Dual Primal

- ✓ Idea of duality is to associate with each linear programming problem called the primal, another linear programming problem called its dual.

Primal & Dual

Consider the linear programme in its canonical form :

Maximize

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$$
$$x_1 \geq 0\ ,\ x_2 \geq 0\ ,\ \ldots,\ x_n \geq 0$$

Dual of the above problem as the linear programming problem :

Minimize

$$D = b_1y_1 + b_2y_2 + \ldots + b_my_m$$

Subject to

$$a_{11}y_1 + a_{21}y_2 + \ldots + a_{m1}y_m \geq c_1$$
$$a_{12}y_1 + a_{22}y_2 + \ldots + a_{m2}y_m \geq c_2$$
$$\vdots$$
$$a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \geq c_n$$
$$y_1 \geq 0\ ,\ y_2 \geq 0\ ,\ \ldots,\ y_m \geq 0$$

- ✓ Original problem is called primal and the other is called the dual. In fact it is interesting to note in general, any of the two problems considered above can be called as the primal then the other is its dual.

- ✓ As a rule of thumb, an additional constraint requires much more computational effort than an additional variable. Thus, if the primal should have a large number of constraints and relatively few variables, its dual will probably require less computational effort since the number of variables and constraints are interchanged in the two problems.

Categories of Primal Dual:

Dual of a primal Maximization problem with '≤' type of constraints

Dual of a primal Minimization problem with '≥' type of constraints

Dual of a primal Maximization problem with mix of '≤' and '≥' type of constraints

Dual of a primal Minimization problem with mix of '≤' and '≥' type of constraints

Dual of a primal Maximization problem with mix of '≤', '≥' and "=" type of constraints

Dual of a primal Minimization problem with mix of '≤', '≥' and "=" type of constraints

Special Case of Dual/Primal

- ✓ If any of the constraints in the primal problem is a perfect equality, then the corresponding dual variable is unrestricted in sign.

- ✓ If any variable of the primal problem is unrestricted in sign, then the corresponding constraint of the dual will be an equality.

- ✓ Given the LPP Max $Z = 2x_1 + 3x_2 + 4x_3$

subject to :
$$x_1 - 5x_2 + 3x_3 = 7$$
$$2x_1 - 5x_2 \le 3$$
$$3x_2 - x_3 \ge 5$$
$$x_1 , x_2 \ge 0;\ x_3 \text{ is unrestricted in sign}$$

Formulate the dual of the LPP.

Solution: Here we see that the primal variable x_3 is unrestricted in sign and the first constraint is an equation.

First of all, let us rewrite the problem in its equivalent form with all positive variables by writing $x_3 = x'_3 - x''_3$ as

Max $Z = 2x_1 + 3x_2 + 4(x'_3 - x''_3)$

subject to:

$$x_1 - 5x_2 + 3(x'_3 - x''_3) \le 7$$
$$x_1 - 5x_2 + 3(x'_3 - x''_3) \ge 7$$
$$2x_1 - 5x_2 \le 3$$
$$3x_2 - (x'_3 - x''_3) \ge 5$$

$$x_1, x_2, x'_3, x''_3 \ge 0$$

Now, the problem in the required standard form as

Max $Z = 2x_1 + 3x_2 + 4(x'_3 - x''_3)$

Subject to:

$x_1 - 5x_2 + 3(x'_3 - x''_3) \le 7$
$-x_1 + 5x_2 - 3x'_3 + x''_3 \le -7$
$2x_1 - 5x_2 \le 3$
$3x_2 - x'_3 + x''_3) \ge 5$
$x_1, x_2, x'_3, x''_3 \ge 0$

Hence the dual problem is given by

Min $Z = 7y_1' - 7y_1'' + 3y_2 - 5y_3$

subject to

$y_1' - y_1'' + 2y_2 \ge 2$

$-5y_1' + 5y_1'' - 5y_2 - 3y_3 \ge 3$

$3y_1' - 3y_1'' + y_3 \ge 4$

$-3y_1' + 3y_1'' - y_3 \ge -4$

$y_1, y_2, y'_3, y''_3 \ge 0$

The above problem can be written as

Min $Z = 7y_1 + 3y_2 - 5y_3$

subject to

$y_1 + 2y_2 \ge 2$
$-5y_1 - 5y_2 - 3y_3 \ge 3$
$3y_1 + y_3 = 4$

y_2, y_3, y_1 ($= y'_3 - y''_3$) is unrestricted in sign.