Department of Computer Applications

23MX11 – MFCS-TWO Phase Simplex Method

Steps involved:

1. Modify the constraints so that the right-hand side of each constraint is nonnegative.

2. Identify each constraint with ≥ or = constraint, add an artificial variable to each constraint.

3. Convert each inequality constraint to the standard form. If constraint is ≤ constraint, then add a slack variable. If constraint is ≥ constraint, subtract an excess variable.

   [The above steps are very similar to BIG M Method]

4. For now, ignore the original LP's objective function. Instead solve an LP whose objective function is

   Minimize W = (sum of all the artificial variables).

   This is called the Phase I LP. The act of solving the Phase I LP will force the artificial variables to be zero. [Eliminate the artificial variable in the objective function row [W] by converting it to 0, before starting the simplex method for the above LPP]

Because each a_i [artificial variables] ≥ 0, solving the Phase I LP will result in one of the following cases:

Case 1: If the optimal value of W is greater than zero, then the original LP has no feasible solution.

Case 2: If the optimal value of W is equal to zero, and no artificial variables are in the optimal Phase I basis; then drop all columns in the optimal Phase I tableau that corresponds to the artificial variables.

We now combine the original objective function with the constraints from the optimal Phase I tableau.

This yields the Phase II LP. The optimal solution to the Phase II LP is the optimal solution to the original LP.

Sample Problem

Maximize $Z = 4x+5y$

Subject to: $2x+3y \leq 6$

$3x+y \geq 3$

Covert the given LPP to standard form as shown below [adding surplus, slack and artificial variables – following the steps discussed]

Maximize Z = 4x+5y

Subject to: 2x+3y+s1 = 6
3x+y -s2+a= 3

s1 and s2 are the slack variables; “a” is the artificial variable.

Now, ignore the original LP’s objective function. Instead solve an LP whose objective function

Minimize W = a [sum of artificial variables, here we have only one artificial variable]

Then, Phase I LP will be as follows

Minimize W = a
Subject to: 2x+3y+s1 = 6
3x+y -s2+a= 3

Initial Simplex table for the Minimization problem is

	x	y	s1	s2	a	RHS	Ratio
s1	2	3	1	0	0	6	
a	3	1	0	-1	1	3	
W	**0**	**0**	**0**	**0**	**1**	**0**	

Now, eliminate the artificial variable in the objective function W row by converting it into 0. The resultant table is as given below. Solve this using the usual simplex method.

$$\text{New W row} = [\,0 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0\,]$$
$$-(-1)\,[\,3 \quad 1 \quad 0 \quad -1 \quad 1 \quad 3\,]$$

$$= [\,3 \quad 1 \quad 0 \quad -1 \quad 0 \quad 3\,]$$

The revised table for Phase I LPP is

	x	y	s1	s2	a	RHS	Ratio
s1	2	3	1	0	0	6	
a	3	1	0	-1	1	3	
W	**3**	**1**	**0**	**-1**	**0**	**3**	

Apply simplex table for this as usual. Identify pivotal column by choosing the most Positive Coefficient – corresponds to x column and then pivotal element.

	x	y	s1	s2	a	RHS	Ratio
s1	2	3	1	0	0	6	3
a	3	1	0	-1	1	3	1
W	**3**	**1**	**0**	**-1**	**0**	**3**	

Pivotal Element

Most Positive Coefficient

New “a” row

$$[3 \quad 1 \quad 0 \quad -1 \quad 1 \quad 3] \div 3$$

$$= [1 \quad \frac{1}{3} \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3} \quad 1] \quad [x - row]$$

New “s1” row

$$[2 \quad 3 \quad 1 \quad 0 \quad 0 \quad 6]$$

$$-(2)\,[1 \quad \frac{1}{3} \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3} \quad 1]$$

$$= [0 \quad \frac{7}{3} \quad 1 \quad \frac{2}{3} \quad \frac{-2}{3} \quad 4]$$

New “W” row

$$= [3 \quad 1 \quad 0 \quad -1 \quad 0 \quad 3]$$

$$-(3)[1 \quad \frac{1}{3} \quad 0 \quad \frac{-1}{3} \quad \frac{1}{3} \quad 1]$$

$$= [0 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0]$$

Simplex table after this,

	x	**y**	**s1**	**s2**	**a**	**RHS**	**Ratio**
s1	0	$\frac{7}{3}$	1	$\frac{2}{3}$	$\frac{-2}{3}$	**4**	
x	1	$\frac{1}{3}$	0	$\frac{-1}{3}$	$\frac{1}{3}$	**1**	
W	**0**	**0**	**0**	**0**	**-1**	**0**	

Since there is no positive number in the objective row, STOP. The optimal basis s1 = 4 and x = 1. The optimal objective value W = 0 and “a” i not in the optimal Phase I basis. Therefore, it is Case 2

Hence, now generate Phase II initial tableau by changing the objective function row into the original objective function (Z = 4 x + 5 y). Also, drop the artificial column.

	x	y	s1	s2	RHS	Ratio
s1	0	$\frac{7}{3}$	1	$\frac{2}{3}$	**4**	
x	1	$\frac{1}{3}$	0	$\frac{-1}{3}$	**1**	
Z	**-4**	**-5**	**0**	**0**	**0**	

Now, apply the Simplex method for this objective function and find the optimal solution.

	x	y	s1	s2	RHS	Ratio
s1	0	$\frac{7}{3}$	1	$\frac{2}{3}$	**4**	$\frac{12}{\frac{7}{3}}$
x	1	$\frac{1}{3}$	0	$\frac{-1}{3}$	**1**	
Z	**-4**	**-5**	**0**	**0**	**0**	

New "s1" row

$$[0 \quad \frac{7}{3} \quad 1 \quad \frac{2}{3} \quad 4] \times \frac{3}{7}$$

$$= \quad [0 \quad 1 \quad \frac{3}{7} \quad \frac{2}{7} \quad \frac{12}{7}] \quad [y - row]$$

New "x" row

$$[1 \quad \frac{1}{3} \quad 0 \quad \frac{-1}{3} \quad 1]$$

$$-(\frac{1}{3})\ [0 \quad 1 \quad \frac{3}{7} \quad \frac{2}{7} \quad \frac{12}{7}]$$

$$= [1 \quad 0 \quad \frac{-1}{7} \quad \frac{-3}{7} \quad \frac{3}{7}\ ]$$

New "Z" row

$$= [-4 \quad -5 \quad 0 \quad 0 \quad 0]$$

$$-(5)\ [0 \quad 1 \quad \frac{3}{7} \quad \frac{2}{7} \quad \frac{12}{7}]$$

$$= [-4 \quad 0 \quad \frac{15}{7} \quad \frac{10}{7} \quad \frac{60}{7}\ ]$$

	x	**y**	**s1**	**s2**	**RHS**	**Ratio**
y	0	1	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{12}{7}$	
x	1	0	$\frac{-1}{7}$	$\frac{-3}{7}$	$\frac{3}{7}$	
Z	-4	0	$\frac{15}{7}$	$\frac{10}{7}$	$\frac{60}{7}$	

No changes required in the "x" and "y" rows

Changes in the Z row only

New "Z" row

$$= [-4 \quad 0 \quad \frac{15}{7} \quad \frac{10}{7} \quad \frac{60}{7}]$$

$$-(1)\,[1 \quad 0 \quad \frac{-1}{7} \quad \frac{-3}{7} \quad \frac{3}{7}]$$

$$= [0 \quad 0 \quad \frac{11}{7} \quad \frac{-2}{7} \quad \frac{72}{7}]$$

Current simplex table after this iteration is

	x	y	s1	s2	RHS	Ratio
y	0	1	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{12}{7}$	
x	1	0	$\frac{-1}{7}$	$\frac{-3}{7}$	$\frac{3}{7}$	
Z	0	0	$\frac{11}{7}$	$\frac{-2}{7}$	$\frac{72}{7}$	

New y row = $[0 \quad 1 \quad \frac{3}{7} \quad \frac{2}{7} \quad \frac{12}{7}] \times \frac{7}{2}$

$$= [0 \quad \frac{7}{2} \quad \frac{3}{2} \quad 1 \quad 6]$$ [s2 - row]

New x row = $[1 \quad 0 \quad \frac{-1}{7} \quad \frac{-3}{7} \quad \frac{3}{7}]$

$$-(\frac{-3}{7})\,[0 \quad \frac{7}{2} \quad \frac{3}{2} \quad 1 \quad 6]$$

$$= [1 \quad \frac{3}{2} \quad \frac{1}{2} \quad 0 \quad 3]$$

$$\text{New Z row} = \begin{bmatrix} 0 & 0 & \frac{11}{7} & \frac{-2}{7} & \frac{72}{7} \end{bmatrix}$$

$$-\left(\frac{-2}{7}\right) \begin{bmatrix} 0 & \frac{7}{2} & \frac{3}{2} & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 & 0 & 12 \end{bmatrix}$$

	x	**y**	**s1**	**s2**	**RHS**
s2	0	$\frac{7}{2}$	$\frac{3}{2}$	1	6
x	1	$\frac{3}{2}$	$\frac{1}{2}$	0	3
Z	0	1	2	0	12

There is no negative coefficient in the Z row. Stop the iteration. Optimal solution obtained for Z as follows

Z = 12; x = 3 and y = 0; s2 = 6

Any Questions