3. Find the mode of the following distribution.

class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	25	34	50	42	38	14	10

$\hookrightarrow$ maximum frequency

Modal class = 20-30

$l = 20 \quad h = 10 \quad f_1 = 34 \quad f_2 = 42 \quad f_m = 50$

$$\text{Mode} = l + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right] h$$

$$= 20 + \left[\frac{50-34}{2(50)-34-42}\right] 10$$

$$= 20 + \left[\frac{16}{100-76}\right] 10$$

$$= 20 + \frac{\cancel{160}^{\cancel{40}\ 20}}{\cancel{24}_{6\ 3}}$$

$$= 80/3$$

Mode = 26.67

$\therefore$ The Mode value is 26.67

Measures of Dispersion :-

Absolute measures of Dispersion.

Relative measures of Dispersion

Absolute measures of Dispersion :-

Range : Difference between maximum & minimum value

Variance: $\sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$ $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}$

Standard deviation: $\sigma = \sqrt{variance} = \sqrt{\frac{\Sigma(x-\mu)^2}{N}}$

Quartile and Quartile Deviation :

Mean and Mean Deviation :

1. Find the variance of the numbers 3, 8, 6, 10, 12, 9, 11, 10, 12, 7

Soln :-

$$Mean\ (\mu) = \frac{3+8+6+10+12+9+11+10+12+7}{10}$$

$$= 88/10 = 8.8$$

x_i	$x_i - \mu$	$(x_i-\mu)^2$
3	-5.8	33.64
8	-0.8	0.64
6	-2.8	7.84
10	1.2	1.44
12	3.2	10.24
9	0.2	0.04
11	2.2	4.84
10	1.2	1.44
12	3.2	10.24
7	-1.8	3.24

$\Sigma(x_i-\mu)^2 =$ ~~[illegible]~~ 73.6

$$\text{variance},\ \sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$$

$$= \frac{73.6}{10} = 7.36$$

$\therefore$ variance $\sigma^2 = 7.36$

→ standard deviation, $\sigma = \sqrt{\text{variance}} = \sqrt{7.36}$
$= 2.7129$

coefficient of variance :

It is the relative measure of variability that indicates ratio of the SD to the mean

$$CV = \frac{\text{Standard Deviation}}{\text{Mean}}$$

Find the coefficient of variance of the following sample set of numbers {1, 5, 6, 8, 10, 40, 65, 88}

Mean (μ) $= \frac{1+5+6+8+10+40+65+88}{8}$
$= 223/8$
$= 27.875$
$= 27.88$

x_i	$x_i-\mu$	$(x_i-\mu)^2$
1	-26.88	722.53
5	-22.88	523.49
6	-21.88	478.73
8	-19.88	395.21
10	-17.88	319.69
40	12.12	146.89
65	37.12	1377.89
88	60.12	3614.41

$$\sum_{i=1}^{n}(x_i-\mu)^2 = 7576.84$$

$$\text{variance},\ \sigma^2 = \frac{\Sigma(x-\mu)^2}{N} = \frac{7578.84}{8} = 947.35$$

$$SD,\ \sigma = \sqrt{\text{variance}} = \sqrt{947.35} = 30.78$$

$$CV = SD/\text{mean} = 30.78/27.88 = 1.104$$

16/11

→ possibility.

Probability.

Experiment

Deterministic — able to find outcome of Experiment

Ex: 2+3 = 5 → standard output.

Non-Deterministic — prediction of outcome is not known.

Ex: Toss of coin → Head, Tail.

Deterministic :- Perfect prodification of the Experiment's outcome.

cooling water below zero degree - It will Freeze (outcome)

Non-Deterministic :-

It has more than one possible outcomes. Due to uncertainty, outcomes of this experiment does not predicted before performing the experiment.

(outcome)

Rolling a die - {1, 2, 3, 4, 5, 6}

Tossing a coin - {Head, Tail}

Examination result - {PASS, FAIL}

Non-deterministic Experiment

Random Experiment	Haphazard Experiment
has individual outcomes that are not completely predictable but probabilities associated with the possible outcomes a well-defined.	has individual outcomes where the probability associated with the possible outcomes are unknown. there is no structure/ pattern for this.

Random Experiment :-

S - sample space

Example :-

1. Tossing a coin :- $S = \{Head, Tail\}$

2. Rolling a Die :- $S = \{1, 2, 3, 4, 5, 6\}$

BASIC TERMINOLOGIES.

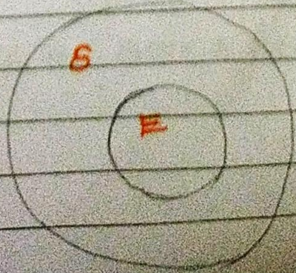
Sample Space (S) :- set of all possible outcomes

The sample space (S) for a random experiment is the set of all possible outcomes.

Event (E) :-

E is any subset of the sample space S.

A set of outcomes (not necessarily all outcomes) of the random Experiment.



Example:

1. Rolling a die:

$S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$

Let E be the event of getting even number is rolled.

$E = \{2, 4, 6\}$ $n(E) = 3$

2. Rolling two die:

$$S = \{(1,1), (1,2), \dots, (1,6)$$
$$(2,1), (2,2), \dots, (2,6)$$
$$\dots \quad \dots \quad \dots$$
$$(6,1), (6,2), \dots, (6,6)\}$$

$n(S) = 36$

Let E be the event of getting the sum as 8 by rolling two die.

$E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$n(E) = 5$

3. Number of customers in a queue - outcomes.

$S = \{0, 1, 2, \dots, n\}$ finite queue.

E = No customers in the queue.

$E = \{0\}$ $n(E) = 1$

Special Events:

The Null Event, the empty Event, ϕ.

Example:

1. In a coin toss.
   The null set is neither heads nor tails is observed.

Null event ϕ - never occurs.
Entire event S - always occurs.

2. In a rolling die
   The null set is expecting 7.

Set operations on Events:

Union:

A & B → Events.
Union of A and B is another event

$A \cup B = \{ O \mid O \in A \text{ or } O \in B \}$

$A \cup B$ occurs if the event A occurs or event B occurs.

Intersection:

A & B → Events
Intersection of A and B is also an event

$A \cap B = \{ O \mid O \in A \text{ and } O \in B \}$

$A \cap B$ occurs if the event A occurs and event B occurs.

Complement

$A \rightarrow$ Event

Complement of A $(\bar{A}) \rightarrow$

$\bar{A} = \{ o \mid o$ does not belongs to $A \}$

$\bar{A}$ event occurs if the event A does not occur.

Mutually Exclusive Event :-

Two events A and B are called mutually Exclusive only if

$A \cap B = \phi$ ↳ null event.

also called as Disjoint.

A and B are independent events

Probability :-

Probability of an Event (E) :-

Suppose that the sample space $[S]$

$S = \{ o_1, o_2, o_3, \dots, o_N \}$ has a finite number (N) of outcomes.

Also each of the outcomes is equally likely.

Then the event E

$$P[E] = \frac{n(E)}{n(S)} = \frac{n(E)}{N} = \frac{\text{no. of outcomes in } E}{\text{total no of outcomes}}$$

Note :- $n(E) \rightarrow$ no of Elements of E.

↳ cardinality of E.

Rules of Probability :

Additive Rule :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the event A and B are mutually exclusive

$$P(A \cap B) = \phi$$

$$P(A \cup B) = P(A) + P(B)$$

city 1 and city 2 are two of cities competing for the world university games. (There are also many others). The organizers are narrowing the competition to the final 5 cities.

There is a 20% chance that city 1 will be amongst the final 5.

There is a 35% chance that city 2 will be amongst the final 5 and

an 8% chance that both city 1 and city 2 will be amongst the final 5.

What is the probability the city 1 or city 2 will be amongst the final 5.

soln :

$C1 \rightarrow$ city 1 $\quad C2 \rightarrow$ city 2

$P(C1) = 20\% \quad P(C2) = 35\% \quad P(C1 \cap C2) = 8\%$

$= 0.20 \quad = 0.35 \quad 0.08$

$$P(C1 \cup C2) = P(C1) + P(C2) - P(C1 \cap C2)$$

$$= 0.20 + 0.35 - 0.08$$

$$P(C1 \cup C2) = 0.47$$

Probability that city 1 or city 2 amongst the final 5 is 47%.

A fair coin is tossed 4 times. Define the sample space corresponding to the random experiment.

$S = \{$ HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTTT, TTTH, THHT, THTH, TTTT, HTHT, THTT, TTHT $\}$

$n(S) = 16$

Write the subsets space corresponding to the following events and also find the respective probabilities.

(i) More heads than tails are obtained.

Let E1 be the event of getting more heads than tails.

$E1 = \{$ HHHH, HHHT, HHTH, HTHH, THHH $\}$

$n(E1) = 5$

$$P(E1) = \frac{n(E1)}{n(S)} = \frac{5}{16} = 0.3125$$

(ii) Tail occur on the even number of tosses.

Let E2 be the event of getting tail on the even number of tosses.

$E2 = \{$ HTHT, TTHT, TTTT, HTTT $\}$

$n(E2) = 4$

$$P(E2) = \frac{n(E2)}{n(S)} = \frac{4}{16} = 1/4 = 0.25$$

sample problem.

3. A letter is chosen at random from the word "Mathematics". Find the probability of getting the letter "I".

$n(I) = 1$ $\quad n(S) = 11$ $\quad P(I) = 1/11$

4. A die is thrown, find the probability of getting a value which is multiple of 2 or 3.

A = Event of getting multiple of 2 $\quad S = \{1, 2, 3, 4, 5, 6\}$

B = Event of getting multiple of 3

$A = \{2, 4, 6\}$ $\quad B = \{3, 6\}$ $\quad A \cap B = \{6\}$

$n(A) = 3$ $\quad n(B) = 2$ $\quad n(A \cap B) = 1$

$P(A) = \frac{n(A)}{n(S)} = 3/6 = 1/2$ $\quad P(B) = \frac{n(B)}{n(S)} = 2/6$ $\quad P(A \cap B) = 1/6$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{3+2-1}{6}$$
$$P(A \cup B) = 4/6 = 2/3$$

6. Find the probability of getting a numbered cards when a card is drawn from the pack of 52 cards.

$$= \frac{9 \times 4}{52} = 9/13$$

Honor cards: K, Q, J, A

Black: 26 cards (spade, club)

Face cards: K, Q, J

Red: 26 cards (Heart, Diamond)

7. What is the probability of getting a sum of 7 when two dice are thrown?

$S = \{(1,1), (1,2) \cdots (6,6)\}$ $\quad n(S) = 36$

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$ $\quad n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = 6/36 = 1/6$$

9. Find the Probability of getting 53 sundays in a leap year.

No. of. days in a leap year = 366

No. of. days in a week = 7

No. of. weeks in a leap year = 52 weeks 2 days.

2 days :-

S = {(sun, mon), (mon, tues), (tue, wed), (wed, thu), (thur, Frid), (Frid, sat), (sat, sun)}

$n(S) = 7$

A = {(sun, mon), (sat, sun)}

$n(A) = 2$

$P(A) = 2/7$.

10. Find the probability of selecting a red card or 2 from a deck of 52 cards.

S = Deck of cards $n(S) = 52$

A = Event of getting red cards $n(A) = 26$

B = Event of getting no. 2 card $n(B) = 4$

$A \cap B$ = both red and 2 no card

$n(A \cap B) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52}$ $P(B) = \frac{4}{52}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$= \frac{2}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{26 + 4 - 2}{52} = \frac{\cancel{28}\,7}{\cancel{52}\,13}$

$= 7/13$.

11. one card in drawn at random from the pack of 52 cards. Find the probability that
(i) it is an honor card
(ii) it is a face card.

(i) Honor card = 4 cards in each category.

$4 \times 4 = 16$

$n(H) = 16 \quad n(S) = 52$

$p(H) = \frac{n(H)}{n(S)} = \frac{16}{52} = \frac{4}{13}$

(ii) Face card = 3 cards in each category.

$3 * 4 = 12$

$n(F) = 12 \quad n(S) = 52$

$p(F) = \frac{n(F)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

12. one card is drawn at random from the pack of 52 cards. Find the probability that is an honor card of red colour.

S = Deck of cards $n(S) = 52$

A = Event of getting honor card of red colour.

$n(A) = 8 \quad p(A) = n(A) / n(S) = 8/52 = 2/13$.

1. A English alphabet is chosen at random. Find the probability that it is letter from "RANDOM".

S = English Alphabet $n(S) = 26$

A = event of chosing letter from "RANDOM".

$A = \{R, A, N, D, O, M\}$

$n(A) = 6$

$P(A) = n(A) / n(S)$

$= 6/26 = 3/13$.

13. A bag contains 3 red balls and 4 black balls. A ball is drawn from the bag. Find the probability the drawn is (i) Black (ii) not black.

(i) S = 3 red Balls, 4 black balls $n(S) = 7$

↳ B1 - event of getting Black ball

$n(B1) = 4$ $P(B1) = n(B1) / n(S)$

$= 4/7$

(ii)

B2 = event of getting not black ball.

$n(B2) = 3$ $P(B2) = n(B2) / n(S)$

$= 3/7$

14. A box containing card numbered 1, 2, 3, ..., 15. A card is drawn from the box. Find the probability of getting a prime numbered card.

S = card numbered from 1 to 15

$n(S)$

A = even of getting prime numbered card.

$A = \{2, 3, 5, 7, 11, 13\}$ $n(A) = 6$

$P(A) = \frac{n(A)}{n(S)} = 6/15$

2. In the word "EDUCATION", find the probability of getting a vowel, if a letter is chosen at random.

$S = \{E, D, U, C, A, T, I, O, N\}$ $n(S) = 9$

A = event of getting vowel character from S.

$A = \{A, E, I, O, U\}$ $n(A) = 5$

$P(A) = n(A) / n(S)$

$P(A) = 5/9$

there are 4 letters and 4 addressed envelope. If the letters are placed in the envelopes at random. find the probability that.

(a) none of the letters is in the correct envelope and.

(b) at least 1 letter is in correct envelope by explicitly writing the space and the event spaces.

5. A coin is thrown 3 times. What is the probability that atleast one head is obtained?

S = { HHH, HHT, HTH, HTT
THH, THT, TTH, TTT }

n(S) = 8

A = event of getting atleast one head.

A = { HHH, HHT, HTH, HTT, THH, THT, TTH }

n(A) = 7

P(A) = n(A)/n(S) = 7/8.

9. What is the probability of getting 3 or 6, if a die is rolled once.

S = {1, 2, 3, 4, 5, 6} n(S) = 6

A = event of getting 3 or 6

A = {3, 6} n(A) = 2

P(A) = n(A) / n(S) = 2/6 = 1/3

15. If a die is thrown once. What is the probability of getting 7?

S = {1, 2, 3, 4, 5, 6} n(S) = 6

A = event of getting 7

A = { } n(A) = 0

P(A) = 0.

23/11

Conditional Probability:

When we are trying to find the probability that one event will happen under the condition that some other event is already known to have occurred - conditional probability.

The probability that one event happens given that another event is already known to have happened is called a conditional probability.

Notation: $P(B|A)$, $P(A|B)$

Event A already occurred

trying to find $P(B)$ after event A occurred.

conditional probability of A given B is :

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \text{if } P[B] \neq 0.$$

conditional probability of B given A is :

$$P(B|A) = \frac{P(B\cap A)}{P(A)} \quad \text{if } P(A) \neq 0.$$

$$\hookrightarrow \frac{P(A\cap B)}{P(A)}$$

conditional probability and independence :

If A and B are independent events, then the probability that A and B both occur is :

$$P(A\cap B) = P(A)\,P(B).$$

conditional prob

In other words, events A and B with respect to independent

if

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \qquad P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$= \frac{P(A)\,P(B)}{P(B)} \qquad = \frac{P(A)\,P(B)}{P(A)}$$

$$P(A|B) = P(A) \qquad P(B|A) = P(B)$$

A coin is fliped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

$P(H) = 1/2$ $\quad P(4) = 1/6$ $\quad$ H and 4 → are independent events.

$P(H \cap 4) = 1/2 \cdot 1/6$

$= 1/12$.

A card is drawn from a deck and replaced; then a 2^{nd} card is drawn. Find the probability of getting a queen and then an ace.

$P(Q) = 4/52$ $\quad P(A) = 4/52$ $\quad$ Q and A are independent event

$P(Q \cap A) = \frac{4}{52} * \frac{4}{52}$

$= 1/13 \times 1/13 = 1/169$

Two dice are thrown together. Let A be the event 'getting 6 on the first die' and B be the event 'getting 2 on the second die'. Are the events A and B are independent?

$S = \{ (1,1), (1,2) \ldots$

$\ldots (6,5), (6,6) \}$ $\quad n(S) = 36$

A = event of getting 6 on the first die

B = event of getting 2 on the second die.

$A = \{ (6,1), (6,2) \ldots, (6,6) \}$ $\quad n(A) = 6$ $\quad A \cap B = \{ (6,2) \}$

$B = \{ (1,2), (2,2), (3,2), \ldots (6,2) \}$ $\quad n(B) = 6$ $\quad n(A \cap B) = 1$

$P(A) = 6/36$ $\quad P(B) = 6/36$ $\quad P(A \cap B) = 1/36$

$P(A \cap B) = P(A) \, P(B)$ $\quad P(A \cap B) = 1/36$.

$= \frac{6 * 6}{36 * 36} = 1/36$

∴ A and B are independent events.

Two cards are selected at random from the standard deck of 52 playing cards. The first card is replaced before the second card is drawn.

What is the probability that the first card is a 10 and the second card is a face card?

$P(A) = 4/52$ $P(B) = 12/52$ A and B are independent events.

no. 10 card face card

$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{4 * 12}{52 * 52}$

$P(A \cap B) = 48/2704 = 6/338 = 3/169$

(7m) -x-

Two cards are selected at random from a deck of 52 cards without replacement.

Find the probabilities for the following events.

(i) Both cards are red.

A - event of getting first card as red.

B - event of getting second card as red

$P(A) = \frac{26}{52}$ $P(B) = \frac{25}{51}$

A and B are independent events.

$P(A \cap B) = P(A) \times P(B)$

$= \frac{26 \times 25}{52 \times 51}$

$P(A \cap B) =$

(ii) The first card is a 7 and second card is a 4.

A - event of getting first card as 7

B - event of getting second card as 4.

$P(A) = \frac{4}{52}$ $P(B) = \frac{4}{51}$

A and B are independent events

$P(A \cap B) = P(A) \times P(B)$

$= \frac{4 \times 4}{52 \times 51}$

$P(A \cap B) =$

(iii) Both cards are five

A - event of getting first cards as 5.

B - event of getting second card also as 5

$p(A) = \frac{4}{52}$ $p(B) = \frac{3}{51}$

A and B are independent events.

$p(A \cap B) = p(A).p(B)$

$= \frac{4 \times 3}{52 \times 51}$

=

(iv) The first card is red and second is black.

A - event of getting first card is red

B - event of getting second card is black.

$p(A) = \frac{26}{52}$ $p(B) = \frac{26}{51}$

A and B are independent events.

$p(A \cap B) = p(A).p(B)$

$= \frac{26 \times 26}{52 \times 51}$

Conditional Probability of Mutually Exclusive Events.

Mutually Exclusive events are events that cannot occur simultaneously.

one event occured already, and the another event cannot occur at the same time.

Therefore, the mutually exclusive events conditional probability is always zero.

$P(B|A) = 0$ and $P(A|B) = 0$

Ex:

(i) coin - event of getting Head and tail.

(ii) Die - event of getting '2' and '5'.

One card is selected at random from a standard deck of 52 playing cards. What is the probability that it is a 10 or a face card?

A - event of getting a card as 10 $P(A) = 4/52$

B - event of getting a face card. $P(B) = 12/52$

A and B are mutually exclusive events.

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{4}{52} + \frac{12}{52}$$

$$= 16/52$$

$$P(A \cup B) = 4/13$$

Three coins are tossed at the same time. Let us say A as the event of receiving at least 2 head. Likewise, B denotes the event of getting no heads and C is the event of getting heads on the second coin.

Which of these events are mutually exclusive?

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

A = event of receiving atleast two heads

$A = \{HHH, HHT, HTH, THH\}$

$n(A) = 4$

B = event of getting no heads.

$B = \{TTT\}$ $n(B) = 1$

C = event of getting heads on second one

$C = \{HHH, HHT, THH, THT\}$

$n(C) = 4$

(i) $A \cap B \cap C = \{\ \}$ $n(A \cap B \cap C) = 0$ A, B and C are mutually exclusive events

(ii) $A \cap B = \{\ \}$ $n(A \cap B) = 0$ A and B are mutually exclusive events

(iii) $A \cap C = \{HHH, HHT, THH\}$ $n(A \cap C) = 3$

(iv) $B \cap C = \{\ \}$ $n(B \cap C) = 0$ B and C are mutually exclusive events.

A card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is a king or an ace.

A - event of getting a king card

B - event of getting a ace card.

$P(A) = \frac{4}{52}$ $P(B) = \frac{4}{52}$ A and B are mutually exclusive events.

$P(A \cup B) = P(A) + P(B)$

$= \frac{4}{52} + \frac{4}{52}$

$P(A \cup B) = \frac{8}{52} = 2/13$.

25/11.

10% of the bulbs produced in a factory are of red color and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

Let A - bulb is redcolor $P(A) = \frac{10}{100} = 1/10$

B - Defective bulb

2% are red and defective

$P(A \cap B) = \frac{2}{100} = 1/50$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(B|A) = \frac{1/50}{1/10} = 10/50 = 1/5$

$\therefore$ the probability of its being defective if it is red is $1/5$.

Two dice are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared atleast once?

$n(S) = 36$

let A - 3 has appeared once

B - sum of numbers is 7

$B = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$P(B) = 6/36$

$A \cap B = \{(3,4),(4,3)\}$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{6/36}$$

$$P(A|B) = \frac{36 \times 2}{6 \times 36} = 1/3$$

The probability the number 3 has appeared atleast once after the sum of numbers obtained is 7 $\Rightarrow$ 1/3.

A single six sided dice is rolled one time. determine the probability that a 2 was rolled, given that even number has been rolled.

$S = \{1,2,3,4,5,6\}$

let A - 2 was rolled

$B = \{2,4,6\}$

B - even number was rolled, $P(B) = 3/6$

$A \cap B = \{2\}$

$P(A \cap B) = 1/6$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{1/3}{3/6} = \frac{6}{9} = 2/3 \qquad \frac{1/6}{3/6} = \frac{6}{}$$

Probability of 2 was rolled, given that an even number has been rolled is 2/3

A director of nursing is selected from a group of 21 nurses, of whom 16 are women and 5 are men. Twelve of the women and two of the men have BSN (Bachelor of Science in Nursing) degrees. What is the probability that the person selected is a woman, given that the person has a BSN degree?

16 - women

5 - men

$\overline{21}$

A - selected person woman

B - The person has a BSN degree.

BSN →

12 - women

2 - men

$$P(B) = \frac{14}{21}$$

$$P(A \cap B) = 12/21$$

woman with BSN

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{12/21}{14/21} = \frac{12}{14} = 6/7.$$

For a specific married couple the probability that the husband watches a dance show is 80%, the probability that his wife watches the same show is 65%, while the probability that they both watch the dance show is 60%.

If the husband is watching the show, what is the probability that his wife is also watching the show.

H → Husband watching dance show

W → Wife watching dance show

Husband - 80%

Wife - 65%

$H \cap W = 60\%$

$$P(H) = \frac{80}{100} = \frac{8}{10}$$

$$P(H \cap W) = \frac{60}{100} = 6/10$$

$$P(W|H) = \frac{P(H \cap W)}{P(H)}$$

$$= \frac{6/10}{8/10} = \frac{6}{8} = \frac{3}{4} = 0.75$$

75%

suppose a box has 3 red marbles and 2 black ones.
select 2 marbles.

What is the probability that second marble is red given that the first one is red ?

A - first marble is red.   Red - 3

B - second marble is red   Black - 2

$$P(A) = 3/5$$

$$P(A \cap B) = 3/5$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{6/20}{3/5}$$

$$= \frac{\cancel{6}^{2}}{\cancel{20}_{10}} \times \frac{\cancel{5}^{1}}{\cancel{3}}$$

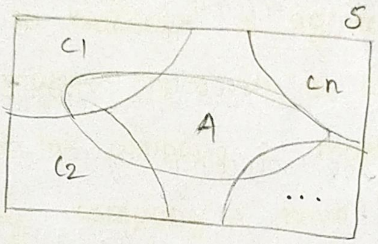
$$P(B|A) = 1/2$$

Law of total probability:

In probability theory, the law of total probability is a useful way to find the probability of some event A when the probability of A is not known, knowing that C1, C2, C3, ... form a partition of the sample space S.

Law of total probability:

$$P(A) = \sum P(A|C_i) * P(C_i)$$



For any event "A" in the sample space S, the event "A" can be denoted as:

A, $C_1, C_2, \ldots, C_n$ are events.

$S = C_1 \cup C_2 \cup C_3 \cup \ldots \cup C_n$

↓

$C_1, C_2, \ldots, C_n$ are disjoint sets

$$A = A \cap S \quad (\because S = C_1 \cup C_2 \cup \cdots \cup C_n)$$

$$= A \cap (C_1 \cup C_2 \cup C_3 \cup \ldots \cup C_n)$$

$$A = (A \cap C_1) \cup (A \cap C_2) \cup \ldots \cup (A \cap C_n)$$

$$P(A) = P[(A \cap C_1) \cup (A \cap C_2) \cup \ldots \cup (A \cap C_n)]$$

$$P(A) = P(A \cap C_1) + P(A \cap C_2) + \cdots + P(A \cap C_n).$$

Multiplication rule: $P(A|C) = \frac{P(A \cap C)}{P(C)}$

$$P(A \cap C_p) = P(C_p) \times P(A|C_p)$$

$$P(A) = P(C_1) . P(A|C_1) + P(C_2) . P(A|C_2) + \cdots + P(C_n) . P(A|C_n)$$

$$P(A) = \sum_{i=1}^{n} P(C_i) . P(A|C_i)$$

suppose there are two bags in a box, which contain the following marbles:

Bag 1: 7 red marbles and 3 green marbles

Bag 2: 2 red marbles and 8 green marbles

If we randomly select one of the bags and then randomly select one marble from that bag, what is the probability that it is a green marble?

B1 → Event of selecting Bag B1 $P(B1) = 1/2$

B2 → Event of selecting Bag B2. $P(B2) = 1/2$

A → Event of selecting a marble from green marble.

$P(A|B1) = 3/10$

$P(A|B2) = 8/10$

Total probability:

$$P(A) = P(B1) * P(A|B1) + P(B2) * P(A|B2)$$

$$= \frac{1}{2} * \frac{3}{10} + \frac{1}{2} * \frac{8}{10}$$

$$= 3/20 + 8/20$$

$$P(A) = 11/20$$

Bayes theorem:-

It calculates the probability of an event A conditional on another event B, given the prior probabilities of A and B and the conditional probability of B on A.

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)} \quad \text{Where } P(B) \neq 0.$$

An aircraft Emergency Locator Transmitter (ELT) is a device designed to transmit a signal in the case of crash. The company A makes 80 % of the ELTs, the company B makes 15 % of them, and the company C makes the other 5 %. The ELTs made by company A have a 4 % of defects, the company B ELTs have a 6 % of defects, and the third ELTs have a 9 % of defects.

(a) If an ELT is randomly selected from the general population of all ELTs, find the probability that it was made by the company A.

(b) If a randomly selected ELT is then tested and is found to be defective, find the probability that it was made by company A.

A – event of getting

B –

C –

$P(A) = 80/100$

$P(B) = 15/100$

$P(C) = 5/100$

$P(D|A) = \frac{4}{100}$

$P(D|B) = 6/100$

$P(D|C) = 9/100$

$$\frac{P(A) * P(D|A)}{P(D)}$$

$$\frac{0.80 * 0.04}{\frac{80}{100} \times \frac{4}{100} + \frac{15}{100} * \frac{6}{100} + \frac{5}{100} * \frac{9}{100}}$$

5/12/22

Random Variables.

A random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcome.

→ designated by capital letters

Random variables

- Discrete → specific values
- continuous → any values within a continuous range

They are required to be measurable and are typically real numbers.

For example, the letter x may be designated to represent the sum of the resulting numbers after three dice are rolled.

In this case, x could be 3 (1+1+1), 18 (6+6+6), or somewhere between 3 and 18.

The variable in an algebraic equation is an unknown value that can be calculated. The equation $10 + x = 13$ shows that we can calculate the specific value for $x = 3$.

Discrete vs continuous

For instance, a random variable representing the no of automobiles sold at a particular dealership on one day would be discrete.	While a random variable representing the weight of a person in kg would be continuous.

Discrete random variable :-

↳ countable no of distinct values

consider an experiment where a coin is tossed three times.

If x represent the no of times that the coin comes up heads, then x is a discrete random variables that can only have the values 0, 1, 2, 3

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

No other values for x random variable.

continuous Random variable :-

continuous random variables can represent any value within a specified range or interval and can take on an infinite number of possible values.

An example of a continuous random variable would be an experiment that involves measuring the amount of rainfall in a city over a year or the average height of a random group of 25 people.

Probability Distribution :- (Discrete Random Variable) :

the probability distribution for a random variable describes how the probabilities are distributed over the values of the random variable.

For a discrete random variable, x, the probability distribution is defined by a probability mass function, denoted by f(x). This function provides the probability for each value of the random variable.

In the development of the probability function for a discrete random variable, two conditions must be satisfied:

(a) $f(x)$ must be non negative for each value of the random variable

(b) the sum of the probabilities for each value of the random variable must equal one.

A random variable has a probability distribution that represents the likelihood that any of the possible values would occur.

Let the random variable, x, is the number on the top face of a die when it is rolled once.

x	1	2	3	4	5	6
$P(X=x)$	1/6	1/6	1/6	1/6	1/6	1/6

→ all the values are non negative

→ sum = 1

Probability Mass Function (PMF):

consider an experiment where a coin is tossed two times

$x \rightarrow$ represents the number of times that the coin comes up heads.

x is a discrete random variable that can only have the values 0, 1, 2

$S = \{TT, TH, HT, HH\}$

No other value is possible for x.

x	0	1	2
$P(X = x)$	1/4	2/4	1/4

An experiment where a coin is tossed three times.

$x \rightarrow$ represents the no of times that coin comes up head

$x \rightarrow$ discrete random variable that can only have the values 0, 1, 2, 3.

No other values is possible for x.

$S = \{HHH, THH, HHT, THT, HTH, TTH, HTT, TTT\}$

x	0	1	2	3
$P(X = x)$	1/8	3/8	3/8	1/8

1. If x represents the total no of heads obtained when a fair coin is tossed 5 times. Find the probability distribution of x.

5	HHHHH	THHHH	4
4	HHHHT	THHHT	3
4	HHHTH	THHTH	2
3	HHHTT	THHTT	2
4	HHTHH	THTHH	3
3	HHTHT	THTHT	2
3	HHTTH	THTTH	2
2	HHTTT	THTTT	1
4	HTHHH	TTHHH	3
3	HTHHT	TTHHT	2
3	HTHTH	TTHTH	2
2	HTHTT	TTHTT	1
3	HTTHH	TTTHH	2
2	HTTHT	TTTHT	1
2	HTTTH	TTTTH	1
1	HTTTT	TTTTT	0

Probability distribution of x

x	0	1	2	3	4	5
$P(X=x)$	$1/32$	$5/32$	$11/32$	$9/32$	$5/32$	$1/32$

2. If the probability distribution of x is given as

x	1	2	3	4
$P(X=x)$	0.4	0.3	0.2	0.1

find $P(1/2 < X < 7/2 \mid X > 1)$.

Required probability =

$$\frac{P\{P(1/2 < X < 7/2) \cap P(X>1)\}}{P(X>1)}$$

3. Find the probability distribution if $Y = X^2 + 2X$.

The probability distribution of X is given below

X	0	1	2	3
$P(X=x)$	0.1	0.3	0.5	0.1

$\Rightarrow X = 0$
$Y = 0^2 + 2(0)$
$Y = 0$

$\Rightarrow X = 1$
$Y = (1)^2 + 2(1)$
$Y = 1 + 2 = 3$

$\Rightarrow X = 2$
$Y = (2)^2 + 2(2)$
$Y = 4 + 4 = 8$

$\Rightarrow X = 3$
$Y = (3)^2 + 2(3)$
$= 9 + 6$
$Y = 15$

Y	0	3	8	15
$P(Y=y)$	0.1	0.3	0.5	0.1

---

4. X has the following probability distribution.

X	-2	-1	0	1
$P(X=x)$	0.4	K	0.2	0.3

Find K.

The sum of the probabilities for each value of the random variable must be equal to one.

$\Rightarrow 0.4 + K + 0.2 + 0.3 = 1$

$0.9 + K = 1$

$K = 1 - 0.9$

$K = 0.1$.

$\therefore \Rightarrow P(X = -1) = 0.1$

6/12/22

(1) PMF - for the event to get sum of the scores after rolling two dice.

36 possibilities :-

Event - sum of the scores on two dice.

sample space (S) = {2, 3, 4, 5, ..., 11, 12}

(1+1) ... (6+6)

1st die

2nd die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(i)

What is the probability that the sum of the scores is 5, 6, 7 or 8?

$= P(5) + P(6) + P(7) + P(8)$

$= \frac{4+5+6+5}{36}$

$= 20/36 = 10/18 = 5/9$

(ii) If $P(X=x) = 1/12$, what is the value of x?

We can have two values,

when $x=4$ and $x=10 \Rightarrow P(X=x) = 1/12$

---

(2) If x is a discrete random variable with a probability mass function as given below.

(3m)

x	0.1	0.2	0.3	0.4
$P(X=x)$	0.2	0.3	0.1	0.4

Find $P(X<0.4)$

$P(X<0.4) = P(X=0.1) + P(X=0.2) + P(X=0.3)$

$= 0.2 + 0.3 + 0.1$

$P(X<0.4) = 0.6$

(5m)

A random variable X has the following distribution.

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find (a) the value of K

(b) $P(1.5 < X < 4.5 \mid X>2)$

(c) the smallest value of λ for which $P(X \le \lambda) \ge 1.2$

(a) Sum of the probabilities of random variable is equal to 1.

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$9k + 10k^2 = 1$$

$$\cancel{k(9+10k) = 1}$$

$$\cancel{9 + 10k = \frac{1}{k}}$$

$$\cancel{10k =}$$

$$10k^2 + 9k - 1 = 0$$

$$(k+1)(k - 1/10) = 0$$

$$k + 1 = 0 \quad | \quad k - 1/10 = 0$$

$$k = -1 \quad | \quad k = 1/10$$

$$-10 \wedge \quad \frac{10}{10} \frac{-1}{10} = 9$$

$k = 1/10$ and $k = -1$.

(b) probability distribution function ::

x	0	1	2	3	4	5	6	7
$P(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$\frac{2}{100}$	$17/100$

$$P(1.5 < x < 4.5 \mid x > 2) = \frac{P(x=3) + P(x=4)}{\sum_{i=3}^{7} P(x = x_i)}$$

$$= \frac{2/10 + 3/10}{2/10 + 3/10 + 1/100 + 2/100 + 17/100}$$

$$= \frac{5/10}{\frac{20+30+1+2+17}{100}} = \frac{5/10}{70/100}$$

$$P(1.5 < x < 4.5 \mid x > 2) = 5/7$$

(c) the smallest value of λ for which $P(x \leq \lambda) > 1/2$

By trials:

$P(x \leq 0) = 0 \qquad P(x \leq 1) = 1/10$

$P(x \leq 2) = 3/10 \qquad P(x \leq 3) = 5/10$

$P(x \leq 4) = 8/10$

$= 4/5$

$\therefore$ The smallest value of λ satisfying the condition is 4.

Mean and variance of Random variables :

Mean of a random variable :

If x is the random variable and p is the respective probabilities, the mean of a random variable is defined as

$$\text{Mean}(\mu) = \Sigma xp$$

$$\sum_{i=1}^{n} x_i \, p(X = x_i)$$

Where variable x consists of all possible values and p consist of respective probabilities.

The mean is also known as the expected value. It is generally denoted by $E[x]$. Where x is the random variable.

$$\text{Mean}, \mu = E(X) = \sum_{i=1}^{n} x_i \, (p(X = x_i))$$

Variance of a Random variable :

The variance tells how much is the spread of random variable x around the mean value. The formula for the variance of a random variable is defined as

$$var(X) = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

where $E(x^2) = \sum_{i=1}^{n} x_i^2 \, p(x = x_i)$ and

$$E(X) = \sum_{i=1}^{n} x_i \, p(x = x_i)$$

Standard deviation of random variable :

$$\sigma = \sqrt{\sigma^2} = \sqrt{var(X)}$$

What is the Expected value (Mean) of a dice roll?

soln:

x	1	2	3	4	5	6
$P(X=x)$	1/6	1/6	1/6	1/6	1/6	1/6

Mean:
$$E(X) = \mu = \sum_{i=1}^{6} x_i \cdot P(X=x_i)$$

$$= (1 * 1/6) + (2 * 1/6) + (3 * 1/6) + (4 * 1/6) + (5 * 1/6) + (6 * 1/6)$$

$$= 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$$

$$E(X) = 21/6$$

Variance:

$$var(X) = \sigma^2 = E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{i=1}^{6} x_i^2 \cdot P(X=x_i)$$

$$= (1^2 * 1/6) + (4 * 1/6) + (9 * 1/6) + (16 * 1/6) + (25 * 1/6) + (36 * 1/6)$$

$$= 1/6 + 4/6 + 9/6 + 16/6 + 25/6 + 36/6$$

$$= \frac{1+4+9+16+25+36}{6}$$

$$= \frac{55+36}{6} = \frac{91}{6}$$

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{91}{6} - \frac{441}{36}$$

$$= \frac{546 - 441}{36}$$

$$var(x) = \sigma^2 = \frac{105}{36}$$

---

calculate the mean, variance and SD for a random variable, x defined as the number of tails in four tosses of a coin. Also, show the probability distribution.

soln:

x	0	1	2	3	4
$P(X=x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

H	H	H	H	→	0
H	H	H	T	→	1
H	H	T	H	→	1
H	H	T	T	→	2
H	T	H	H	→	1
H	T	H	T	→	2
H	T	T	H	→	2
H	T	T	T	→	3
T	H	H	H	→	1
T	H	H	T	→	2
T	H	T	H	→	2
T	H	T	T	→	3
T	T	H	H	→	2
T	T	H	T	→	3
T	T	T	H	→	3
T	T	T	T	→	4

Mean :

$$E(X) = \mu = \sum_{i=0}^{4} x_i \cdot P(X = x_i)$$

$$= (0 * 1/16) + (1 * 4/16) + (2 * 6/16) + (3 * 4/16) + (4 * 1/16)$$

$$= \frac{0 + 4 + 12 + 12 + 4}{16}$$

$$E(X) = \mu = 32/16 = 2$$

variance :

$$var(X) = \sigma^2 = E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{i=0}^{4} x_i^2 \cdot P(X = x_i)$$

$$= (0^2 * 1/16) + (1^2 * 4/16) + (2^2 * 6/16) + (3^2 * 4/16) + (4^2 * 1/16)$$

$$E(x^2) = \frac{0 + 4 + 24 + 36 + 16}{16} = \frac{80}{16} = 5$$

$$var(x) = \sigma^2 = E(x^2) - (E(x))^2$$
$$= 5 - (2)^2$$
$$= 5 - 4 = 1$$

standard deviation , $\sigma = \sqrt{\sigma^2} = \sqrt{var(x)}$

$$= \sqrt{1} = 1$$

(2)

The probability function for the number of seizures, x, of a typical epileptic person in any given year is given in the following table. Find Mean, variance and standard deviation

x	0	2	4	6	8	10
$P(X=x)$	0.17	0.21	0.18	0.11	0.16	0.17

Ans:

$\mu = 4.78$

$\sigma^2 = 12.07$

$\sigma = 3.47$

(3) Calculate the mean and variance for a random variable, x defined as the sum of two rolls of a pair of dice. Also, show the probability distribution.

$\mu = 7$

$var(x) = \sigma^2 = 5.83$

$\sigma = 2.41$

soln::

(2)

x	0	2	4	6	8	10
$p(X=x)$	0.17	0.21	0.18	0.11	0.16	0.17

Mean::

$$\text{Mean}, \mu = E(X) = \sum_{i=1}^{6} x_i \cdot p(X=x_i)$$

$$= (0 * 0.17) + (2 * 0.21) + (4 * 0.18) + (6 * 0.11) + (8 * 0.16) + (10 * 0.17)$$

$$= 0 + 0.42 + 0.72 + 0.66 + 1.28 + 1.7$$

$$\mu = 4.78$$

variance::

$$var(X), \sigma^2 = E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{i=1}^{6} x_i^2 \cdot p(X=x_i)$$

$$= (0 * 0.17) + (4 * 0.21) + (16 * 0.18) + (36 * 0.11) + (64 * 0.16) + (100 * 0.17)$$

$$= 0 + 0.84 + 2.88 + 3.96 + 10.24 + 17$$

$$E(X^2) = 34.92$$

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= 34.92 - (4.78)^2$$

$$= 34.92 - 22.85$$

$$\sigma^2 = 12.07$$

standard Deviation:

$$SD, \sigma = \sqrt{\sigma^2} = \sqrt{var(X)}$$

$$= \sqrt{12.07} = 3.47$$

Mean, $\mu = 4.78$

variance, $\sigma^2 = 12.07$

SD, $\sigma = 3.47$

(3) sum of two rolls of a pair of Die.

x	2	3	4	5	6	7	8	9	10	11	12
$p(X=x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Mean :

$$\text{Mean},\ \mu = \sum_{i=1}^{11} x_i \cdot p(X=x_i)$$

$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36}$$

$$= \frac{70+82+66+34}{36}$$

$$= 252/36$$

$E(x) = \mu = 7$

variance :

$$var(x) = \sigma^2 = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum_{i=1}^{11} x_i^2 p(X=x_i)$$

$$= \frac{4+18+48+100+180+294+320+324+300+242+144}{36}$$

$$= 1974/36$$

$E(x^2) = 54.83$

$var(x) = E(x^2) - (E(x))^2$

$= 54.83 - (7)^2$

$= 54.83 - 49$

$var(x), \sigma^2 = 5.83$

$\therefore$ Mean μ, $E(x) = 7$

variance, $\sigma^2 = 5.83$

SD, $\sigma = 2.41$

standard deviation,

$$\sigma = \sqrt{\sigma^2} = \sqrt{var\,x} = \sqrt{5.83} = 2.41$$

7/10

Moment Generating Function : (MGF)

The expected values $E(X)$, $E(X^2)$, $E(X^3)$, ..., and $E(X^n)$ are called moments. As you have already experienced in some cases, the mean :

$$\mu = E(X)$$

and the variance.

$$\sigma^2 = var(X) = E(X^2) - (E(X))^2$$
$$= E(X^2) - \mu^2$$

Moment generating functions (MGFs) are function of t. MGFs are calculated by using the definition of expectation of function of a random variable.

The moment generating function of X is

$$M_X(t) = E[e^{tX}]$$
$$= E[\exp(tX)]$$
$$= \sum e^{tx} f(x)$$

$$M_X(t) = \sum_{i=1}^{n} e^{tx_i} P_i \quad \text{(Random Variable)}$$
$$\downarrow$$
$$P_{x_i} = P(X = x_i)$$

If a moment generating function exists for a random variable X; then,

The mean of X can be found by evaluating the first derivative of the moment generating function at $t=0$

$$\mu = E(X) = M'(x=0) = M'_{t=0}$$

The variance of x can be found by Evaluating the first and second derivatives of the moment generating function at $t=0$.

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= M''(0) - [M'(0)]^2$$

$$= M''_{t=0} - (M'_{t=0})^2$$

Example :

If x represents the outcome, when a fair die is tossed, find the MGF of x, hence find $E(X)$ and $var(X)$.

$$S = \{1, 2, 3, 4, 5, 6\}$$

Probability Distribution Function :

x_i	1	2	3	4	5	6
$P(X = x_i)$ or p_i	1/6	1/6	1/6	1/6	1/6	1/6

Moment generating Function (MGF) :

$$M(t) = \sum_{i=1}^{6} e^{tx_i} p_i$$

$$= \frac{e^t}{6} + \frac{e^{2t}}{6} + \frac{e^{3t}}{6} + \frac{e^{4t}}{6} + \frac{e^{5t}}{6} + \frac{e^{6t}}{6}$$

$$M(t) = 1/6 \left(e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right)$$

Mean :

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} e^{2x} = 2e^{2x}$

$$E(X) = \mu = [M'(t)]_{t=0}$$

$$= 1/6 \left(e^t + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right) \text{ at } t$$

$= \frac{1}{6} (e^{0} + 2e^{2(0)} + 3e^{3(0)} + 4e^{4(0)} + 5e^{5(0)} + 6e^{6(0)})$

$(\because e^{0} = 1)$

$= \frac{1}{6} (1+2+3+4+5+6)$

$\mu = E(X) = 21/6 = 7/2$

variance:

$var(x) = E(x^2) - (E(x))^2$

$E(x^2) = [M''(t)]_{t=0}$

$= \frac{1}{6} (e^{t} + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t})$ at $t=0$

$= \frac{1}{6} (e^{0} + 4e^{2(0)} + 9e^{3(0)} + 16e^{4(0)} + 25e^{5(0)} + 36e^{6(0)})$

$= \frac{1}{6} (1+4+9+16+25+36)$

$E(x^2) = 91/6$

$var(x) = E(x^2) - [E(x)]^2$

$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$

$= \frac{91}{6} - \frac{49}{4}$

$= \frac{182 - 147}{12}$

$var(x) = \frac{35}{12}$

suppose that you have a fair 4-sided die, and let x be the random variable representing the value of the number rolled.

(a) Write down the moment generating function of x.

(b) Use this moment generating function to compute the first and second moments of x.

$S = \{1, 2, 3, 4\}$

Probability Distribution Function,

x	1	2	3	4
$P(X = x_i)$	$1/4$	$1/4$	$1/4$	$1/4$

(a)

Moment generating function (MGF):

$$M(t) = \sum_{i=1}^{4} e^{t x_i} p_i$$

$$= \frac{e^t}{4} + \frac{e^{2t}}{4} + \frac{e^{3t}}{4} + \frac{e^{4t}}{4}$$

$$M(t) = 1/4 \left(e^t + e^{2t} + e^{3t} + e^{4t}\right)$$

(b) First moment of x.

$$E(x) = [M'(t)]$$

$$= \frac{1}{4}\left(e^t + 2e^{2t} + 3e^{3t} + 4e^{4t}\right)$$

second moment of x.

$$E(x^2) = [M''(t)]$$

$$= \frac{1}{4}\left(e^t + 4e^{2t} + 9e^{3t} + 16e^{4t}\right)$$

$E(X) = [M'(t)]_{t=0}$

$= \frac{1}{4}(e^{0} + 2e^{2(0)} + 3e^{3(0)} + 4e^{4(0)}) = \frac{1}{4}(1+2+3+4)$

$= 10/4$

$E(X^2) = [M''(t)]_{t=0}$

$= \frac{1}{4}(e^{0} + 4e^{2(0)} + 9e^{3(0)} + 16e^{4(0)})$

$= \frac{1}{4}(1+4+9+16) = 30/4$

$\therefore E(X) = 10/4$ and $E(X^2) = 30/4$

Suppose that a mathematician determines that the revenue the Uconn dairy Bar makes in a week is a random variable, x, with the moment generating function

$M_x(t) = \frac{1}{(1-2500t)^4}$ $\quad M_x(t) = (1-2500t)^{-4}$

Find the standard deviation of the revenue the Uconn dairy bar makes in a week.

First moment of x. $\quad \frac{d}{dx} x^n = n x^{n-1}$

$E(X) = [M'(t)]_{t=0}$

$= -4 * (1-2500t)^{-5} * (-2500)$ at $t=0$

$= 10000 (1-2500t)^{-5}$ at $t=0$

$= 10000 (1-2500(0))^{-5} = 10000(1-0)^{-5}$

$= 10000(1)^{-5} = 10000(1)$

$E(x) = \mu = 10000$

Mean, $E(X)$, $\mu = 10000$

second moment of x:

$$E(X^2) = -5(10000)(1-2500t)^{-6} \cdot -2500 \quad \text{at } t=0$$

$$= 12500(10000)(1-2500t)^{-6} \quad \text{at } t=0$$

$$= 125000000\,(1-2500t)^{-6} \quad \text{at } t=0$$

$$= 125000000\,(1-2500(0))^{-6}$$

$$= 125000000\,(1-0)^{-6} = 125000000(1)$$

$$E(X^2) = 125000000.$$

$$\text{variance }(x) = E(X^2) - (E(X))^2$$

$$= 125000000 - (10000)^2$$

$$= 125000000 - 100000000$$

$$\text{var}(x) = \sigma^2 = 25000000$$

standard Deviation, $\sigma = \sqrt{\text{var}(x)}$

$$= \sqrt{25000000}$$

$$\sigma = 5000$$

$\therefore$ standard Deviation of the revenue is 5000.