TWO Phase Simplex Method

Steps involved:

1. Modify the constraints so that the right-hand side of each constraint is nonnegative.

2. Identify each constraint with ≥ or = constraint, add an artificial variable to each constraint.

3. Convert each inequality constraint to the standard form. If constraint is ≤ constraint, then add a slack variable. If constraint is ≥ constraint, subtract an excess variable.

   [The above steps are very similar to BIG M Method]

4. For now, ignore the original LP's objective function. Instead solve an LP whose objective function is

   Minimize W = (sum of all the artificial variables).

   This is called the Phase I LP. The act of solving the Phase I LP will force the artificial variables to be zero. [Eliminate the artificial variable in the objective function row [W] by converting it to 0, before starting the simplex method for the above LPP]

Because each a_i [artificial variables] ≥ 0, solving the Phase I LP will result in one of the following cases:

Case 1: If the optimal value of W is greater than zero, then the original LP has no feasible solution.

Case 2: If the optimal value of W is equal to zero, and no artificial variables are in the optimal Phase I basis; then drop all columns in the optimal Phase I tableau that corresponds to the artificial variables.

We now combine the original objective function with the constraints from the optimal Phase I tableau.

This yields the Phase II LP. The optimal solution to the Phase II LP is the optimal solution to the original LP.

Example – without solutions

Maximize $Z = 4x + 5y$

Subject to

$$2x + 3y \leq 6$$

$$3x + y \geq 3$$

$$x, y \geq 0$$

First, convert the constrains to equations

Maximize $Z = 4x + 5y$

Subject to

$$2x + 3y + s_1 = 6$$

$$3x + y - e + a = 3$$

$$x, y, s_1, e, a \geq 0$$

S_1 is the slack variable; e is the surplus and a is the artificial variable.

Now, ignore the original LP's objective function. Instead solve an LP whose objective function

Minimize W = a [sum of artificial variables, here we have only one artificial variable]

Then, Phase I LP will be as follows

Minimize $W = a$

Subject to

$$2x + 3y + s + 0e + 0a + 0W = 6$$

$$3x + y + 0s - e + a + 0W = 3$$

$$x, y, s, e, a \geq 0$$

Initial Simplex table for the Minimization problem is

	x	y	s	e	a	W	
x	2	3	1	0	0	0	6
y	3	1	0	-1	1	0	3
W	0	0	0	0	-1	1	0

Now, eliminate the artificial variable in the objective function row by converting it to 0. The resultant table is as given below. Solve this using the usual simplex method.

	x	y	s	e	a	W	
s	2	3	1	0	0	0	6
a	$\boxed{3}$	1	0	-1	1	0	3
W	3	1	0	-1	0	1	3

After applying simplex method, the final table in the PHASE I simplex method is

	x	y	s	e	a	W	
s	0	7/3	1	2/3	-2/3	0	4
x	1	1/3	0	-1/3	1/3	0	1
W	0	0	0	0	-1	1	0

Since there is no positive number in the objective row, STOP. The optimal basis s = 4 and x = 1. The optimal objective value W = 0 and a is not in the optimal Phase I basis. Therefore, it is Case 2

Hence, now generate Phase II initial tableau by changing the objective function row into the original objective function ($Z = 4x + 5y$). Also, drop the artificial column.

	x	y	s	e	Z	
s	0	7/3	1	2/3	0	4
x	1	1/3	0	-1/3	0	1
Z	-4	-5	0	0	1	0

Now, apply the Simplex method for this objective function and find the optimal solution.