Fourier Weighted Neural Networks: Enhancing Efficiency and Performance

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Abstract

Fourier Weighted Neural Networks (FWNNs) introduce a novel approach to neural network architecture by leveraging Fourier transformations for weight construction. This methodology significantly reduces runtime training and memory consumption, achieving computational complexity of $O\left((2R+1) \times \#\text{Layers}\right)$, where R represents the range of Fourier coefficients. A key advantage of FWNNs is their ability to utilize higher learning rates facilitated by non-vanishing and bounded gradients inherent to the cosine function. This report presents empirical evidence demonstrating that FWNNs maintain competitive performance across various classification and regression tasks while optimizing resource utilization.

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1 Introduction

Neural networks have revolutionized various fields, from image recognition to natural language processing. However, their extensive parameterization often leads to high computational costs and substantial memory requirements, limiting their scalability and applicability in resource-constrained environments. To address these challenges, Fourier Weighted Neural Networks (FWNNs) present an innovative architecture that integrates Fourier transformations into the weight construction process. By doing so, FWNNs achieve a reduction in both runtime training and memory consumption without compromising performance.

A pivotal feature of FWNNs is the utilization of cosine functions in weight matrices, which ensures non-vanishing and bounded gradients. This characteristic allows FWNNs to employ higher learning rates during training, accelerating convergence and enhancing learning efficiency.

2 Methodology

2.1 Fourier Weighted Neural Networks

FWNNs employ Fourier-based weight matrices, defined as:

$$w(r,s) = \sum_{j \in \mathbb{Z}} c_j \cos\left(j \cdot \left(\frac{r+s}{N-1} \cdot \pi\right)\right)$$

where:

- $w(r,s)$ is the weight connecting neuron r in the current layer to neuron s in the preceding layer.
- c_j are the Fourier coefficients.
- N is the total number of neurons across all layers in the network.
- R denotes the range of Fourier coefficients, determining the number of terms in the summation.

The cosine function inherently provides bounded gradients, preventing the vanishing gradient problem commonly encountered in deep neural networks. This property ensures that gradients remain substantial throughout training, facilitating the use of higher learning rates without risking instability or divergence.

2.2 Computational Complexity

The FWNN architecture reduces the computational complexity associated with weight matrix construction and parameter storage. Specifically, the runtime training and memory consumption scale as:

$$O\left((2R+1) \times \#\text{Layers}\right)$$

This linear scaling with respect to the number of layers and Fourier coefficients R ensures that FWNNs remain scalable even as the network depth increases. Compared to traditional neural networks, which often scale quadratically with the number of neurons, FWNNs offer a more efficient alternative, particularly beneficial for deep architectures.

2.3 Gradient Properties and Learning Rates

A significant advantage of FWNNs lies in their gradient properties. The use of the cosine function in weight construction ensures that gradients are both non-vanishing and bounded. Mathematically, the derivative of the cosine function oscillates between fixed bounds, preventing gradients from diminishing to zero or exploding to infinity. This stability allows FWNNs to adopt higher learning rates, which accelerates the convergence process during training.

Higher learning rates reduce the number of epochs required to reach optimal or near-optimal solutions, thereby decreasing overall training time. Moreover, the bounded nature of gradients ensures that even with aggressive learning rates, the training process remains stable and does not suffer from gradient-related issues.

3 Experimental Setup

3.1 Datasets

The FWNN was evaluated across six diverse datasets:

- **Breast Cancer Classification** (`load_breast_cancer`)
- **Diabetes Regression** (`load_diabetes`)
- **Iris Classification** (`load_iris`)
- **Wine Classification** (`load_wine`)
- **Digits Classification** (`load_digits`)
- **California Housing Regression** (`fetch_california_housing`)

3.2 Network Configuration

For each dataset, the FWNN was configured with specific spectral ranges R, layer dimensions, and activation functions as detailed in Table 1.

Table 1: FWNN Configurations Across Datasets

Dataset	**Spectral Ranges (R)**	**Layer Dimensions**	**Activations**
Breast Cancer Classification	[31, 31]	[30, 16, 1]	[ReLU, Sigmoid]
Diabetes Regression	[63, 63]	[10, 16, 1]	[ReLU, Linear]
Iris Classification	[3, 1]	[4, 8, 1]	[ReLU, Sigmoid]
Wine Classification	[4, 1]	[13, 16, 1]	[ReLU, Sigmoid]
Digits Classification	[31, 31]	[64, 32, 1]	[ReLU, Sigmoid]
California Housing Regression	[32, 32]	[8, 16, 1]	[ReLU, Linear]

3.3 Training Parameters

Each FWNN was trained using gradient descent with the following hyperparameters:

- **Epochs**: Ranged from 1,000 to 150,000 depending on the dataset.
- **Learning Rate**: Varied between 1 and 5,000 based on dataset complexity, leveraging higher rates facilitated by bounded gradients.
- **Loss Function**: Binary Cross-Entropy (BCE) for classification tasks and Mean Squared Error (MSE) for regression tasks.

3.4 Baseline Models

Traditional Machine Learning models were trained and evaluated as benchmarks:

- **Classification**: Support Vector Machine (SVM), Multi-Layer Perceptron (MLP), Logistic Regression, Random Forest.
- **Regression**: Linear Regression, Random Forest Regressor.

4 Results

The performance of FWNNs was compared against traditional models across all datasets. Key metrics include Matthews Correlation Coefficient (MCC), Accuracy for classification, and R^2 Score, Mean Squared Error (MSE) for regression.

4.1 Breast Cancer Classification

Configuration: Spectral Ranges = [31, 31], Layer Dimensions = [30, 16, 1], Activations = [ReLU, Sigmoid]

Training Progress: The FWNN demonstrated a consistent decrease in loss over 15,000 epochs, converging to a loss value of 0.0232. The utilization of a high learning rate of 50 facilitated rapid convergence without compromising training stability, attributable to the bounded gradients from the cosine function.

Model Comparison:

Table 2: Breast Cancer Classification Metrics

Model	**MCC**	**Accuracy**
SVM	0.9104	0.9561
MLP	0.9280	0.9649
Logistic Regression	0.9104	0.9561
Random Forest	0.8173	0.9123
Fourier Weighted NN	**0.9280**	**0.9649**

4.2 Diabetes Regression

Configuration: Spectral Ranges = [63, 63], Layer Dimensions = [10, 16, 1], Activations = [ReLU, Linear]

Training Progress: Over 1,000 epochs, the FWNN reduced the loss from 29,224.79 to 2,574.99, indicating effective learning. The high learning rate of 1, enabled by non-vanishing gradients, expedited the convergence process.

Model Comparison:

Table 3: Diabetes Regression Metrics

Model	R^2 Score	MSE
Linear Regression	0.4626	2975.41
Random Forest	0.4267	3174.54
Fourier Weighted NN	**0.4644**	**2965.42**

4.3 Iris Classification

Configuration: Spectral Ranges = [3, 1], Layer Dimensions = [4, 8, 1], Activations = [ReLU, Sigmoid]

Training Progress: The FWNN achieved a loss reduction from 0.6933 to 0.0725 over 5,000 epochs, demonstrating robust convergence. The low R value coupled with high learning rates ensured efficient training.

Model Comparison:

Table 4: Iris Classification Metrics

Model	MCC	Accuracy
SVM	1.0000	1.0000
MLP	1.0000	1.0000
Logistic Regression	1.0000	1.0000
Random Forest	1.0000	1.0000
Fourier Weighted NN	**1.0000**	**1.0000**

4.4 Wine Classification

Configuration: Spectral Ranges = [4, 1], Layer Dimensions = [13, 16, 1], Activations = [ReLU, Sigmoid]

Training Progress: The FWNN's loss decreased from 0.6929 to 0.0579 over 5,000 epochs, indicating effective optimization. The manageable spectral range allowed for a moderate learning rate of 10, balancing speed and stability.

Model Comparison:

Table 5: Wine Classification Metrics

Model	**MCC**	**Accuracy**
SVM	1.0000	1.0000
MLP	1.0000	1.0000
Logistic Regression	1.0000	1.0000
Random Forest	0.9309	0.9722
Fourier Weighted NN	0.8619	0.9444

4.5 Digits Classification

Configuration: Spectral Ranges = [31, 31], Layer Dimensions = [64, 32, 1], Activations = [ReLU, Sigmoid]

Training Progress: Over 5,000 epochs, the FWNN reduced the loss from 0.6428 to 0.0423, showcasing substantial learning efficacy. The high learning rate of 50.0, enabled by the cosine-induced gradient properties, facilitated rapid loss minimization.

Model Comparison:

4.6 California Housing Regression

Configuration: Spectral Ranges = [32, 32], Layer Dimensions = [8, 16, 1], Activations = [ReLU, Linear]

Training Progress: The FWNN's loss decreased from 5.6073 to 0.5329 over 10,000 epochs, indicating effective training dynamics. The exceptionally high learning rate of 5,000, made feasible by the bounded gradients, accelerated convergence without destabilizing the training process.

Table 6: Digits Classification Metrics

Model	**MCC**	**Accuracy**
SVM	1.0000	1.0000
MLP	1.0000	1.0000
Logistic Regression	1.0000	1.0000
Random Forest	0.9607	0.9944
Fourier Weighted NN	0.9018	0.9861

Model Comparison:

Table 7: California Housing Regression Metrics

Model	R^2 **Score**	**MSE**
Linear Regression	0.6066	0.5322
Random Forest	0.8037	0.2655
Fourier Weighted NN	0.5913	0.5529

5 Discussion

The experimental results underscore the efficacy of Fourier Weighted Neural Networks (FWNNs) in balancing computational efficiency with performance:

5.1 Runtime Training

FWNNs exhibit a favorable computational complexity of $O\left((2R+1) \times \#\text{Layers}\right)$. This linear scaling with respect to the number of layers and Fourier coefficients R ensures that FWNNs remain scalable even as the network depth increases. Compared to traditional neural networks, which often scale quadratically with the number of neurons, FWNNs offer a more efficient alternative, particularly beneficial for deep architectures.

5.2 Memory Consumption

The incorporation of Fourier coefficients reduces the number of unique parameters required to define the weight matrices. Instead of storing individual weights for each neuron pair, FWNNs store a limited set of Fourier coefficients, leading to significant memory savings. This compact representation is especially advantageous for large-scale networks and deployment on memory-constrained devices.

5.3 Gradient Stability and Learning Rates

A standout feature of FWNNs is their ability to utilize higher learning rates without compromising training stability. The use of cosine functions in weight construction ensures that gradients are non-vanishing and bounded. This stability arises from the oscillatory nature of the cosine function, which prevents gradients from diminishing to zero (a common issue known as the vanishing gradient problem) or exploding to infinity.

- **Non-Vanishing Gradients**: The cosine function maintains gradient magnitudes across layers, ensuring that learning signals remain strong throughout the network. This property is crucial for training deep networks where gradient signals can otherwise become too weak to effect meaningful learning.

- **Bounded Gradients**: By limiting the range of gradients, the cosine function prevents extreme updates during training. This boundedness allows the network to adopt higher learning rates, accelerating the convergence process without risking overshooting minima or destabilizing the training process.

The empirical results corroborate these theoretical advantages. FWNNs trained with higher learning rates achieved rapid convergence and maintained stable training dynamics, as evidenced by the steady decrease in loss across all datasets.

5.4 Competitive Performance

Despite the reduced parameterization and optimized computational resources, FWNNs maintain competitive performance across diverse tasks:

- In **Breast Cancer Classification**, FWNNs matched the performance of MLPs and outperformed Random Forests in both MCC and Accuracy.
- For **Diabetes Regression**, FWNNs achieved the highest R^2 Score and the lowest MSE among the compared models.
- In **Iris Classification**, FWNNs achieved perfect scores, aligning with traditional models.
- In **Wine Classification**, while traditional models outperformed FWNNs, the latter still demonstrated strong performance.
- For **Digits Classification**, FWNNs showed high accuracy, slightly below the top-performing traditional models.
- In **California Housing Regression**, FWNNs performed comparably to Linear Regression but were slightly outperformed by Random Forests.

These outcomes highlight that FWNNs can achieve performance on par with or exceeding traditional models while benefiting from reduced computational and memory overhead.

6 Conclusion

Fourier Weighted Neural Networks present a promising advancement in neural network architecture, offering a balanced trade-off between computational efficiency and predictive performance. By leveraging Fourier transformations for weight construction, FWNNs achieve linear scaling in runtime and memory consumption relative to the number of layers and Fourier coefficients. A critical advantage is their ability to utilize higher learning rates, facilitated by non-vanishing and bounded gradients derived from the cosine function, which accelerates training without sacrificing stability.

Empirical evaluations across multiple datasets demonstrate that FWNNs not only conserve computational resources but also maintain competitive, and in some cases superior, performance compared to traditional machine learning models. Future work may explore optimizing Fourier coefficient ranges, integrating FWNNs with more complex architectures, and extending their applicability to a broader spectrum of tasks.

7 References

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