21MX225

Sree Varshini B

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Computer Applications

M.C.A - Semester I

Tutorial 1 Date: 30-DEC-2021

20MX11 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Duration: 50 mins **Maximum Marks: 25**

(5 × 5 = 25)

1. Determine the domain and range of the relation R defined by R={ (x, x+5): x∈{0, 1, 2, 3, 4, 5 } }

2. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L1, L2): L1 is parallel to L2}. Show that R is an Equivalence Relation. Find the set of all lines related to the line y = 2x + 4

3. Let A = {9, 10, 11, 12, 13} and let f: A→N be defined by f (n) = the highest prime factor of n. Find the range of f.

4. Prove that for every positive integer n, $n^2 - n + 17$ is a prime number using mathematical induction.

5. ***Prove using mathematical induction that for all $n \geq 1$,***

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$$

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Computer Applications

M.C.A - Semester 1

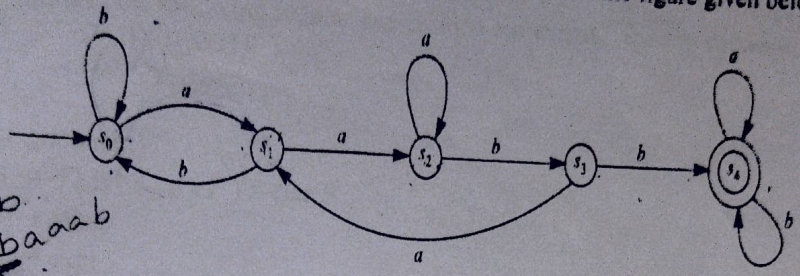
Tutorial 2 Date: 15-FEB-2022

20MX11 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Duration: 50 mins **Maximum Marks: 25**

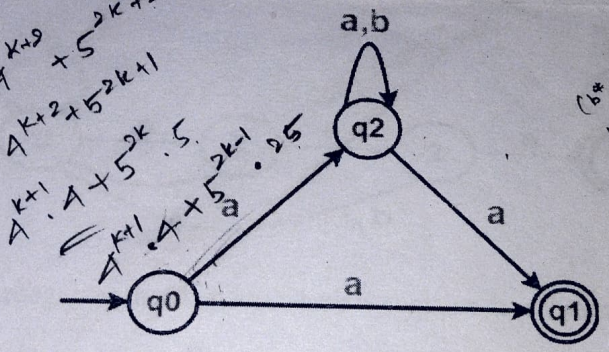
Answer any 5 questions **(5 × 5 = 25)**

1. Find the language L(G) generated by the grammar G, whose productions are $S \to aB$, $B \to b$, $B \to bA$, $A \to aB$.
2. Find the language $L(M)$ accepted by the automaton M in the figure given below.

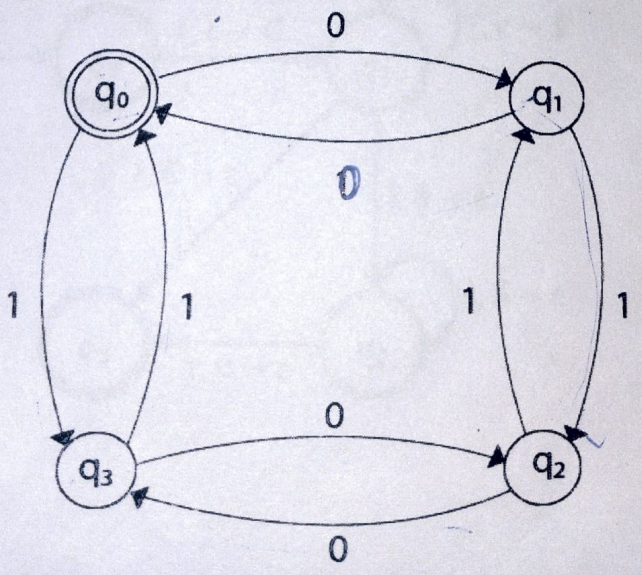


bbbaaabbbb
babaaaabaaab

3. Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is positive integer.
4. Convert the following Non-Deterministic Finite Automata (NFA) to Deterministic Finite Automata (DFA)



5. Draw DFA machine which generates set of strings starts with 1 and ends with 0.
6. Consider the below FA with $\sum = \{0, 1\}$. Draw transition table. Identify the language generated by the given machine.



21MX225

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Computer Applications

MCA - I Semester

Tutorial 3 Date: 28-02-2022

20MX11 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

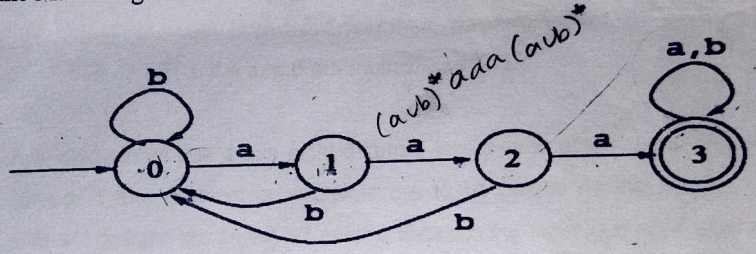
Duration: 1 hr **Maximum Marks: 25**

1. Find the language generated by the Grammar G
   G: ({S, A, B}, {a, b}, S, {S → aA, A → aA | B, B → ε | bB }) (3 marks)

2. Which languages the following regular expressions represents? State in common English.
   i. (0∪1)*01(0∪1)*
   ii. (aa)*(bb)*b (3 marks)

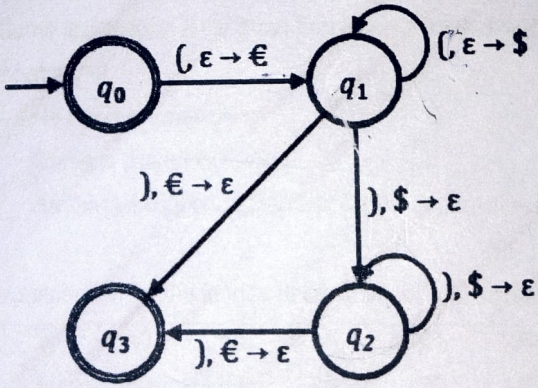
3. Given the grammar:
   S → if (E) S | if (E) S else S
   S → other
   E → expr
   Prove that it represents ambiguous **if-else** grammar using derivation trees. (5 marks)

4. Identify the Machine given below and the Language recognized by M. (5 marks)



5. Draw the NFA that recognizes the language where w contains the substring 0101. Do this using 5 states. (4 marks)

6. Prove that the given PDA checks the Matching Parentheses (by taking any two examples) (5 marks)



PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Computer Applications

M.C.A - Semester 1

CONTINUOUS ASSESSMENT TEST 1 Date: 16-DEC-2021

20MX11 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time: 1 Hour 30 mins. **Maximum Marks: 50**

INSTRUCTIONS:

1. Answer **ALL** questions. Each question carries 25 Marks.
2. Course Outcome Table :

Qn.1	CO1	Qn.2	CO2

1. a) (i) Find whether the given below statements are true or false. Defend your answer with justification (6 marks)

A. For a particular data (say weight of students in a class), the arithmetic mean, geometric mean and harmonic mean are 60.5, 61.5, 59.5 respectively

B. The daily rainfalls (in mm) of a city for a week are 110, 108, 109, 110, 107, 108, 109. This data reveals that the rainfall is inconsistent

C. For a particular data (say weight of students in a class), ogive can be symmetric (bell shaped)

D. In a city, probability of having no corona virus cases is 0.4 and probability of having atleast one corona case is 0.55

E. Weighted mean is always lesser than arithmetic mean

F. P(B/A) = P(B) if A and B are mutually exclusive

(ii) A printed circuit board has eight different locations in which a component can be placed. If four different components are to be placed on the board, how many different designs are possible? What is the counting technique to be used here? (3 marks)

b) A study finds that racism in cricket event more often takes place when the game is played in England or Australia or New Zealand (Say EAN). Given that

(a) Racism takes place or Game is played in EAN is 7/10

(b) Racism takes place and Game is played in EAN is 2/5

(c) Game is played in EAN given that Racism takes place is 2/3

Find the probability of

i. No Racism takes place

ii. Game is played in EAN

iii. Racism takes place given that Game is played in EAN (6 marks)

c) The frequency distribution for the length, in seconds, of 100 telephone calls was:

Time (seconds)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140	140 – 160	160 – 180
Frequency	0	5	7	14	28	21	13	9	3

Draw Histogram. Compute mean, mode and standard deviation. (10 marks)

2. a) (i) Prove that $(A \cup B)' \equiv A' \cap B'$ using Venn diagram.

(3 marks)

(ii) The Cartesian product of $A \times A$ has 9 elements among which are (-1, 0) and (0, 1). Find the set A and the remaining elements of $A \times A$

(3 marks)

(iii) A service organization in a large town organizes a raffle each month. One thousand raffle tickets are sold. Each has an equal chance of winning. First prize is \$300, second prize is \$200, and third prize is \$100. Let the random variable X denote the gain from the purchase of one ticket. Construct the probability distribution of X.

(3 marks)

b) Prove that $(A \cap B) \cup (A - B) = A \cap (B \cup (A - B))$ using Set Equivalence (laws) or membership table.

(6 marks)

c) (i) In a fruit feast among 200 students, 88 chose to eat durians, 73 ate mangoes, and 46 ate litchis. 34 of them had eaten both durians and mangoes, 16 had eaten durians and litchis, and 12 had eaten mangoes and litchis, while 5 had eaten all 3 fruits. Determine, how many of the 200 students ate none of the 3 fruits, and how many ate only mangoes?

(4 marks)

(ii) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax+3a & 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

A. Determine the constant a

B. Compute $P(X \le 1.5)$

C. Compute E(X)

(6 marks)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Computer Applications

M.C.A - Semester 1 CONTINUOUS ASSESSMENT TEST 2 Date: 03-MAR-2022

20MX11 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time: 1 Hour 30 mins. **Maximum Marks: 50**

INSTRUCTIONS:

1. Answer ALL questions. Each question carries 25 Marks.
2. Course Outcome Table : | Qn.1 | CO3 | | Qn.2 | CO4 |

1. a) (i) Consider regular expression 0(0+1)*0 | 1(0+1)*1. What it represents?

(ii) By induction prove that $n^2 - 3n + 4$ is even and it is true for all positive integers

(iii) Find the language generated by the Grammar G

G : < {S}, {a, b}, S, {S → aSb, S → ba } >

(3+3+3 = 9 marks)

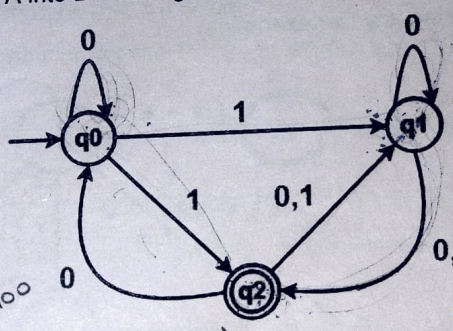
b) Consider the method PatternS(int n, int m, String S), it prints all strings of the form S+P, where P consists of n |'s and m *'s. (6 marks)

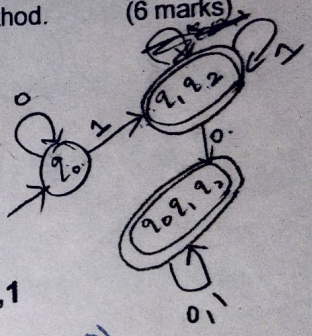
```
public static long PatternS(int n, int m, String S)
{
        if(n==0 && m==0)
            System.out.println(S);   // It is like printf() in C
        else  {
            if(n!=0)  PatternS(n-1, m, S+'|');
            if(m!=0)  PatternS(n, m-1, S+'*');
        }
}
```

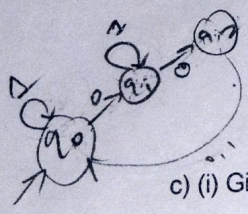
Prove that this recursive method is correct using mathematical induction. (Hint: Base step is P(0) = PatternS(0, 0, S). Assume inductive hypothesis P(k-1) is true where n+m=k-1)

(OR)

Convert the below NFA into DFA using subset construction method. (6 marks)







c) (i) Given the grammar: (4 marks)

S → if (E) S | if (E) S else S

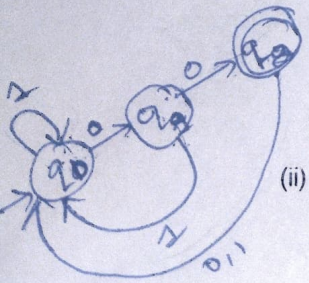
S → other

E → expr

Prove that it represents ambiguous **if-else** grammar using derivation trees.

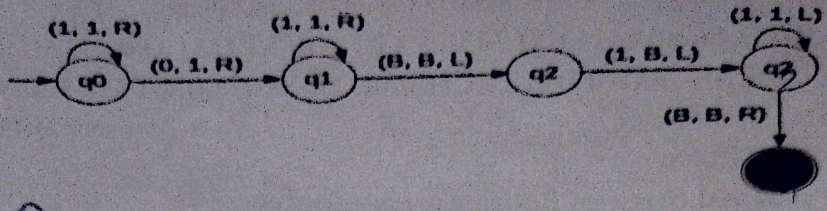
(ii) Exemplify in detail 4 different types of grammars with relevant examples.

(6 marks)



2. a) (i) Draw the simplest possible DFA (in terms of number of states and arcs) that describes the language of all strings that end in "00"

(ii) Write the Transition (δ) table for the machine given below.

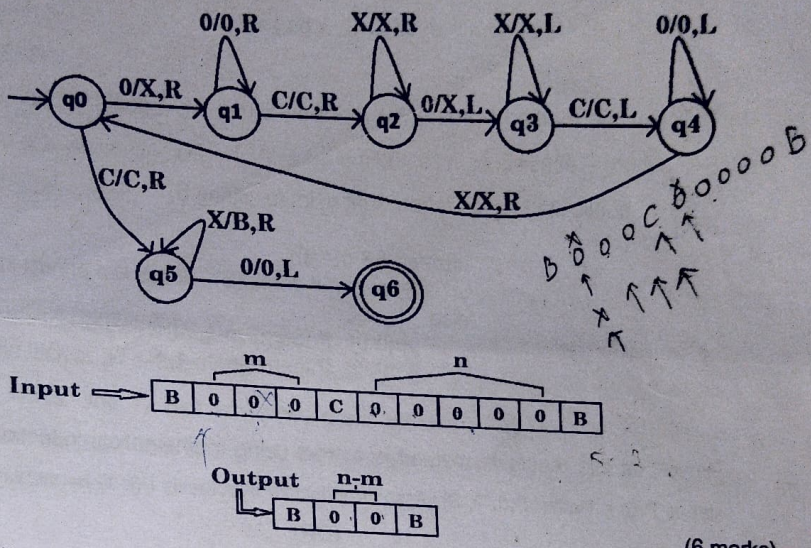


(iii) Give pushdown automata that recognize the following language

L={w ∈ {0,1}* | w contains at least three 1's}

(3+3+3 = 9 marks)

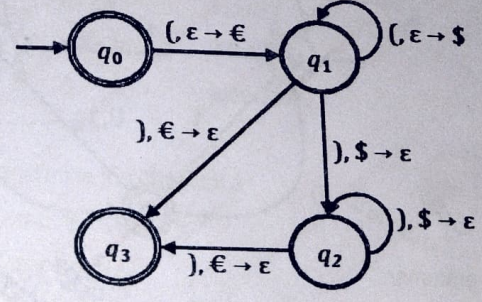
b) Prove that the following Turing machine is doing subtraction of numbers. (Here note that the representation 0/X, R means 0 → X, R)



(6 marks)

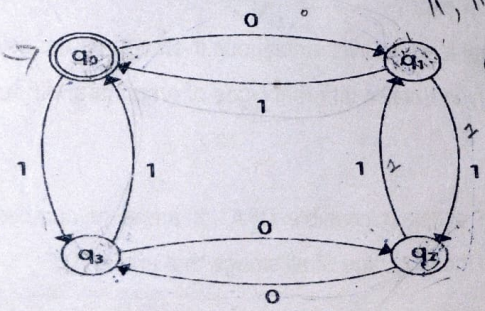
c) (i) Prove that the given PDA checks the Matching Parentheses (by taking any two examples)

(5 marks)



(ii) Consider the below FA with ∑ = {0, 1}. Draw transition table. Identify the language generated by the given machine.

(5 marks)



No of Pages : 4

Course Code : 20MX11

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, MARCH 2022

MCA Semester : 1

20MX11 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:

1. Answer **ALL** questions. Each question carries 25 Marks.
2. Course Outcome : Table

Qn.1	CO1

Qn.2	CO2

Qn.3	CO3

Qn.4	CO4

1. a) What counting techniques to be used for the below scenarios? Find answers and defend it. (6)

   (i) We need to create a team of 5 players for the competition out of 10 team members. How many different teams is it possible to create?

   (ii) In a supermarket, each product will be represented by a product code which is string of five-digit code (A, B, C, D, E) with repetition allowed. (For eg: AABDE). How many different possible product codes can be generated?

   (iii) How many three letter "words" can be made from the letters a, b, and c with no letters repeating?

   b) Assume that the word 'offer' occurs in 80% of the spam messages in my account. Also, let's assume 'offer' occurs in 10% of my desired e-mails. If 30% of the received e-mails are considered as a scam, and I will receive a new message which contains 'offer', what is the probability that it is spam? (5)

   c) i) The frequency distribution for the length, in seconds, of 100 telephone calls was:

Time (seconds)	0–20	20–40	40–60	60–80	80–100	100–120	120–140	140–160	160–180
Frequency	0	5	7	14	28	21	13	9	3

Draw Histogram, and ogive. Compute mean, mode, median and standard deviation. (14)

(OR)

ii) The time one has to wait for a bus at a bus stop is observed to a random phenomenon (X) with probability density function

$$f_X(x) = \begin{cases} 0, & \text{for } x < 0 \\ \frac{1}{9}(x+1), & \text{for } 0 \le x < 1 \\ \frac{4}{9}(x-\frac{1}{2}), & \text{for } 1 \le x < \frac{3}{2} \\ \frac{4}{9}(\frac{5}{2}-x), & \text{for } \frac{3}{2} \le x < 2 \\ \frac{1}{9}(4-x), & \text{for } 2 \le x < 3 \\ \frac{1}{9}, & \text{for } 3 \le x < 6 \\ 0, & \text{for } x \ge 6 \end{cases}$$

Compute the following

- Let A be the event of person waits between 0 to 2 minutes, B be the event of person waits between 0 to 3 minutes. Show that P(B/A) = 2/3
- Probability of person waits more than 2 minutes
- The expected waiting time of a person

2. a) Prove the following using Set Equivalence (laws) or membership table. (14)

(i) $(A \cap B) \cup (A - B) = A \cap (B \cup (A - B))$ (7)

(ii) $((A - B) - (B - C))^c = A^c \cup B$

b) i) In a fruit feast among 200 students, 88 chose to eat durians, 73 ate mangoes, and 46 ate litchis. 34 of them had eaten both durians and mangoes, 16 had eaten durians and litchis, and 12 had eaten mangoes and litchis, while 5 had eaten all 3 fruits. Determine, how many of the 200 students ate none of the 3 fruits, and how many ate only mangoes? (6)

(OR)

ii) Let R be a relation from N (set of all natural numbers) to N defined by

$R = \{ (a, b) : a = b^2 \text{ and } a, b \in N \}$.

Prove that R is not reflexive, not symmetric, not transitive. (6)

c) Use the Principle of Mathematical Induction to prove the following statements. (12)

(i) For any positive integer n, $6^n - 1$ is divisible by 5

(ii) Find formula for the following sum of series by examining values and prove your formula is correct.

$1/2 + 1/4 + 1/8 + \ldots\ldots + 1/2^n$

(iii) Devise a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b with a < b using the fact that gcd(a, b) = gcd(a, b − a). Prove your algorithm is correct

3. a) Find the languages generated by the following grammars or regular expressions.

(i) G = ({S}, {a,b,c}, {S→aSa, S→bSb, S→c}, S)

(ii) G: ({S, A, B}, {a, b}, S, {S → aA, A → aA | B, B → ε | bB })

(iii) Regular expression 0(0+1)*0 | 1(0+1)*1

(iv) Regular expression (011 + 1)* (8)

b) Prove that the following Grammar generates all even integers up to 998. Let G = (N, T, P, S), where N = {S, S_1, A, B}, T = {0,1,2,3,...,9} and P consists of S → 0|2|4|6|8, S→AS_1, A→1|2|3|4|5|6|7|8|9, S_1→0|2|4|6|8, S→ABS_1, B→0|1|2|3|4|5|6|7|8|9 (5)

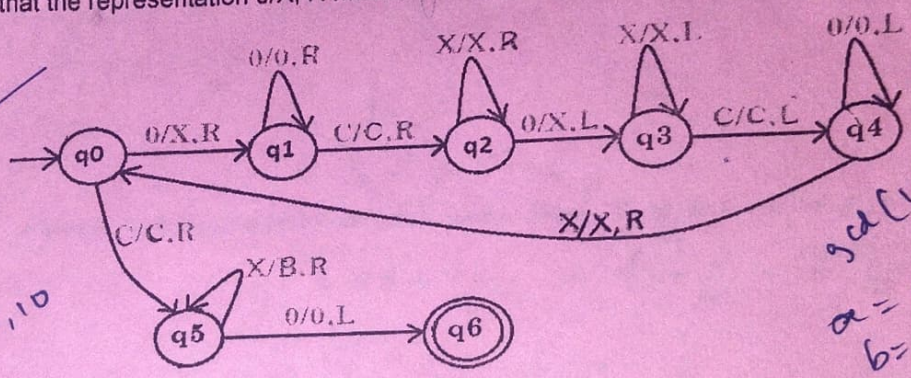
c) Exemplify in detail 4 different types of grammars. Construct grammars for the following languages. (12)

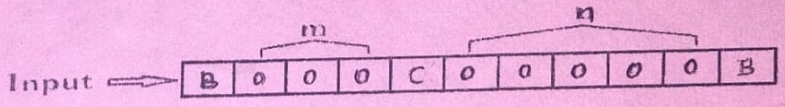
(i) Context-free grammar for the language $L = a^n b^{2n}$ where n>=1

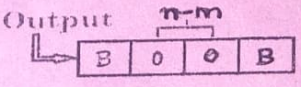
(ii) Parentheses matching in programming language using any grammar

(iii) Context sensitive grammar for the language L = { w ∈ {a, b, c}* | Number of a's in w = Number of b's in w = Number of c's in w , Number of a's in w ≥ 1}

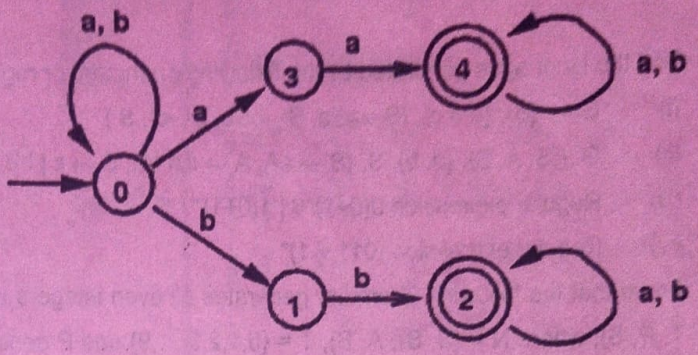
4. a) Prove that the following Turing machine is doing subtraction of numbers. (Here note that the representation 0/X, R means 0 → X, R) (7)







b) Convert the given NFA machine (M1) into a DFA machine (M2) and then identify the language accepted by both M1 and M2. (8)



c) Illustrate about Pushdown Automata. Construct a PDA that accepts

$$L = \{ ww^R \mid w = (a+b)^* \}$$

Here w is a word and w^R is its reverse (10)

/END/

FD/RL