

Department of Computer Applications

23MX11 – MFCS – Optimization Techniques – Problem sheet

1. Write the duals of the following problem

1.1 $\max z = 2x_1 + 4x_2 - 4x_3$

subject to

$$3x_1 + 2x_2 + 4x_3 \geq 1$$

$$4x_1 - 3x_2 = 2$$

$$2x_1 + x_2 + 6x_3 \leq 3$$

$$x_1, x_2 \geq 0, x_3 : \text{unrestricted}$$

1.2

$$\min z = 2x_1 - 3x_2 + x_3$$

subject to

$$x_1 - 5x_2 + 6x_3 \geq 8$$

$$x_1 - 4x_2 \leq -12$$

$$2x_1 - x_2 + 4x_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

1.3 $\min z = 2x_1 + 2x_2 - 4x_3$

subject to

$$2x_1 + 2x_2 + 2x_3 = 10$$

$$-2x_1 + 6x_2 - x_3 \leq -10$$

$$-x_1 + 3x_2 \geq 3$$

$$x_1 \leq 0, x_2, x_3 \geq 0$$

1.4

$$\max z = 3x_1 - 7x_2 + 5x_3$$

subject to

$$x_2 - x_3 \leq -9$$

$$-x_1 - 2x_3 \geq 5$$

$$4x_1 - x_2 = 6$$

$$x_1 \leq 0, x_2 \geq 0, x_3 : \text{unrestricted}$$

2. "The dual of the dual is the primal" – Justify this statement considering one of the problems from the above.
3. Solve the following LPP model using the Graphical and Simplex Method. Also, check the chance for alternate solution. If so, find the same.

$$\max z = x_1 + x_2$$

subject to

$$-x_1 + x_2 \leq 4$$

$$2x_1 + 5x_2 \leq 20$$

$$2x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

4. Mathematically model the following LPP.

i) A steel company has two mills. Mill 1 costs \$70,000 per day to operate, and it can produce 400 tons of high-grade steel, 500 tons of medium-grade steel, and 450 tons of low-grade steel each day. Mill 2 costs \$60,000 per day to operate, and it can produce 350 tons of high-grade steel, 600 tons of medium-grade steel, and 400 tons of low-grade steel each day. The company has orders totalling 100,000 tons of high-grade steel, 150,000 tons of medium-grade steel, and 124,500 tons of low-grade steel. How many days should the company run each mill to minimize its costs and still fill the orders?

ii) A small petroleum company owns two refineries. Refinery 1 costs \$25,000 per day to operate, and it can produce 300 barrels of high-grade oil, 200 barrels of medium-grade oil, and 150 barrels of low-grade oil each day. Refinery 2 is newer and more modern. It costs \$30,000 per day to operate, and it can produce 300 barrels of high-grade oil, 250 barrels of medium-grade oil, and 400 barrels of

low-grade oil each day. The company has orders totalling 35,000 barrels of high-grade oil, 30,000 barrels of medium-grade oil, and 40,000 barrels of low-grade oil. How many days should the company run each refinery to minimize its costs and still meet its orders?

- iii) A company has three production plants, each of which produces three different models of a particular product. The daily capacities (in thousands of units) of the three plants are as follows.

	Model 1	Model 2	Model 3
Plant 1	8	4	8
Plant 2	6	6	3
Plant 3	12	4	8

The total demand for Model 1 is 300,000 units, for Model 2 is 172,000 units, and for Model 3 is 249,500 units. Moreover, the daily operating cost for Plant 1 is \$55,000, for Plant 2 is \$60,000, and for Plant 3 is \$60,000. How many days should each plant be operated in order to fill the total demand, and keep the operating cost at a minimum?

5. One of the main products of P & T Company, a large canning industry is canned beans. There are three canneries which send products to four distribution centers. Since transportation costs are particularly increased due to large distances, the administration decided to reduce it. There have been some estimates of the quantity production and transportation to distribution centers and as for the transportation costs of each load (fully loaded truck company) it is denominated in U.S. dollars.

		Distribution centers			
		1	2	3	4
Canneries	1	464	513	654	867
	2	352	416	690	791
	3	995	682	388	685
		80	65	70	85

Find the combination of the transportation load that minimizes the total transportation cost.

6. An airline company buys fuel for the plane from three vendors. The company needs for each of the three airports that uses and for the next month's 100000 gallons for the first, 180000 gallons for the second and 35000 gallons for the third airport. Each vendor can supply fuel to any airport in the price (dollars per gallon) given in the table below

	Airport 1	Airport 2	Airport 3
Seller 1	0.92	0.89	0.90
Seller 2	0.91	0.91	0.95
Seller 3	0.87	0.90	0.92

Each seller has a restriction on the total amount of fuel that can be supplied each month. The potential is 320000 gallons for the seller 1, 1270000 gallons for the seller 2 and 190000 gallons for the seller 3. Find the right fuel purchasing policy so as to minimize the total cost of feeding the three airports