No of Pages : 4 **Course Code : 18MX12**

Roll No:
(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, JANUARY 2020

MCA Semester: 1

18MX12 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:

1. Answer **ALL** questions. Each question carries 25 Marks.
3. Course Outcome Table

Qn.1	CO1	Qn.2	CO2	Qn.3	CO3	Qn.4	CO4

1. a) Prove the following Distributive laws using Venn diagram.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

b) i) How many numbers from 1 to 1000 are not divisible by 3 and 5? [4]

ii) Consider the functions f : R→R and g : R→R defined by f(x) = 2x+1, g(x) = x/3. Verify $(g \bullet f)^{-1} = f^{-1} \bullet g^{-1}$ [3]

c) i) Find (g•f•h)(t) for g, f, and h as $g(t) = \sqrt{x}$; $f(t) = x^2$; $h(t) = 5x^9$ [4]

ii) Prove that $(ab)^n = a^n b^n$ for every natural number n. [4]

iii) There are 350 farmers in a large region. 260 farm beetroot, 100 farm carrot, 70 farm radish, 40 farm beetroot and radish, 40 farm carrot and radish, and 30 farm beetroot and carrot. Let B, C, and R denote the set of farms that farm beetroot, carrot and radish respectively. Determine the number of farmers that farm beetroot, carrot, and radish. [7]

2. a) Consider the relation R = {(a, a), (a, b), (b, c), (c, c)} on the set A = {a, b, c}.

Find: (i) reflexive(R); (ii) symmetric(R); (iii) transitive(R).

b) Obtain the DNF and CNF of $P \wedge (Q \leftrightarrow R)$.

c) i) Given A={1,2,3,4} and B={x,y,z}. Let R be the following relation from A to B: R = {(1, y), (1, z), (3, y), (4, x), (4, z)} [5]

1. Determine the matrix of the relation.
2. Draw the arrow diagram of R.
3. Find the inverse relation R^{-1} of R.
4. Determine the domain and range of R.

ii) Let A be a set of nonzero integers and let R be the relation on A × A defined by xRy whenever x+y is even. Prove that R is an equivalence relation. Check for equivalence relation if x + y is odd. [5]

iii) Determine the validity of the following argument: [5]

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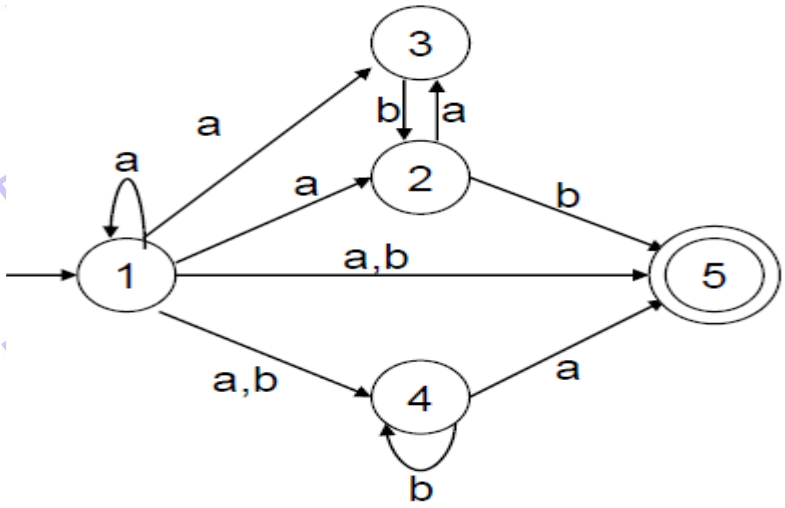
If I study, then I will not fail in mathematics
If I do not play basketball, then I will study
But, I failed in mathematics.

Therefore, I must have played basketball.

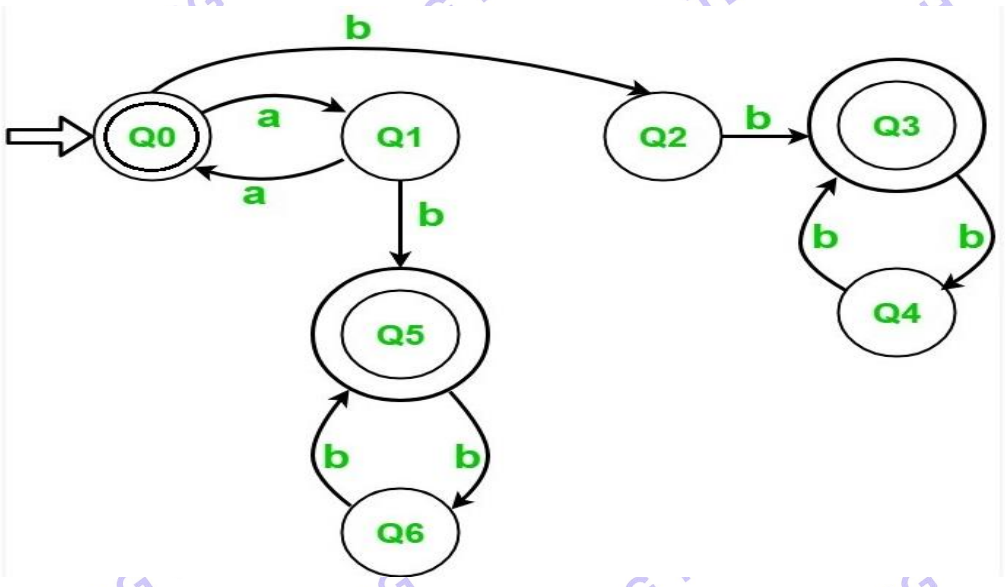
3. a) Find the language L(G) generated by the grammar G, whose productions are [3]

$$S \to aSb,\ aS \to Aa,\ Aab \to c.$$

b) Convert the given NFA into a DFA [7]



c) i) Identify the machine and prove that the following machine M recognizes **only** the Language L = {$a^n b^m$ | n + m is even and m,n ≥ 1} using appropriate examples.[15]



(OR)

ii) 1. Differentiate between DFA and NDFA. [4]

2. Prove that the following Grammar generates all even integers up to 999. [4]

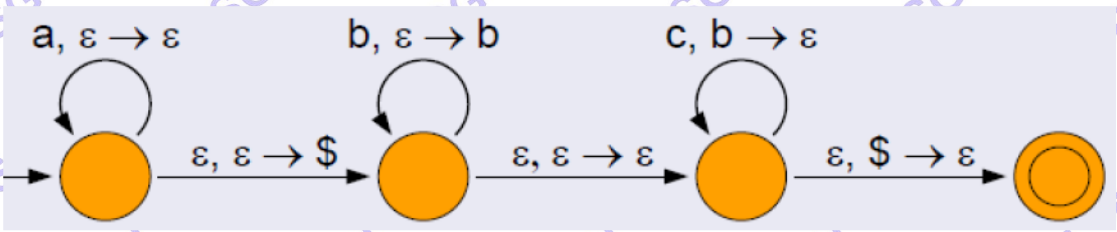
Let G = (N, T, P, S), where N = {S, S_1, A, B}, T = {0,1,2,3,…,9} and

P : S → 0|2|4|6|8, S→AS_1, A→1|2|3|4|5|6|7|8|9, S_1→0|2|4|6|8, S→ABS_1, B→0|1|2|3|4|5|6|7|8|9

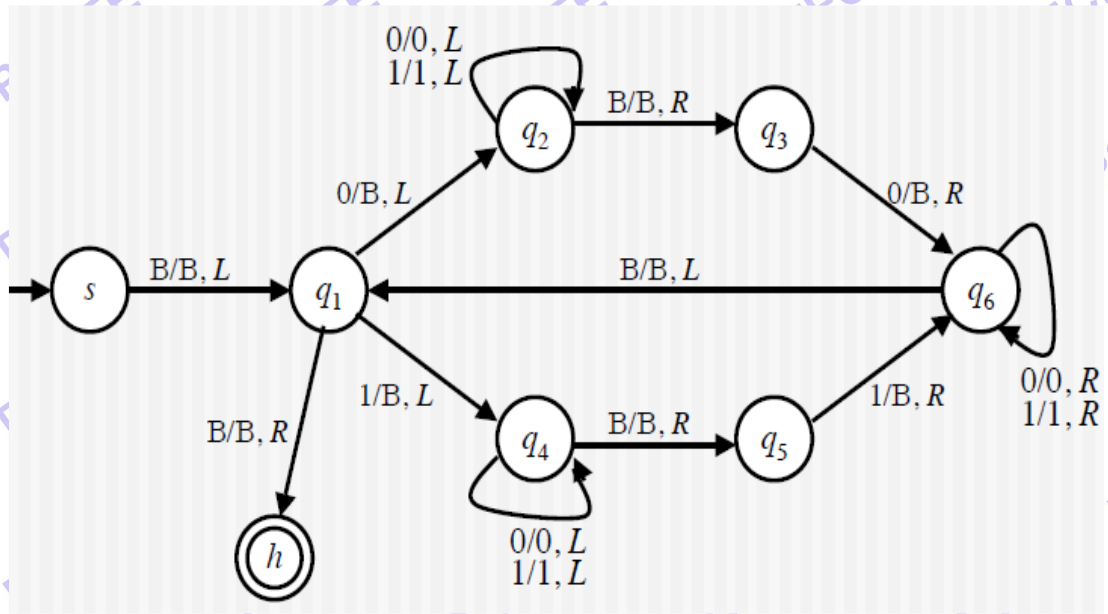
3. Modify the above grammar G, to generate all positive integers 1 to 9999. Verify the grammar with few sample positive integers. [4+3]

4. a) Draw the machine diagram **alone** to show the Push Down Automata (PDA) derived from the Context Free Grammar (CFG) : S → aTXb, T → XTS | ε and X → a | b

b) Identify the Language accepted by the machine given below. Justify your answer with suitable example.

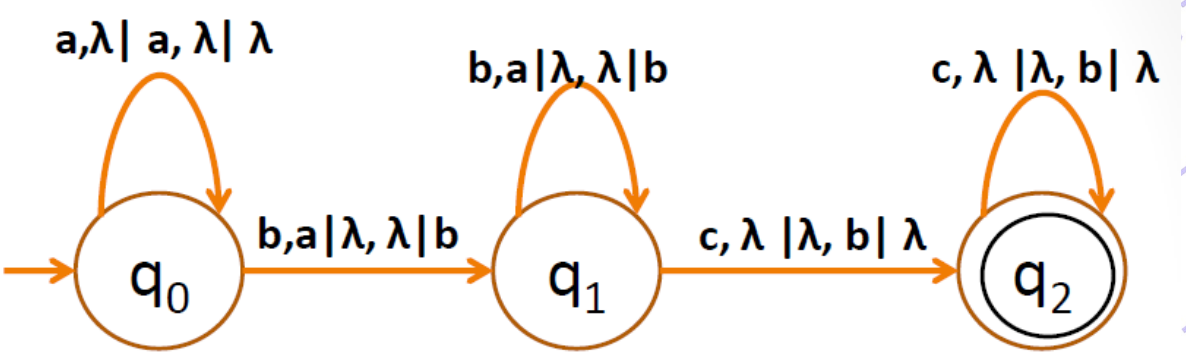


c) i) Define the Turing Machine M shown below. Also, prove that the Machine M recognizes the language L = {ww^R | w ϵ {0,1}* }

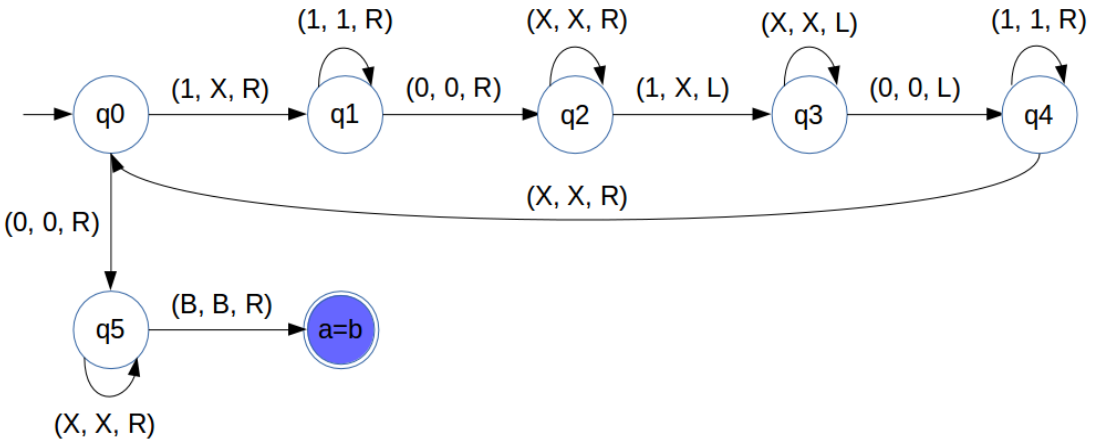


(OR)

ii) 1. Prove that the following PDA machine recognizes L = {$a^n b^n c^n$ / n ≥1} [5]



2. Prove that, the following Turing machine is designed to check whether any two numbers "a" and "b" are equal. Illustrate with appropriate examples. [10]



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