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MTH 280

Attendance - 5 MT-20 (MCQ 40) ETE-50 (MIX) CA-25

$N = \{1, 2, 3, ...\}$
$W = \{0, 1, 2, 3, ...\}$
$\mathbb{Z} = \{0, \pm 1, \pm 2, ...\}$
$\mathbb{Q} = \{p/q \mid q \neq 0,\ p, q \in \mathbb{Z}\}$

Rational Number: If a number either terminates or repeats after converting into a decimal is called as rational no.

$10/3 = 3.3333....$
$10/20 = 0.5$

$\mathbb{R} - \mathbb{Q}$ = Set of irrational Number.

Irrational Number: A no. which neither terminate nor repeat after converting into a decimal then it is called an irrational Number.
$\sqrt{2}, 5\sqrt{7}, \pi, e .....$

$\mathbb{R} = \mathbb{Q} \cup (\mathbb{R} - \mathbb{Q})$
$\mathbb{C}$ = Set of complex number.
$= \{a + ib : a, b \in \mathbb{R},\ i = \sqrt{-1}$

Is 2 a complex number or not.
$2 = 2 + 0i$

Algebric Operations:-

	Addition	Subtraction	Multiplication	Division
N	✓	X	✓	X
W	✓	X	✓	X
Z	✓	✓	✓	X
Q	✓	✓	✓	X
R	✓	✓	✓	✓
C	✓	✓	✓	✓

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$\mathbb{Q}, \mathbb{R}, \mathbb{C}$ are called field. We write them by the symbol F.

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Matrix:-

A matix is a rectangular array of numbers and functions. A matrix is always denoted by capital letters.

$$A = \begin{array}{c c} C_1 & C_2 \\ \left[\begin{array}{cc} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{array}\right] & \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \end{array}$$

Order of matrix = row x column
$= 4 \times 2$

Number of element = row x column
$= 4 \times 2$
$= 8$

Vector: A single row or column of number denoted with bold small letters.

(i) Row Vector:

$$a = [1 \;\; 2 \;\; 3 \;\; 4 \;\; 5]_{1\times 5}$$

(ii) Column Vector:

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$$

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Square Matrix:- If a number of rows and number of column of a matrix are same then it is called as square matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad , \quad A = [a_{ij}]_{n\times n}$$

Q: Construct a matrix of 2x2 order where $a_{ij} = i^2+j$.

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$$

$a_{11} = 1+1$ $a_{12} = 1+2$

$a_{21} = 5$ $a_{22} =$

Zero Matrix or Null Matrix:- If all the element of a matrix is zero.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} , \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A zero matrix can be square or rectangular matrix.

Diagonal Matrix:- A square matrix with its all non diagonal elements as zero. i.e. if $A = [a_{ij}]$ is a diagonal matrix, then $a_{ij} = 0$ where $i \neq j$.
Diagonal elements are the elements a_{ij} of the square matrix A for which $i = j$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{or} \quad A = \text{Diag}[1 \ 2 \ 3]$$

$$B = \text{Diag}[2, 0, 0, 4] \quad \text{or} \quad B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Scalar Matrix: It's a diagonal matrix whose all elements are same.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Unit Matrix:- It's a scalar matrix whose all diagonal elements are equal to unity.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{100} = \begin{bmatrix} 1 & 0 & \cdots \\ 0 & 1 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Triangular Matrix:- If every element above or below the diagonal is zero.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 5 & 6 \end{bmatrix}$$

Upper T.M   Lower T.M

Equal Matrix:-

- Two matrices A & B are said to be equal if and only if A & B are of same order.
- All the elements of A are equal as that of corresponding elements of B

Q:- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, If A & B are equal

$x = 1, \; y = 2, \; z = 3, \; w = 4.$

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Q:- If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

$$\begin{array}{r} x-y=-1 \\ \underline{\overset{-}{2x}\overset{+}{-}y=\overset{-}{0}} \\ -x=-1 \\ x=1 \end{array}$$

$$x-y=-1$$
$$y=x+1$$
$$\boxed{y=2}$$

$$2x+z=5$$
$$2\times1+z=5$$
$$\boxed{z=3}$$

$$3z+w=13$$
$$3\times3+w=13$$
$$\boxed{w=4}$$

Trace of a matrix:- Trace of a square matrix is the sum of the diagonal elements.

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 7 & 6 & 1 \end{bmatrix}, \quad tr\,A = 1+4+1=6$$

$$B=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad tr\,B = 1+4=5$$

* trace of I_n $(tr\,I_n)=n$

Addition & Subtraction:-

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} \\ a_{21}-b_{21} & a_{22}-b_{22} \end{bmatrix}$$

Q $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 5 \\ 6 & 8 \end{bmatrix}$, find $(A+B)$ & $(A-B)$

$$A+B = \begin{bmatrix} 6 & 6 \\ 9 & 10 \end{bmatrix} \qquad A-B = \begin{bmatrix} -2 & -4 \\ -3 & -6 \end{bmatrix}$$

also find $2A+3B$

$$2A = \begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}, \quad 3B = \begin{bmatrix} 12 & 15 \\ 18 & 24 \end{bmatrix}$$

$$2A+3B = \begin{bmatrix} 16 & 17 \\ 24 & 28 \end{bmatrix}$$

Q:- $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 4 & 8 \\ 3 & -2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 5 \\ 0 & 1 & 6 \end{bmatrix}$

find $5A+2B$

$$5A = \begin{bmatrix} 5 & 15 & 20 \\ -10 & 20 & 40 \\ 15 & -10 & -5 \end{bmatrix}, \quad 2B = \begin{bmatrix} 8 & 2 & 0 \\ 2 & 6 & 10 \\ 0 & 2 & 12 \end{bmatrix}$$

$$5A+2B = \begin{bmatrix} 13 & 17 & 20 \\ -8 & 26 & 50 \\ 15 & -8 & 7 \end{bmatrix}$$

Q:- $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -2 & 2 & 3 \end{bmatrix}$

find a 2×4 matrix "X" such that
$A - 2X = 3B$.

$$A\;X = \begin{bmatrix} x & \cancel{y} & z & p \\ a & \cancel{b} & c & d \end{bmatrix}$$

$$A - \cancel{2X =} \begin{bmatrix} 1- \end{bmatrix}$$

$$\begin{bmatrix} 1-6 & 2-3 & 0-0 & 4-9 \\ 2-3 & 4+6 & -1-6 & 3-9 \end{bmatrix}$$

Sol:- $A - 2X = 3B$

$$A - 3B = 2X$$

$$2X = \begin{bmatrix} -5 & -1 & 0 & -5 \\ -1 & 10 & -7 & -6 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -5 & -1 & 0 & -5 \\ -1 & 10 & -7 & -6 \end{bmatrix}$$

Multiplication Matrix:-

Q:- Find AB & BA

Condition: No. of column of A = No. of row of B.

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2\times4+1\times1 & 2\times1+1\times3 \\ 3\times4+1\times1 & 3\times1+1\times3 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 13 & 6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4\times2+1\times3 & 4\times1+3\times1 \\ 1\times2+3\times3 & 1\times1+3\times1 \end{bmatrix} = \begin{bmatrix} 11 & 7 \\ 11 & 4 \end{bmatrix}$$

$$\boxed{AB \neq BA}$$

* It is not commutative.

Property of Matrix:-

(i) Is $(A+B)^2 = A^2 + 2AB + B^2$?

$$(A+B)^2 = (A+B)(A+B)$$
$$= A^2 + AB + BA + B^2 \quad \because (BA \neq AB)$$
$$\neq A^2 + 2AB + B^2$$

(ii) If $A \neq 0$, $B \neq 0$, then $AB = 0$.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = 0$$

(iii) $A+B = B+A$ (Commutative law)

(iv) $A+(B+C) = (A+B)+C$ (associative law)

(v) $\lambda(A+B) = \lambda A + \lambda B$, where λ is a scalar (distributive law)

(vi) $A(B+C) = AB + AC$ (Left distributive law)

(vii) $(A+B)C = AC + BC$ (Right distributive law)

(viii) $A(BC) = (AB)C$ (Associative property of matrices under multiplication)

(ix) $AB \neq BA$ in general

(x) $AB = 0$ Not necessary imply $A = 0$ or $B = 0$

(xi) $AB = AC$ Not necessarily imply $B = C$.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = 0, \quad AC = 0$$

$$AB = AC$$

But $B \neq C$

Q.1 Find "x"

$$[1 \quad x \quad 1]_{1\times 3}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$$

$$[1+2x+15 \quad 3+5x+3 \quad 2+x+2]\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$$

$$1+2x+15+6+10x+6+2x+x^2+2x=0$$

$$x^2+16x+28=0$$

$$x^2+14x+2x+28=0$$

$$x(x+14)+2(x+14)=0$$

$$(x+2)(x+14)=0$$

$$x=-2 \quad \text{or} \quad x=-14 \quad \underline{\text{Ans}}.$$

Q.1 $A=\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 7\\ 6 & 4 & 8\end{bmatrix}$ find $A^2+7A+3I$.

$$A^2=\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 7\\ 6 & 4 & 8\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 7\\ 6 & 4 & 8\end{bmatrix}=\begin{bmatrix}1+0+12 & 3+15+8 & 2+21+16\\ 42 & 25+28 & 35+56\\ 6+48 & 18+20+32 & 12+28+64\end{bmatrix}$$

$$=\begin{bmatrix}13 & 26 & 39\\ 42 & 53 & 91\\ 54 & 70 & 104\end{bmatrix}$$

$$7A=\begin{bmatrix}7 & 21 & 14\\ 0 & 35 & 49\\ 42 & 28 & 56\end{bmatrix}+\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

$$A^2+7A+3I=\begin{bmatrix}23 & 47 & 53\\ 42 & 91 & 140\\ 96 & 98 & 163\end{bmatrix}$$

Q:- $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Prove that $A^2 = 4A + 5I$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$4A + 5I = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$\underline{A^2 = 4A + 5I}$$

Transpose of a matrix:- let $A = [a_{ij}]_{m \times n}$ then on interchanging rows by columns and columns by rows we get the transpose of a matrix.
The transpose of a matrix is denoted by A^T, A', A^t.

$$A^T = [a_{ji}]_{n \times m}$$

Ex:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ $(A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \end{bmatrix}$

$$\boxed{A = (A^T)^T}$$

Application:-

→ $(A+B)^T = A^T + B^T$

→ $(AB)^T = B^T A^T$ (reversal law)

→ $(kA)^T = kA^T$

Symmetric Matrix:- A square matrix $A=[a_{ij}]_{n\times n}$ is called a symmetric matrix. if $a_{ij} = a_{ji}\ \forall\ i,j$.

$$A = \begin{bmatrix} a & g & h \\ g & b & f \\ h & f & c \end{bmatrix}, \quad a_{12} = a_{21},\ a_{13} = a_{31},\ a_{23} = a_{32}$$

$$A^T = \begin{bmatrix} a & g & h \\ g & b & f \\ h & f & c \end{bmatrix}$$

$$\boxed{A = A^T}$$

Skew-Symmetric Matrix:- A square matrix $A = [a_{ij}]_{n\times n}$ is called a skew-symmetric matrix if $a_{ij} = -a_{ji}$

$$A = \begin{bmatrix} 0 & g & h \\ -g & 0 & f \\ -h & -f & 0 \end{bmatrix}, \quad a_{23} = -a_{32},\ a_{13} = -a_{31},\ a_{21} = -a_{12}$$

→ Diagonal element must be zero.

$$A^T = \begin{bmatrix} 0 & -g & -h \\ g & 0 & -f \\ h & f & 0 \end{bmatrix} = -\begin{bmatrix} 0 & g & h \\ -g & 0 & f \\ -h & -f & 0 \end{bmatrix} = -A$$

$$\boxed{A^T = -A}$$

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Determinant:- It is just a number.
$\parallel$: $D \to R$ or C.

Ex:- $|1| = 1$
$|-1| = -1$
Determinant of a 1×1 order matrix is the element itself.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{12}(a_{22}) - a_{21}a_{12}$$

2x2 Matrix :->

(i) $\begin{vmatrix} 1 & 3 \\ 5 & 9 \end{vmatrix} = 9 - 15$
$= -6$

(ii) $\begin{vmatrix} 1 & -4 \\ 8 & 3 \end{vmatrix} = 3 + 32$
$= 35$

3x3 Matrix :->

(iii) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 1 \end{vmatrix} = 1(2-12) - 2(1-4) + 1(3-2)$
$= -10 + 6 + 1$
$= -3$ Ans.

Q (i) $\begin{vmatrix} 3 & -5 & 4 \\ 7 & 6 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 4 & 7 \\ -2 & 3 & 4 \\ 1 & 4 & -4 \end{vmatrix}$

Sol (i):- $3(18-2) - (-5)(21-1) + 4(14-6)$
$= 3\times16 + 5\times20 + 4\times8$
$= 48 + 100 + 32$
$= 80 + 100$
$= 180$ Ans.

(ii) $1(-12-16)-4(8-4)+7(-8-3)$
$= -28-16+7\times(-11)$
$= -28-16-77$
$= -121$ Ans.

Properties of determinants

(i) If all the element of ~~all~~ a row or column are zero then value of determinant is zero.

(ii) $|A^T| = |A|$

(iii) $|AB| = |A||B|$
$det(AB) = det(A)\,det(B)$

(iv) If two rows or column are identical then the value of $D = 0$.

Ex:- $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 3 \\ 2 & 2 & 4 \end{vmatrix} = 0$ $\qquad \begin{vmatrix} 2 & 4 & 2 \\ 2 & 4 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 0$

Note: In case of upper triangular, lower triangular, diagonal matrix, scalar matrix & Identity matrix the determinant is equal to the product of diagonal element.

$\begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & 0 \\ 5 & 6 & 7 \end{vmatrix} = 1\times3\times7 = 21.$ $\qquad \begin{vmatrix} 1 & 4 & 5 \\ 0 & 3 & 6 \\ 0 & 0 & 7 \end{vmatrix} = 1\times3\times7 = 21$

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Q:- Without expanding the determinant find the value of determinant.

(i) Prove that $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$

Sol:- $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$R1 + R2$

Changes are appear in R1 b/c it is written first.

$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$= 0$ ($\because$ Two rows are identical)

(ii) Show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Sol:-

$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

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$C_3 + C_2$

$$= \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & ab+ac+bc \\ 1 & ab & ab+ac+bc \end{vmatrix}$$

$$= (ab+ac+bc) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$= 0$ ($\because$ two column are identical)

(iii) Show that

$$\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

Sol $\Rightarrow$ $R_1 + R_2 + R_3$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$C_2 - C_1$, $C_3 - C_1$

$$= (5x+4) \begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

$= (5x+4)(4-x)^2$ {We know the property of det. that if upper Δ, or lower Δ then value of det. is product of diagonal.}

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Show that

(i) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(iii) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(ii) $C_1 - C_2$, $C_3 - C_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ a-b & b & c-a \\ a^2-b^2 & b^2 & c^2-a^2 \end{vmatrix}$$

$$= (a-b)\begin{vmatrix} 0 & 1 & 0 \\ 1 & b & c-a \\ a+b & b^2 & c^2-a^2 \end{vmatrix}$$

$$= (a-b)(c-a)\begin{vmatrix} 0 & 1 & 0 \\ 1 & b & 1 \\ a+b & b^2 & c+a \end{vmatrix}$$

$C_1 - C_3$

$$= (a-b)(c-a)\begin{vmatrix} 0 & 1 & 0 \\ 0 & b & 1 \\ b-c & b^2 & c+a \end{vmatrix}$$

$$= (a-b)(c-a)(b-c)\begin{vmatrix} 0 & 1 & 0 \\ 0 & b & 1 \\ 1 & b^2 & c+a \end{vmatrix} = (a-b)(c-a)(b-c)$$

(i) $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2x+2y+2z & x & y \\ 2x+2y+2z & y+z+2x & y \\ 2x+2y+2z & x & z+x+2y \end{vmatrix}$$

$$= (2x+2y+2z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

$R_2 - R_1 \quad , R_3 - R_1$

$$= (2x+2y+2z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)(x+y+z)^2 = 2(x+y+z)^3.$$

(iii) Take a common from C_1, b common from C_2 & c common from C_3.

$$= (abc) \begin{vmatrix} \frac{1}{a}+1 & 1/b & 1/c \\ 1/a & 1/b+1 & 1/c \\ 1/a & 1/b & 1/c+1 \end{vmatrix}$$

$C_1 + C_2 + C_3$

$$= abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1/b & 1/c \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1/b+1 & 1/c \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1/b & 1/c+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1/b & 1/c \\ 1 & 1/b+1 & 1/c \\ 1 & 1/b & 1/c+1 \end{vmatrix}$$

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$R_2 - R_1,\ R_3 - R_1$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1/b & 1/c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \text{ Ans.}$$

(IV) $$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0,$$

{Standard result of $1+\omega+\omega^2 = 0$}
its a pre defined.

$R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega^2+\omega & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

We know that $1+\omega+\omega^2$ is equal to zero

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

We know that if any row or column are zero then value of determinent will be zero

$$\boxed{\Delta = 0}$$

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Q:- Find the minor & ~~adjoint~~ cofactor of each element of $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

$$\left.\begin{aligned} M_{11} &= \cancel{(-1)^{i+j}}\, 6 = 6 \\ M_{12} &= \cancel{(-1)^{3}}\, 5 = +5 \\ M_{21} &= \cancel{(-1)^{3}}\, 4 = +4 \\ M_{22} &= \cancel{(-1)^{4}}\, 2 = 2 \end{aligned}\right\} \text{Minor}$$

$$\left.\begin{aligned} A_{11} &= (-1)^2 6 = 6 \\ A_{12} &= (-1)^3 5 = -5 \\ A_{21} &= (-1)^3 4 = -4 \\ A_{22} &= (-1)^4 2 = 2 \end{aligned}\right\} \begin{matrix} \cancel{\text{Adjoint}} \\ \text{Cofactor} \end{matrix}$$

$$\text{Adj } A = \cancel{A} \begin{bmatrix} 6 & -5 \\ -4 & 2 \end{bmatrix}^T$$

$$\boxed{\text{Adj } A = \begin{bmatrix} 6 & -4 \\ -5 & 2 \end{bmatrix}}$$

$$\boxed{A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{-8} \begin{bmatrix} 6 & -4 \\ -5 & 2 \end{bmatrix}}$$

Q:- find adjoint of $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

~~adj~~ = $A_{11} = -1 \qquad A_{21} = -1$

$A_{12} = -4 \qquad A_{22} = 2$

$$\text{Adj } A = \begin{bmatrix} -1 & -4 \\ -1 & 2 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix} \quad \underline{\text{Ans.}}$$

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Q Find the adjoint and Inverse of the following matrix:-

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

$$|A| = 1(4-3) - 1(0-1) + 2(-2)$$
$$= 1 + 1 - 4 = -2$$

~~Adj A =~~

$A_{11} = 4-3 = 1$
$A_{12} = -(-1) = 1$
$A_{13} = -2$
$A_{21} = -(2-6) = 4$
$A_{22} = 0$
$A_{23} = -(3-1) = -2$
$A_{31} = 1-4 = -3$
$A_{32} = -(1) = -1$
$A_{33} = 2$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 & -2 \\ 4 & 0 & -2 \\ -3 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & -3 \\ 1 & 0 & -1 \\ -2 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & 4 & -3 \\ 1 & 0 & -1 \\ -2 & -2 & 2 \end{bmatrix}$$

Q:- $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -1 \\ 3 & -2 & 1 \end{bmatrix}$ find A^{-1}.

$$|A| = 1(5-2) + 2(4+3) + 3(-8-15)$$
$$= 3 + 14 - 469$$
$$= 17 - 469$$
$$= -52$$

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$A_{11} = (5-2) = 3$
$A_{12} = -(4+3) = -7$
$A_{13} = (-8-15) = -23$
$A_{21} = -(-2+6) = -4$
$A_{22} = (1-9) = -8$
$A_{23} = -(-2+6) = -4$
$A_{31} = (2-15) = -13$
$A_{32} = -(-1-12) = 13$
$A_{33} = (5+8) = 13$

$$\text{Adj } A = \begin{bmatrix} 3 & -4 & -13 \\ -7 & -8 & 13 \\ -23 & -4 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-52}\begin{bmatrix} 3 & -4 & -13 \\ -7 & -8 & 13 \\ -23 & -4 & 13 \end{bmatrix}$$

Q. $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ find A^{-1}.

$|A| = -1(-4-3) \cancel{+}1(12+1) + 2(9\cancel{+}1)$
$= 7 - 13 + 16$
$= 10$

$A_{11} = -7$ $\quad A_{21} = 2$ $\quad A_{31} = 3$
$A_{12} = -13$ $\quad A_{22} = -2$ $\quad A_{32} = 7$
$A_{13} = +8$ $\quad A_{23} = 2$ $\quad A_{33} = -2$

$$\text{Adj } A = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$ Ans.

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Imp Result :->

$$\boxed{A(\text{adj } A) = |A| I}$$

What is the value of $|\text{adj } A|$?

~~Adj A~~ $A \text{ adj } A = |A| I$

$$= |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$|A| \, |\text{adj} A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$|A| \, |\text{adj} A| = |A|^3$$

$$\boxed{|\text{adj} A| = |A|^2 = |A|^{n-1}} \left\{ \because n = \text{order of matrix} \right\}$$

If $|A| = 3$ and A is 4×4 order matrix.

$$|A \text{ adj } A| = |A| \cdot I$$

$$|\text{adj } A| = |A|^3$$

$$= 3^3 = 27 \text{ Ans}$$

Properties of Inverse :-

① $(A^{-1})^{-1} = A$

② $(A+B)^{-1} \neq A^{-1} + B^{-1}$

③ $(AB)^{-1} = B^{-1} A^{-1}$ (Inversal law)

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④ $A^{-1} = \frac{1}{|A|}(\text{adj} A)$

⑤ $|A^{-1}| = \frac{1}{|A|}$

Q:- If $|A^{-1}| = 10$. what is the value of det(A).

$|A^{-1}| = \frac{1}{|A|}$

$\boxed{|A| = \frac{1}{10}} = \det(A)$

Rank of Matrix :-

A rectangular matrix $A = [a_{ij}]_{m\times n}$ is said to have rank r. If
(i) $\exists$ a minor of $r\times r$ order which doesn't vanish
(ii) Every Submatrix of $(r+1)\times(r+1)$ order vanishes.
Rank of the matrix is denoted by $\rho(A)$.

Q:- find the rank of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$|A| = 1(2-1) - 1(1-2) + 2(1-4)$$
$$= 1 + 1 - 6$$
$$= 2 - 6$$
$$= -4 \neq 0$$

Rank $\Rightarrow \boxed{\rho(A) = 3}$

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(ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = 0$. As two rows are identical.
$\rho(A) \neq 3$

Value of determinant is zero. So, we take sub matrix whose determinant is not zero.

$$\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1 \neq 0$$

$$\boxed{\rho(A) = 2}$$

Q: find the rank of following matrix!-

(i) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = 0$ $\{\because$ All row are identical$\}$
$\rho(A) \neq 3$.
So, we take 2×2 order matrix.
All the minor of 2×2 order they also vanish.
$\rho(A) \neq 2$.

$[1] = 1 \neq 0$
$\boxed{\rho(A) = 1}$

(1) Rank of a non-zero matrix is atleast 1.
$\rho(A) \geq 1$.
(2) Rank of the zero matrix is zero.

Row Echelon form of the Matrix :-

This element must be 1.

$$A = \begin{array}{l} \\ \text{atleast one zero on } R_{21} \\ \text{atleast two zero on } R_{31}, R_{32} \\ \text{atleast three zero on } R_{41}, R_{42}, R_{43} \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ \boxed{0} & 5 & 6 & 7 \\ \boxed{0} & \boxed{0} & 8 & 9 \\ \boxed{0} & \boxed{0} & \boxed{0} & 10 \end{bmatrix} \checkmark \qquad A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ \boxed{0} & 0 & 5 & 6 \\ \boxed{0} & \boxed{0} & 0 & 3 \\ \boxed{0} & \boxed{0} & \boxed{0} & 0 \end{bmatrix} \checkmark$$

Q :- $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$

$R_2 - 2R_1 , \quad R_3 - 3R_1 , \quad R_4 - 4R_1$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$|A| = 1 \qquad \rho(A) = 1$

Rank of matrix is 1.

Q Find the Rank of following matrix

{Always prefer first row to change other row

(i) $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_2$ ~~$R_1 + R_2$~~ , ~~$R_2 - R_4$~~

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & +1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 - 3R_1$, $R_4 - R_1$

$$[A] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 - R_2$, $R_4 - R_2$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\rho(A) = 2}$ {Two rows are non-zero}

Q:- $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_2 - R_3$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 - 4R_2$, $R_4 - 9R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

$R_4 - 2R_3$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

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(ii) $A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 2 & 5 & 1 \\ -4 & -8 & 1 & -3 & 1 \end{bmatrix}$

$R_2 - 2R_1 \,,\; R_3 - 3R_1 \,,\; R_4 + 4R_1$

$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 2 & 2 & -2 \\ 0 & 0 & 1 & 1 & 5 \end{bmatrix}$

$R_4 - R_2$

$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 2 & 2 & -2 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

$R_3 - 2R_2$

$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

$\boxed{\rho(A) = 4}$

Q:- 30 head, 100 legs. How many cows & crows are there?

Let No. of cows be $= x$

No. of crows be $= y$

$x + y = 30$

$4x + 2y = 100$

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$$4(30-y)+2y=100$$
$$120-4y+2y=100$$
$$20=2y$$
$$\boxed{y=10}$$
$$\boxed{x=20}$$

Consider a liner System of Equations:-

$$a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1$$
$$a_{21}x_2+a_{22}x_2+\cdots+a_{2n}x_n=b_2$$
$$\vdots$$
$$a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n=b_n$$

$b_1, b_2, b_3 \ldots b_n \neq 0$ then it is non-homogeneous System of Equations.

$b_1, b_2, b_3 \ldots b_n = 0$ cann't be the sol^n of the the non-homogeneous System of Equations.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$AX=B$$

This System has three types of Sol^n:-

1. Unique Sol^n
2. Infinitely many Sol^n
3. No Sol^n

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(i) Unique Sol^n:-
$\rho(A) = \rho(A|B) =$ no. of unknowns.
System has a unique Sol^n.

(ii) Infinitely Many Sol^n:-

$\rho(A) = \rho(A|B) <$ no. of unknowns.
The System has a infinitely many sol^n.

(iii) No Sol^n :-
$\rho(A) \neq \rho(A|B)$
The System has no Sol^n.

Q:- Solve the following System of Equation:-

$x+y+z = 2$
$6x-4y+5z = 31$
$5x+2y+2z = 13$

Sol:-
$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 6 & -4 & 5 & 31 \\ 5 & 2 & 2 & 13 \end{array}\right]$$

$R_2 - 6R_1$, $R_3 - 5R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -10 & -1 & 19 \\ 0 & -3 & -3 & 3 \end{array}\right]$$

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$R_3\left(-\frac{1}{3}\right)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -10 & -1 & 19 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & -10 & -1 & 19 \end{array}\right]$$

$R_3 + 10 \times R_2$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 9 & 9 \end{array}\right] \end{array}$$

A

$\rho(A) = 3$, $\rho(A|B) = 3$, no. of unknowns $= 3$

$\rho(A) = \rho(A|B) =$ no. of unique sol^n.

System has a unique solution.

$x + y + z = 9$

$0 + 0 + 9z = 9$

$\boxed{z = 1}$

$x + y + z = -1$

$0 + y + z = -1$

$\boxed{y = -2}$

$x + y + z = 2$

$x - 2 + 1 = 2$

$\boxed{x = 3}$

Sol:- $[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -2 & -15 \\ 2 & 1 & -5 & -21 \\ 1 & -1 & 1 & 18 \end{array}\right]$

$R_2 - 2R_1 \ , \ R_3 - R_1$

$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & -15 \\ 0 & -3 & -1 & 9 \\ 0 & -3 & 3 & 33 \end{array}\right]$

$R_3 - R_2$

$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & -15 \\ 0 & -3 & -1 & 9 \\ 0 & 0 & 4 & 24 \end{array}\right]$

$\rho(A) = \rho(A|B) =$ no. of unknowns.

This is unique solⁿ.

$x = 7 \quad y = -5 \ , \ z = 6$

Q:- find the value of lamda and u. for following

(i) No Sol^n

(ii) unique Sol^n

(iii) Infinite Sol^n.

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 1 & -6 & -17 & -19 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 - 2R_1 \ , \ R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 0 & 15 & 39 & 47 \\ 0 & 15 & \lambda+34 & \mu+38 \end{array}\right]$$

$R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 0 & 15 & 39 & 47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

(1) The system has no solution if

$$\rho(A) \neq \rho(A|B)$$

$\boxed{\lambda = 5}$ $\boxed{\mu \neq 9}$

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(2) The System has a unique Sol^n.

$\rho(A) = \rho(A|B) = $ No. of unknowns (3)

$\boxed{\lambda \neq 5}$ $\boxed{\cancel{\mu \neq 9}}$

μ can take any value.

(3) Infinite Many Sol^n

$\rho(A) = \rho(A|B) <$ no. of unknown

$\boxed{\lambda = 5 \quad , \mu = 9}$

Q'→ $2x - y + 3z = 9$
$x - 3y - 2z = 0$
$3x + 2y - z = -1$

$\left\{\begin{array}{l} y = -1 \\ x = 1 \\ z = 2 \end{array}\right\}$ Ans

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 9 \\ 1 & -3 & -2 & 0 \\ 3 & 2 & -1 & -1 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 2 & -1 & 3 & 9 \\ 3 & 2 & -1 & -1 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 5 & & \\ & & & \end{array}\right]$$

Homogeneous System of Equations:-

$a_{11}x + a_{12}y + a_{13}z = 0$
$a_{21}x + a_{22}y + a_{23}z = 0$
$a_{31}x + a_{32}y + a_{33}z = 0$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = 0$

If we set $X = 0$

$\boxed{0 = 0}$

$X = 0$ is always a solⁿ of the homogeneous System of Equation.

Homogeneous System of Equations has only two types of solⁿ:-

1. Zero Solution (trivial solⁿ)
2. Non-zero solⁿ (Non-trivial solⁿ)
   ↓
   Infinitely Many Solution.

$AX = 0$

case 1:- $|A| \neq 0$ then A^{-1} exists.

$A^{-1}(AX) = A^{-1} \cdot 0$

$(A^{-1}A)X = 0$

$XI = 0$

$\boxed{X = 0}$

$x = 0, y = 0, z = 0$ is the solⁿ.

This solⁿ is called a trivial solⁿ.

case-2:- If |A ...
System ...
i.e. Sy...

Q:- Solve ...

x + ...
3x ...
7x ...

|A| ...

|A ...

Th...

Q:- D...

case-2 ⇒ If $|A| = 0$
System has non-trivial sol^n.
i.e. System has infinitely many sol^n.

Q:- Solve the following system of equation.

$x + 2y + 3z = 0$
$3x + 4y + 4z = 0$
$7x + 10y + 12z = 0$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX = 0$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{vmatrix} = 1(48-40) - 2(36-28) + 3(30-28)$$
$$= 8 - 16 + 6$$
$$= -2 \neq 0$$

This system has only zero sol^n.
$x = 0, \; y = 0, \; z = 0.$

Q:- Determine the value of K for which the following System of Equation.

$(3k-8)x + 3y + 3z = 0$
$3x + (3k-8)y + 3z = 0$
$3x + 3y + (3k-8)z = 0$

has a non-trivial sol^n.

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Sol (Correct)

$$\begin{bmatrix} 3k-8 & 3 & 3 \\ 3 & 3k-8 & 3 \\ 3 & 3 & 3k-8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As this System has a non-trivial Soln.

$$\begin{vmatrix} 3k-8 & 3 & 3 \\ 3 & 3k-8 & 3 \\ 3 & 3 & 3k-8 \end{vmatrix} = 0$$

$R_1 + R_2 + R_3$

$$\begin{vmatrix} 3k-2 & 3k-2 & 3k-2 \\ 3 & 3k-8 & 3 \\ 3 & 3 & 3k-8 \end{vmatrix} = 0$$

$$(3k-2) \begin{vmatrix} 1 & 1 & 1 \\ 3 & 3k-8 & 3 \\ 3 & 3 & 3k-8 \end{vmatrix} = 0$$

$C_1 - C_2 , C_2 - C_3$

$$(3k-2) \begin{vmatrix} 0 & 0 & 1 \\ 11-3k & 3k-11 & 3 \\ 0 & 11-3k & 3k-8 \end{vmatrix} = 0$$

$$(3k-2)(11-3k)(11-3k) = 0$$

$3k-2=0$

$k = 2/3$, ~~3/11, 3/11~~

$11-3k=0$

$3k = 11$

$k = 11/3, 11/3$

Ans $\boxed{k = 2/3, \frac{11}{3}, \frac{11}{3}}$

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Q:- A following System has non-trivial Sol^n then prove that either $a+b+c=0$ or $a=b=c$

$$ax+by+cz=0$$
$$bx+cy+az=0$$
$$cx+ay+bz=0$$

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX = 0$$

The System has a Non-Zero Sol^n.

$$\therefore |A| = 0$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = 0$$

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$

$R_1 - R_2,\ R_2 - R_3$

$$(a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix} = 0$$

Multiply by (-1) on above equation:-

$$(a+b+c)(-1)\{(b-c)(a-b) - \{(c-a)(c-a)\}\} = 0$$

$$(a+b+c)(-1)[ba - b^2 - \cancel{ca} + bc - c^2 + \cancel{ac} + ac - a^2] = 0$$

$$\frac{1}{2}(a+b+c)[2a^2+2b^2+2c^2-2ab-2bc-2ca] = 0$$

$$\frac{1}{2}(a+b+c)[(a^2+a^2)+(b^2+b^2)+(c^2+c^2)-2ab-2bc-2ca] = 0$$

$$\frac{1}{2}(a+b+c)[(a^2+b^2+\cancel{c^2}-2ab)+(b^2+c^2-2bc)+(a^2+c^2-2ac)] = 0$$

$$\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 0$$

either $(a+b+c) = 0$ or $(a-b)^2+(b-c)^2+(c-a)^2 = 0$

$a-b=0, \quad b-c=0, \quad c-a=0$

$a=b, \quad b=c, \quad c=a$

$$\boxed{a=b=c}$$