2 Fractional Cubic Root Elimination

This section addresses an obstacle that will appear shortly. Here we eliminate the fractional roots in this equation:

$c_1\, k^{1/3} + c_2\, k^{2/3} = z$

(I'm not sure if this transformation has a more official name, but I'll refer to it here simply as "Fractional Cubic Root Elimination".)

$c_1\, k^{1/3} + c_2\, k^{2/3} + c_3 = 0$	(Given.)
$c_1\, k^{1/3}\,(1 + \frac{c_2}{c_1}\, k^{1/3}) = -c_3$	Factor the left-hand side by $c_1\, k^{1/3}$.
$1 + \frac{c_2}{c_1}\, k^{1/3} = \frac{-c_3}{c_1 k^{1/3}}$	Divide both sides by $c_1\, k^{1/3}$.
$1 + 3\,\frac{c_2}{c_1}\, k^{1/3} + 3\,\frac{{c_2}^2}{{c_1}^2}\, k^{2/3} + \frac{{c_2}^3}{{c_1}^3}\, k = \frac{(-c_3)^3}{{c_1}^3\, k}$	Cube both sides of the equation.
${c_1}^3\, k + 3\,{c_1}^2\, c_2\, k^{4/3} + 3\, c_1\, {c_2}^2\, k^{5/3} + {c_2}^3\, k^2 = -{c_3}^3$	Multiply both sides by ${c_1}^3\, k$.
$3\,{c_1}^2\, c_2\, k^{4/3} + 3\, c_1\, {c_2}^2\, k^{5/3} = -{c_3}^3 - {c_1}^3\, k - {c_2}^3\, k^2$	Isolate fractional exponents.
$c_1\, k^{1/3} + c_2\, k^{2/3} = \frac{-{c_3}^3 - {c_1}^3\, k - {c_2}^3\, k^2}{3\, c_1\, c_2\, k}$	Divide by $3\, c_1\, c_2\, k$.

The left-hand side of the last equation resembles to the left-hand of the first equation, so it follows that:

$\frac{-{c_3}^3 - {c_1}^3\, k - {c_2}^3\, k^2}{3\, c_1\, c_2\, k} = -c_3$

So when we encounter the first equation, we can replace it with this equivalent equation:

$-{c_3}^3 - {c_1}^3\, k - {c_2}^3\, k^2 + 3\, c_1\, c_2\, c_3\, k = 0$