

General Topology

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Topology Space

Definition 1.1. (open set)

Definition 1.2. (closed set) Complement of open set.

Definition 1.3. (topology space)

Definition 1.4. (neighborhood)

Definition 1.5. (limit point)

Definition 1.6. (isolation point)

Definition 1.7. (interior point)

Definition 1.8. (boundary point)

Theorem 1.9. *Given topology space (X, \mathcal{T}) and $Y \subseteq X$. Y is closed if and only if Y contains all of its limit points.*

Proof. Proof by contradiction. □

Definition 1.10. (closure)

Theorem 1.11. *Given topology space (X, \mathcal{T}) , the following statements are equivalent:*

Definition 1.12. (open cover)

Definition 1.13. (compact space)

Definition 1.14. (subspace)

Continuous and Homeomorphism

Definition 2.1. (continuous map)

Definition 2.2. (homeomorphism)

Metric Space

1. Metric Space

Definition 3.1. (metric)

Definition 3.2. (metric space)

Definition 3.3. (cauchy sequence)

Definition 3.4. (convergence of sequence)

Definition 3.5. (complete space)

Definition 3.6. (bounded space)

Definition 3.7. (totally bounded space)

Warning 3.8. Not every bounded space is a totally bounded space.

Theorem 3.9. *Every metric space (X, d) can generate a topology space (X, \mathcal{T}_d) .*

Theorem 3.10. *Given a compact metric space (X, d) and $Y \subseteq X$. If Y is closed, then Y is compact.*

Theorem 3.11. *Given metric space (X, d) , the following statements are equivalent:*

Theorem 3.12. *Given a metric space (X, d) and $Y \subseteq X$. Y is a compact space if and only if Y is complete and totally bounded.*

Definition 3.13. (dense set)

Definition 3.14. (separable set)

2. Normed vector space

Definition 3.15. (norm)

Definition 3.16. (normed vector space)

Definition 3.17. (Banach space)

Theorem 3.18. *Every normed vector space $(X, \|\cdot\|)$ can generate a metric space $(X, \|\cdot\|_d)$.*

3. map and function

Definition 3.19. (pointwise continuity)

Definition 3.20. (uniformly continuity)

Theorem 3.21. *Uniformly continuity implies pointwise continuity.*

Theorem 3.22. (*Dini's theorem*)

Bibliography

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