PDE and Martingale Methods in Option Pricing

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Elements of Malliavin calculus

1. Stochastic derivative

proposition 16.1. *$\forall n \in \mathbb{N}$, $W_T \in \mathcal{S}_n$.*

Proof. Given n, if $\varphi(\Delta_n) = \varphi(x_1, \ldots, x_{2^n}) \coloneqq x_1 + \cdots + x_{2^n}$, then $\varphi(\Delta_n) \in \mathcal{S}_n$. Therefore $W_T \in \mathcal{S}_n$. □

proposition 16.2. *$\forall n \in \mathbb{N}$, $\mathcal{S}_n \subseteq \mathcal{S}_{n+1}$.*

Proof. The partition length is $\frac{1}{2^n}$ for each $n \in \mathbb{N}$. □

proposition 16.3. *$\mathcal{S}$ is dense in $L^p(\Omega, \mathcal{F}_T^W)$.*

Proof. To-do. □

Note 16.4. Given a filtered measure space $(\Omega, \mathcal{A}, \mathcal{A}_n, \mu)$, for each $X \in \mathcal{S}_n$, X and $D_t X$ are both measurable functions: $\Omega \to \mathbb{R}$.