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2:38 PM

Property-(ii)

Poll Que: *If one of the regression coefficients is greater than unity, the other must be…………?*

Less than unity > 1

(a) equal to unity (b) greater than unity (c) less than unity (d) *None of these.*

Remark: The converse of the above property may not be true i.e. if one of the regression coefficients is less than unity then the other regression coefficient may be greater than or less than unity. In other words, both the regression coefficients can be less than unity, but both cannot be greater than unity.

$$b_{XY} \times b_{YX} = r^2 \le 1$$

$$b_{XY} > 1 \qquad b_{XY} < 1$$

$$b_{YX} \le \frac{1}{b_{XY}} < 1 \qquad \frac{1}{b_{XY}} < 1 \qquad \frac{1}{b_{XY}} > 1$$

$$b_{XY} \times b_{YX} \le 1 \Rightarrow b_{YX} \le \frac{1}{b_{XY}} \qquad 1.3$$

$$0.9$$

Property-(iii) Poll Que: *If $r > 0$, then the arithmetic mean of the regression coefficients is…………..the correlation coefficient r.*

(a) equal to (b) greater than or equal to (c) less than or equal to (d) *None of these.*

Regression Coefficients.

$$\frac{\mu_{11}}{\sigma_X^2} = \frac{Cov.(X,Y)}{\sigma_X^2} = \frac{r\sigma_Y}{\sigma_X} = \text{Regression coefficient of } Y \text{ on X} = b_{YX}$$

$$\frac{\mu_{11}}{\sigma_Y^2} = \frac{Cov.(X,Y)}{\sigma_Y^2} = \frac{r\sigma_X}{\sigma_Y} = \text{Regression coefficient of } X \text{ on Y} = b_{XY}$$

$$r < 0$$

$$(\sigma_X - \sigma_Y)^2 \ge 0$$

$$(\sigma_X - \sigma_Y)^2 \le 0$$

$$\frac{b_{XY} + b_{YX}}{2} \ge r$$

$$\frac{1}{2}\left[\frac{\cancel{r}\sigma_X}{\sigma_Y} + \frac{\cancel{r}\sigma_Y}{\sigma_X}\right] \ge \cancel{r} \Rightarrow \frac{1}{2}\left[\frac{\sigma_X^2 + \sigma_Y^2}{\sigma_X\sigma_Y}\right] \ge 1 \Rightarrow \sigma_X^2 + \sigma_Y^2 \ge 2\sigma_X\sigma_Y$$

$$\Rightarrow \boxed{(\sigma_X - \sigma_Y)^2 \ge 0}$$

Property-(iv) Poll Que: *Regression coefficients are independent of:*

(a) change of origin and scale (b) change of scale only (c) change of origin only (d) *None of these.*

$$U = \frac{X-a}{h} \qquad b_{XY} = \frac{Cov.(X,Y)}{\sigma_Y^2} = \frac{hk\,Cov.(U,V)}{k^2\sigma_V^2} = \frac{h}{k} b_{UV}$$

$$V = \frac{Y-b}{}$$

$$V = \frac{Y-b}{k}$$

$$X = a + hU$$

$$E(X) = a + hE(U)$$

$$\sigma_Y^2 = k^2\sigma_V^2$$

$$Cov.(X,Y) = E\left[(X-E(X))(Y-E(Y))\right]$$

$$= E\left[h(U-E(U)) \times k(V-E(V))\right]$$

$$b_{YX} = \frac{k}{h} b_{VU}$$

Angle Between Two Lines of Regression

Poll Que: The angle between two Lines of Regression is given by:

(a) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X\sigma_Y}{\sigma_X^2+\sigma_Y^2}\right)\right]$ (b) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X^2\sigma_Y^2}{\sigma_X^2+\sigma_Y^2}\right)\right]$

(c) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X\sigma_Y}{\sigma_X+\sigma_Y}\right)\right]$ (d) *None of these.*

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$m_1 = \frac{r\sigma_Y}{\sigma_X}$$

$$m_2 = \frac{\sigma_Y}{r\sigma_X}$$

(i) $Y - \bar{y} = \frac{\mu_{11}}{\sigma_X^2}(X - \bar{x})$ Or $Y - \bar{y} = \frac{Cov.(X,Y)}{\sigma_X^2}(X - \bar{x})$ Or $Y - \bar{y} = \frac{r\sigma_Y}{\sigma_X}(X - \bar{x})$ is the line of regression of 'Y' on 'X' which passes through the point $(\bar{x}, \bar{y})$ and with *slope* $= \frac{Cov.(X,Y)}{\sigma_X^2} = \frac{r\sigma_Y}{\sigma_X}$.

(ii) $(X - \bar{x}) = \frac{\mu_{11}}{\sigma_Y^2}(Y - \bar{y})$ Or $(X - \bar{x}) = \frac{Cov.(X,Y)}{\sigma_Y^2}(Y - \bar{y})$ Or $(X - \bar{x}) = \frac{r\sigma_X}{\sigma_Y}(Y - \bar{y})$ is the line of regression of 'X' on 'Y' which passes through the point $(\bar{x}, \bar{y})$ and with *slope* $= \frac{\sigma_Y^2}{Cov.(X,Y)} = \frac{\sigma_Y}{r\sigma_X}$.

$$\tan\theta = \left|\frac{\frac{r\sigma_Y}{\sigma_X} - \frac{\sigma_Y}{r\sigma_X}}{1 + \frac{r\sigma_Y}{\sigma_X} \times \frac{\sigma_Y}{r\sigma_X}}\right| = \left|\frac{\frac{r^2\sigma_Y - \sigma_Y}{r\sigma_X}}{\frac{\sigma_X^2 + \sigma_Y^2}{\sigma_X^2}}\right|$$

Angle Between Two Lines of Regression

Poll Que: The angle between two Lines of Regression is given by:

$$= \left|\frac{r^2 - 1}{r}\right| \frac{\sigma_X\sigma_Y}{\sigma_X^2 + \sigma_Y^2}$$

(a) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X\sigma_Y}{\sigma_X^2+\sigma_Y^2}\right)\right]$ (b) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X^2\sigma_Y^2}{\sigma_X^2+\sigma_Y^2}\right)\right]$

(c) $\tan^{-1}\left[\left|\frac{1-r^2}{r}\right|\left(\frac{\sigma_X\sigma_Y}{\sigma_X+\sigma_Y}\right)\right]$ (d) *None of these.*

$$r = 0$$

$$\theta = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Poll Que: If the two variables X and Y are uncorrelated then the lines of regression are:

(a) *parallel* (b) *perpendicular* (c) *coincide* (d) *None of these*

$$\theta = \tan^{-1}(0) = 0$$

Poll Que: If the two variables X and Y are perfectly correlated $(r = \pm 1)$ then the lines of regression are:

Poll Que: If the two variables X and Y are perfectly correlated ($r = \pm 1$) then the lines of regression are:

(a) *parallel* (b) *perpendicular* (c) *coincide* (d) *None of these*

$\to l_1$

$\to l_2$

$(\bar{x}, \bar{y}) \to l_1, l_2$

Poll Que. Which of the following is the equation of lines of regression of Y on X for the given data? Also, what is the estimate of X for $Y = 70$?

X :	65	66	67	67	68	69	70	72
Y:	61	68	65	68	72	72	69	71

(a) $Y = (0.54)X + 30.74$ (b) $Y = (0.54)X + 23.78$ (c) $Y = (0.665)X + 23.78$ (d) *None of these*

(SHORT-CUT METHOD)

X	Y	U = X − 68	V = Y − 69	U²	V²	UV
65	67	−3	−2	9	4	6
66	68	−2	−1	4	1	2
67	65	−1	−4	1	16	4
67	68	−1	−1	1	1	1
68	72	0	3	0	9	0
69	72	1	3	1	9	3
70	69	2	0	4	0	0
72	71	4	2	16	4	8
Total		0	0	36	44	24

$$Y - \bar{y} = \frac{Cov(X,Y)}{\sigma_X^2}(X - \bar{x})$$

$\bar{y} = 69$, $\bar{x} = 68$

$$\boxed{Cov(X,Y) = hk\, Cov(U,V)}$$

$$= \frac{1}{n}\sum u_i v_i - \bar{u}\bar{v}$$

$$= \frac{1}{8}(24) = 3$$

$$\sigma_X^2 = h^2 \sigma_U^2 = \frac{1}{n}\sum u_i^2 - (\bar{u})^2 = \frac{1}{8}(36) = 4.5$$

$$Y - 69 = \frac{3}{4.5}(X - 68)$$

Poll Que. Which of the following is the equation of lines of regression of Y on X for the given data? Also, what is the estimate of X for $Y = 70$?

X :	65	66	67	67	68	69	70	72
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Poll Que. In a partially destroyed laboratory record of analysis of correlation data, the following results only are legible: Variance of $X = 9$, Regression equations $8X - 10Y + 66 = 0$, $40X - 18Y = 214$. What were (i) The mean values of X & Y. (ii) The correlation coefficient between X & Y and (iii) standard deviation of Y?

(a) $\bar{X} = 17, \bar{Y} = 13$ (b) $\bar{X} = 13, \bar{Y} = 17$ (c) $\bar{X} = \frac{1}{13}, \bar{Y} = \frac{1}{17}$ (d) *None of these.*