

Referee report: Anomalous waves triggered by abrupt depth changes: laboratory experiments and truncated KdV statistical mechanics

The paper deals with the statistics of the 1D evolution of a surface-gravity-wave field in shallow water, encountering an abrupt depth reduction. The authors perform experimental investigations carried out in a water tank and in parallel they develop a theory based on the statistical mechanics of the Korteweg de Vries equation, used to interpret the experimental results. The two aspects, experimental and theoretical, have been considered before, in References [5] and [25], but here they are put together in an expanded and synergetic way that makes the manuscript clear and interesting. Moreover, although many of the results have been presented before, the analysis of the experimental data of surface slope is new. This is performed in light of the recently derived relationship linking the ratio between displacement skewness and variations of slope variance to the nonlinearity in the system. The agreement of the experimental and theoretical scalings of the ratio is remarkable over a wide range of amplitudes, providing compelling evidence that the theory proposed captures the fundamental, leading-order aspects of the phenomenon.

The authors should address the following concerns:

1. Equilibrium statistical mechanics, i.e. ultimately the use of the canonical ensemble, is indeed a powerful tool. Though, too often the relaxation conditions required for the machinery to apply are overlooked. Here, the 1D KdV equation is an integrable system, whose trajectories are constrained by an infinite number of conserved quantities and whose highly nonlinear regime is governed by coherent structures such as solitons. The description of these structures is out of reach for the Gibbs measure – see the recent rise of an entire subject, namely *integrable turbulence*, to describe the statistics of integrable systems such as KdV or NLS. I do believe that some *weak ergodicity*, especially for the statistics of physical-space variables, may justify a sort of *relaxation* to the Gibbs measure before the ADC is encountered, but all of this deserves being discussed, and should at least be given careful consideration. Consider for instance the most linear cases in the paper: at the wave maker the measure in Fourier space is the one induced by equation (1). Since the modes do not interact (linear limit), the measure at the ADC will also be the one with the spectrum of eq. (1); i.e., relaxation to the Gibbs measure does not take place (simply because the necessary randomizing interactions are absent in the system), in which case the canonical ensemble is justified if and only if the initial condition at the wave maker is itself distributed according to the Gibbs measure. This fact should be discussed and may explain why the theory does not match the experimental points for low amplitudes in figure (8.c).
2. The authors may find it useful to compare their equation (53) to [*On the origin of heavy-tail statistics in equations of the Nonlinear Schrödinger type. Onorato et al., Phys. Lett. A, 380, 89 (2016)*], where an analogous relationship is derived for NLS. It should be possible to relate the skewness to spectral variations, which may lead to further interesting investigations.
3. From the experimental point of view, have the authors considered corrections due to the second-order of the Stokes series? When the nonlinearity is important, the bound-modes

impact on the skewness may be non-negligible. Is it possible to filter out such corrections in the experimental signal and then compare with the theory? May this explain why the ADC skewness jump is so much larger in the experiment than in the theory?

Upon satisfying consideration of the remarks above, I am ready to consider the manuscript suitable for publication in *Journal of Nonlinear Science*. The manuscript is well written, clear, instructive and original. The methods presented here, for the first time in such a complete and pedagogical way, tackle a problem of fluid mechanics with ideas from dynamical systems and statistical physics. The resulting method is general and of broad applicability in the realm of nonlinear physics.