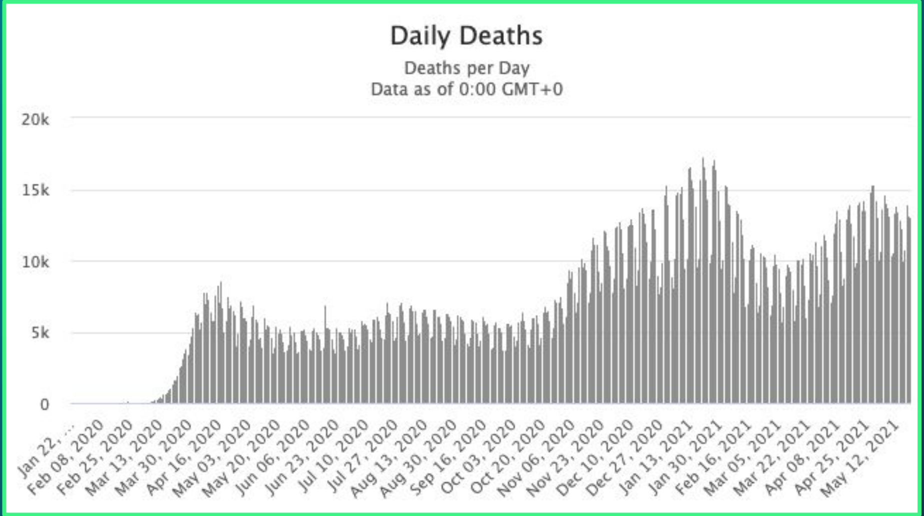
Airborne

- Viral particles can live in the air for up to 3 hours

Surface

- Surfaces touched by those infected can maintain viral particles for up to 3 days





Source: worldometers.info/coronavirus

Daily Cases and Daily Deaths

170,000,000

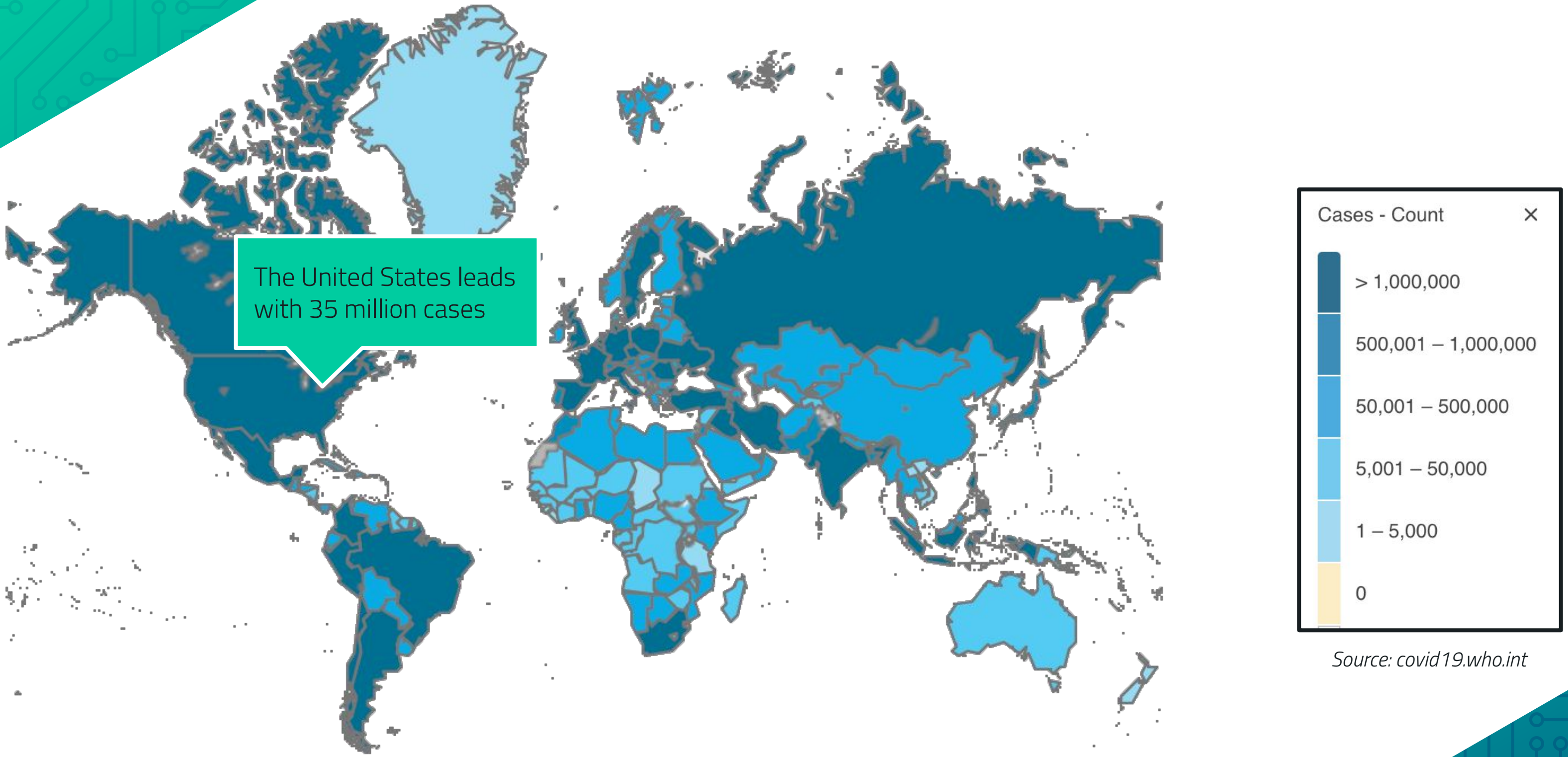
Total COVID-19 Cases

3,500,000

Total COVID-19 Related deaths

2.06%

Fatality Rate



Source: covid19.who.int

Quantum Walk Modeling

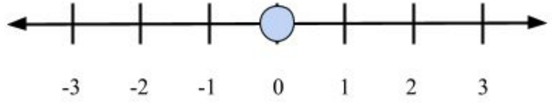
Quantum walks within systems demonstrate faster spreading of the final probability distributions

But First, Random Walks

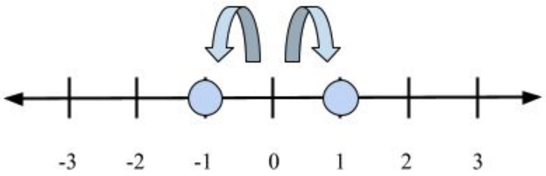
- Mathematical processes that describe sequences of random steps in a mathematical space
- Use mathematical tools called a coin, walker, and shift operator to perform each step
- Most studied model: random walk on a line

Random Walk On A Line

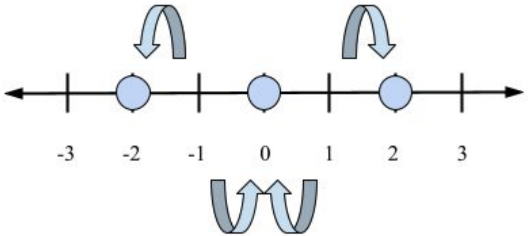
- Imagine having a coin wherein heads directs you to walk (shift) forwards and tails directs you to walk (shift) backwards
- After a certain number of time steps (coin flips), where can you expect to find yourself?



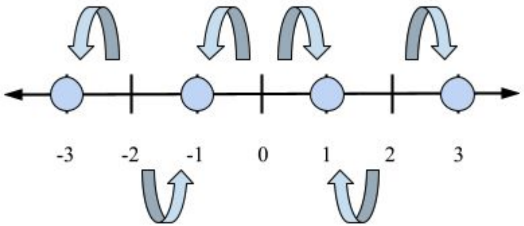
Time Step 0: Dot at starting Position 0



Time Step 1: Dot at either Position -1 or 1



Time Step 2: Dot at either Positions -2, 0 or 2

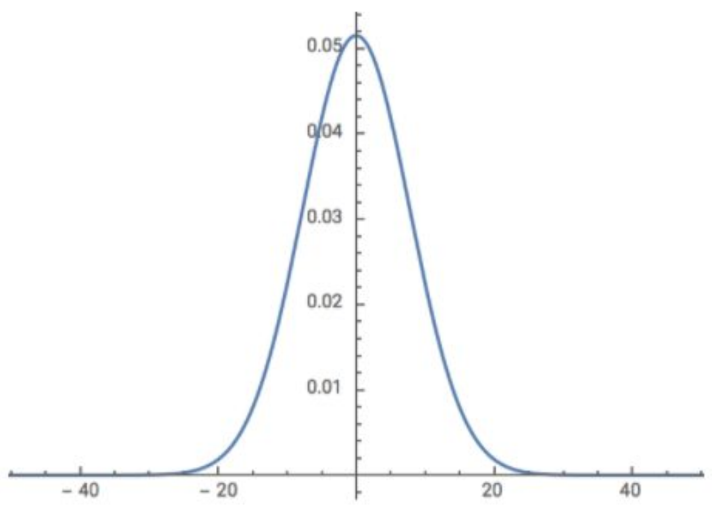


Time Step 3: Dot at either Positions -3, -1, 1, or 3

⋮

Probability Distribution for a Random Walk On a Line

Time Step (T)	Positions										
	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					$\frac{1}{2}$		$\frac{1}{2}$				
2				$\frac{1}{4}$		$\frac{1}{2}$		$\frac{1}{4}$			
3			$\frac{1}{8}$		$\frac{3}{8}$		$\frac{3}{8}$		$\frac{1}{8}$		
4		$\frac{1}{16}$		$\frac{1}{4}$		$\frac{3}{8}$		$\frac{1}{4}$		$\frac{1}{16}$	



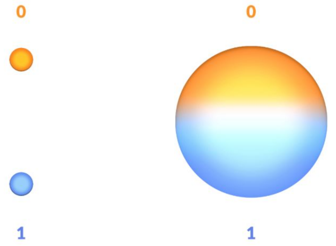
Random Walks vs. Quantum Walks

Random Walks

- Referred to as "classical" because they use logic methods corresponding to classical computing
- All classical information is represented as binary units, or 0's and 1's
- Classical computing limited by physical capacities of transistors

Quantum Walks

- Rely on qubits, or quantum bits, which can be represented as 0's, 1's, both, neither, or anything in between
- The uncertainty of qubits allow quantum machines to handle information in a much faster way
- Quantum particles possess characteristics of both wave functions and particles



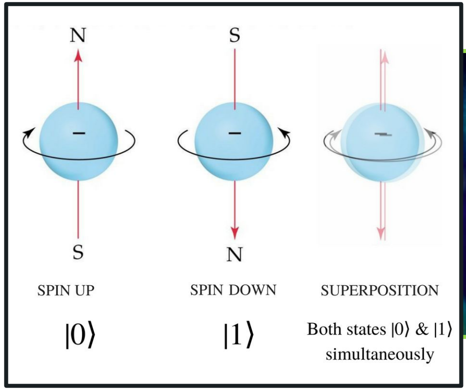
Fundamental Quantum Principles

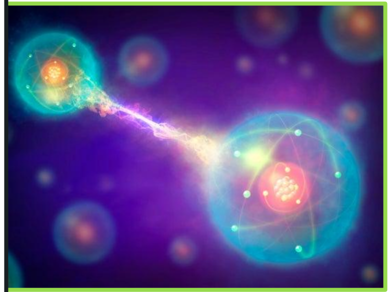
Quantum Superposition

- The quantum mechanical phenomenon where a quantum particle can be in multiple states at the same time until a measurement is made

Quantum Entanglement

- The linkage of two quantum particles that occurs after they interact with each other and can span an infinite distance





Different Models of Quantum Walks

Discrete Quantum Walks

- Consists of quantum walker and coin
- Shifting only happens in discrete time steps
- Can be separated into repeated applications of the coin flip and position shift operator
- Useful for computer scientists

Continuous Quantum Walks

- Consists of quantum walker and coin
- Evolution of system can be applied with no timing restrictions
- Used to model physical phenomena because time is continuous

This project utilizes a discrete quantum walk to carefully examine each time step

Quantum Walk On a Line Theory

- Must define infinite dimensional Hilbert space to translate coin, walker, and shift operator to the quantum world
- Simply define the walker in Hilbert space A and the coin in Hilbert space B, and then create a Hilbert space C so operations can be performed on both A and B together

Quantum Walk On a Line Theory (cont.)

Hadamard Coin Application

- Dictate the coin in Hilbert space B to be a Hadamard coin
- The Hadamard coin is essentially a quantum coin flip operator
- Mathematically, Hadamard coin is used by applying the Hadamard matrix

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Quantum Walk On a Line Theory (cont.)

Hadamard Coin's Effect On Computational Basis

- Essentially, below mathematically shows that the coin is now in a superposed state

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \times |0> = \frac{|0> \ +|1>}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \times |1> = \frac{|0> \ -|1>}{\sqrt{2}}$$

Quantum Walk On a Line Theory (cont.)

Define Shift Operator and Apply Tensor to Hilbert Space

- Per the figure on the left, defining a shift operator dictates how the position walker can move on a line after a coin flip
- Figure on the right displays the process of mathematically applying a tensor elevates our Hilbert space C to a superposed state (as opposed to just the coin being in a superposed state before)

$$S\,|n>|0> = |n+1>$$

$$S\,|n>|1> = |n-1>$$

$$I \otimes \mathrm{H}$$

Quantum Walk On a Line Theory (cont.)

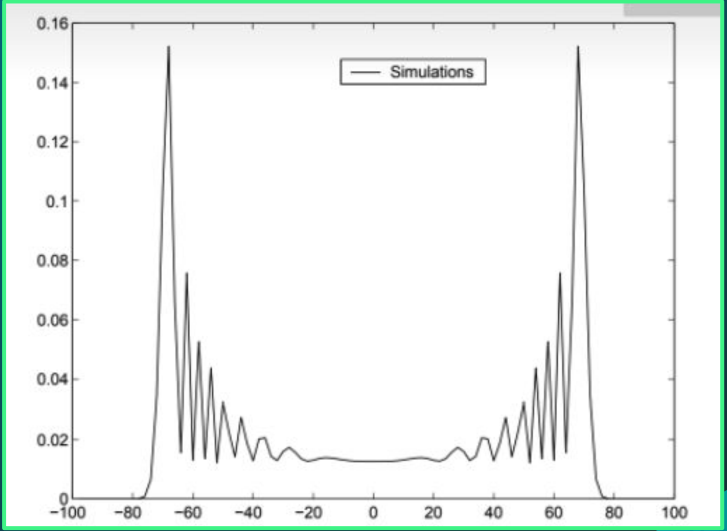
Hadamard Coin Utilization and Distribution

- To use the Hadamard coin, represent the H in the previous tensor operation using the computational bases affected by the coin, shown on the left figure
- Simply distribute and apply the shift operator to find the final resulting figure on the right

$$|n\rangle \otimes \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \qquad \frac{1}{\sqrt{2}}\left(|n+1\rangle \otimes |0\rangle + |n-1\rangle \otimes |1\rangle\right)$$

Probability Distribution for a Quantum Walk On a Line

Time Step (T)	Positions										
	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					$\frac{1}{2}$		$\frac{1}{2}$				
2				$\frac{1}{4}$		$\frac{1}{2}$		$\frac{1}{4}$			
3			$\frac{1}{8}$		$\frac{3}{8}$		$\frac{3}{8}$		$\frac{1}{8}$		
4		$\frac{1}{16}$		$\frac{3}{8}$		$\frac{1}{8}$		$\frac{3}{8}$		$\frac{1}{16}$	

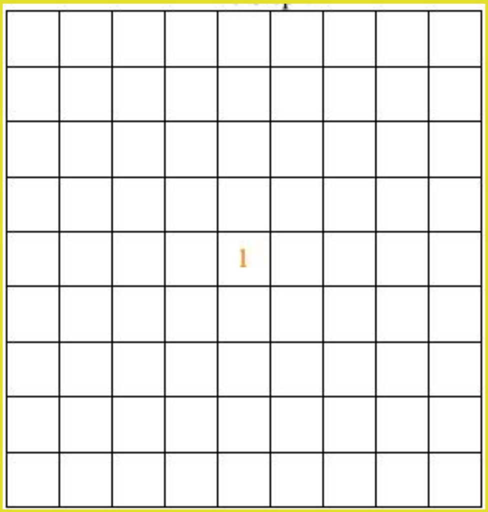


COVID-19 Exposure and Propagation Quantum Walk Models

- To simulate the spatial spread of the virus through direct aerosol transmission, must visualize the quantum walk on a two dimensional scale, or matrix
- Exposure refers to probability that a person is exposed to the virus, propagation refers to how the virus can spread outwards

COVID-19 Exposure and Propagation Quantum Walk Models (cont.)

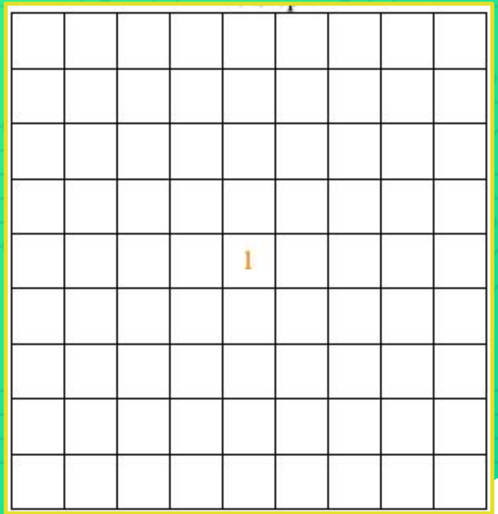
COVID-19 Exposure Probability Distribution



Time Step

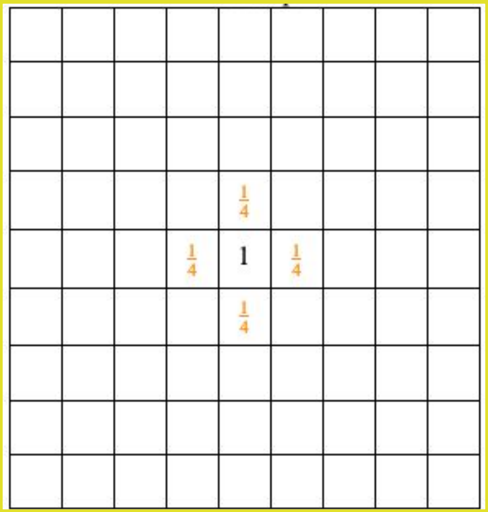
0

COVID-19 Propagation Probability Distribution



COVID-19 Exposure and Propagation Quantum Walk Models (cont.)

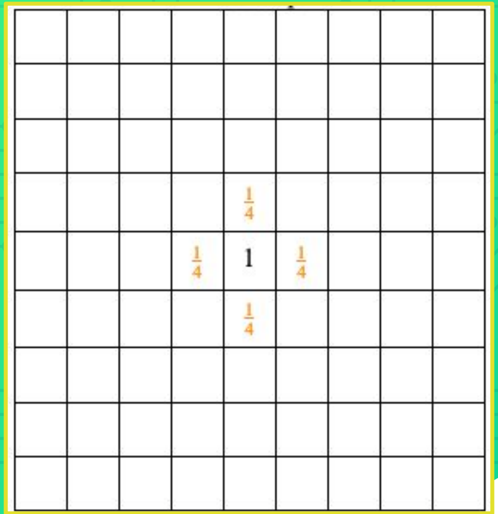
COVID-19 Exposure Probability Distribution



Time Step

1

COVID-19 Propagation Probability Distribution



COVID-19 Exposure and Propagation Quantum Walk Models (cont.)

COVID-19 Exposure Probability Distribution

				$\frac{1}{16}$				
			$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$			
		$\frac{1}{16}$	$\frac{1}{4}$	$\frac{4}{16}$	$\frac{1}{4}$	$\frac{1}{16}$		
			$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$			
				$\frac{1}{16}$				

Time Step

2

COVID-19 Propagation Probability Distribution

				$\frac{1}{12}$				
			$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$			
		$\frac{1}{12}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{12}$		
			$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$			
				$\frac{1}{12}$				

COVID-19 Exposure and Propagation Quantum Walk Models (cont.)

COVID-19 Exposure Probability Distribution

				$\frac{1}{64}$				
			$\frac{3}{64}$	$\frac{1}{16}$	$\frac{3}{64}$			
		$\frac{3}{64}$	$\frac{1}{8}$	$\frac{9}{64}$	$\frac{1}{8}$	$\frac{3}{64}$		
	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{9}{64}$	$\frac{4}{16}$	$\frac{9}{64}$	$\frac{1}{16}$	$\frac{1}{64}$	
		$\frac{3}{64}$	$\frac{1}{8}$	$\frac{9}{64}$	$\frac{1}{8}$	$\frac{3}{64}$		
			$\frac{3}{64}$	$\frac{1}{16}$	$\frac{3}{64}$			
				$\frac{1}{64}$				

Time Step

3

COVID-19 Propagation Probability Distribution

				$\frac{1}{36}$				
			$\frac{4}{36}$	$\frac{1}{12}$	$\frac{4}{36}$			
		$\frac{4}{36}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{4}{36}$		
	$\frac{1}{36}$	$\frac{1}{12}$	0	0	0	$\frac{1}{12}$	$\frac{1}{36}$	
		$\frac{4}{36}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{4}{36}$		
			$\frac{4}{36}$	$\frac{1}{12}$	$\frac{4}{36}$			
				$\frac{1}{36}$				

COVID-19 Exposure and Propagation Quantum Walk Models (cont.)

COVID-19 Exposure Probability Distribution

				$\frac{1}{256}$				
			$\frac{4}{256}$	$\frac{1}{64}$	$\frac{4}{256}$			
		$\frac{4}{256}$	$\frac{3}{64}$	$\frac{36}{256}$	$\frac{3}{64}$	$\frac{4}{256}$		
	$\frac{4}{256}$	$\frac{3}{64}$	$\frac{14}{256}$	$\frac{9}{64}$	$\frac{14}{256}$	$\frac{3}{64}$	$\frac{4}{256}$	
$\frac{1}{256}$	$\frac{1}{64}$	$\frac{36}{256}$	$\frac{9}{64}$	$\frac{4}{256}$	$\frac{9}{64}$	$\frac{36}{256}$	$\frac{1}{64}$	$\frac{1}{256}$
	$\frac{4}{256}$	$\frac{3}{64}$	$\frac{14}{256}$	$\frac{9}{64}$	$\frac{14}{256}$	$\frac{3}{64}$	$\frac{4}{256}$	
		$\frac{4}{256}$	$\frac{3}{64}$	$\frac{36}{256}$	$\frac{3}{64}$	$\frac{4}{256}$		
			$\frac{4}{256}$	$\frac{1}{64}$	$\frac{4}{256}$			
				$\frac{1}{256}$				

Time Step

4

COVID-19 Propagation Probability Distribution

				$\frac{1}{108}$				
			$\frac{11}{108}$	$\frac{1}{36}$	$\frac{11}{108}$			
		$\frac{4}{108}$	$\frac{4}{36}$	0	$\frac{4}{36}$	$\frac{4}{108}$		
	$\frac{11}{108}$	$\frac{4}{36}$	0	0	0	$\frac{4}{36}$	$\frac{11}{108}$	
$\frac{1}{108}$	$\frac{1}{36}$	0	0	0	0	0	$\frac{1}{36}$	$\frac{1}{108}$
	$\frac{11}{108}$	$\frac{4}{36}$	0	0	0	$\frac{4}{36}$	$\frac{11}{108}$	
		$\frac{4}{108}$	$\frac{4}{36}$	0	$\frac{4}{36}$	$\frac{4}{108}$		
			$\frac{11}{108}$	$\frac{1}{36}$	$\frac{11}{108}$			
				$\frac{1}{108}$				

Analysis and Conclusion

- Probability distributions begin to differ at time step 2
- Up until time step 3, both distributions can be mathematically deduced
- At time step 4, the logical process breaks down to produce seemingly random probabilities
- Distributions at time step 4 begin to display effects of quantum interference
- In time step 4, the probability distribution in the middle ring is far more significant than any other ring of probabilities
- Values in matrices roughly proportional to one-dimensional counterparts of quantum walk on a line

Analysis and Conclusion (cont.)

- Propagation process ignores the ability of quantum walks to inhabit previous positions, realistically representing the inability of infected individuals to become directly reinfected
- Propagation method normalizes the distributions on the basis of the possible number of infected people
- Matrices of COVID-19 exposure signify probabilities that people are exposed to the virus regardless of infection status
- Exposure method resets the normal distributions at each step

Future Applicable Concepts

- **Reproduction Number**: the United States Center for Disease Control estimates that an infected person will infect an average of 5.7 others; this consideration may be useful to estimate the number of time steps the virus requires to propagate on average
- **Immunizations**: in the United States, nearly 40% of the population is vaccinated; the factorization of vaccinated individuals may significantly decrease the probabilities of propagation while leaving exposure untouched

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THANKS!

Any questions?