Department of Computer Applications

23MX11 – Exercise Problems in Propositional Calculus

1. Show that $p \wedge (q \vee r)$ & $(p \wedge q) \vee r$ are not logically equivalent

2. Let p stand for the proposition "I bought a lottery ticket" and q for "I won the jackpot". Express the following as natural English sentences:

   (a) $\neg p$ (b) $p \vee q$ (c) $p \wedge q$ (d) $p \Rightarrow q$ (e) $\neg p \Rightarrow \neg q$ (f) $\neg p \vee (p \wedge q)$

3. Check for the validity of the following arguments
   If $x > 2$, then $x^2 > 4$. $x > 2$. Therefore, $x^2 > 4$

4. Determine the validity of the argument: $p \rightarrow q \vee \sim r \; q \rightarrow p \wedge r \therefore p \rightarrow r$

5. If 18,486 is divisible by 18, then 18,486 is divisible by 9. If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9. ∴ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9. Determine the validity of the given arguments.

6. State whether the following are true or false, where x , y and z range over the integers.

   (a) $\forall x, \exists y, (2x - y = 0)$ (b) $\exists y, \forall x, (2x - y = 0)$ (c) $\forall x, \exists y, (x - 2y = 0)$
   (d) $\forall x, x < 10 \Rightarrow \forall y, (y < x \Rightarrow y < 9)$ (e) $\exists y, \exists z, y + z = 100$
   (f) $\forall x, \exists y, (y > x \wedge \exists z, y + z = 100)$

7. Show that $\neg(p \wedge q) \vee (\neg p \wedge q) \equiv \neg p$ without constructing the truth tables

8. Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.

9. Show that the following proposition is a tautology : $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

10. Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent ?
