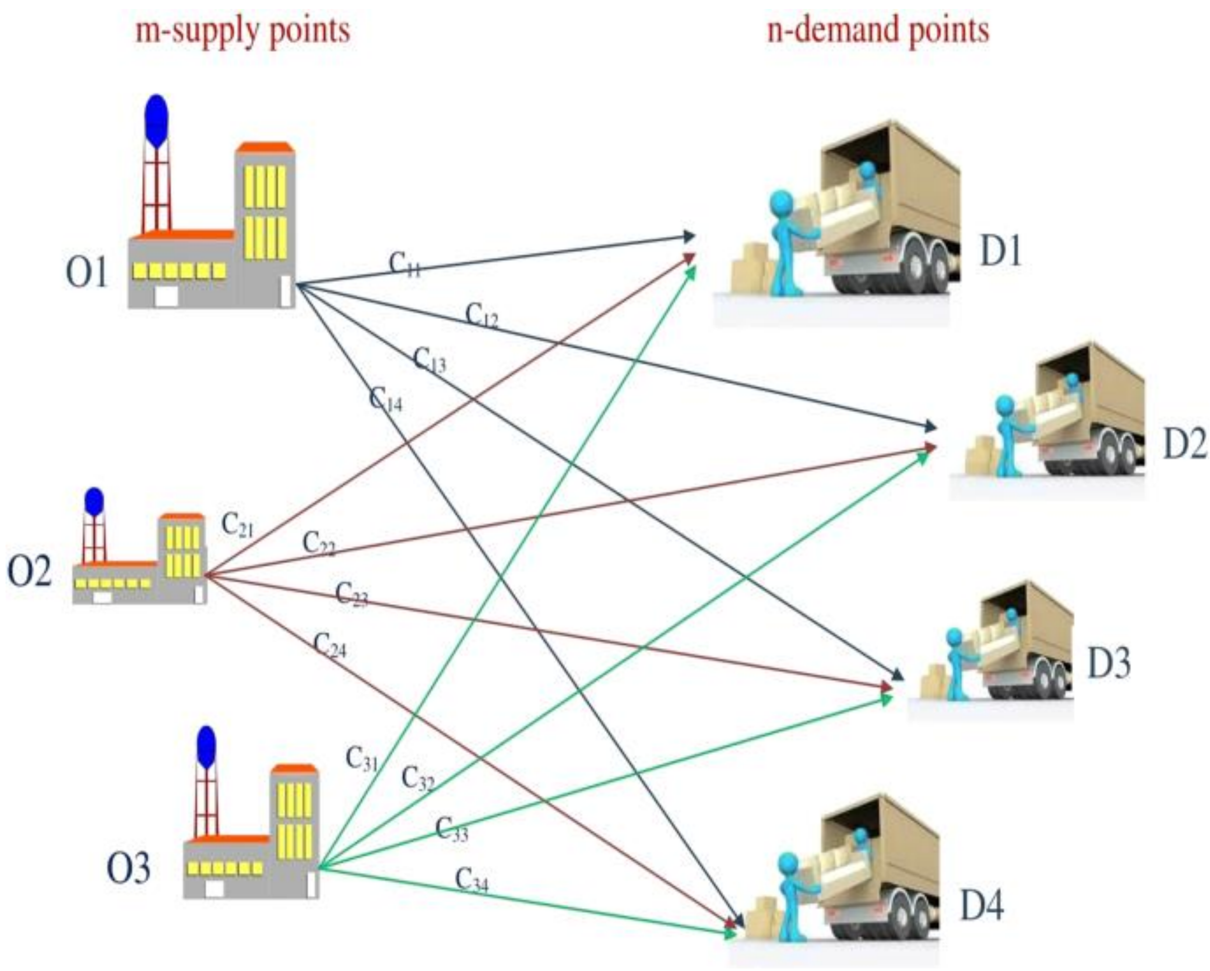
TRANSPORTATION PROBLEM

- Transportation problem talks about transporting items from a given set of Supply centres to Demand centres. [Supplier to Customer]

- A transportation problem when expressed in terms of an LP model can also be solved by the simplex method.

- The structure of transportation problem involves a large number of shipping routes from several supply centres to several demand centres.



- Thus, objective is to determine shipping routes between supply centres and demand centres in order to satisfy the required quantity of goods or services at each demand centre, with available quantity of goods or services at each supply centre at the minimum transportation cost and / time.

- Transportation problem is of two types:

  - Balanced transportation problem:
    (total demand = total supply)

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

  - Unbalanced transportation problem:
    (total demand ≠ total supply)

Destination

Source	D1	D2	D3	D4	D5	Supply(S_i)
O1	5	1	8	7	5	15
O2	3	9	6	7	8	25
O3	4	2	7	6	5	42
O4	7	11	10	4	9	35
Demand(d_j):	30	20	15	10	20	95 \ 117

MATHEMATICAL MODEL OF TRANSPORTATION PROBLEM

Example 1:

A company has three production factories S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product respectively.

These units are to be shipped to four warehouses D1, D2, D3 and D4 with requirement of 5, 8, 7 and 14 units (in 100s) per week respectively.

The transportation costs (in rupees) per unit between factories to warehouses are given in the table below:

	D1	D2	D3	D4	Supply (availability)
*S*1	19	30	50	10	7
*S*2	70	30	40	60	9
*S*3	40	8	70	20	18
Demand (requirement)	5	8	7	14	34

Formulate this transportation problem as an LP model to minimize the total transportation cost.

The transportation problem is stated as an LP model as follows:

Minimize (total transportation cost)

$Z = 19\,x11 + 30\,x12 + 50\,x13 + 10\,x14 + 70\,x21 + 30\,x22 + 40\,x23 + 60\,x24 + 40\,x31 + 8\,x32 + 70\,x33 + 20\,x34$

Subject to the constraints

$x11 + x12 + x13 + x14 = 7$
$x21 + x22 + x23 + x24 = 9$
$x31 + x32 + x33 + x34 = 18$ (Supply)

$x11 + x21 + x31 = 5$
$x12 + x22 + x32 = 8$
$x13 + x23 + x33 = 7$
$x14 + x24 + x34 = 14$ (Demand)

$x_{ij} \geq 0$ for i = 1, 2, 3 and j = 1, 2, 3, and 4.

In the above LP model, there are m × n = 3 × 4 = 12 decision variables, xij and m + n = 7 constraints, where m are the number of rows and n are the number of columns in a general transportation table.

Solving this type of LPP with 12 decision tables and 7 constraints will be very difficult.

Alternate Method: **Transportation Algorithm**

The objective is to minimise the cost associated with such transportation from places of supply to places of demand within given constraints of availability of supply and level of demand. These distribution problems are amenable to solution by a special type of linear programming model known as **"Transportation Model".**

Transportation Algorithm

The algorithm for solving a transportation problem may be summarized into the following steps:

Step 1: Formulate the problem and arrange the data in the matrix form. The formulation of the transportation problem is similar to the LP problem formulation. In transportation problem, the objective function is the total transportation cost and the constraints are the amount of supply and demand available at each source and destination, respectively.

Step 2: Obtain an initial basic feasible solution. In this chapter, following three different methods are discussed to obtain an initial solution:

✓ North-West Corner Method,

✓ Least Cost Method, and

To / Source	D	E	F	Supply
A	5	8 50	4	50
B	6	6 5	3 35	40
C	3 20	9 40	6	60
Demand	20	95	35	150

✓ Vogel's Approximation (or Penalty) Method

The initial solution obtained by any of the three methods must satisfy the following conditions:

- ✓ The solution must be feasible, i.e. it must satisfy all the supply and demand constraints.
- ✓ The number of positive allocations must be equal to m + n – 1, where m is the number of rows and n is the number of columns.
- ✓ Any solution that satisfies the above conditions is called <u>non-degenerate basic feasible solution, otherwise, degenerate solution.</u>

North-West Corner Method

To / From	1	2	3	4	supply
A	12 400	13 100	4	6	500
B	6	4 700	10	11	700
C	10	9 100	12 200	4 500	800
demand	400	900	200	500	

	D1	D2	D3	D4	Supply	
S1	19 5	30	50	10	~~7~~	2
S2	70	30	40	60	9	
S3	40	8	70	20	18	
Demand	~~5~~	8	7	14	34	

	D2	D3	D4	Supply
~~S1~~	~~30~~ 2	~~50~~	~~10~~	~~2~~
S2	30	40	60	9
S3	8	70	20	18
Demand	~~8~~	7	14	
	6			

	D2	D3	D4	Supply	
S2	30 6	40	60	~~9~~	3
S3	8	70	20	18	
Demand	~~6~~	7	14		

	D3	D4	Supply
~~S2~~	~~40~~ 3	~~60~~	~~3~~
S3	70	20	18
Demand	~~7~~	14	
	4		

	D3	D4	Supply
S3	70 4	20 14	~~18~~ ~~14~~
Demand	~~4~~	~~14~~	

Once the procedure is over, count the number of positive allocations. These allocations (occupied cells) should be equal to m + n – l = 3 + 4 – l = 6. If yes, then solution is non-degenerate feasible solution, otherwise degenerate solution.

The total transportation cost of the initial solution is obtained by multiplying the quantity xij in the occupied cells with the corresponding unit cost cij and adding all the values together. Thus, the total transportation cost of this solution is

Total cost =

5 × 19 + 2 × 30 + 6 × 30 + 3 × 40 + 4 × 70 + 14 × 20 = Rs 1,015

Final Solution:

Total cost = $67,000

Optimal cost = $42,000

		Destination				Total supply	Used supply	Unused supply
		1	2	3	4			
Supply sources	1	*80* 200	*50* 100	*30*	*60*	300	300	0
	2	*20*	*60* 200	*40* 200	*50*	400	400	0
	3	*50*	*40*	*70* 200	*40* 300	500	500	0
	Demand	200	300	400	300			
	Satisfied demand	200	300	400	300			
	Unsatisfied demand	0	0	0	0			