Vogel's Approximation Method

23MX11 – MFCS

Vogel's Approximation Method (VAM) is one of the methods used to calculate the initial basic feasible solution to a transportation problem. However, VAM is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost. Learn how to find the initial basic feasible solution to a transportation problem such that the total cost is minimized.

VAM (Vogel's Approximation Method) is the best method of computing the initial basic feasible solution to a transportation problem. As it provided better results when compared with other methods.

Vogel's Approximation Method Steps

Below are the steps involved in Vogel's approximation method of finding the feasible solution to a transportation problem.

Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min (supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Solve the given transportation problem using Vogel's approximation method.

Factories	Destination centers				Supply
	D_1	D_2	D_3	D_4	
F_1	3	2	7	6	50
F_2	7	5	2	3	60
F_3	2	5	4	5	25
Demand	60	40	20	15	

For the given cost matrix,

Total supply = 50 + 60 + 25 = 135 & Total demand = 60 + 40 + 20 + 25 = 135

Thus, the given problem is balanced transportation problem.

Apply the Vogel's approximation method to minimize the total cost of transportation.

Step 1: Identify the least and second least cost in each row and column and then write the corresponding absolute differences of these values. For example, in the first row, 2 and 3 are the least and second least values, their absolute difference is 1.

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	3	2	7	6	50	1
F_2	7	5	2	3	60	1
F_3	2	5	4	5	25	2
Demand	60	40	20	15		
Column difference	1	3	2	2		

These row and column differences are called penalties.

Step 2: Now, identify the maximum penalty and choose the least value in that corresponding row or column. Then, assign the min (supply, demand).

Here, the maximum penalty is 3 and the least value in the corresponding column is 2. For this cell, min (supply, demand) = min (50, 40) = 40

Allocate 40 in that cell and strike the corresponding column since in this case demand will be satisfied, i.e., 40 – 40 = 0.

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	3	$\boxed{40}$ 2	7	6	50 - 40=10	1
F_2	7	5	2	3	60	1
F_3	2	5	4	5	25	2
Demand	60	40-40=0	20	15		
Column difference	1	3	2	2		

Step 3: Now, find the absolute row and column differences for the remaining rows and columns. Then repeat step 2.

Here, the maximum penalty is 3 and the least cost in that corresponding row is 3. Also, the min(supply, demand) = min(10, 60) = 10

Thus, allocate 10 for that cell and write down the new supply and demand for the corresponding row and column.

Supply = 10 – 10 = 0

Demand = 60 – 10 = 50

As supply is 0, strike the corresponding row.

	D_1	D_2	D_3	D_4	Supply	Row difference	
F_1	$\boxed{10}$ 3	$\boxed{40}$ 2	7	6	10-10=0	1	3
F_2	7	5	2	3	60	1	1
F_3	2	5	4	5	25	2	2
Demand	60-10=50	0	20	15			
Column difference	1	3	2	2			
	1	-	2	2			

Step 4: Repeat the above step, i.e., step 3. This will give the following result.

	D_1	D_2	D_3	D_4	Supply	Row difference		
F_1	$\boxed{10}$ 3	$\boxed{40}$ 2	7	6	0	1	3	-
F_2	7	5	2	3	60	1	1	1
F_3	$\boxed{25}$ 2	5	4	5	25-25= 0	2	2	2
Demand	50-25=25	0	20	15				
Column difference	1	3	2	2				
	1	-	2	2				
	5	-	2	2				

In this step, the second column vanishes and the min(supply, demand) = min(25, 50) = 25 is assigned for the cell with value 2.

Step 5: Again repeat step 3, as did for the previous step.

	D_1	D_2	D_3	D_4	Supply	Row difference			
F_1	10 3	40 2	7	6	0	1	3	-	-
F_2	25 7	5	2	3	60-25=35	1	1	1	1
F_3	25 2	5	4	5	0	2	2	-	-
Demand	25-25=0	0	20	15					
Column difference	1	3	2	2					
	1	-	2	2					
	5	-	2	2					
	7	-	2	3					

In this case, we got 7 as the maximum penalty and 7 as the least cost of the corresponding column.

Step 6: Now, again repeat step 3 by calculating the absolute differences for the remaining rows and columns.

	D_1	D_2	D_3	D_4	Supply	Row difference				
F_1	10 3	40 2	7	6	0	1	3	-	-	-
F_2	25 7	5	2	15 3	35-15=20	1	1	1	1	1
F_3	25 2	5	4	5	0	2	2	-	-	-
Demand	25-25=0	0	20	15-15=0						
Column difference	1	3	2	2						
	1	-	2	2						
	5	-	2	2						
	7	-	2	3						
	-	-	2	3						

Step 7: In the previous step, except for one cell, every row and column vanishes. Now, allocate the remaining supply or demand value for that corresponding cell.

	D_1	D_2	D_3	D_4	Supply	Row difference				
F_1	$\boxed{10}$ 3	$\boxed{40}$ 2	7	6	0	1	3	-	-	-
F_2	$\boxed{25}$ 7	5	$\boxed{20}$ 2	$\boxed{15}$ 3	20-20=0	1	1	1	1	1
F_3	$\boxed{25}$ 2	5	4	5	0	2	2	-	-	-
Demand	25-25=0	0	20-20=0	0						
Column difference	1	3	2	2						
	1	-	2	2						
	5	-	2	2						
	7	-	2	3						
	-	-	2	3						

Total cost = (10 × 3) + (25 × 7) + (25 × 2) + (40 × 2) + (20 × 2) + (15 × 3)

= 30 + 175 + 50 + 80 + 40 + 45

= 420

Vogel's Approximation Method Problems

1. Consider the transportation problems given below. Solve the problems by Vogel's approximation method.

Origin	Destination				Supply
	D_1	D_2	D_3	D_4	
O_1	3	1	7	4	300
O_2	2	6	5	9	400
O_3	8	3	3	2	500
Demand	250	350	400	200	

From	To			Supply
	D_1	D_2	D_3	
A_1	6	8	10	150
A_2	7	11	11	175
A_3	4	5	12	275
Demand	200	100	350	

Sources	Destinations			Supply
	D_1	D_2	D_3	
S_1	4	5	1	40
S_2	3	4	3	60
S_3	6	2	8	70
Demand	70	40	60	