General Topology

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Definition 1.9. (adherent point)

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Example 1.14. to-do

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Proof. Proof by contradiction. $\square$

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Theorem 3.11. *Given metric space (X, d), the following statements are equivalent:*

Theorem 3.12. *Given a metric space (X, d) and $Y \subseteq X$. Y is a compact space if and only if Y is complete and totally bounded.*

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