Mathematical Foundations of computer sciences.

CA1 → CO1, CO2

CA2 → CO3, CO4

28/09 2DMX11 MFCS

CO1. Statistics (CAI) CO2 Sets
Probability. Relations
Random variables Mathematical Induction.

CO3 Grammars (CAII) CO4 Automata & its types
Languages Turing machine.

CO2 Sets :

↳ collection of Elements / Objects

→ sets represented in uppercase Alphabet. (or) meaningful name.

Ex :- A | 22MX1ANS =
{ 22MX1D1, 22MX1D2, ... }

↳ Each element in the set are seperated by comma (,)

A = {a, b, c, d, ..., z} set of Alphabets.

B = { 1, 3, 5, 7, 9, ... } set of odd integers

↳ another notation

B = { x | x is an odd integer } → Infinite set.

A = { y | y is an English Alphabet }

Finite set

D = { x | x is an odd integer & x ≤ 20 }

↳ D = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

Finite & Infinite set is defined based on the no of elements present in the set.

Finite ← countable

Infinite ← uncountable

cardinality :

cardinality of a set is defined as no of elements present in the set.

$A = \{1, 3, 5, 7, 9\}$ → no of elements present in the set

$n(A) = |A| = 5$

C = Infinite set → cardinality of Infinite set

$n(C) = |C| = \infty$

Membership : ($\in$, $\notin$)

whether the element is belongs to the set.

$A = \{a, b, c, d, e, f\}$

$a \in A$ $\quad \in$ → belongs to

$z \notin A$ $\quad \notin$ → not belongs to

subset : one set is the set of another set.

$A = \{0, 1, 2, 3\}$ $\quad B = \{2, 3, 1, 4, 5, 6, 7, 0\}$

If all the elements of set A is present in the elements of another set B.

A is a subset of B

$\subseteq$ → subset of $\quad A \subseteq B$ $\quad B \nsubseteq A$

$\nsubseteq$ → not subset of $\quad$ subset $\quad$ superset

B is not a subset of A.

Equal sets :

$A = \{1, 2, 3, 4\}$ $\quad n(A) = 4$

$B = \{3, 1, 2, 4\}$ $\quad n(B) = 4$

$\left(\begin{array}{c} A \subseteq B \\ B \subseteq A \end{array} \right)$

$A = B$.

The elements of set A & set B are equal (or) same

Proper subset :-

all the elements of set 1 is present in set 2, but should not be equal

atleast one excess element should be present in the superset

$A = \{0,1,2,3\}$

$B = \{0,1,2,3,4\}$

$4 \in B$

$4 \notin B$.

so A is a propersset of B.

$\subset \rightarrow$ propersubset of

$\not\subset \rightarrow$ not a propersubset

$A \subset B$

A is a propersset of B.

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Null Set :- $\{\ \}$ (or) ϕ calling notation as pi

A set with no elements.

cardinality: $n(\phi) = |\phi| = 0$

"Nullset is the subset of Every set"

zero set :-

$B = \{0\}$ A set contains only one element $= 0$

$n(B) = |B| = 1$.

$A = \{1,2\}$

All possible subsets of A $B = \{\{1\}, \{2\}, \{1,2\}, \phi\}$

$n(A) = |A| = 2$

$n(B) = |B| = 4$ A is a set & B is the collection of subsets of A.

A with cardinality $n(A)$ if B is all possible sub sets of A.

then the cardinality of $B = 2^{n(A)}$

$x = \{1,2,3,4\}$ derive y. where y is the set of all subsets of x.

$y = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,2,3,4\}, \{1,3,4\}$

$n(x) = 4$ $n(y) = 2^{n(x)} = 2^4 = 16$

Power set :

If X is the set then Y = set of all possible sub sets of X

power(X) = $P(X)$ = set of all possible subsets of X.

$|X| = n$ then $|P(X)| = 2^n$

Ex :

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$A = \{x \mid x$ is an even integer & $1 \le x \le 20\}$

$B = \{x \mid x$ is a prime number between 1 and 20$\}$

$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$n(B) = |B| = 8$

Operations on set : SET OPERATIONS.

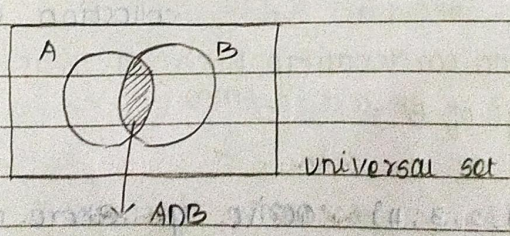
Intersection ($\cap$) : common elements in Both set

If A and B are set

then $A \cap B = \{x \mid x \in A$ and $x \in B\}$

$A = \{1, 2, 4, 5\}$
$B = \{6, 7, 5, 1\}$ } $A \cap B = \{1, 5\}$

$A = \{a, b, c, d\}$ $B = \{1, 2, 3, 4\}$ $A \cap B = \phi$



If A and B are sets then if $A \cap B = \phi$

then A and B are called Disjoint set

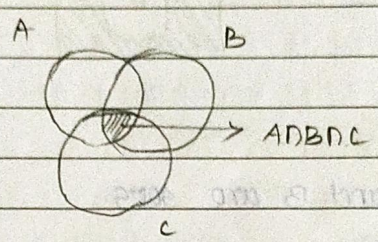
3 sets → If A and B and C are sets

then $A \cap B \cap C = \{x \mid x \in A, x \in B \text{ and } x \in C\}$

$A = \{1, 2, 3, 6, 8\}$ $B = \{3, 4, 7, 8\}$ $C = \{1, 2, 7, 8\}$

$A \cap B \cap C = \{8\}$

$n(A \cap B \cap C) = |A \cap B \cap C| = 1$.



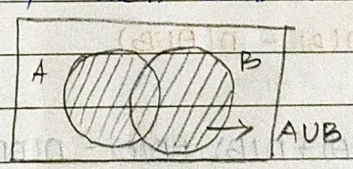
Union ($\cup$)

If A and B are set

then $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

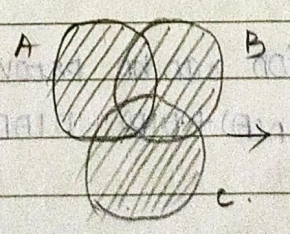
$A = \{a, b, c, d\}$
$B = \{x, y, z, a\}$ } $A \cup B = \{a, b, c, d, x, y, z\}$

If $A \cup B = \phi$ then Both A and B is also a null set



If A and B and C are sets

then $A \cup B \cup C = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\}$

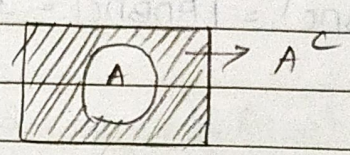


Complement :: A^c elements not in A but present in universal set.

$U = \{1, 2, 3, \ldots, \infty\}$

$A = \{1, 2, 3, \ldots 10\}$

$A^c = \{11, 12, 13, \ldots, \infty\}$



Difference :: (−)

If A and B are sets

then

$A - B = \{x \mid x \in A, x \notin B\}$

$B - A = \{x \mid x \notin A, x \in B\}$

$A = \{a, b, c, d\}$ $B = \{x, y, z, a\}$

$A - B = \{b, c, d\}$

$B - A = \{x, y, z\}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

3/10 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Prove the above equation :

The above equation can be proved using

$n(A \cup B) = n(A) + n(B) - n(A \cap C)$

Laws on sets :

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$

} commutative law

3. Associative Laws :
   (i) $A \cup (B \cup C) = (A \cup B) \cup C$
   (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

4. Distributive Laws :
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof :
(i) sample set
(ii) venn diagram
(iii) using definitions

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Prove that $A \cup B = B \cup A$

Let $x \in A \cup B$ $\quad A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
$\Rightarrow x \in A$ or $x \in B$
$\Rightarrow x \in B$ or $x \in A$
$\Rightarrow x \in B \cup A$
$\Rightarrow A \cup B = B \cup A$
Hence proved.

Prove that $A \cap B = B \cap A$

Let $x \in A \cap B$ $\quad A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
$\Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in B$ and $x \in A$
$\Rightarrow x \in B \cap A$
$\therefore A \cap B = B \cap A$
Hence proved.

Prove that $A \cap (B \cap C) = (A \cap B) \cap C$

Let $x \in A \cap (B \cap C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in A \cap B$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

$\Rightarrow A \cap (B \cap C) = (A \cap B) \cap C$

Hence proved

Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A \cup B$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\Rightarrow \therefore A \cup (B \cup C) = (A \cup B) \cup C$

Hence proved

Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cup C$

$\Rightarrow x \in A$ and ($x \in B$ or $x \in C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

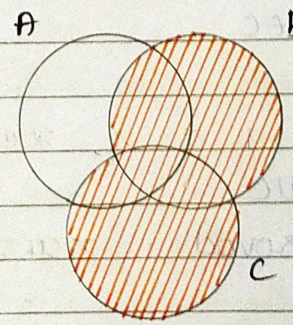
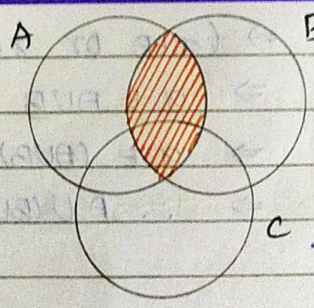
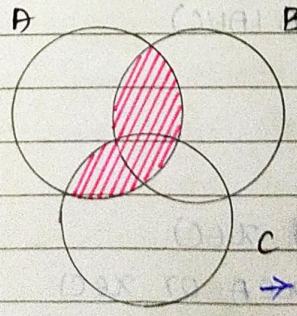
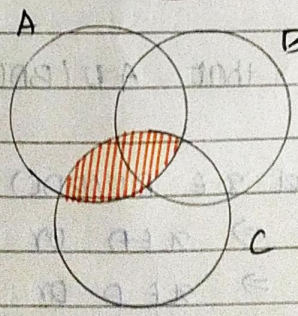
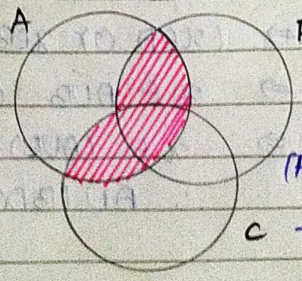
$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence proved

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using Venn Diagram.

LHS : $A \cap (B \cup C)$	RHS : $(A \cap B) \cup (A \cap C)$
 $B \cup C$ → (1)	 $A \cap B$ → (3)
 → (2) $A \cap (B \cup C)$	 $A \cap C$ → (4)
	 $(A \cap B) \cup (A \cap C)$ → (5)

From (2) & (5) LHS = RHS $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup A^c = U$

$A \cap A^c = \phi$

$A \cup \phi = A$

$A \cup A = A$

$A \cap A = A$

$(A \cup B)^c$ can also be denoted as $(A \cup B)'$

Demorgan's Law :

(i) $(A \cup B)^c = A^c \cap B^c$

$(A \cup B)' = A' \cap B'$

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

all are same different types of notations.

(ii) $(A \cap B)^c = A^c \cup B^c$

$(A \cap B)' = A' \cup B'$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Prove that $(A \cup B)^c = A^c \cap B^c$

Let $x \in (A \cup B)^c$ $\qquad A^c = \{x \mid x \notin A\}$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$ $\qquad$ or → and under complement rules.

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

$\therefore (A \cup B)^c = A^c \cap B^c$

Hence proved.

NOTE :-

(i) $x \in A \cup B$

$x \in A$ or $x \in B$

(ii) $x \notin A \cup B$

$x \notin A$ and

$x \notin B$

Prove that $(A \cap B)^c = A^c \cup B^c$

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\therefore (A \cap B)^c = A^c \cup B^c$

Hence proved.

$(A \cap B) \cup (A \cap B^c) = ?$

$\because$ By using Distributive Law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

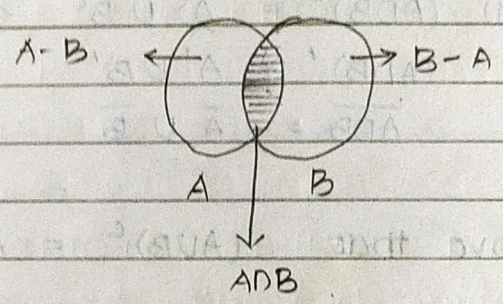
$\Rightarrow (A \cap B) \cup (A \cap B^c) = A \cap (B \cup B^c)$

$= A \cap U$

$= A$

$\therefore (A \cap B) \cup (A \cap B^c) = A$

7/10



$\because$ $A - B$, $B - A$ and $A \cap B$ are disjoint sets.

$\Rightarrow (A - B) \cap (B - A) \cap (A \cap B) = \phi$

$\Rightarrow (A - B) \cup (B - A) \cup (A \cap B) = A \cup B$

$\Rightarrow n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

Prove that

$n(A - B) + n(B - A) + n(A \cap B) = n(A) + n(B) - n(A \cap B)$

From the venn diagram $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow n(A - B) + n(B - A) + n(A \cap B)$

$\Rightarrow n(A) - \cancel{n(A \cap B)} + n(B) - \cancel{n(A \cap B)} + \cancel{n(A \cap B)}$

$\Rightarrow n(A) + n(B) - n(A \cap B)$

$\therefore n(A - B) + n(B - A) + n(A \cap B) = n(A) + n(B) - n(A \cap B)$

Hence proved.

Inclusion & Exclusion rule : 2 sets $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3 sets

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

n no of terms n - finite no of sets

$n(A \cup B \cup C \cup \ldots \cup n)$

(or)

wrong.

$$\cancel{n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + n(A_3) + \cdots + n(A_n) - n(A_1 \cap A_2) - n(A_1 \cap A_3) - \cdots - n(A_1 \cap A_n) - n(A_2 \cap A_3) - n(A_2 \cap A_4) - \cdots - n(A_2 \cap A_n) - \cdots - n(A_{n-1} \cap A_n) + n(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

$$n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + \cdots + n(A_n) - n(A_1 \cap A_2) - n(A_1 \cap A_3) - \cdots - n(A_{n-1} \cap A_n) + n(A_1 \cap A_2 \cap A_3) + \cdots + n(A_{n-2} \cap A_{n-1} \cap A_n) - \cdots + (-1)^{n+1} n(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n)$$

4 sets

$$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) - n(A \cap B) - n(A \cap C) - n(A \cap D) - n(B \cap C) - n(B \cap D) - n(C \cap D) + n(A \cap B \cap C) + n(A \cap B \cap D) + n(B \cap C \cap D) - n(A \cap B \cap C \cap D)$$

no of terms in RHS $= 2^n - 1$

2 sets $= 2^2 - 1 = 3$

3 sets $= 2^3 - 1 = 7$

4 sets $= 2^4 - 1 = 15$

1. In a school, there 20 teachers who teach mathematics (or) physics. of which 12 teach mathematics & 4 teach both mathematics & physics. How many teachers teach physics?

$n(\text{mathematics} \cup \text{physics}) = 20$

$n(\text{mathematics}) = 12$ $\quad n(\text{mathematics} \cap \text{physics}) = 4$

$n(\text{Physics}) = ?$

$n(\text{maths} \cup \text{physics}) = n(\text{maths}) + n(\text{Physics}) - n(\text{maths} \cap \text{physics})$

$n(\text{physics}) = n(\text{maths} \cup \text{physics}) - n(\text{maths}) + n(\text{maths} \cap \text{Physics})$

$= 20 - 12 + 4$

$= 12$

$\therefore n(\text{physics}) = 12$

2. In a class of 35 students, 24 like to play cricket, & 16 like to play football. Also each student likes to play atleast one of the two games. How many students like to play both cricket & football.

$C \rightarrow$ cricket $\quad F \rightarrow$ Football

$n(C) = 24$ $\quad n(F) = 16$ $\quad n(C \cup F) = 35$ $\quad n(C \cap F) = ?$

$n(C \cup F) = n(C) + n(F) - n(C \cap F)$

$n(C \cap F) = n(C) + n(F) - n(C \cup F)$

$= 24 + 16 - 35$

$= 5$

$n(C \cap F) = 5$

$\therefore$ 5 students among the class likes to play both cricket & Football.

3. There are 200 individuals with a skin disorder. Out of which 120 had been exposed to chemical C_1 & 50 to chemical C_2 & 30 to both the chemicals C_1 & C_2. Find the no of individuals exposed to

(i) chemical C_1 but not to chemical C_2
(ii) chemical C_2 but not to chemical C_1
(iii) chemical C_1 or chemical C_2

$n(U) = 200$

$n(C_1) = 120 \quad n(C_2) = 50 \quad n(C_1 \cap C_2) = 30$

(i) $n(C_1 - C_2) = n(C_1) - n(C_1 \cap C_2)$
$= 120 - 30$
$= 90$

(ii) $n(C_2 - C_1) = n(C_2) - n(C_1 \cap C_2)$
$= 50 - 30$
$= 20$

(iii) $n(C_1 \cup C_2) = n(C_1) + n(C_2) - n(C_1 \cap C_2)$
$= 120 + 50 - 30$
$= 140$

10.10.2022

Department of Computer Applications
20mx11 – MFCS – SET Theory – Practice Questions

1. Let A = {a, e, i, o, u} and B = {a, b, c, d}. Is A a subset of B? Yes/No (Why?). Is B a subset of A? Yes/No (Why?)
2. How many elements has P(A), if A is a null set?
3. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}. Find A^c, B^c, $A^c \cap B^c$, $A \cup B$ and $(A \cup B)^c$
4. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.
5. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?
6. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?
7. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?
8. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?
9. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find i) the number of people who read at least one of the newspapers and ii) the number of people who read exactly one newspaper
10. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Practice Questions :

1. $A = \{a, e, i, o, u\}$ $B = \{a, b, c, d\}$

A is not a subset of B $A \not\subset B$

Because the elements e, i, o, u is not present in the set A

inside the ~~subs.~~ set B.

B is also ~~a~~ not a subset of A $B \not\subset A$

Because the elements b, c, d in set B is not present in the set A.

2. A is a null set ϕ, $\{\phi\}$

so the $|A| = n(A) = 0$

Power set, $P(A) = 2^{n(A)}$

$= 2^0 = 1$

$P(A) = 1$

3. $U = \{1, 2, 3, 4, 5, 6\}$ $A = \{2, 3\}$ $B = \{3, 4, 5\}$

$A^c = \{1, 4, 5, 6\}$

$B^c = \{1, 2, 6\}$

$A^c \cap B^c = \{1, 6\}$

$A \cup B = \{2, 3, 4, 5\}$

$(A \cup B)^c = \{1, 6\}$

4. Let set A denotes the students taking apple juice.

set B denotes the students taking orange juice.

set U be the Universal set

Given data :

$n(U) = 400$ $n(A) = 100$ $n(B) = 150$

$n(A \cap B) = 75$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 100 + 150 - 75$

$n(A \cup B) = 175$

Students were taking neither apple juice nor orange juice.

$n(A \cup B)^c = n(U) - n(A \cup B)$

$= 400 - 175$

$= 225$ students

5. Let set A denotes the people like coffee.
set B denotes the people like tea.
set U be the universal set

Given Data : $n(U) = 70$

$n(A) = 37$ $n(B) = 52$ $n(A \cup B) = 70$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 37 + 52 - 70$

$n(A \cap B) = 19$

19 People like both coffee and tea.

6. Let set A be people speak French
set B be people speak Spanish

Given Data : $n(A) = 50$ $n(B) = 20$ $n(A \cap B) = 10$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 50 + 20 - 10$

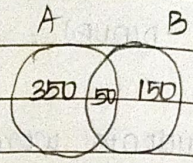
$= 60$

$\therefore$ 60 people speak atleast one of these two languages.

7.

$n(A) = 400 \quad n(B) = 200$

$n(A \cap B) = 50$



Statement: Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B car.

According to the venn diagram,

The given statement is Incorrect. The totally investigated owners is 500. But the venn diagram says universe contains 550 so the given statement is incorrect.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 400 + 200 - 50$

$n(A \cup B) = 550$

8. Let set T be students taking tea

set C be students taking coffee

set U be the Universal set

Given data: $n(U) = 600$

$n(T) = 150 \quad n(C) = 225 \quad n(T \cap C) = 100$

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$= 150 + 225 - 100$

$= 275$

students were taking neither tea nor coffee

$n(T \cup C)^c = n(U) - n(T \cup C)$

$= 600 - 275$

$= 325$ students.

$\therefore$ 325 students were taking neither tea nor coffee.

9. Let set H be the people read newspaper H
set T be the people read newspaper T
set I be the people read newspaper I
U be the universal set.

Given data :-

$n(U) = 60$ $n(H) = 25$ $n(T) = 26$ $n(I) = 26$

$n(H \cap I) = 9$ $n(H \cap T) = 11$ $n(T \cap I) = 8$

$n(H \cap T \cap I) = 3$

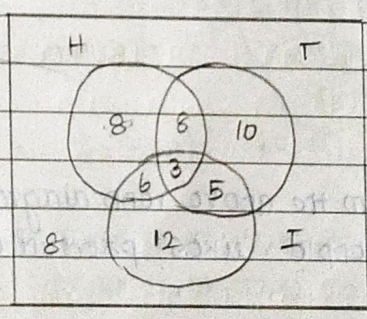
(i) $n(H \cup T \cup I) = n(H) + n(T) + n(I) - n(H \cap T) - n(H \cap I) - n(T \cap I) + n(H \cap T \cap I)$

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$

$= 80 - 28$

$= 52$

(ii)



By using venn diagram,

The number of people who read exactly one newspaper $\Big\} = 8 + 10 + 12$
$= 30.$

Thus 30 people reads exactly one newspaper.

10. Let A be the set of people who likes product A
B be the set of people who likes product B
C be the set of people who likes product C

Given data :

$n(A) = 21$ $n(B) = 26$ $n(C) = 29$

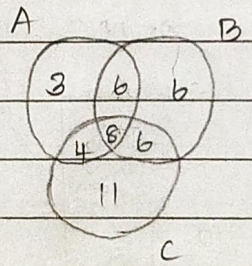
$n(A \cap B) = 14$ $n(A \cap C) = 12$ $n(B \cap C) = 14$

$n(A \cap B \cap C) = 8$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 21 + 26 + 29 - 14 - 12 - 14 + 8$

$= 44.$



$n(A \cup B \cup C) = 44$

Thus from the above venn diagram, the number of people likes product C is 11 peoples.

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also called as
↑ cross product

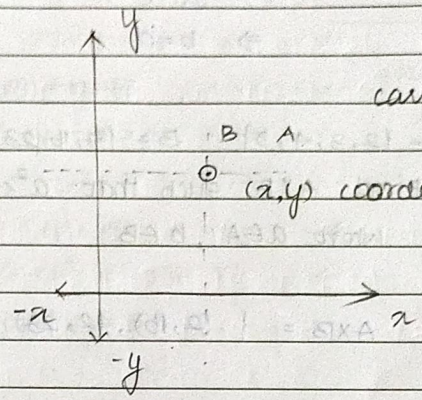
Prerequisite for Relations:

Cartesian Product of Two sets :

↓

two dimensional plane, intersection of two lines.

× → cartesian product notation.



cartesian plane

(x,y) - ordered pair

General definition :-

set of all ordered pairs.

$$A \times B = \{ (x,y) \mid x \in A, y \in B \}$$

$A \times A = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) \}$

Example :-

$A = \{1,2,3\}$ $B = \{4,5,6,7\}$

$A \times B = \{ (1,4), (1,5), (1,6), (1,7), (2,4), (2,5), (2,6), (2,7), (3,4), (3,5), (3,6), (3,7) \}$

$B \times A = \{ (4,1), (5,1), (6,1), (7,1), (4,2), (5,2), (6,2), (7,2), (4,3), (5,3), (6,3), (7,3) \}$

no of elements present in $A \times B$

$|A \times B| = n(A) * n(B)$

$A \times B \neq B \times A$

$|A \times B| = n(A) * n(B)$

$= 3 * 4 = 12$

$|B \times A| = n(B) * n(A)$

$= 4 * 3 = 12$

Note :-

only if the set $A = B$ then only we can get $A \times B = B \times A$.

When can we say two different ordered pairs are same

$(a,b) = (c,d)$

only when

$\Rightarrow a = c$

$\Rightarrow b = d$

1. $A = \{2,3,4,5\}$ $B = \{4,16,23\}$

Find $A \times B$ such that $a^2 < b$

where $a \in A$, $b \in B$.

$(5,23) \notin A \times B$

where $a^2 < b$

$a \in A, b \in B$

$A \times B = \{(2,16), (2,23), (3,16), (3,23), (4,23)\}$

2. Let ~~A~~ = $A = \{1,2,3\}$

$B = \{x \mid x$ is a prime number less than $10\}$

Find $A \times B$, $B \times A$, $|A \times B|$

$A = \{1,2,3\}$

$B = \{2,3,5,7\}$

$A \times B = \{(1,2), (1,3), (1,5), (1,7),$
$(2,2), (2,3), (2,5), (2,7),$
$(3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{(2,1), (2,2), (2,3),$
$(3,1), (3,2), (3,3), (5,1), (5,2), (5,3),$
$(7,1), (7,2), (7,3)\}$

$|A \times B| = |A| * |B|$

$= 3 * 4 = 12$

$|A \times B| = 12$

3. Find x & y if $(\underset{x}{2x-y}, \underset{y}{25}) = (\underset{x}{15}, \underset{y}{2x+y})$

$2x - y = 15 \rightarrow (1)$

$2x + y = 25 \rightarrow (2)$

$$\begin{aligned} 2x - \cancel{y} &= 15 \\ 2x + \cancel{y} &= 25 \\ \hline 4x &= 40 \\ x &= 40/4 = 10. \end{aligned}$$

$x = 10$

$2(10) - y = 15$

$-y = 15 - 20$

$-y = -5$

$y = 5$

$\therefore x = 10$ and $y = 5$

4. if $A \times B = \{(a,x), (a,y), (b,x), (b,y), (a,z), (b,z)\}$
Find A and B.

$A = \{a, b\}$ $B = \{x, y, z\}$

Standard results :

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

~~$A \cap (B \times C) = A$~~

$(A \cup B) \times C = (A \times C) \cup (B \times C)$

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \times (B - C) = (A \times B) - (A \times C)$

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A \times B = \{(x, y) \mid x \in A, y \in B\}$

Prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

LHS :

Let $(x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A, y \in B \cap C$

$\Rightarrow x \in A, \{y \in B \text{ and } y \in C\}$

$\Rightarrow \{x \in A, y \in B\}$ and $\{x \in A, y \in C\}$

$\Rightarrow (x, y) \in A \times B$ and $(x, y) \in A \times C$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\Rightarrow A \times (B \cap C) = (A \times B) \cap (A \times C)$

$\therefore$ Hence proved.

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PROBLEM SHEET

1. Let A be the set of even numbered students opt for physics
B be the set of students whose numbers are divisible by 5 opt for chemistry
C be the set of students whose numbers are divisible by 7 opt for Math.

Given data: $n(U) = 120$

$n(A) = \frac{120}{2} = 60$

$\frac{120}{10}$

$n(B) = \left\lfloor \frac{120}{5} \right\rfloor = 24$ $\quad n(C) = \left\lfloor \frac{120}{7} \right\rfloor = 17$

$n(A \cap B) = 12$ $\quad n(A \cap C) = 8$ $\quad n(B \cap C) = 3$

$n(A \cap B \cap C) = 1$.

Solution:

~~$n(A \cap B)$~~

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= 60 + 24 + 17 - 12 - 8 - 3 + 1$$

$$= 101 - 23 + 1$$

$$n(A \cup B \cup C) = 102 - 23 = 79$$

The students opt for none of the three subjects,

$$n(A \cup B \cup C)^c = n(U) - n(A \cup B \cup C)$$

$$= 120 - 79$$

$$= 41 \text{ students.}$$

$\therefore$ 41 of the students not opted for any of the three subjects.

2. Let V be the set of people having a VCD
C be the set of people having a CD player or
T be the set of people having cable TV

Given Data :

$n(V) = 40 \quad n(C) = 60 \quad n(T) = 50$

$n(V \cap C) = 25 \quad n(V \cap T) = 35 \quad n(C \cap T) = 30$

$n(V \cap C \cap T) = 10$

Solution :

$n(V \cup C \cup T) = n(V) + n(C) + n(T) - n(V \cap C) - n(V \cap T) - n(C \cap T) + n(V \cap C \cap T)$

$= 40 + 60 + 50 - 25 - 35 - 30 + 10$

$= 150 - 90 + 10$

$n(V \cup C \cup T) = 160 - 90 = 70$

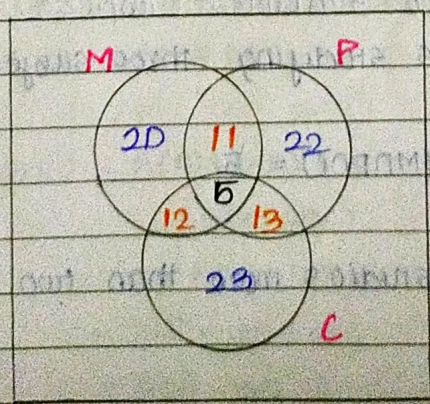
$\therefore$ 70 people had at least one of the three households.

3. Let M be the set of students studies Maths
P be the set of students studies Physics
C be the set of students studies chemistry

Given Data : $n(U) = 106$

$n(M) = 48 \quad n(P) = 51 \quad n(C) = 53$

$n(M \cap P) = 16 \quad n(M \cap C) = 17 \quad n(P \cap C) = 18$



since, from the statement each student studies atleast one of the three subjects.

$$n(M \cup P \cup C) = n(U) = 160.$$

(iii) The number of students who study all the three subjects.

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$$

$$106 = 48 + 51 + 53 - 16 - 17 - 18 + n(M \cap P \cap C)$$

$$106 = 101 + n(M \cap P \cap C)$$

$$n(M \cap P \cap C) = 106 - 101 = 5$$

$\therefore$ 5 students studies all the three subjects.

(i) The number of students who study exactly two subjects.

$$= 11 + 12 + 13$$

$$= 36$$

$\therefore$ 36 students studies ~~all the~~ exactly two subjects.

(ii) The number of students who study more than two subjects.

The above statement implies that students studying three subjects.

From (iii) $n(M \cap P \cap C) = 5$

$\therefore$ 5 students studies more than two subjects

iv) The number of students who exactly study one subject.

From the venn diagram,

Exactly 1 subject = 20 + 22 + 23
= 65

∴ 65 students studies exactly one subject.

(v) The number of students who study maths and physics but not chemistry.

= (M∩P) − (M∩P∩C)
= 16 − 5 = 11

∴ 11 students studies maths & physics but not chemistry.

4. Let A be the set of people had two wheeler
B be the set of people had credit card
C be the set of people had mobile phone

Given Data : n(U) = 200
n(A) = 100 n(B) = 70 n(C) = 140
n(A∩B) = 40 n(B∩C) = 30 n(A∩C) = 60
n(A∩B∩C) = 10

n(A∪B∪C) = n(A) + n(B) + n(C) − n(A∩B) − n(A∩C) − n(B∩C) + n(A∩B∩C)
= 100 + 70 + 140 − 40 − 30 − 60 + 10
= 310 − 130 + 10
= 320 − 130
n(A∪B∪C) = 190

No. of. candidates having none of the three.

$$(n(A \cup B \cup C))^c = n(U) - n(A \cup B \cup C)$$
$$= 200 - 190$$
$$= 10.$$

$\therefore$ 10 candidates had none of the three.

5. Let B be the set of Farmers, farm Beetroot
C be the set of Farmers, farm carrot
R be the set of Farmers, farm Raddish.

Given Data :- $n(U) = 350$

$n(B) = 260$ $n(C) = 100$ $n(R) = 70$

$n(B \cap C) = 30$ $n(B \cap R) = 40$ $n(C \cap R) = 40$

$n(B \cup C \cup R) = 350$

$$n(B \cup C \cup R) = n(B) + n(C) + n(R) - n(B \cap C) - n(B \cap R) - n(C \cap R) + n(B \cap C \cap R)$$
$$350 = 260 + 100 + 70 - 40 - 40 - 30 + n(B \cap C \cap R)$$
$$350 = 430 - 110 + n(B \cap C \cap R)$$
$$350 = 320 + n(B \cap C \cap R)$$
$$n(B \cap C \cap R) = 350 - 320$$
$$= 30$$

$\therefore$ 30 Farmers, farms all the three vegetables.

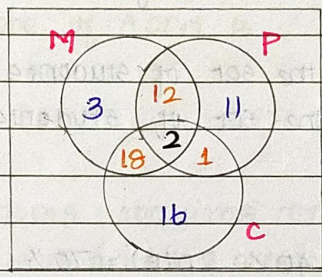
6. Let M be the students liked to study Mathematics
C be the students liked to study chemistry.
P be the students liked to study Physics.

Given Data: $n(U) = 85$

$n(M) = 35$ $n(C) = 37$ $n(P) = 26$

$n(M \cap C) = 20$ $n(M \cap P) = 14$ $n(C \cap P) = 3$

$n(M \cap C \cap P) = 2$



From the Venn Diagram,

(i) students like Mathematics or Physics.

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$= 35 + 26 - 14$

$= 47$

$\therefore$ 47 students like Mathematics or Physics.

(ii) students did not like any of the three subjects.

$n(M \cup P \cup C)^c = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$

$= 35 + 37 + 26 - 20 - 14 - 3 + 2$

$= 98 - 37 + 2 = 100 - 37$

$n(M \cup P \cup C) = 63$

Did not like any of the three,

$n(M \cup P \cup C)^c = n(U) - n(M \cup P \cup C)$

$= 85 - 63$

$n(M \cup P \cup C)^c = 22$

$\therefore$ 22 students did not like any of the three subject.

(iii) students likes mathematics & chemistry but not physics.

$= n(M \cap C) - n(M \cap C \cap P)$

$= 20 - 2 = 18$

$\therefore$ 18 students like mathematics & chemistry but not physics.

7. Let M be the set of students enrolled for Maths
E be the set of students enrolled for Economics

Given data :

$n(M) = 40\%$ $n(E) = 70\%$

$n(M \cap E) = 15\%$

$n(M \cup E) = n(M) + n(E) - n(M \cap E)$

$= 40 + 70 - 15$

$= 95\%$

No. of. students did not enroll for either of the two subjects.

$n(M \cup E)^c = 100\% - n(M \cup E)$

$= 100\% - 95\%$

$= 5\%$

$\therefore$ 5% of students did not enroll for either of the two subjects.

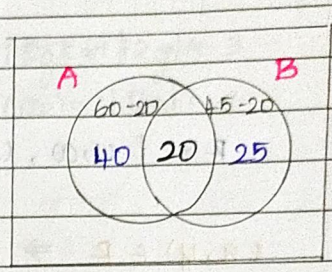
8. Let A be the set of students require Mathematics
B be the set of students require science.

Given data :

$n(U) = 140$

$n(A) = 60 \quad n(B) = 45$

$n(A \cap B) = 20$



(i) At least one of A and B

$= 40 + 20 + 25$

$= 85$

∴ 85 students requires atleast one of the subject Mathematics & science.

(ii) exactly one of A or B

$= 40 + 25$

$= 65$

∴ 65 students requires exactly one of the subject Mathematics & science.

(iii) neither A nor B

$= 140 - (40 + 20 + 25)$

$= 140 - 85$

$= 55$

∴ 55 students neither requires Mathematics nor requires science subject.

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Denotion :

RELATIONS (R, S, T)

$R = \{ (x,y) \mid x \in A, y \in B \ \& \ (x,y) \subseteq A \times B \}$

$A = \{1, 2, 3\}$ $B = \{a, b, c\}$ $A \times B = \{(1,a), (1,b), (2,a), (2,b), (2, \ldots), (3,a), (3,b), (3, \ldots)\}$

set of ordered pairs

$R = \{ (1,a), (2,b), (3,c), (3,b) \}$

$(x,y) \in R \Rightarrow x R y$. $(x,y) \in R \Rightarrow (y,x) \notin R$

Range :: (R_g)

$R_g = \{ x \mid (x,y) \in R \}$ $R_g = \{1, 2, 3\}$

Domain :: (D_R)

$D_R = \{ y \mid (x,y) \in R \}$ $D_R = \{a, b, c\}$

Example::

Set Representation :

(i) $R = \{ (1,1), (2,4), (3,9), (4,16) \}$

(x,y) $\longrightarrow$ x - Range & y - Domain

(ii) Matrix representation : (M_R)

		1	4	9	16 ← Domain
	1	1	0	0	0
	2	0	1	0	0
x	3	0	0	1	0
	4	0	0	0	1

↑ Range

(iii) Graph representation :: (R_G) → directed graph.

R D

1 ⟶ 1

2 ⟶ 4

3 ⟶ 9

4 ⟶ 16

something wrong here please refer pg no :: 40

Ex :

$R = \{(0,a), (1,b), (2,c), (1,c), (0,b), (1,d)\}$

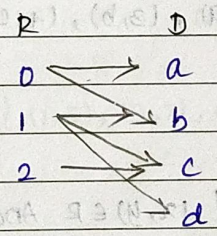
$R_g = \{0, 1, 2\}$ $\quad n(R_g) = |R_g| = 3$

$D_R = \{a, b, c, d\}$ $\quad n(D_R) = |D_R| = 4$

M_R :

	a	b	c	d
0	1	1	0	0
1	0	1	1	1
2	0	0	1	0

R_G :



1. $A = \{a, b, c, d\}$ $\quad B = \{5, 6, 7, 8, 9\}$

$R = \{(a,5), (c,7), (d,8), (b,7), (b,9), (c,9), (d,6)\}$

Represent the Relation using Matrix and Graph.

(i) M_R :

	5	6	7	8	9
a	1	0	0	0	0
b	0	0	1	0	1
c	0	0	1	0	1
d	0	1	0	1	0

← Domain.

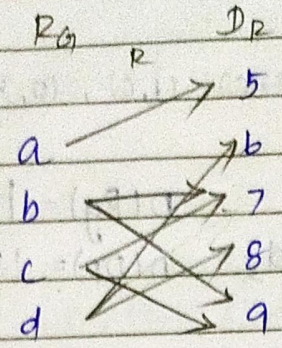
4×5

↑ Range.

$R_G = \{a, b, c, d\}$ $\quad |R_G| = 4$

$D_R = \{5, 6, 7, 8, 9\}$ $\quad |D_R| = 5$

(ii) R_G :



Inverse Relation : R^{-1}

$$R^{-1} = \{ (y, x) \mid (x, y) \in R \}$$

Ex :- $R = \{ (a,1), (b,3), (c,4), (d,3), (b,5) \}$

$$R^{-1} = \{ (1,a), (3,b), (4,c), (3,d), (5,b) \}$$

Operations :-

$$R \cap S = \{ (x,y) \mid (x,y) \in R \text{ And } (x,y) \in S \}$$

$$R \cup S = \{ (x,y) \mid (x,y) \in R \text{ or } (x,y) \in S \}$$

$$R^{-1} = \{ (y, x) \mid (x,y) \in R \}$$

$$R - S = \{ (x,y) \mid (x,y) \in R, (x,y) \notin S \}$$

Composition of Two Relations :- $(\circ)$

composition

$$R = \{ (x,y) \mid x \in A, y \in B \}$$

$$S = \{ (y,z) \mid y \in B, z \in C \}$$

$$R \subseteq A \times B \qquad S \subseteq B \times C$$

Then

$R \circ S \subseteq A \times C$

$\Rightarrow$

$R \subseteq A \times B \quad (a,b) \in R \mid a \in A,\ b \in B$

$S \subseteq B \times C \quad (b,c) \in S \mid b \in B,\ c \in C$

$\Rightarrow \quad (a,b) \in R \ \& \ (b,c) \in S$

$\Rightarrow \quad (a,c) \in R \circ S$

Example 1:

$A = \{1,2,3,4\} \quad B = \{1,3,9,10\} \quad C = \{5,6,7,8\}$

$ARB \rightarrow R = \{(1,1), (1,3), (2,9), (2,10), (3,3), (4,10)\}$

(a, b)

$BRS \rightarrow S = \{(1,5), (3,7), (9,7), (10,8)\}$

(b, c)

Find $R \circ S$?

(a, c)

$R \circ S = \{(1,5), (1,7), (2,7), (2,8), (3,7), (4,8)\}$

$\subseteq A \times C$

$S \circ R = \phi$

Example 2:

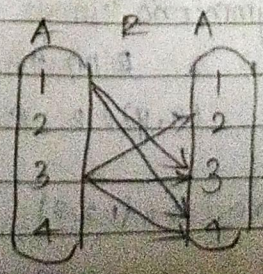
$R = \{(1,3), (1,4), (3,2), (3,3), (3,4)\}$ on $A = \{1,2,3,4\}$

soln:

$R \circ R = R^2$ (R power 2) (not R square)

$R \circ R = \{(1,2), (1,3), (1,4), (3,2), (3,4), (3,3)\}$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 1 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array} \quad 4 \times 4$$



see page: 49

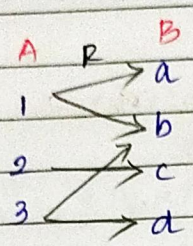
Domain :

$D(R) = \{ x \mid (x,y) \in R, \text{ for some } x \in A, y \in B \}$

Range :

$Rg(R) = \{ y \mid (x,y) \in R, \text{ for some } x \in A, y \in B \}$

RG :



$D(R) = \{1, 2, 3\}$

$Rg(R) = \{a, b, c, d\}$

Types of RELATIONS :

→ Reflexive
→ symmetric
→ Transitive.

Reflexive :

R on A

$(x,x) \in R \mid x \in A$

Ex : $A = \{1, 2, 3\}$

$R = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (3,3) \}$

all the elements in the A satisfies reflexive property.

So the Relation R is Reflexive

Symmetric :

R on A

$(x,y) \in R \Rightarrow (y,x) \in R$

(x,y) alternate available (y,x) is

Ex : $A = \{1, 2, 3\}$ $R = \{ (1,2), (1,3), (2,1), (3,1), (1,1), (2,2), (3,3) \}$

R is symmetric & Reflexive.

Transitive :- R on A

$(x, y) \in R$ & $(y, z) \in R \Rightarrow (x, z) \in R$

then it is transitive.

Ex :- $A = \{1, 2, 3\}$

$R = \{(1,2), (1,3), (2,1), (3,1), (1,1), (2,2), (3,3)\}$

(3,1), (1,2)
(3,2)

(i) $(1,2), (2,1) \Rightarrow (1,1)$ ✓

(ii) $(1,2), (2,2) \Rightarrow (1,2)$ ✓

(iii) $(3,3), (3,1) \Rightarrow (3,1)$ ✓

(iv) $(2,1), (1,1) \Rightarrow (2,1)$ ✓

× (v) $(3,1), (1,2) \Rightarrow (3,2) \notin R$

× (vi) $(2,1), (1,3) \Rightarrow (2,3) \notin R$

The given relation R is Reflexive, symmetric & not Transitive.

$(2,3) \notin R$, $(3,2) \notin R$ so the given R is not Transitive.

Transitive closure :-

Non transitive relation into transitive by adding the ordered pair to make the Relation as transitive.

For above Ex :-

Transitive closure = $\{(1,2), (1,3), (2,1), (3,1), (1,1), (2,2), (3,3), (3,2), (2,3)\}$.

The Relation R is transitive closure.

Equivalence Relation :-

R is a Relation.

If R satisfies all the three properties namely Reflexive, symmetric & transitive, the R is called as Equivalence Relation.

Ex :- $R = \{(1,2), (1,3), (2,1), (3,1), (1,1), (2,2), (3,3), (3,2), (2,3)\}$

R satisfies all the three properties. ⇒ Equivalence Relation.

Problem :

1. Let $A = \{1,2,3,4,5,6,7\}$ SEE DN PG: 47

R on A.

$R = \{(x,y) \mid x-y \text{ is divisible by } 3\}$

Prove that R is equivalence ?

Solution :

$R = \{(1,4), (1,7), (2,5), (3,6), (4,1), (4,7), (5,2), (6,3), (7,1), (7,4)\}$

(i) Reflexive :

The given relation R, where $(x,x) \notin R$ because $0 = 0 \div 3$ 1-1 does not divisible by 3.

So R is not reflexive.

So R is not a equivalence relation.

2. Let N be the Natural No

R on N

$R = \{(x,y) \mid x+y \text{ is even } \& \ x,y \in N\}$

Check for equivalence relation?

Solution :

(i) R is Reflexive only if

$\forall (x,x) \in R$, for all $x \in N$.

$(x+x)$ is even

$2x$ is even.

For all $x \in N$, sum of x is even.

$\therefore$ R is reflexive.

(ii) R is symmetric only if

$\forall (x,y) \in R \Rightarrow (y,x) \in R$

$x+y$ is even $\Rightarrow y+x$ is even.

$\Downarrow$

$(y,x) \in R$.

$\therefore$ R is symmetric.

(iii) R is transitive only if
$\forall (x,y) \in R, (y,z) \in R \Rightarrow (x,z) \in R$

$\Rightarrow$ $x+y$ is even & $y+z$ is even
$\Rightarrow$ $(x+y)+(y+z)$ is even
$\Rightarrow$ $x+2y+z$ is also even.
$\Rightarrow$ $(x,z) \in R$
$\therefore$ R is transitive.

Since R is Reflexive, symmetric & transitive.
$\therefore$ R is a Equivalence relation.

3. Let $R = \{(a,b) \mid a,b \in Z$ & $(a-b)$ is divisible by $5\}$
check for R is transitive?

R is transitive only if
$\forall (a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$

$\Rightarrow$ $a-b$ is divisible by 3 and $b-c$ is divisible by 3.
$\Rightarrow$ $(a-b+b-c)$ is divisible by 3
$\Rightarrow$ $(a-c)$ is divisible by 3
$\Rightarrow$ $(a,c) \in R$

$\therefore$ R is transitive

↑ If it is asked for divisible by k.

4. Let $R = \{(x,y) \mid (x-y) \text{ is divisible by } 3\}$

3 can b

(i) Let $(x,x) \in R$

$\Rightarrow x-x$ is divisible by 3

$\Rightarrow 0$ is divisible by 3 ✓

$\therefore R$ is reflexive.

(ii) Let $(x,y) \in R$

$\Rightarrow x-y$ is divisible by 3 $\quad (x-y) \quad (-y+x)$

$\Rightarrow -(y-x)$ is divisible by 3

$\Rightarrow (y,x) \in R$

$\therefore R$ is symmetric.

(iii) $\forall (x,y) \in R \; (y,z) \in R \Rightarrow (x,z) \in R$

$\Rightarrow (x-y)$ is divisible by 3 & $(y-z)$ is divisible by 3

$\Rightarrow (x-y+y-z)$ is divisible by 3

$\Rightarrow (x-z)$ is divisible by 3

$\Rightarrow (x,z) \in R$

$\therefore R$ is transitive.

R is reflexive, symmetric, & transitive.

$\therefore R$ is an equivalence relation.

(*) 5. If R is an equivalence Relation. Prove that R^{-1} is also an equivalence Relation.

$R = \{(x,y) \mid (x,y) \subseteq A \times B, \; x \in A, y \in$

$R^{-1} = \{(y,x) \mid (x,y) \subseteq A \times B, \; x \in A, y \in$

(i) Let $(x,x) \in R^{-1}$ since R is an equivalence relation

$\Rightarrow (x,x) \in R$ R is reflexive.

$\therefore R^{-1}$ is also reflexive.

(ii) Let $(x,y) \in R^{-1} \Rightarrow (y,x) \in R$

$\Rightarrow (x,y) \in R$ since R is an equivalence relation, R is symmetric

$\Rightarrow (y,x) \in R^{-1}$

$(x,y) \in R^{-1} \Rightarrow (y,x) \in R^{-1}$

$\therefore R^{-1}$ is also symmetric.

(iii) Let $(x,y) \in R^{-1}$ and $(y,z) \in R^{-1}$

$(x,y) \in R^{-1} \Rightarrow (y,x) \in R$

$(y,z) \in R^{-1} \Rightarrow (z,y) \in R$

$(z,y) \in R$ and $(y,x) \in R$

$(z,x) \in R$

$(x,z) \in R^{-1}$

$(z,y) \in R$ and $(y,x) \in R$

$\Rightarrow (z,x) \in R$ since R is an equivalence relation, R is transitive.

$\Rightarrow (x,z) \in R^{-1}$

$\therefore R^{-1}$ is also transitive.

$\therefore R^{-1}$ is reflexive, symmetric & transitive.

$\therefore R^{-1}$ is also an equivalence relation.

Hence proved.

6. If R and S are equivalence relation. Check whether $R \cap S$ is an equivalence relation?

(i) Let $(x,x) \in R \cap S$

$\Rightarrow (x,x) \in R$ and $(x,x) \in S$

since $R \cap S$ is an equivalence relation. R and S is reflexive.

$\Rightarrow (x,x) \in R \cap S$

$\therefore R \cap S$ is reflexive.

(ii) Let $(x,y) \in R \cap S$

$\Rightarrow (x,y) \in R$ and $(x,y) \in S$

since R and S is an equivalence relation. R and S are symmetric

$\Rightarrow (y,x) \in R$ and $(y,x) \in S$

$\Rightarrow (y,x) \in R \cap S$

$\therefore R \cap S$ is symmetric.

(iii) Let $(x,y) \in R \cap S$, $(y,z) \in R \cap S$

$\Rightarrow (x,y) \in R$ and $(x,y) \in S$ $\Rightarrow (y,z) \in R$ and $(y,z) \in S$

$\Rightarrow (x,y) \in R$ and $(y,z) \in R$ $\Rightarrow (x,y) \in S$ and $(y,z) \in S$

~~$\Rightarrow (y,z) \in R$~~ $\Rightarrow (x,z) \in S$

$\Rightarrow (x,z) \in R$

$\Rightarrow (x,z) \in R$ and $(x,z) \in S$

$\Rightarrow (x,z) \in R \cap S$

$\therefore R \cap S$ is transitive.

$\therefore R \cap S$ is an equivalence relation.

7. Let R be the relation defined on $(Z \times Z \times Z)$

$(a,b,c) \, R \, (d,e,f)$

$\Rightarrow b = e$ and $c = f$

Prove that R is an equivalence.

(i) Let $(a,b,c) \, R \, (a,b,c)$

$b = b$ and $c = c$

$\therefore R$ is reflexive.

(ii) Let $(a,b,c) R (d,e,f)$ $\Rightarrow (d,e,f) R (a,b,c)$
$\rightarrow$ R is symmetric

$(a,b,c) R (d,e,f)$
$\Rightarrow b=e$ and $c=f$
$\Rightarrow e=b$ and $f=c$
$\Rightarrow (d,e,f) R (a,b,c)$
$\therefore$ R is symmetric

(iii) Let $(a,b,c) R (d,e,f)$ and $(d,e,f) R (g,h,i)$

$\Rightarrow (a,b,c) R (d,e,f)$ $\Rightarrow (d,e,f) R (g,h,i)$
$\Rightarrow b=e$ and $c=f \rightarrow (1)$ $\Rightarrow e=h$ and $f=i \rightarrow (2)$
From (1) and (2)
$\Rightarrow b=e=h$ and $c=f=i$
$\Rightarrow b=h$ and $c=i$
$\Rightarrow (a,b,c) R (g,h,i)$
$\therefore$ R is transitive

$\therefore$ R is an equivalence relation.

1. Let $A = \{1,2,3,4,5,6,7\}$
R on A.
$R = \{(x,y) \mid (x-y)$ is divisible by $3\}$
Prove that R is an equivalence relation?
Solution:
$R = \{(1,1), (1,4), (1,7), (2,2), (2,5), (3,3), (3,6), (4,4), (4,1), (4,7), (5,5), (5,2), (6,6), (6,3), (7,7), (7,1), (7,4)\}$

(i) Reflexive.
R is reflexive only if
$\forall (x,x) \in R$, for all $x \in N$.
$(x-x)$ is divisible by 3
$\Rightarrow (x,x) \in R$ all the elements in A satisfies reflexive.
$\therefore$ R is reflexive.

MATHEMATICAL INDUCTION /

Principle of Mathematical Induction (PMI)

$A = \{1, 2, 3\}$

$P(A) = \{ \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{\phi\} \}$

$n(P(A)) = 2^3 = 8$ ↳ cardinality of null set $= 0$ ↑ even number

$\left.\begin{array}{l}\text{no. of subsets} \\ \text{having even cardinality}\end{array}\right\} = 4 = 2^{n-1} = 2^{3-1} = 2^2 = 4$

$A = \{1, 2, 3, 4, 5\}$

$P(A) = \{ \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{2,3,4\}, \{2,3,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{2,3,4,5\}, \{1,2,3,4,5\}, \{1,3,4,5\}, \{1,4,5\}, \{2,4,5\}, \{1,2,4,5\}, \{1,3,4\}, \{1,3,5\}, \phi \}$

$\left.\begin{array}{l}\text{no of subsets} \\ \text{having even cardinality}\end{array}\right\} = 16 = 2^{n-1} = 2^{5-1} = 2^4 = 16.$

2^{n-1} subsets are going to have even cardinality.

$A = \{1, 2, 3, \ldots, n\}$

$n(P(A)) = 2^n$ no of all possible subsets of A.

no of subsets having even cardinality $= 2^{n-1}$

continuation of this is available at pg: no :: 66

$k+1$

consecutive check.

1 3 5 $n=k$ $k+1$ n

Step 1: Whether the statement is true for $n=1$ (BOSIS value)

Step 2: Assume that the statement is true for $n=k$.

Step 3: Check for whether the statement is true for $n=k+1$ with the assumption of step 2

$\Rightarrow$ If yes then the statement is true for all n values.

Example:

Problem 1 $1+2+3+4+\cdots+n = \frac{n(n+1)}{2}$

Prove by using PMI/MI.

Step 1: Check for $P(1)$

$LHS = 1$ $RHS = \frac{1(1+1)}{2} = 2/2 = 1$

True for $n=1$

Step 2: Assume that $P(n)$ is true for $n=k$.

$\Rightarrow P(k)$ is true.

$LHS = 1+2+3+\cdots+k$ $RHS = \frac{k(1+k)}{2}$

$P(k) = \frac{k(k+1)}{2}$ $P(k)$ is true.

Step 3: Check for $P(k+1)$ using $P(k)$

$LHS = \underbrace{1+2+3+\cdots+k}_{\text{from step 2}} + (k+1)$ $RHS = \frac{(k+1)(k+2)}{2}$

$P(k+1) = \frac{(k+1)(k+2)}{2}$

$= \frac{k(k+1)}{2} + (k+1)$

$= \frac{k(k+1)+2(k+1)}{2}$

$\therefore P(k+1)$ is true

$\therefore P(n)$ is true for all values of n.

$LHS = \frac{(k+1)(k+2)}{2}$

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closure relation

1. Let R be the relation on the set $\{0,1,2,3\}$ containing the ordered pairs $(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)$. Find all the closures.

solution:

Let $A = \{0,1,2,3\}$

$R = \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\}$

$\rightarrow (1,0)$

Reflexive closure:

R is not reflexive because $(0,0)$ is $\notin R$

$$\text{Reflexive}(R) = R \cup \Delta_A$$
$$= \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\} \cup \{(0,0), (1,1), (2,2), (3,3)\}$$
$$\text{Reflexive}(R) = \{(0,0), (0,1), (1,1), (1,2), (2,0), (2,2), (3,0), (3,3)\}$$

Symmetric closure:

R is not symmetric because $(0,1) \in R$ but $(1,0) \notin R$

$$\text{symmetric}(R) = R \cup R^{-1}$$
$$= \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\} \cup \{(1,0), (1,1), (2,1), (0,2), (2,2), (0,3)\}$$
$$\text{symmetric}(R) = \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0), (1,0), (2,1), (0,2), (0,3)\}$$

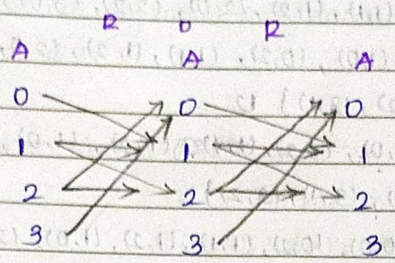
Transitive closure:

R is not transitive because $(0,1) \in R$ & $(1,2) \in R$ but $(0,2) \notin R$.

$R^* = R \cup R^2$

$\rightarrow R^2 = R \circ R$ → main graph

RG: (Graphic representation): R^2



$R^2 = R \cup R^2$

$\downarrow$

$R^2 \circ R$

$R^2 = R \circ R$

$= \{(0,1), (1,0), (0,2), (1,1), (1,2), (2,1), (2,0), (2,2), (3,1)\}$

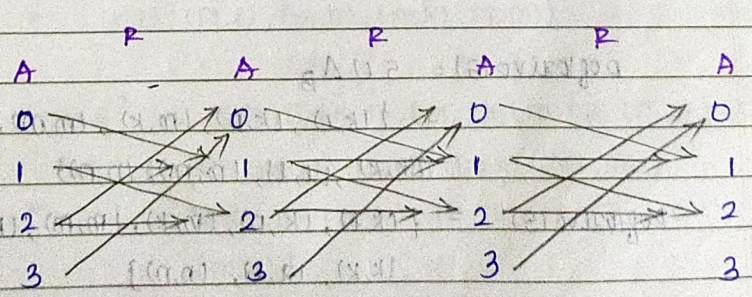
$R^* = R \cup R^2$

$= \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\} \cup \{(0,1), (1,0), (0,2), (1,1), (1,2), (2,1), (2,0), (2,2), (3,1)\}$

$R^* = \{(0,1), (1,1), (1,0), (1,2), (0,2), (2,0), (2,2), (3,0), (2,1), (3,1)\}$

R^* is not transitive because $(0,1) \in R^*$ & $(1,0) \in R^*$ but $(0,0) \notin R^*$

RG: (Graphic representation): R^3



$R^3 = R^2 \circ R$

$= \{(0,1), (0,0), (0,2), (1,1), (1,2), (1,0), (2,1), (2,2), (2,0), (3,1), (3,2)\}$

$R^* = R \cup R^2 \cup R^3$

$= \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\} \cup$
$\{(0,1), (1,0), (0,2), (1,1), (1,2), (2,1), (2,0),$
$(2,2), (3,1)\} \cup$
$\{(0,1), (0,0), (0,2), (1,1), (1,2), (1,0), (2,1), (2,2),$
$(2,0), (3,1), (3,2)\}$

$R^* = \{(0,1), (0,0), (0,2), (1,1), (1,2), (1,0), (2,1), (2,2),$
$(2,0), (3,1), (3,2), (3,0)\}$

R^* is transitive.

$\therefore$ transitive $(R) = \{(0,1), (0,0), (0,2), (1,1), (1,2), (1,0),$
$(2,1), (2,2), (2,0), (3,1), (3,2), (3,0)\}$

2. Let $S = \{(k,l), (k,n), (m,k), (m,m), (n,l)$ be a relation defined on set $B = \{k,l,m,n\}$. Find all closures of S.

Solution :

Let $B = \{k,l,m,n\}$
$S = \{(k,l), (k,n), (m,k), (m,m), (n,l)\}$

Reflexive closure.

S is not reflexive because $(k,k) \notin S$

$\text{Reflexive}(S) = S \cup \Delta_B$
$= \{(k,l), (k,n), (m,k), (m,m), (n,l)\} \cup$
$\{(k,k), (l,l), (m,m), (n,n)\}$

$\text{Reflexive}(S) = \{(k,l), (k,n), (m,k), (m,m), (n,l),$
$(k,k), (l,l), (n,n)\}$

Symmetric closure:

S is not symmetric because $(k, l) \in S$ but $(l, k) \notin S$.

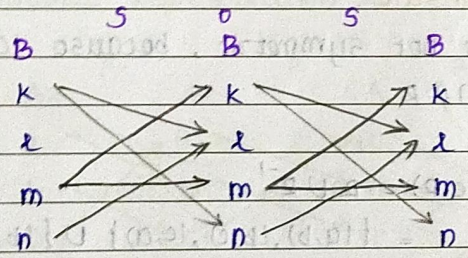
$$\text{symmetric}(S) = S \cup S^{-1}$$
$$= \{(k,l), (k,n), (m,k), (m,m), (n,l)\} \cup \{(l,k), (n,k), (k,m), (m,m), (l,n)\}$$

$$\text{Symmetric}(S) = \{(k,l), (k,n), (m,k), (m,m), (n,l), (l,k), (n,k), (k,m), (l,n)\}$$

Transitive closure:

S is not transitive because $(m,k) \in S$ & $(k,n) \in S$ but $(m,n) \notin S$

Sol: Graphic Representation S^2



$$S^2 = S \circ S$$
$$= \{(k,l), (m,l), (m,n), (m,k), (m,m)\}$$

$$S^* = S \cup S^2 = \{(k,l), (k,n), (m,k), (m,m), (n,l)\} \cup \{(k,l), (m,l), (m,n), (m,k), (m,m)\}$$

$$S^* = \{(k,l), (k,n), (m,k), (m,m), (n,l), (m,l), (m,n)\}$$

S^* is transitive

$\therefore$ Transitive$(S) = \{(k,l), (k,n), (m,k), (m,m), (n,l), (m,l), (m,n)\}$

3. Let $R = \{(a,b),(b,c),(c,a)\}$ be a relation defined on the set $A = \{a,b,c\}$. Find all the closures of R.

Solution :

$A = \{a,b,c\}$

$R = \{(a,b),(b,c),(c,a)\}$

Reflexive closure :

R is not reflexive, because $(a,a) \notin R$

$\text{Reflexive}(R) = R \cup \Delta_A$

$= \{(a,b),(b,c),(c,a)\} \cup \{(a,a),(b,b),(c,c)\}$

$\text{Reflexive}(R) = \{(a,a),(a,b),(b,b),(b,c),(c,c),(c,a)\}$

Symmetric closure :

R is not symmetric, because $(a,b) \in R$ but $(b,a) \notin R$.

$\text{symmetric}(R) = R \cup R^{-1}$

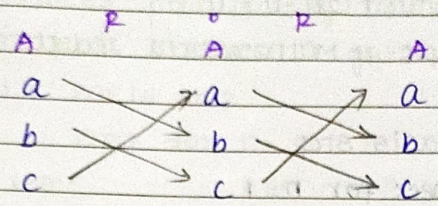
$= \{(a,b),(b,c),(c,a)\} \cup \{(b,a),(c,b),(a,c)\}$

$\text{symmetric}(R) = \{(a,b),(b,c),(c,a),(b,a),(c,b),(a,c)\}$

Transitive closure :

R is not transitive, because $(a,b) \in R$ & $(b,c) \in R$ but $(a,c) \notin R$.

RG :- Graphic Representation R^2



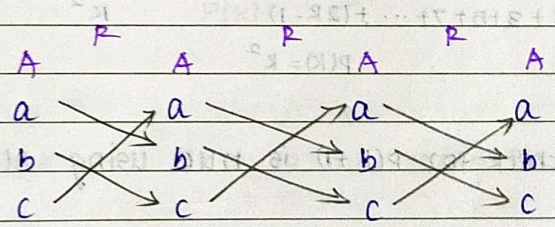
$R^2 = R \circ R = \{(a,c), (b,a), (c,b)\}$

$R^* = R \cup R^2 = \{(a,b), (b,c), (c,a)\} \cup \{(a,c), (b,a), (c,b)\}$

$R^* = \{(a,b), (b,c), (c,a), (a,c), (b,a), (c,b)\}$

R^* is not transitive because $(a,b) \in R^*$ & $(b,a) \in R^*$ but $(a,a) \notin R^*$

RG :- Graphic Representation R^3



$R^3 = R^2 \circ R = \{(a,a), (b,b), (c,c)\}$

$R^* = R \cup R^2 \cup R^3 = \{(a,b), (b,c), (c,a)\} \cup \{(a,c), (b,a), (c,b)\} \cup \{(a,a), (b,b), (c,c)\}$

$R^* = \{(a,a), (a,b), (a,c), (b,b), (b,c), (b,a), (c,c), (c,a), (c,b)\}$

R^* is transitive.

Transitive $(R) = \{(a,a), (a,b), (a,c), (b,b), (b,c), (b,a), (c,c), (c,a), (c,b)\}$

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continuation of page 58.

Mathematical Induction :

PT $\downarrow P(n)$

Problem 2. $1+3+5+7+\dots+(2n-1) = n^2$

using Principle of Mathematical Induction :

Step 1 :: Basis step

check for $n=1$

LHS : RHS :

1 $1^2 = 1$.

LHS = RHS

true for $n=1$

Step 2 :: Induction step.

Let us assume that

$P(n)$ is true for $n=k$.

i.e., $P(k)$ is true

LHS :: RHS ::

$1+3+5+7+\dots+(2k-1)$ k^2

$P(k) = k^2$

Step 3 :

check for $P(k+1)$ is true using $P(k)$

LHS := $\underbrace{1+3+5+7+\dots+2(k-1)}_{k^2 \ (\because \text{ from step 2})} + (2(k+1)-1)$ RHS : $(k+1)^2$

$= k^2 + (2k+2-1)$

$= k^2 + 2k+1$

LHS $= (k+1)^2$

LHS = RHS

$P(k+1) = (k+1)^2$

$P(k+1)$ is true for $n = k+1$

$\therefore P(n)$ is true for all values of n.

Hence proved

Problem 3 Prove that $2+6+10+\cdots+(4n-2) = 2n^2$ using principle of mathematical induction.

Let $P(n) = 2+6+10+\cdots+(4n-2)$

Step 1 :. Basis step

check for $n=1$

LHS :

$= 4(1)-2$

$= 4-2 = 2$

RHS :

$= 2(1)^2$

$= 2$

LHS = RHS

True for $n=1$

Step 2 :. Induction step

Let us assume that $P(n)$ is true for $n=k$

i.e., $P(k)$ is true

LHS :

$2+6+10+\cdots+(4k-2)$

RHS :

$2k^2$

$P(k) = 2k^2$

Step 3 :

check for $P(k+1)$ is true using $P(k)$

LHS :. $\underbrace{2+6+10+\cdots+(4k-2)}_{2k^2 \ (\because \text{from step 2})} + (4(k+1)-2)$

RHS :. $2(k+1)^2$

$= 2k^2 + (4k+4-2)$

$= 2k^2 + 4k-2$

$= 2(k^2+2k+1)$

LHS $= 2(k+1)^2$

LHS = RHS

$P(k+1) = 2(k+1)^2$

$P(k+1)$ is true is true for $n=k+1$

$\therefore P(n)$ is true for all values of n

Hence proved.

Problem 4. Prove that $2+4+6+\cdots+2n = n(n+1)$ using principles of mathematical induction.

Let $P(n) = 2+4+6+\cdots+2n$

Step 1. Basis step

check for $n=1$

LHS:

$= 2(1) = 2$

RHS:

$= 1(1+1) = 2$

LHS = RHS

True for $n=1$

Step 2: Induction step

Let us assume that $P(n)$ is true for $n=k$

i.e., $P(k)$ is true.

LHS:

$2+4+6+\cdots+2k$

RHS:

$k(k+1)$

$P(k) = k(k+1)$

Step 3:

check for $P(k+1)$ is true using $P(k)$

LHS:

$= \underbrace{2+4+6+\cdots+2k}_{k(k+1)} + 2(k+1)$ ∵ from step (2)

$= k(k+1) + 2(k+1)$

$= (k+1)(k+2)$

RHS:

$= (k+1)(k+1+1)$

$= (k+1)(k+2)$

LHS = RHS

$P(k+1) = (k+1)(k+2)$

$P(k+1)$ is true for $n=k+1$

∴ $P(n)$ is true for all values of n.

Problem 5 Prove that $3^n - 1$ is divisible by 2 using principles of mathematical induction.

Let $P(n) = 3^n - 1$

Step 1 : Basis step

check for $n = 1$

$= 3^{(1)} - 1$

$= 3 - 1 = 2$ statement : 2 is divisible by 2

True for $n=1$

Step 2 :: Induction step

Let us assume that $P(n)$ is true for $n = k$

i.e., $P(k)$ is true

$3^k - 1$ is divisible by 2

Step 3 : .

check for $P(k+1)$ is true using $P(k)$

statement to be checked $3^{k+1} - 1$ is divisible by 2 or not.

$$3^{k+1} - 1 = (3^k . 3^1) - 1$$

$$= 3^k * (2+1) - 1$$

$$= \underbrace{(3^k . 2)}_{} + \underbrace{3^k - 1}_{}$$

a number multiplied by 2 is always divisible by 2

∵ from step 2. $3^k - 1$ is divisible by 2

sum of (2 numbers divisible by 2) is always divisible by 2.

∴ $P(k+1)$ is true for $n = k+1$

∴ $P(n)$ is true for all values of n

Hence proved.

Problem 6: Prove that $5^n - 1$ is divisible by 4 using principles of Mathematical Induction

Let $P(n) = 5^n - 1$

Step 1: Basis Step

check for $n=1$

$= 5^{(1)} - 1$

$= 4$ Statement: 4 is divisible by 4

$\therefore$ True for $n=1$

Step 2: Induction Step

Let us assume that $P(n)$ is true for $n=k$

i.e., $P(k)$ is true

$5^k - 1$ is divisible by 4

Step 3:

check for $P(k+1)$ is true using $P(k)$

Statement to be checked $5^{k+1} - 1$ is divisible by 4 or n

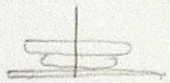
$$5^{k+1} - 1 = (5^k \cdot 5) - 1$$
$$= 5^k \cdot (4+1) - 1$$
$$= \underbrace{(5^k * 4)}_{\text{a number multiplied by 4 is always divisible by 4.}} + \underbrace{(5^k - 1)}_{\text{from step (2) } 5^k - 1 \text{ is divisible by 4}}$$

sum of 2 numbers which are divisible by 4 is always divisible by 4

$\therefore P(k+1)$ is true for $n=k+1$

$\therefore P(n)$ is true for all values of n.

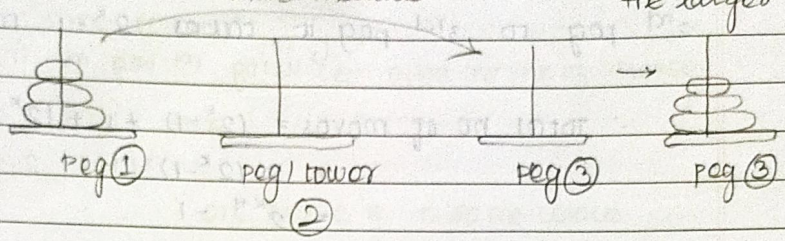
Hence proved.



→ Invalid

rule :- no smaller disks are placed under the larger disk.

Towers of Hanoi



Problem 7 :- In the tower of Hanoi problem, it takes $2^n - 1$ moves to move n no of disk from the first peg/tower to third peg having second peg as an intermediate peg. prove this using principles of Mathematical Induction.

Let $P(n) = 2^n - 1$.

Step 1 :- Basis Step

check for $n = 1$

$2^{(1)} - 1 = 2 - 1 = 1$.

True for $n = 1$

Step 2 :- Induction Step

Let us assume that $P(n)$ is true for $n = k$.

i.e., $P(k)$ is true where k is the no of disks.

$2^n - 1 = 2^k - 1$

∴ k disks can be placed in $2^k - 1$ moves.

Step 3 :

check for $P(k+1)$ is true using $P(k)$

To move k disks to the 2nd peg using 3rd peg as intermediate, it takes $2^k - 1$ moves.

To move the $(k+1)^{th}$ disks from 1st peg to the 3rd peg it takes 1 move.

k

To move the 2^k ~~disks~~ disks from the 2nd peg to 3rd peg, it takes $2^k - 1$ moves.
↳ using 1st peg as intermediate

$\therefore$ Total no of moves $= (2^k - 1) + 1 + (2^k - 1)$

$= 2 \cdot (2^k - 1) + 1 = 2 \cdot 2^k - 2 + 1$

$= 2^{k+1} - 1$

$\therefore$ It takes $2^{k+1} - 1$ moves to move all the

k+1 2^{k+1} disks from 1st peg to the 3rd peg, using the 2nd peg as the intermediate.

$\therefore P(k+1)$ is true for all $n = k+1$ where $(k+1)$ is the no of disks.

$\therefore P(n)$ is true for all n.

$\therefore$ It takes $2^n - 1$ moves to move n pegs from 1st peg to 3rd peg using the 2nd one as intermediate.

Problem 9 "All natural numbers are interesting". Prove by PMI.

Problem8: Prove by MI the following statement "All horses are in the same colour"

Let n be the no of horses

Step1 : Basis step

$n=1$

One horse is in some colour

P(1) is true $n=1$ is true.

Step 2 : Induction step

Let us assume that P(n) is true for $n=k$

i.e, P(k) is true.

k no of horses are in same colour.

Step3 :

check for P(k+1) is true using P(k).

Statement to be checked k+1 horses are in same colour

Totally k+1 horses are available.

(i) $\underbrace{H_1, H_2, H_3, \ldots, H_k}_{\text{from step 2 true}}, H_{k+1}$.

(ii) $\underbrace{H_1, H_2, H_3, \ldots, H_k, H_{k+1}}_{n=k \text{ from step (2)}}$

From (i) and (ii) true.

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Tutorial class practice sheet.

3. Is $3^n - 1$ is multiple of 2 ? check your answer using Mathematical Induction.

Let $P(n) = 3^{n-1}$ $3^n - 1$

Step 1: Basis Step:

check for $n = 1$

LHS RHS:

$3^1 - 1 = 3 - 1$

$= 2$ Statement: 2 is multiple of 2

True for $n = 1$

Step 2: Induction Step:

Let us assume that $P(n)$ is true for $n = k$.

i.e., $P(k)$ is true.

$3^k - 1$ is multiple of 2.

Step 3:

check for $P(k+1)$ is true using $P(k)$

Statement to be checked $3^{k+1} - 1$ is multiple of 2 or not

$$3^{k+1} - 1 = (3^k \cdot 3^1) - 1$$
$$= 3^k \cdot (2+1) - 1$$
$$= \underbrace{(3^k \cdot 2)}_{\text{any number multiplied by 2 is always multiple of 2}} + \underbrace{(3^k - 1)}_{\text{from step(2) } 3^k - 1 \text{ is multiple of 2}}$$

sum of 2 numbers that are multiples of 2 is always the multiples of 2

$\therefore$ $P(k+1)$ is true for $n = k+1$

$\therefore$ $P(n)$ is true for all values of n

Hence proved

8. Prove by Mathematical Induction:

$P(n) = 1+2+2^2+2^3+\dots+2^n = 2^{n+1}-1$.

Let $P(n) = 2^{n+1}-1$

Step 1: Basis step:

Check for $n=0$

LHS: $2^0 = 1$

RHS: $2^{0+1}-1 = 2^1-1 = 2-1 = 1$

LHS = RHS

True for $n=1$

Step 2: Induction step:

Let us assume that $P(n)$ is true for $n=k$

i.e., $P(k)$ is true

LHS: $\underline{1+2+2^2+\dots+2^k}$

RHS: $2^{k+1}-1$

$P(k) = 2^{k+1}-1$ ✓

Step 3:

Check for $P(k+1)$ is true using $P(k)$

LHS: $\underbrace{1+2+2^2+\dots+2^k}_{\text{from step (2)}} + 2^{k+1}$

RHS: $= 2^{k+1+1}-1$

$= 2^{k+2}-1$

$= 2^{k+1}+2^{k+1}-1$

$= 2 \cdot 2^{k+1}-1$

$= 2^1 \cdot 2^{k+1}-1$

$= 2^{k+2}-1$

LHS = RHS

$P(k+1) = 2^{k+2}-1$

$P(k+1)$ is true for $n=k+1$

$\therefore$ $P(n)$ is true for all values of n.

10. Let $A = \{(x,y) \mid x, y \text{ are integers}\}$. Define a relation R on A by the rule $(a,b) R (c,d)$ as $a \le c$ and $b \le d$. Determine whether R is transitive.

Transitive :

Let $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow (a,b) R (c,d)$ $\quad\quad$ $\Rightarrow (c,d) R (e,f)$

$a \le c$ and $b \le d \rightarrow (1)$ $\quad\quad$ $c \le e$ and $d \le f \rightarrow (2)$

From (1) and (2)

$\Rightarrow a \le c \le e$ and $b \le d \le f$

$\Rightarrow a \le e$ and $b \le f$

$\Rightarrow (a,b) R (e,f)$

$\therefore$ R is transitive.

11. Let A be a set of nonzero integers and let $\approx$ be the relation on $A \times A$ defined by $(a,b) \approx (c,d)$ whenever $ad = bc$. Prove that $\approx$ is an equivalence relation.

$(a,b) \approx (c,d) \Rightarrow ad = bc$

Reflexive :

Let $(a,b) \approx (a,b)$

$\Rightarrow ab = ab$

$\therefore \approx$ is reflexive.

Symmetric :

Let $(a,b) \approx (c,d) \Rightarrow$ If $(c,d) \approx (a,b)$ then symmetric

$\Rightarrow (a,b) \approx (c,d)$

$\Rightarrow ad = bc$

$\Rightarrow da = cb$

$\Rightarrow cb = da$

$\Rightarrow (c,d) \approx (a,b)$

$\therefore \approx$ is symmetric

Transitive : To be check $(a,b) \approx (e,f) \Rightarrow af = be$

Let $(a,b) \approx (c,d)$ and $(c,d) \approx (e,f)$

$\Rightarrow (a,b) \approx (c,d)$

$ad = bc \rightarrow (1)$

$c = \frac{ad}{b} \rightarrow (3)$

$\Rightarrow (c,d) \approx (e,f)$

$cf = de \rightarrow (2)$

$c = \frac{de}{f} \rightarrow (4)$

From (3) and 4

$c = \frac{ad}{b}$ and $c = \frac{de}{f} \Rightarrow \frac{ad}{b} = \frac{de}{f}$

$af = be \Rightarrow (a,b) \approx (e,f)$

$\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

12. Let A be a set of non zero integers and let R be the relation on A×A defined by $x R y$ whenever $x+y$ is even. Prove that R is an equivalence relation. check for equivalence relation if $x+y$ is odd.

Reflexive :

CO 1. Probability & Statistics

Basic concepts of statistics.

It is a science of collection, presentation, analysis & reasonable interpretation of data.

Ex: Weight of 100 patients in a study.

Important task of statistic is to give a meaningful inferences of data.

Data :

It is a group of facts or figures which are numerical or otherwise, collected with a definite purpose.

Types of data :

Quantitative data → Nominal data
→ Ordinal data

Qualitative data → Discrete data
→ continuous data.

Primary data : (First hand Data)

It is the data that is collected for the first time through the personal experiences or evidences, particularly for research.

Secondary Data :

secondary data is second-hand data that is already collected & recorded by some researchers for their purpose.

↳ measurable

Discrete / continuous data :

↳ countable

↳ Data that can take any value

Ex: Height, weight

It is the info that can only take certain values. those values need not be whole number but they are fixed.

Ex: shoe size, no of tooth etc.,

Data Presentation :

Types of statistical presentation of data.

Graphical &

Numerical Data.

Graphical data presentation :

Bar diagram & pie charts are used for categorical variables.

Histogram, stem & leaf and Box-plot are used for numerical variable.

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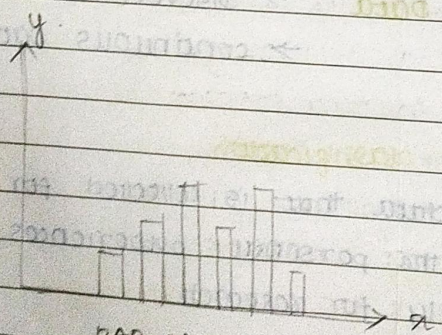

BAR DIAGRAM

Histogram :

It is a graphical display of data using bars of different heights. In a histogram, each bar groups numbers into ranges. Taller bars show that more data falls in that range. A histogram displays the shape and spread of continuous sample data.

similar to Bar chart

classification of data :-

classification is the process of arranging data into homogeneous (similar) groups according to their common characteristics.

Raw data, is unfit for further data analysis & interpretation.

Geographical classification → basis of location.
chronological classification → on order
qualitative classification.
↳ data are classified on the basis of some attributes or quality such as gender, colour of hair, literacy & region.
quantitative classification.
↳ classification of data according to some characteristics, measured such as height, weight, income, profit etc.,

Quantitative classification:

Discrete frequency distribution
continuous frequency distribution.

Two elements:

(i) variable

It refers to the characteristics that varies in magnitude or quantity.

Ex: weight of student → A variable may be discrete or continuous

(ii) Frequency.

Frequency refers to the no of times each variable gets repeated.

Ex: 50 students having weight of 50 kg.
Frequency = 50.

continuous Frequency Distribution:

weights (in kg)	No. of students
31 - 40	14
41 - 50	5
51 - 60	11
61 - 70	12
71 - 80	7
81 - 90	8
91 - 100.	10
Total:	67.

25
30
12
7

Terms of continuous Frequency Distribution :

class limits :

class limits are the lowest and highest values that can be included in class.

Ex : 41 - 50 → lower class limit = 41
upper class limit = 50

class interval :

Difference between lower limit & upper limit of class is called class interval.

class frequency :

Discrete Frequency Distribution :

The values of the variable are determined individually. discrete frequency distribution is also known as ungrouped Frequency Distribution.

Measures of central tendency:

The central tendency is the descriptive summary of a data set.

central tendency
- Mean
- Median
- Mode

Mean : (Arithmetic mean)

Average value of data set.

CO(1) 1(b) (7m) ※

$$\text{Mean} = \frac{\text{sum of values}}{\text{no. of values}}$$

may be

Geometric mean : n^{th} root of the product of n numbers

Harmonic mean : the reciprocal of the averages of the reciprocals of the data values.

weighted mean

(or) weighted harmonic mean.

Harmonic mean :

Harmonic mean is the reciprocal of the averages of the reciprocals of the data values.

If $x_1, x_2, x_3, \ldots, x_n$ are the individual items up to n terms.

then,

$$\text{Harmonic mean} = \frac{n}{(1/x_1) + (1/x_2) + \cdots + (1/x_n)}$$

$$\Rightarrow n / \sum_{i=1}^{n} \frac{1}{x_i}$$

$$7\overline{)10}\ 0.142 \quad \frac{7}{30}\ \frac{28}{20}\ \frac{16}{4}$$

$$9\overline{)10}\ 0.11 \quad \frac{9}{10}$$

Harmonic mean of 2, 5, 7 and 9

$$\text{Harmonic mean} = \frac{4}{(1/2)+(1/5)+(1/7)+(1/9)}$$

$$= \frac{4}{0.5+0.2+0.143+0.111}$$

$$= \frac{4}{0.954} = 4.1929$$

Weighted Harmonic Mean:

weighted mean

A man travel from Jaipur to Agra by a car and takes 4 hrs to cover the whole distance. In the first hour, he travels at a speed of 50 km/hr., in the second hour his speed is 65 km/hr., in the third hour his speed is 80 km/hr., and in the fourth hour he travels at the speed of 55 km/hr. Find the Harmonic mean of motorist.

$$\text{Harmonic mean} = \frac{4}{(1/50)+(1/65)+(1/80)+(1/55)}$$

$$= \frac{4}{0.066}$$

Harmonic mean = 60.61

$$\text{Mean (Average)} = \frac{50+65+80+55}{4}$$

Arithmetic mean = 62.5

Speed of the car (x)	50	65	80	55
1/x	1/50	1/65	1/80	1/55
	= 0.02	0.015	0.025	0.0181

Weighted Harmonic mean :- (Weighted mean)

Simple harmonic mean is special case of harmonic mean where all the weights are equal to 1.

If the set of weights such as $f_1, f_2, \dots, f_n$ with the sample space $x_1, x_2, \dots, x_n$ then the weighted harmonic mean is defined by.

Weighted Harmonic mean

$$HM_W = \frac{N}{(f_1/x_1) + (f_2/x_2) + \dots + (f_n/x_n)}$$

where N is the sum of all weights

$$N = f_1 + f_2 + \dots + f_n$$

$$HM_W = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} (f_i/x_i)}$$

calculate the weighted Harmonic mean

x	1	3	5	7	9	11
weights f	2	4	6	8	10	12

Soln :-

x	f	$1/x$	f_i/x_i
1	2	$1/1 = 1$	$2*1 = 2$
3	4	$1/3 = 0.333$	$4*0.333 = 1.332$
5	6	$1/5 = 0.2$	$6*0.2 = 1.2$
7	8	$1/7 = 0.143$	$8*0.143 = 1.144$
9	10	$1/9 = 0.111$	$10*0.111 = 1.11$
11	12	$1/11 = 0.091$	$12*0.091 = 1.092$
	$N = 42$		$\Sigma f/x = 7.878$

Weighted Harmonic mean :

$$HM_W = \frac{N}{\sum_{i=1}^{n} f_i/x_i}$$

$$= \frac{42}{7.878}$$

$$HM_W = 5.331$$

Find the Harmonic mean of 0, 2, 4, 6.

x	$1/x$
0	$1/0$
2	$1/2$
4	$1/4$
6	$1/6$

$1/0$ tends to infinity.
$\therefore$ we cannot compute Harmonic mean.

The following data is obtained from the survey. compute Harmonic mean $\Rightarrow$ weighted harmonic mean

speed of car	130	135	140	145	150
No. of. cars	3	4	8	9	2

Soln :

Speed of cars (x)	No of. cars (f)	$1/x$	f/x
130	3	$1/130$	$3/130 = 0.023$
135	4	$1/135$	$4/135 = 0.029$
140	8	$1/140$	$8/140 = 0.057$
145	9	$1/145$	$9/145 = 0.062$
150	2	$1/150$	$2/150 = 0.013$
	$N = 26$		$\Sigma f/x = 0.184$

Weighted Harmonic mean

$$HM_W = \frac{N}{\sum_{i=1}^{n} f_i/x_i} = \frac{26}{0.184} = 141.304$$

Geometric mean :

The Geometric mean (GM) is the average value or mean which signifies the central tendency of the set of numbers by finding the product of their value.

Basically, we multiply the numbers altogether & take out of the nth root of the multiplied number where n is the total number of values.

Consider, if $x_1, x_2, x_3, x_4, \dots, x_n$ are the observations, then GM is defined as GM = nth root $(x_1 * x_2 * x_3 * \dots * x_n)$ or $(x_1 * x_2 * \dots * x_n)$ ^ (1/n)

$$GM = (x_1 * x_2 * \dots * x_n) \wedge (1/n)$$

→ calculate the GM of the annual % growth rate of profits in a corporate from 2000 to 2004 is given below
50, 72, 54, 82, 93

$$GM = (x_1 * x_2 * x_3 * x_4 * x_5) \wedge (1/5)$$
$$= (50 * 72 * 54 * 82 * 93) \wedge (1/5)$$
$$= (1482494400) \wedge (1/5)$$
$$= 1482494400 \wedge 0.2$$
$$GM = 68.265$$

→ Geometric mean of 4, 10, 16, 24

$$GM = (x_1 * x_2 * x_3 * x_4) \wedge (1/4)$$
$$= (4 * 10 * 16 * 24) \wedge (1/4)$$
$$= 15360 \wedge 0.25$$
$$GM = 11.132$$

(*) Relation between AM, GM and HM. (3m)

Assume that "a" and "b" are the two numbers & the no of values = 2 then

$$AM = \frac{(a+b)}{2} \Rightarrow 1/AM = 2/(a+b) \rightarrow (1)$$

$$GM = \sqrt{ab} \Rightarrow GM^2 = ab \rightarrow (2)$$

$$HM = \frac{2}{1/a + 1/b}$$

$$HM = \frac{2ab}{a+b} \rightarrow (3)$$

Now substitute (1) and (2) in 3, we get.

$$HM = \frac{GM^2}{AM}$$

$$GM^2 = HM * AM$$

$$GM = \sqrt{(HM \times AM)}$$

Formulas :-

Arithmetic Mean

$$\frac{\Sigma x_i}{n} \qquad \frac{\Sigma f_i x_i}{N} \quad \text{where } N = \Sigma f_i$$

Harmonic mean.

$$\frac{n}{\Sigma 1/x_i} \qquad \frac{N}{\Sigma f_i/x_i} \quad \text{where } N = \Sigma f_i$$

Geometric mean.

$$\sqrt[n]{\prod_{i=1}^{n} x_i} \qquad \sqrt[N]{\prod_{i=1}^{n} x_i^{f_i}} \quad \text{where } N = \Sigma f_i$$

9/11

1. calculate the average number of children per family from the following data.

NO. of. children	0	1	2	3	4	5	6
NO. of. families	30	52	60	65	18	10	5

Soln:

NO. of. children (x_i)	NO. of. families (f_i)	$f_i x_i$	f_i/x_i
0 7	30	210	30/7 = 4.29
1	52	52	52/1 = 52
2	60	120	60/2 = 30
3	65	195	21.67
4	18	72	4.5
5	10	50	2
6	5	30	0.83
	N = 240	$\Sigma f_i x_i = 729$	$\Sigma f_i/x_i = 115.29$

arithmetic mean $= \frac{\sum_{i=1}^{n} f_i x_i}{N}$

$= \frac{729}{7}$ $= \frac{729}{240}$

$AM_W = 104.14$ $AM_W = 3.04$

Harmonic mean $= \frac{N}{\sum_{i=1}^{n} f_i/x_i}$

$HM_W = \frac{240}{115.29} = 2.08$

The following is the distribution of marks obtained by 109 students in a subject in an institution.

2. The following table shows the daily income distribution of 500 workers. Find the average income.

Note :- If x is given in range take the middle value as x.

$$\text{Midpoint } x_i = \frac{\{\text{lower limit}\} + \{\text{upper limit}\}}{2}$$

Marks	4-8	8-12	12-16	16-20	20-24	24-28
No. of students	6	10	18	30	15	12

28-32	32-36	36-40
10	6	2

Marks	x	No. of students f	$f_i x_i$	f_i/x_i
4-8	6	6	36	1
8-12	10	10	100	1
12-16	14	18	252	1.29
16-20	18	30	540	1.67
20-24	22	15	330	0.68
24-28	26	12	312	0.46
28-32	30	10	300	0.33
32-36	34	6	204	0.18
36-40	38	2	76	0.05

$N = 109$ $\quad \Sigma f_i x_i = 2150$ $\quad \Sigma f_i/x_i = 6.66$

$$\text{Arithmetic mean} = \frac{\Sigma f_i x_i}{N}$$

$$AM_W = \frac{2150}{109} = \cancel{238.89}\ 19.724$$

$$\text{Harmonic mean} = \frac{N}{\Sigma f_i/x_i}$$

$$HM_W = \frac{109}{6.66} = 16.37$$

11/11

Geometric Mean :-

$$GM = \sqrt[n]{\prod x_i}$$

↓ taking log on both sides

$$\log GM = \log\left(\sqrt[n]{\prod x_i}\right)$$

$$= 1/n \log\left(\prod x_i\right)$$

$$= 1/n\left(\log x_1 + \log x_2 + \cdots + \log x_n\right)$$

$$\log GM = 1/2 \sum \log x_i$$

$$GM = \text{antilog}\left(1/n \sum \log x_i\right) \Rightarrow \text{antilog}\left(\frac{\sum \log x_i}{n}\right)$$

↓

$$\text{antilog}\left(\frac{\log x_1 + \log x_2 + \cdots + \log x_n}{n}\right)$$

$$GM_W = \sqrt[N]{\prod x_i^{f_i}}$$

taking log on both sides

$$\log GM_W = \log\left(\sqrt[N]{\prod x_i^{f_i}}\right)$$

$$= 1/N \log\left(\prod x_i^{f_i}\right)$$

$$= 1/N\left(f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n\right)$$

$$\log GM_W = 1/N \sum_{i=1}^{n} f_i \log x_i$$

$$GM_W = \text{antilog}\left(\frac{1}{N} \sum_{i=1}^{n} f_i \log x_i\right)$$

↓

$$\text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right)$$

1. Find the Geometric mean of the following data.

x	13	14	15	16	17
f	2	5	13	7	3

x_i	f_i	$\log x_i$	$f_i \log x_i$
13	2	1.114	2.228
14	5	1.146	5.73
15	13	1.176	15.288
16	7	1.204	8.428
17	3	1.230	3.69

$N = 30$ $\qquad \Sigma f_i \log x_i = 35.364$

$$\text{Weighted Geometric Mean} = \text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right)$$

$$= \text{antilog}\left(\frac{35.364}{30}\right)$$

$$= \text{antilog}(1.1788)$$

$$GMW = 15.094$$

2. Find the Geometric mean of the following data.

Marks	0-30	30-50	50-80	80-100
No. of. students	20	30	40	10

$x_i = \frac{\text{Low lim} + \text{up. lim}}{2}$

Marks	x_i	No. of. students f_i	$\log x_i$	$f_i \log x_i$
0-30	15	20	1.176	23.52
30-50	40	30	1.602	48.06
50-80	65	40	1.813	72.52
80-100	90	10	1.954	19.54

$N = 100$ $\qquad \Sigma f_i \log x_i = 163.64$

$$GM_W = \text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right) = \text{antilog}\left(\frac{163.64}{100}\right)$$

$$GM_W = \text{antilog}(1.6364) = 43.291$$

Median :-

Median is the middle value of the dataset in which the dataset is arranged in the ascending order or descending order.

When the dataset contains an even number of values, then the median value of the dataset can be found by taking the mean of the middle two values.

If you have skewed distribution, the best measure of finding the central tendency is the median.

1. Find the median of 14, 63 and 55?

14, 55, 63
↳ Median

2. 4, 17, 77, 25, 22, 23, 92, 82, 40, 24, 14, 12, 67, 23, 29.

4, 12, 14, 17, 22, 23, 23, 24, 25, 29, 40, 67, 77, 82, 92
↑
Median.

Even :-

1 2 3 4 5 6 7 8

$\underbrace{4+5}$ = 9/2 = 4.5 median.

→ Rahul's family drove through 7 states on summer vacation. The prices of gasoline differ from state to state. Calculate the median of gasoline cost.

1.79, 1.61, 2.09, 1.84, 1.96, 2.11, 1.75

ordered data :- 1.61, 1.75, 1.79, 1.84, 1.96, 2.09, 2.11

2 3 4 5 6 7

↑
Median : 1.84

Median of gasoline cost is 1.84.

14.11.2022

Tutorial sheet - 3

22MX221
Santhosh.R

DEPARTMENT OF COMPUTER APPLICATIONS

20MX11-MFCS-Tutorial Class-14th November, 2022

1. Two weeks before Mark opened Technology Titans, he launched his company Web site. During those 14 days, Mark had an average of 24.5 hits on his Web site per day. In the first two days that Technology Titans was open for business, the Web site received 42 and 53 hits respectively. Determine the new average for hits on the Web site.

2. In Tim's office, there are 25 employees. Each employee travels to work every morning in his or her own car. The distribution of the driving times (in minutes) from home to work for the employees is shown in the table below. Calculate the mean of the driving times.

Driving times	0-10	10-20	20-30	30-40	40-50
Number of Employees	3	10	6	4	2

3. Stephen has been working on programing and updating a Web site for his company for the past 15 months. The following numbers represent the number of hours Stephen has worked on this Web site for each of the past 7 months: 24, 25, 31, 50, 53, 66, 78. What is the mean (average) number of hours that Stephen worked on this Web site each month?

4. A die is thrown 20 times and the following scores were recorded 6, 3, 2, 4, 5, 5, 6, 1, 3, 3, 5, 6, 6, 1, 3, 3, 5, 6, 6, 2. Prepare the frequency table of scores on the upper face of the die and find the mean score.

5. If the mean of the following distribution is 9, find the value of p.

X	4	6	p + 7	10	15
f	5	10	10	7	8

6. The following table shows the number of plants in 20 houses in a group.

Number of Plants	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of Houses	1	2	2	4	6	2	3

Find the mean number of plants per house.

7. Determine the mean of the first 5 prime numbers.

8. What is the Harmonic Mean if the Arithmetic mean is 10 and the Geometric mean is 7?

9. What is the Geometric Mean if the Arithmetic mean is 20 and the Harmonic mean is 15?

10. Calculate the harmonic mean for the data given below:

Marks	30–39	40–49	50–59	60–69	70–79	80–89	90–99
f	2	3	11	20	32	25	7

11. Find the Harmonic Mean of distribution of weights of 75 students at virtual University in the table:

Weight	110-119	120-129	130-139	140-149	150-159
Frequency	1	4	17	28	25

12. Given the following frequency distribution of weights of 60 apples, calculate the Arithmetic and Harmonic Mean for grouped data.

Weights (grams)	65--84	85--104	105--124	125--144	145--164	165--184	185--204
Frequency	09	10	17	10	05	04	05

13. Find the G.M for the following data, which gives the defective screws obtained in a factory.

Diameter (CM)	5	15	25	35
# of Defective screws	5	8	3	4

14. If the AM and HM of the data sets are 9 and 49 respectively. Find the Geometric Mean of the following data.

Doubt 15. Find the geometrical mean of the number $3, 3^2, \ldots, 3^n$.

16. The arithmetic mean and the geometric mean of two positive integers are 10 and 8. Determine the two numbers.

17. Find the geometric mean of the following grouped data for the frequency distribution of weights.

Data	60-80	80-100	100-120	120-140	140-160
Frequency	22	38	45	35	20

18. The mean of 10 observation is 20. If each observation is added by '5'. Find the mean of new observation.

19. The mean of 100 observations is 50. If one of the observations which was 50 is replaced by 150; find the new mean.

20. The geometric mean of data containing 3 terms is 10. If each value is doubled, what will be the new mean?

↳ geometric mean

14.11.22

Tutorial sheet - 3

1. Average of hits on his website per day for past 14 days } = 24.5 hits

No of hits on his website per day for past 14 days } = 14 × 24.5

= 343

In addition to past 14 days the newly opened two days data } = 343 + 42 + 53

= 438

Average of hits on his website per day for total 16 days } = $\frac{438}{16}$

= 27.375

∴ The new average for hits on the website is 27.375.

2.

Driving times	0-10	10-20	20-30	30-40	40-50
No. of Employees	3	10	6	4	2

Driving times	x_i	f_i	$x_i f_i$
0-10	5	3	15
10-20	15	10	150
20-30	25	6	150
30-40	35	4	140
40-50	45	2	90
		$N = 25$	$\Sigma f_i x_i = 545$

Weighted Arithmetic mean = $\frac{\Sigma f_i x_i}{N}$

= 545/25 = 21.8

∴ The mean of the driving time is 21.8 minutes.

5. Mean = 9

x	4	6	P+7	10	15
f	5	10	10	7	8

Soln:

x_i	f_i	$x_i f_i$
4	5	20
6	10	60
P+7	10	10P+70
10	7	70
15	8	120
	N = 40	$\Sigma f_i x_i = 10P + 340$

$$\text{Weighted Arithmetic Mean} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$$

$$9 = \frac{10P+340}{40}$$

$$360 = 10P + 340$$

$$10P = 360 - 340$$

$$P = 20/10$$

$$P = 2$$

∴ The value of P = 2.

6. Harmonic Mean = ? Arithmetic mean = 10

Geometric mean = 7

$$GM^2 = HM \times AM$$

$$HM = \frac{GM^2}{AM} = \frac{(7)^2}{10}$$

$$= 49/10$$

$$HM = 4.9$$

∴ Harmonic Mean = 4.9

16. Arithmetic mean of two positive integers = 10
Geometric mean " " = 8
two numbers, = ?

Let x and y be the two positive integers.

Arithmetic mean, $\frac{x+y}{2} = 10$

$$x+y = 20 \rightarrow (1)$$

Geometric mean, $(x*y)^{1/2} = 8$

$$xy = 8^2$$
$$xy = 64$$
$$x = \frac{64}{y} \rightarrow (2)$$

Substituting $x = 64/y$ in equation (1)

$$\frac{64}{y} + y = 20$$
$$y^2 + 64 = 20y$$
$$y^2 - 20y + 64 = 0$$
$$(y-16)(y-4) = 0$$

$y - 16 = 0$ $\quad$ $y - 4 = 0$

$y = 16$ $\quad$ $y = 4$

64

$-16 \quad -4 = -20$

When $y = 16$ in eqn (1)

$$x + 16 = 20 \Rightarrow x = 4$$

When $y = 4$ in eqn (1)

$$x + 4 = 20 \Rightarrow x = 16$$

$\therefore$ two positive integers are 16 and 4.

18. Mean of 10 observation is 20.

Each observation added by '5'.

So for 10 observations = 50

$$\frac{x_1+x_2+\cdots+x_{10}}{10}=20$$

$$x_1+x_2+\cdots+x_{10}=200$$

$x_1+5+x_2+5+x_3+5+\cdots+x_{10}+5$

$x_1+x_2+\cdots x_{10}+10(5)$

Added value of '5' for each observation for 10 times add up with 50

$$x_1+x_2+\cdots+x_{10}=200+50$$

$$x_1+x_2+\cdots+x_{10}=250$$

$$\text{Average}=\frac{x_1+\cdots+x_{10}}{10}$$

$$=250/10=25$$

$\therefore$ the mean of new observation is 25

20. Geometric mean of 3 terms = 10

Let x, y and z be the 3 terms.

Given : geometric mean = 10

$$\sqrt[3]{xyz}=10$$

$$(xyz)^{1/3}=10$$

$$\sqrt[3]{2x\,2y\,2z}=?$$

$$\sqrt[3]{8\,xyz}=?$$

$$\sqrt[3]{8}\,\sqrt[3]{xyz}=?$$

$$(2^3)^{1/3}\,10=2(10)=20$$

$\therefore$ the new geometric mean is 20

19. The mean of 100 observation is 50. one observation which was 50 is replaced by 150.

Mean of 100 observation = 50

sum of total of 100 observation = 50 × 100
= 5000

In one observation 50 is replaced by 150, so in addition 100 is added to total.

sum of total of 100 observation = 5000 + 100
= 5100

$$\text{Average} = \frac{5100}{100} = 51$$

∴ Now mean = 51

3. Past 7 months data: 24, 25, 31, 50, 53, 66, 78

$$\text{Arithmetic mean} = \frac{\Sigma x_i}{n}$$

$$= \frac{24+25+31+50+53+66+78}{7}$$

$$= 327/7$$

AM = 46.714

∴ the average no of hours stephen worked on web site each month is 46.714 hrs

4. Frequency table :

	Die X	1	2	3	4	5	6	
acc of	scores of die f	2	2	5	1	4	6	= 20

x_i	f_i	$x_i f_i$
1	2	2
2	2	4
3	5	15
4	1	4
5	4	20
6	6	36
	$N = 20$	$\Sigma f_i x_i = 81$

Weighted Arithmetic mean, $= \frac{\Sigma x_i f_i}{N}$

$= 81/20 = 4.05$

Mean score $= 4.05$

6.

Number of plants	x_i	No. of Houses f_i	$x_i f_i$
0 - 2	1	1	1
2 - 4	3	2	6
4 - 6	5	2	10
6 - 8	7	4	28
8 - 10	9	6	54
10 - 12	11	2	22
12 - 14	13	3	39
		$N = 20$	$\Sigma f_i x_i = 160$

Weighted Arithmetic mean $= \frac{\Sigma x_i f_i}{N}$

$= 160/20$

$AM_W = 8$

$\therefore$ Mean number of plants per house $= 8$

7. First 5 prime Numbers : 2, 3, 5, 7, 11

Mean of first 5 prime Numbers $= \frac{2+3+5+7+11}{5}$

$= 28/5$

$= 5.6$

∴ Mean of first 5 prime numbers 5.6

9. Geometric Mean = ? Arithmetic mean = 20

Harmonic mean = 15

$GM^2 = AM \times HM$

$= 20 \times 15$

$GM^2 = 300$

$GM = \sqrt{300}$

$GM = 17.320$

10.

Marks	x_i	f_i	f_i/x_i
30-39	34.5	2	0.116 → wrong answer
40-49	44.5	3	0.067
50-59	54.5	11	0.202
60-69	64.5	20	0.310
70-79	74.5	32	0.430
80-89	42.25	25	0.592
90-99	94.5	7	0.074
	$N = 100$		$\Sigma f_i/x_i = 1.791$

Weighted Harmonic Mean $= N / \Sigma f_i/x_i$

$= 100/1.791$

$HM_W = 55.835$

∴ Weighted Harmonic Mean = 55.835

11. Harmonic mean

Weight	x_i	f_i	f_i/x_i
110-119	114.5	1	0.0087
120-129	124.5	4	0.0321
130-139	134.5	17	0.1264
140-149	144.5	28	0.1938
150-159	154.5	25	0.1618
		$N = 75$	$\Sigma f_i/x_i = 0.5228$

Weighted Harmonic Mean $HM_W = \frac{N}{f_i/x_i}$

$= 75/0.5228$

$HM_W = 143.459$

∴ Harmonic Mean of distribution of weights of 75 students at virtual University is 143.459

12. Arithmetic and Harmonic mean:

Weight (grams)	x_i	Frequency f_i	$f_i x_i$	f_i/x_i
65-84	74.5	9	670.5	0.1208
85-104	94.5	10	945	0.1058
105-124	114.5	17	1946.5	0.1485
125-144	134.5	10	1345	0.0743
145-164	154.5	05	772.5	0.0323
165-184	174.5	04	698	0.0230
185-204	194.5	05	972.5	0.0257
		$N = 60$	$\Sigma f_i x_i = 7350$	$\Sigma f_i/x_i = 0.530$

Weighted Arithmetic mean, $AM_W = \frac{\Sigma f_i x_i}{N}$

$AM_W = 7350/60 = 122.5$

Weighted Harmonic Mean $HM_W = N / \Sigma f_i/x_i$

$= 60 / 0.5304$

$HM_W = 113.122$

Arithmetic Mean $AM_W = 122.5$

Harmonic Mean $HM_W = 113.122$

14. Arithmetic Mean $AM = 9$

Harmonic Mean $HM = 49$

Geometric Mean = ?

$GM^2 = AM \times HM$

$= 9 \times 49$

$= 441$

$GM = \sqrt{441}$

$GM = 21$

Geometric Mean = 21

13. Geometric Mean :

Diameter (CM) x_i	# of defective screws f_i	$\log x_i$	$f_i \log x_i$
5	5	0.6989	3.4945
15	8	1.1761	9.4088
25	3	1.3979	4.1937
35	4	1.5440	6.176
	$N = 20$		$\sum_{i=1}^{n} f_i \log x_i = 23.273$

weighted geometric mean, $GM_W = \text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right)$

$= \text{antilog}\left(\frac{23.273}{20}\right) = \text{antilog}(1.16365)$

$GM_W = 14.5763$

17. Geometric Mean :

Data	x_i	Frequency f_i	$\log x_i$	$f_i \log x_i$
60-80	70	22	1.845	40.59
80-100	90	38	1.954	74.252
100-120	110	45	2.041	91.845
120-140	130	35	2.114	73.99
140-160	150	20	2.176	43.52
		$N = 160$		$\Sigma f_i \log x_i = 324.197$

$$\left.\begin{array}{c}\text{weighted Geometric} \\ \text{Mean, GMW}\end{array}\right\} = \text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right)$$

$$= \text{antilog}\left(\frac{324.197}{160}\right)$$

$$= \text{antilog}(2.026)$$

$$GMW = 115.345$$

Geometric Mean $= 115.345$

15. Geometric Mean : $3, 3^2, \ldots, 3^n$

$$= \sqrt[n]{3 \times 3^2 \times 3^3 \times \cdots \times 3^n}$$

$$= \sqrt[n]{3^{1+2+3+\cdots+n}}$$

$$= \sqrt[n]{3^{n(n+1)/2}}$$

$$= \left(3^{n(n+1)/2}\right)^{1/n}$$

$$(3^a)^b = 3^{ab}$$

$$\left(3^{n(n+1)/2}\right)^{1/n} = 3^{\frac{n(n+1)}{2n}}$$

$$GM = 3^{(n+1)/2}$$

15/11

MODE

The mode represents the frequently occurring value in the data set.

Sometimes the data set may contain multiple values of mode.

5, 4, 3, 2, 1, 5, 4, 2, 1, 5 → Mode = 5

Median of Grouped Data:

$$Me = l + \{ h \times (N/2 - cf) / f \}$$

where

l = lower limit of Median class

h = width of median class

f = frequency of median class

cf = cumulative frequency of class preceding the median class

$N = \Sigma f_i$

$$\text{Median} = l + \left(\frac{N/2 - c_f}{f} \right) h$$

Problem: calculate Median.

Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	10	16	24	15	7

	Class Interval	Frequency	Cumulative Frequency.
	0-8	8	8
	8-16	10	18
	16-24	16	34
Median class →	24-32	24	58
	32-40	15	73
	40-48	7	80
		N = 80	

Identify Median class: N/2 gives a hint to find Median class

N/2 nearer value. locate median class whose cf > N/2

class ~~Frequency~~

Median class : 24 - 32

$l = 24$ $h = 8$ $N/2 = 40$

$cf = 34$ (cumulative frequency of class preceding to the median class)

$f = 24$

$$\text{Median} = l + \left(\frac{N/2 - cf}{f}\right) h$$

$$= 24 + \left(\frac{40-34}{24}\right) 8$$

$$= 24 + \left(\frac{6}{24}\right) 8$$

$$= 24 + 2$$

Median $= 24 + 2 = 26$

2. The following data represents the survey regarding the heights (in cm) of 51 girls of class X. Find the median height.

Height in cm	<140	140-145	145-150	150-155	155-160	160-165
# of girls	4	7	18	11	6	5

Height in cm	# of girls	cumulative Frequency
# <140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

$N = 51$

$N/2 = 25.5$

nearest greater value.

Median class :: 145-150

$l = 145$ $h = 5$ $N/2 = 25.5$

$cf = 11$

$f = 18$

$$\text{Median} = l + \left(\frac{N/2 - cf}{f}\right) h$$

$$= 145 + \left(\frac{25.5 - 11}{18}\right) 5$$

$$= 145 + \left(\frac{14.5}{18}\right) 5$$

$$= 145 + \frac{72.5}{18}$$

$$= 145 + 4.028$$

$$\text{Median} = 149.028$$

Mode of grouped Data :

A student recorded her scores on weekly English quizzes that were marked out of a possible 10 points.

Her scores were as follows :

8, 5, 8, 5, 7, 6, 7, 7, 5, 7, 5, 5, 6, 6, 9, 8, 9, 7, 9, 9, 6, 8, 6, 6, 6

What is the mode of her scores on the weekly English quizzes?

Frequency table :

x	5	6	7	8	9
f	5	7	5	4	4

maximum frequency

$\therefore$ Mode = 6

Finding the mode of Grouped Data (class intervals)

$$\text{Mode} = l + \left[\frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)}\right] \times h$$

where

l = lower limit of the modal class

f_m = frequency of modal class

f_1 = frequency of class preceding the modal class

f_2 = frequency of class succeeding the modal class

h = width of the modal class.

class with the maximum frequency is called the modal class.

Problem :-

The height in cm of 50 students are recorded. compute the Mode.

Height (in cm)	125 - 130	130 - 135	135 - 140	140 - 145	145 - 15
# of students	7	14	10	10	9

maximum frequency.

Modal class :- 130 - 135

$l = 130$ $h = 5$ $f_1 = 7$ $f_2 = 10$ $f_m = 14$

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) h$$

$$= 130 + \left(\frac{14 - 7}{2(14) - 7 - 10}\right) 5$$

$$= 130 + \left(\frac{7}{11}\right) 5 = 130 + 3.1818$$

Mode = 133.1818

Fact about Mean, Median & Mode:

The distance between the mean and median is about one-third the distance between the mean and the mode.

$$\text{Mean} - \text{Median} = (1/3) * (\text{Mean} - \text{Mode})$$

$$\text{Mode} = 3 \times \text{Median} - 2\,\text{Mean}$$

1. The mean and median of distribution are 44.6 and 44.05. Find the mode.

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$
$$= 3(44.05) - 2(44.6)$$
$$= 132.15 - 89.2$$
$$\text{Mode} = 42.95$$

Note:

case (i)

If modal class is the very first column in the table, then there is no possibility for preceding class then ⇒

$f_1 = 0$

case (ii)

If modal class is the very last column in the table, then there is no possibility for succeding class then ⇒

$f_2 = 0$

case (iii)

If there is two modal class then we have to compute, two mode values.

↳ Bimodal distribution.

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Inclusive Form :

An inclusive series includes the upper limit and the lower limit in the class interval.

Exclusive method :

Includes the lower limit and do not include the upper limit in the distribution table.

Studying time (in minutes)	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50
No. of. students	2	10	6	4	3

↑ Inclusive form

↓ Exclusive form

Finding Difference of 1 class upper limit and 1 class lower limit

$h = 11 - 10 = 1$

$h/2 = 0.5$

So the $h/2$ value is subtracted from lower limit and the $h/2$ value is added to the upper limit.

Assume that the marks (out of 40) obtained by 15 students of a class are given as: 5, 10, 21, 29, 9, 6, 20, 16, 18, 25, 30, 25, 35, 37, 40. Prepare the frequency distribution table, considering the inclusive method of distribution.

Soln:

Inclusive method of distribution

Marks	0 - 10	11 - 20	21 - 30	31 - 40
No. of students (Frequency)	4	3	5	3

Exclusive method of distribution

Marks	0 - 10	10 - 20	20 - 30	30 - 40
No. of students	3	3	5	4

Converted table from inclusive to exclusive.

Studying time (in minutes)	0.5 - 10.5	10.5 - 20.5	20.5 - 30.5	30.5 - 40.5	40.5 - 50.5
No. of students	2	10	6	4	3

Exclusive method - includes the lower limit and do not include the upper limit in distribution table

0.5 is deducted from lower limit and 0.5 is added to upper limit.

1. If the difference between mode and mean of data is 24, what is the difference between the median and the mean?

mode - mean = 24

Median - mean = 1/3 * (~~mode~~ mean - mode - ~~mean~~)

Mean - Median = 1/3 * (mode - mean)

= 1/3 * 24

= 8

∴ difference between median and mean is 8.

2. The temperature in °F on 20 days during the month of June was as follows:

70°F, 76, 76, 74, 70, 70, 72, 74, 78, 80, 74, 74, 78, 76, 78, 76, 74, 78, 80, 76

What is the mode of temperatures for the month of June?

Soln:

Frequency table:

calcius	70°	72°	74°	76°	78°	80°
frequency	3	1	5	5	4	2

Mode = 5

∴ The mode of temperature for month of June is 5.