BIG M Method

Small Recap in Simplex Method

Variables in Simplex Method

- Simplex method is the method to solve (LPP) models which contain two or more decision variables.
- Basic variables: Are the variables which coefficients One in the equations and Zero in the other equations.
- Non-Basic variables: Are the variables which coefficients are taking any of the values, whether positive or negative or zero.
- Slack, surplus & artificial variables:
  a) If the inequality be less than or equal, then we add a slack variable + S to change to =.
  b) If the inequality be greater than or equal, then we subtract a surplus variable - S to change to =.
  c) If we have = we use artificial variables.

BIG – M method focuses on the LPP with = constraints.

Steps in BIG – M method

Step 1: Express the problem in the standard form.

- Modify the constraints so that the RHS of each constraint is nonnegative

- Convert each inequality constraint to standard form: (If constraint is a $\leq$ constraint, add a slack variable s_i; and if constraint is a $\geq$ constraint, subtract an excess/surplus variable e_i).

Step 2: Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type $\geq$ or =.

Step 3: Assign a very large penalty cost (-M for Maximization and M for Minimization) with artificial variables in the objective function.

- If the LP is a MAX problem, add (for each artificial variable) -Mai to the objective function where M denote a very large positive number.

- If the LP is a MIN problem, add (for each artificial variable) Mai to the objective function where M denote a very large positive number.

Step 4: Solve the transformed problem by the simplex. Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex. Now (In choosing the entering variable, remember that M is a very large positive number!).

- If all artificial variables are equal to zero in the optimal solution, we have found the optimal solution to the original problem.

$$
\begin{array}{llll}
\textbf{Minimize} & z = & 4x_1 + x_2 & \\
\textbf{Subject to:} & & & \\
& & 3x_1 + x_2 = 3 & \\
& & 4x_1 + 3x_2 >= 6 & \\
& & x_1 + 2x_2 <= 4 & \\
& & & \\
& & x_1, x_2 >= 0 &
\end{array}
$$

Add a slack variable [S_3] in the third constraint and subtract a surplus variable [S_2] in the second constraint as shown below.

$$
\begin{array}{lll}
\textbf{Minimize} & z = & 4x_1 + x_2 \\
\textbf{Subject to:} & & \\
& & 3x_1 + x_2 = 3 \\
& & 4x_1 + 3x_2 - S_2 = 6 \\
& & x_1 + 2x_2 + s_3 = 4 \\
& & \\
& & x_1, x_2, S_2, s_3 >= 0
\end{array}
$$

Now, [as mentioned in Step 2] add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type ≥ or = [First and second constraint] as shown below.

$$
\begin{array}{lll}
\textbf{Minimize} & z = & 4x_1 + x_2 \\
\textbf{Subject to:} & & \\
& & 3x_1 + x_2 + R_1 = 3 \\
& & 4x_1 + 3x_2 - S_2 + R_2 = 6 \\
& & x_1 + 2x_2 + s_3 = 4 \\
& & \\
& & x_1, x_2, S_2, s_3, R_1, R_2 >= 0
\end{array}
$$

Now, assign a very large penalty cost (-M for Maximization and M for Minimization) with artificial variables in the objective function.

Since the LP is a MIN problem, add (for each artificial variable) MR_1 and MR_2 to the objective function where M denote a very large positive number as shown here.

Minimize $z = 4x_1 + x_2 + MR_1 + MR_2$

Subject to:

$$
\begin{aligned}
3x_1 + x_2 + R_1 &= 3 \\
4x_1 + 3x_2 - S_2 + R_2 &= 6 \\
x_1 + 2x_2 + s_3 &= 4
\end{aligned}
$$

$$x_1, x_2, S_2, s_3, R_1, R_2 >= 0$$

We can now set x_1, x_2 and S_2 to zero and use R_1, R_2 and s_3 as the starting basic feasible solution.

Now, construct the SIMPLEX table as usual and solve the LPP.

Basic	z	x_1	x_2	S_2	R_1	R_2	s_3	Solution
z	1	-4	-1	0	-M	-M	0	0
R_1	0	3	1	0	1	0	0	3
R_2	0	4	3	-1	0	1	0	6
s_3	0	1	2	0	0	0	1	4

With this, it is not possible to proceed with the simplex method. Modify the above table into a table suitable to apply simplex method as shown below.

Basic	z	x_1	x_2	S_2	R_1	R_2	s_3	Solution
z	1	-4+7M	-1+4M	-M	0	0	0	9M
R_1	0	3	1	0	1	0	0	3
R_2	0	4	3	-1	0	1	0	6
s_3	0	1	2	0	0	0	1	4

After this, it is similar to the normal Simplex Method. Since this is a minimization problem, select the entering variable with the **most positive objective row coefficient.** In the case of Maximization problem, select the entering variable with the most negative objective row coefficient.