

Real Analysis

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Contents

Chapter 0. Preliminary	1
1. Calculus	1
2. Fundamental Theorem of Calculus	1
3. Integration by Substitution	1
Chapter 1. σ Algebra	3
1. σ Algebra	3
2. Semi Ring	3
3. π System	3
4. Product σ -algebra	3
Chapter 2. Measure	5
1. Measure	5
2. Existence	5
3. Uniqueness	5
4. Inner Measure	5
5. Outer Measure	5
6. Measurable Space	5
7. Product Measure	5
8. Independence	5
Chapter 3. Measurable Map & Function	7
1. Measurable Map	7
2. Measurable Function	7
3. Random Variable	7
Chapter 4. Integration	9

1. Integral of Simple Function	9
2. Integral of Measurable Function	9
3. Measurable Function	9
4. Lebesgue Integration	9
5. Substitution	9
6. Fubini's Theorem	9
Chapter 5. Differentiation	11
Chapter 6. Conditional Expectation	13
1. Conditional Probability	13
2. Conditional Expectation	13
Bibliography	15

Preliminary

1. Calculus

Theorem 0.1 (extreme value theorem).

Theorem 0.2 (intermediate value theorem).

Theorem 0.3 (Rolle's theorem).

Theorem 0.4 (mean value theorem).

Theorem 0.5 (mean value theorem for definite integral).

2. Fundamental Theorem of Calculus

3. Integration by Substitution

σ Algebra

1. σ Algebra
2. Semi Ring
3. π System
4. Product σ -algebra

Measure

1. Measure
2. Existence
3. Uniqueness
4. Inner Measure
5. Outer Measure
6. Measurable Space
7. Product Measure
8. Independence

Measurable Map & Function

- 1. Measurable Map**
- 2. Measurable Function**
- 3. Random Variable**

Integration

- 1. Integral of Simple Function**
- 2. Integral of Measurable Function**
- 3. Measurable Function**
- 4. Lebesgue Integration**
- 5. Substitution**
- 6. Fubini's Theorem**

Differentiation

Conditional Expectation

1. Conditional Probability
2. Conditional Expectation

Bibliography