DEPARTMENT OF COMPUTER APPLICATIONS

23MX11 – LPP – BIG – M METHOD

Find the optimal solution for the following LPP using BIG – M Method.

Maximize $Z = 5x1+12x2+4x3$

Subject to:

$x1+2x2+x3 \le 5$

$2x1-x2+3x3 = 2$

$x1, x2, x3 \ge 0$

First convert the given LPP into standard form

Maximize $Z = 5x1+12x2+4x3-Ma$

Subject to: $x1+2x2+x3+s = 5$

$2x1-x2+3x3+a = 2$

$x1, x2, x3, s, a \geq 0$

a is the artificial and s is the slack variable.

The objective function is

$$\mathbf{Z-5x1-12x2-4x3+0s+Ma = 0}$$

Now, eliminate the artificial variable from Z.

<u>New Z row:</u>

$$[-5 \quad -12 \quad -4 \quad 0 \quad M \quad 0]$$

$$-(M)[\,2 \quad -1 \quad 3 \quad 0 \quad 1 \quad 2]$$

$$= \quad \mathbf{[-5-2M \quad -12+M \quad -4-3M \quad 0 \quad 0 \quad -2M]}$$

	x1	x2	x3	s	a	RHS	Ratio
s	1	2	1	1	0	5	5
a	2	-1	3	0	1	2	$\frac{2}{3}$
Z	-5-2M	-12+M	-4-3M	0	0	-2M	

Most Negative Coefficient

Pivotal Element

New "a" row

$$[2 \quad -1 \quad 3 \quad 0 \quad 1 \quad 2\,] \div 3$$

$$= \quad [\frac{2}{3} \quad \frac{-1}{3} \quad 1 \quad 0 \quad \frac{1}{3} \quad \frac{2}{3}] \text{ [x3 row]}$$

New "s" row

$$[1 \quad 2 \quad 1 \quad 1 \quad 0 \quad 5]$$

$$-(1)\ [\frac{2}{3} \quad \frac{-1}{3} \quad 1 \quad 0 \quad \frac{1}{3} \quad \frac{2}{3}]$$

$$= [\frac{1}{3} \quad \frac{7}{3} \quad 0 \quad 1 \quad \frac{-1}{3} \quad \frac{13}{3}]$$

New "Z" row

= [-5-2M   -12+M   -4-3M   0   0   -2M]

-(-4-3M) $[\frac{2}{3} \quad \frac{-1}{3} \quad 1 \quad 0 \quad \frac{1}{3} \quad \frac{2}{3}]$

$=[\frac{-7}{3} \quad \frac{-40}{3} \quad 0 \quad 0 \quad \frac{4+3M}{3} \quad \frac{8}{3}]$

Simplex table after this iteration

	x1	x2	x3	s	a	RHS	Ratio
s	$\frac{1}{3}$	$\frac{7}{3}$	0	1	$\frac{-1}{3}$	$\frac{13}{3}$	$\frac{13}{7}$
x3	$\frac{2}{3}$	$\frac{-1}{3}$	1	0	$\frac{1}{3}$	$\frac{2}{3}$	NO
Z	$\frac{-7}{3}$	$\frac{-40}{3}$	0	0	$\frac{4+3M}{3}$	$\frac{8}{3}$	

New “a” row

$$[\frac{1}{3} \quad \frac{7}{3} \quad 0 \quad 1 \quad \frac{-1}{3} \quad \frac{13}{3}] \text{ X } \frac{3}{7}$$

$$= \quad [\frac{1}{7} \quad 1 \quad 0 \quad \frac{3}{7} \quad \frac{-1}{7} \quad \frac{13}{7}]$$

New “x3” row

$$[\frac{2}{3} \quad \frac{-1}{3} \quad 1 \quad 0 \quad \frac{1}{3} \quad \frac{2}{3}]$$

$$-(\frac{-1}{3})\ [\frac{1}{7} \quad 1 \quad 0 \quad \frac{3}{7} \quad \frac{-1}{7} \quad \frac{13}{7}]$$

$$= [\frac{5}{7} \quad 0 \quad 1 \quad \frac{1}{7} \quad \frac{2}{7} \quad \frac{9}{7}]$$

New “Z” row

$$= \left[\frac{-7}{3} \quad \frac{-40}{3} \quad 0 \quad 0 \quad \frac{4+3M}{3} \quad \frac{8}{3}\right]$$

$$-\left(\frac{-40}{3}\right)\left[\frac{1}{7} \quad 1 \quad 0 \quad \frac{3}{7} \quad \frac{-1}{7} \quad \frac{13}{7}\right]$$

$$= \left[\frac{-3}{7} \quad 0 \quad 0 \quad \frac{40}{7} \quad \frac{21M-12}{21} \quad \frac{192}{7}\right]$$

Simplex table after this iteration

	x1	**x2**	**x3**	**s**	**a**	**RHS**	**Ratio**
x2	$\frac{1}{7}$	1	0	$\frac{3}{7}$	$\frac{-1}{7}$	$\frac{13}{7}$	13
x3	$\frac{5}{7}$	0	1	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{9}{7}$	$\frac{9}{5}$
Z	$\frac{-3}{7}$	0	0	$\frac{40}{7}$	$\frac{21M-12}{21}$	$\frac{192}{7}$	

New “x3” row

$$\left[\frac{5}{7} \quad 0 \quad 1 \quad \frac{1}{7} \quad \frac{2}{7} \quad \frac{9}{7}\right] \text{X } \frac{7}{5}$$

$$= \left[\,\mathbf{1} \quad \mathbf{0} \quad \frac{\mathbf{7}}{\mathbf{5}} \quad \frac{\mathbf{1}}{\mathbf{5}} \quad \frac{\mathbf{2}}{\mathbf{5}} \quad \frac{\mathbf{9}}{\mathbf{5}}\right]$$

New “x2” row

$$\left[\frac{1}{7} \quad 1 \quad 0 \quad \frac{3}{7} \quad \frac{-1}{7} \quad \frac{13}{7}\right]$$

$$-\left(\frac{-1}{7}\right)\left[\,1 \quad 0 \quad \frac{7}{5} \quad \frac{1}{5} \quad \frac{2}{5} \quad \frac{9}{5}\right]$$

$$= \left[\,\mathbf{0} \quad \mathbf{1} \quad \frac{\mathbf{-1}}{\mathbf{5}} \quad \frac{\mathbf{2}}{\mathbf{5}} \quad \frac{\mathbf{-1}}{\mathbf{5}} \quad \frac{\mathbf{8}}{\mathbf{5}}\right]$$

New “Z” row

$$= \left[\frac{-3}{7} \quad 0 \quad 0 \quad \frac{40}{7} \quad \frac{21M-12}{21} \quad \frac{192}{7}\right]$$

$$-\left(\frac{-3}{7}\right)\left[\,1 \quad 0 \quad \frac{7}{5} \quad \frac{1}{5} \quad \frac{2}{5} \quad \frac{9}{5}\right]$$

$$= \left[\,\mathbf{0} \quad \mathbf{0} \quad \mathbf{3} \quad \frac{\mathbf{29}}{\mathbf{5}} \quad \frac{\mathbf{5M-2}}{\mathbf{5}} \quad \frac{\mathbf{141}}{\mathbf{5}}\right]$$

Simplex table after this iteration

	x1	x2	x3	s	a	RHS
x2	0	1	$\frac{-1}{5}$	$\frac{2}{5}$	$\frac{-1}{5}$	$\frac{8}{5}$
x1	1	0	$\frac{7}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{9}{5}$
Z	0	0	3	$\frac{29}{5}$	$\frac{5M-2}{5}$	$\frac{141}{5}$

There is no negative coefficient in the Z row. Therefore, the optimal solution is obtained for Z.

Solution:

x1 = $\frac{9}{5}$ **, x2 =** $\frac{8}{5}$ and $Z = \frac{141}{5}$

Any Questions