Program : **B.Tech**

Subject Name: **Basic Civil Engineering & Mechanics**

Subject Code: **BT-204**

Year: **1st**

Unit V:

Centre of Gravity and moment of Inertia: Centroid and Centre of Gravity, Moment Inertia of Area and Mass, Radius of Gyration, Introduction to product of Inertia and Principle Axes. Support Reactions, Shear force and bending moment Diagram for Cantilever & simply supported beam with concentrated, distributed load and Couple. Centre of Gravity

A single point where the entire weight or mass of a body is concentrated is known as centre of gravity of that body. Centre of gravity is generally denoted by 'G'. In general when a rigid body lies in a field of force acts on each particle of the body. We equivalently represent the system of forces by single force acting at a specific point. This point is known as centre of gravity.

Centroid

Centroid is another term to centre of gravity. It is the Centroid of plane geometrical figures like rectangle, triangle, trapezoid, circle etc. the word Centroid is used when there are only geometrical figures instead of weight or mass. Therefore, centre of gravity of plane geometrical figure is termed as Centroid or centre of area. Two distances are required for each area in evaluation of centre of gravity. One is from reference x axis and the other is from reference Y axis. These distances are denoted as x' and y'. Centroid can be determined by the method of integration.

CENROIDS OF AREAS BY FIRST MOMENT OF AREA

Consider the following lamina. Let's assume that it has been exposed to gravitational field. Obviously every single element will experience a gravitational force towards the centre of earth. Further let's assume the body has practical dimensions, and then we can easily conclude that all elementary forces will be unidirectional and parallel.

Consider G to be the Centroid of the irregular lamina. As shown in first figure we can easily represent the net force passing through the single point G. We can also divide the entire region into let's say n small elements. Let's say the coordinates to be (x_1,y_1), (x_2,y_2), (x_3,y_3)………. (x_n , y_n) as shown in figure . Let W_1, W_2, W_3,……. , W_n be the elementary forces acting on the elementary elements. Clearly,

$W = W_1+ W_2+ W_3 +…………..+ W_n$

Let's consider plate of uniform thickness and a homogenous density. Now weight of small element is directly proportional to its thickness, area and density as:

$$W= \Upsilon\, t\, dA.$$

Where ϒ is the density per unit volume, t is the thickness; dA is the area of the small element. So we can replace W with this relationship in the expression we obtained in the prior topic. Therefore we get:

Centroid of area:

$$x_c=W_1x_1+ W_2x_2+ W_3x_3+\ W_nx_n/ W ,$$

$$y_c= W_1y_1+ W_2y_2+ W_3y_3+\ W_ny_n /W$$ **Substituting for weights we get,**

$$x_c=A_1x_1+ A_2x_2+ A_3x_3+\ A_nx_n/ A ,$$

$$Yc= A_1y_1+A_2y_2+ A_3y_3+\ A_ny_n /A$$

Where x,y are the coordinate of the small element and dA the elemental area. Also A (total area of the plate).(x_c ,y_c,) is called the Centroid of area of the lamina. If the surface is homogenous we conclude that it is the same as centre of gravity.

Thus it follows from the above discussion that Centroid of a area can be determined by dividing first moment of the area with the area itself. If the first moment of area with respect to an axis is zero, it indicates that the point lies on that axis itself. So we can conclude that the first moment about the axis will be zero about the axis of symmetry. Further Centroid also lies on the axis of symmetry. If a body has more than one axis of symmetry then Centroid will lie on the point of intersection of the axes. For some type of surfaces of bodies there lies a probability that the centre of gravity may lie outside the body. Secondly centre of gravity represents the entire lamina; therefore we can replace the entire body by the single point with a force acting on it when needed. From the above discussion we can draw the following differences between centre of gravity and Centroid;

1. The term centre of gravity applies to bodies with weight, and Centroid applies to lines, plane areas and volumes.
2. Centre of gravity of a body is a point through which the resultant gravitational force (weight) acts for any orientation of the body whereas Centroid is a point in a line plane area volume such that the moment of area about any axis through that point is zero.

OBTAINING CENTROIDS BY INTEGRATION

The general expression of Centroid of a body is given by:

$$x_c= \int x\, dA/A, \quad y_c=\int y\, dA/A$$

Where dA is the area of element.

We divide the area into thin rectangular strips or sectors. For rectangle it is pre-known that its centre of gravity lies at the centre of the rectangle.

CENTROID OF SOME STANDARD GEOMETRIC FIGURES:

Following results are obtained by integration which will be explained later. Results for symmetrical objects like square, circle, cylinder, rectangle, ring etc. are omitted. For such cases Centroid can be pre-assumed to be the geometric centre of the body.

1. Centroid of Rectangle of breadth b and depth d:

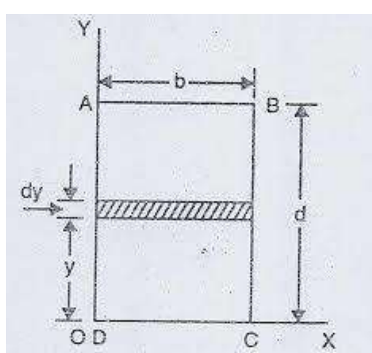


Figure1: Centroid of Rectangle

Area of the element dA = b*dy

From the method of integration we know that Ay' = ∫y dA

(b*d) y' = $\int_0^y y * dA$ = $\int_0^d y * b * dy$ = (b/2) [y^2] = (b/2) [d^2 – 0]

Y' = d/2

Similarly x' = b/2

3. Centroid of Triangle of breadth b and height h:

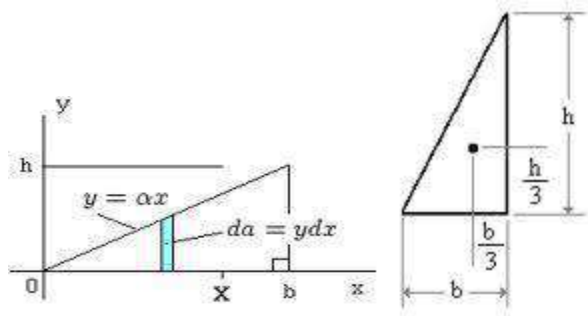


Figure2: Centroid of Triangle

Area of the element dA = y * dx

From similar triangles y/h = x/b

Therefore y = h/b * x When x = 0, y = 0 and when x = b, y = h. It satisfies all values of x.

Substituting above in the area equation we get dA = y * dx = (h/b * x) dx.

From the method of integration we know that Ax' = ∫x dA = $\int_0^x xdA$ = $\int_0^b x * (h/b * x)dx$

(b*h/2) x' = (h/b) (x^3/3) with limits 0 to b.

Therefore x' = 2*b/3

Similarly y' = h/3.

3. Centroid of quarter circle of radius R:

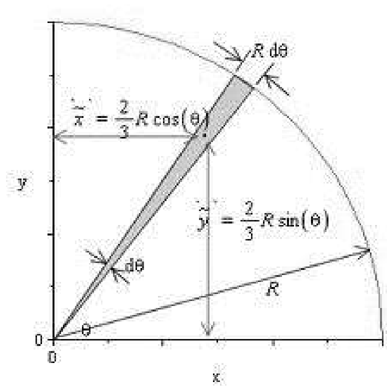


Figure3: Centroid of Quarter Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{\pi/2} \left(\frac{2}{3}\right) R\cos\theta\, R * R\, d\theta/2$

= $2R^3/3 \int_0^{\pi/2} \cos\theta\, d\theta$

= $2R^3/3\ [\sin\theta]$ **with the limits 0 to π/2.**

Therefore x' = (4*R)/ (3*π)

From the method of integration we know that Ay' = ∫x dA = $\int_0^{\pi/2} \left(\frac{2}{3}\right) R \sin\theta\, R * R\, d\theta/2$

= $2R^3/3 \int_0^{\pi/2} \sin\theta\, d\theta$

= $2R^3/3\ [\sin\theta]$ **with the limits 0 to π/2.**

Therefore y' = (4*R)/ (3*π)

4. Centroid of semi-circle of radius R:

(Semi-circle with symmetry about vertical axis)

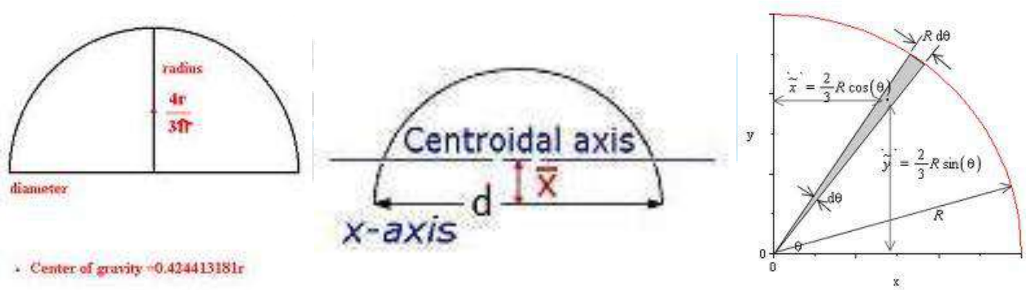


Figure4: Centroid of Semi Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{\pi} \left(\frac{2}{3}\right) R\cos\theta\, R * R\, d\theta/2$

= $2R^3/3 \int_0^{\pi} \cos\theta\, d\theta$

= $2R^3/3\ [\sin\theta]$ **with the limits 0 to π.**

Therefore x' = 0

From the method of integration we know that Ay' = ∫x dA = $\int_0^{\pi} \left(\frac{2}{3}\right) R \sin\theta\, R * R\, d\theta/2$

= $2R^3/3 \int_0^{\pi} \sin\theta\, d\theta$

= $2R^3/3\ [\sin\theta]$ **with the limits 0 to π.**

Therefore y' = (4*R)/ (3*π) = 0.424413181 R

5. Centroid of semi-circle of radius R:

(Semi-circle with symmetry about horizontal axis)

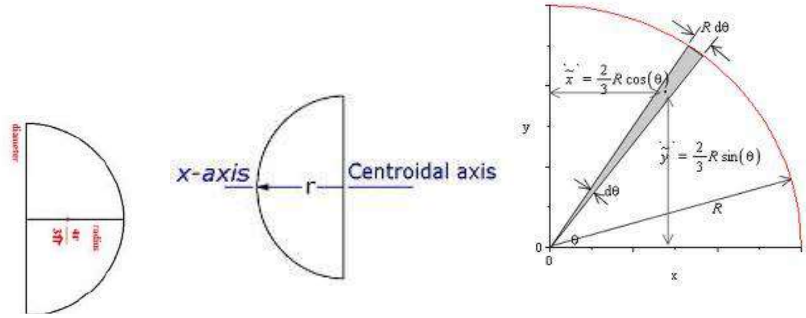


Figure5: Centroid of Semi circle with symmetry about horizontal axis

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_{-\pi/2}^{\pi/2} \left(\frac{2}{3}\right) R\cos\theta \, R * R \, d\theta/2$

$$= 2R^3/3 \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= 2R^3/3 \, [\sin\theta] \text{ with the limits } -\pi/2 \text{ to } +\pi/2.$$

Therefore x' = (4*R)/ (3*π) = 0.424413181 R

From the method of integration we know that Ay' = ∫x dA = $\int_{0}^{\pi} \left(\frac{2}{3}\right) R\, Sin\,\theta \, R * R \, d\theta/2$

$$= 2R^3/3 \int_{0}^{\pi} Sin\,\theta \, d\theta$$

$$= 2R^3/3 \, [Sin\,\theta] \text{ with the limits } -\pi/2 \text{ to } +\pi/2.$$

Therefore y' = 0

Note: Centroid of a semi-circle is always at a distance of (4*R)/ (3*π) from the diameter.

6. Centroid of circle of radius R:

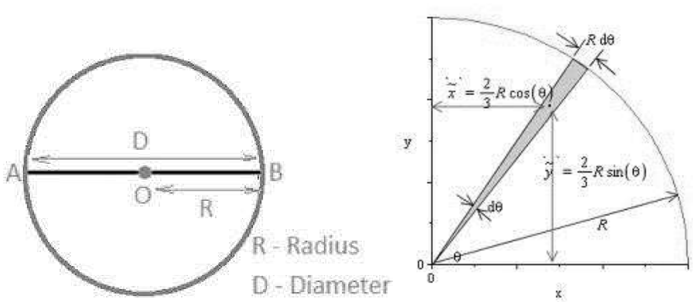


Figure6: Centroid of Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{2\pi}\left(\frac{2}{3}\right)R\cos\theta\, R*R\, d\theta/2$

$= 2R^3/3 \int_0^{2\pi} \cos\theta\, d\theta$

$=$ **$2R^3/3$ [sin θ] with the limits 0 to 2π.**

Therefore x' = 0

From the method of integration we know that Ay' = ∫x dA = $\int_0^{2\pi}\left(\frac{2}{3}\right)R\, Sin\,\theta\, R*R\, d\theta/2$

$= 2R^3/3 \int_0^{2\pi} Sin\,\theta\, d\theta$

$=$ **$2R^3/3$ [Sin θ] with the limits 0 to 2π.**

Therefore y' = 0

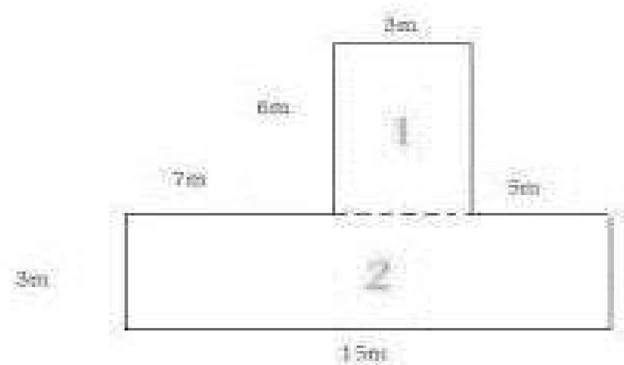
Centroid of composite figure:

Composite figure is the combination of regular figure. Divide the composite figure into number of parts such that centroid of each part is known. Determine the coordinates of centroid of each part from the given XX and YY axis. Let the coordinates of each part be (x_1, y_1), (x_2, y_2)…

Also calculate the area of each part as A_1, A_2, A_3…

Then $x' = \frac{A1*x1+A2*x2+A3*x3+\cdots}{A1+A2+A3+\cdots}$

$y' = \frac{A1*y1+A2*y2+A3*y3+\cdots}{A1+A2+A3+\cdots}$



N the above composite figure it is divided in to two parts 1

In the above composite figure it is divided in to two parts 1 and 2.

Moment of Inertia:

The second moment of area, also known as moment of inertia of plane area, area moment of inertia, or second area moment, is a geometrical property of an area which reflects how its points are distributed with

regard to an arbitrary axis. The second moment of area is typically denoted with either I for an axis that lies in the plane or with J for an axis perpendicular to the plane. Its unit of dimension is length to fourth power, L^4.

In the field of structural engineering, the second moment of area of the cross-section of a beam is an important property used in the calculation of the beam's deflection and the calculation of stress caused by a moment applied to the beam.

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

Where x and y are the distance to some reference plane.

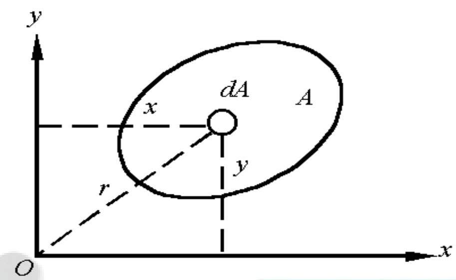


Figure7: Moment of Inertia

The polar second moment of area, $I = \int_A r^2 \, \mathrm{d}A$, where r is the distance to some reference axis. In each case the integral is over all the infinitesimal elements of area, dA, in some two-dimensional cross-section.

<u>Perpendicular Axis Theorem</u>

The moment of inertia (MI) of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.

That means the Moment of Inertia $I_{zz} = I_{xx}+I_{yy}$

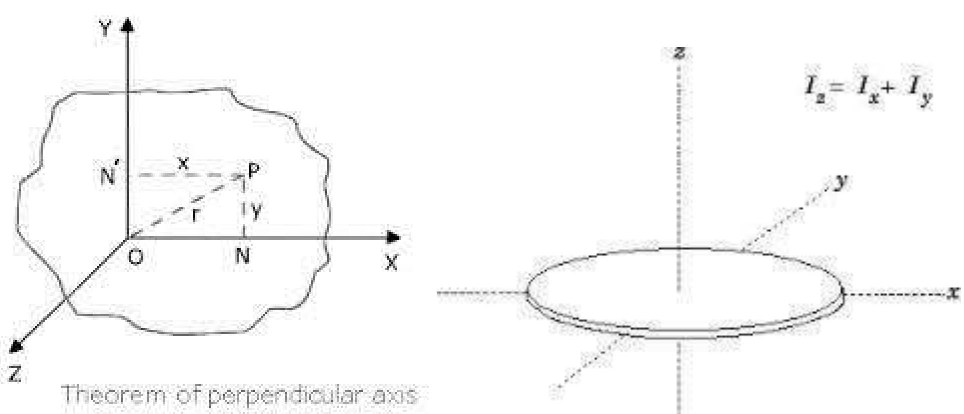


Figure8: Perpendicular Axis Theorem

Parallel Axis Theorem:

The moment of area of an object about any axis parallel to the centroid axis is the sum of MI about its centroid axis and the product of area with the square of distance of from the reference axis.

Essentially, $I_{XX} = I_{X'X'} + A\, y_c^2$

Where A is the cross-sectional area.

Yc is the perpendicular distance between the centroid axis and the parallel axis.

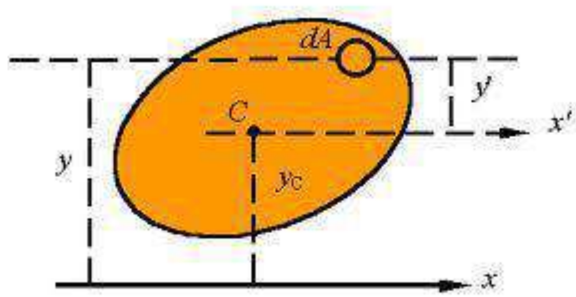


Figure9: Parallel Axis Theorem

$$\begin{aligned} I_x &= \int y^2 dA \\ &= \int (\bar{y} + y_c)^2 dA \\ &= \int \bar{y}^2 dA + 2\int y_c \bar{y}\, dA + \int y_c^2\, dA \\ &= \bar{y}^2 A + 2\bar{y} \underbrace{\int y_c\, dA}_{0} + I_{x_c} \\ &= \bar{y}^2 A + I_{x_c} \end{aligned}$$

Radius of gyration

In structural engineering, the two-dimensional radius of gyration is used to describe the distribution of cross sectional area in a column around its centroidal axis. The radius of gyration is given by the following formula

K = $\sqrt[2]{I/A}$ K_{xx} = $\sqrt[2]{Ixx/A}$ K_{yy} = $\sqrt[2]{Iyy/A}$ K_{zz} = $\sqrt[2]{Izz/A}$

Where I is the second moment of area and A is the total cross-sectional area.

Moment of Inertia of regular geometrical figures.

1. Moment of inertia of semi-circle of radius R:

(Semi-circle with symmetry about horizontal axis)

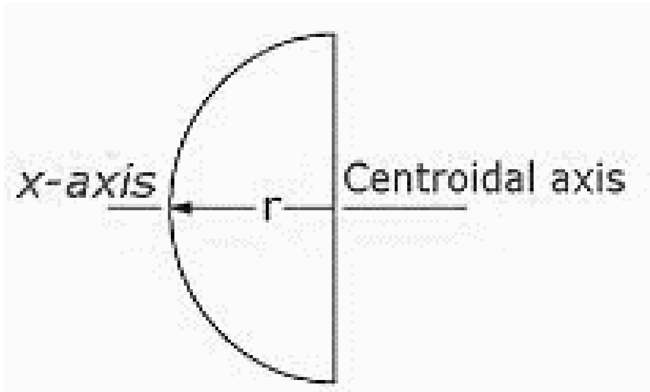


Figure9: Moment of inertia of semi-circle of radius R

$I_{xx} = I_{yy} = \pi R^4/8$, $I_{y'y'} = 0.11 R^4$.

2. Moment of inertia of quarter circle of radius R:

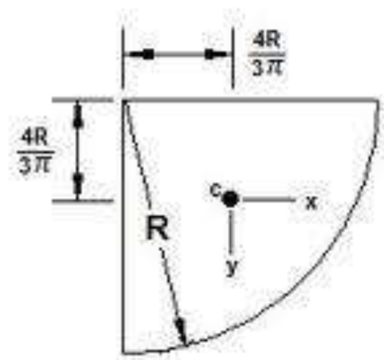
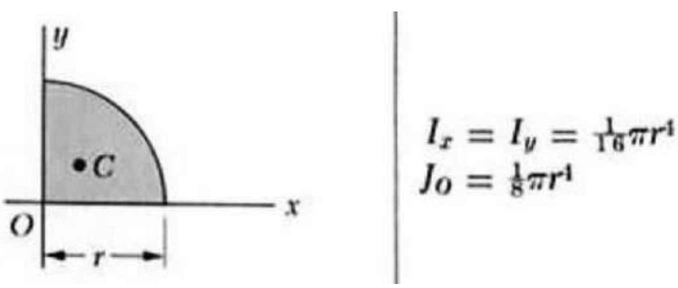


Figure10: Moment of inertia of quarter circle of radius R

$I_{xx} = I_{yy} = \pi R^4/16$, $I_{y'y'} = 0.05488 R^4. = 0.055 R^4$.



4. Moment of inertia of circle of radius R:

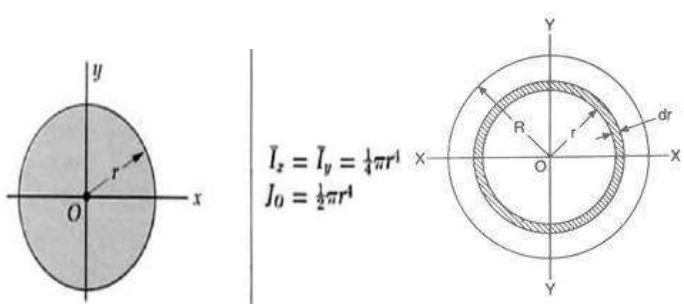


Figure11: Moment of inertia of circle of radius R

Area of Element $dA = 2\pi\, r\, dr$.

$I_{zz} = \int z^2 dA = \int z^2 (2\pi\, r\, dr) = \pi R^4/2$

$I_{zz} = I_{xx} + I_{yy}$ (by perpendicular axis theorem)

We know that for a circle $I_{xx} = I_{yy}$

Therefore $I_{xx} = I_{yy} = I_{zz}/2 = \pi R^4/4$

5. **Moment of inertia of rectangle of breadth b and depth d:**

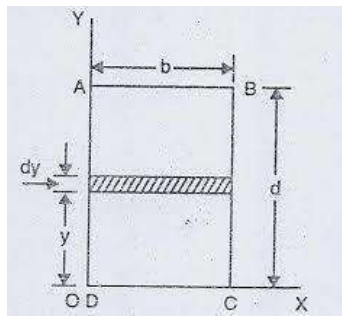


Figure12: Moment of inertia of rectangle

Area of Element $dA = b * dy$.

$I_{zz} = \int y^2 dA = \int_0^d bx^2\, dy = bd^3/12$

Similarly $I_{yy} = \int x^2 dA = b^3 d/12$

6. **Moment of inertia of triangle of breadth b and height h:**

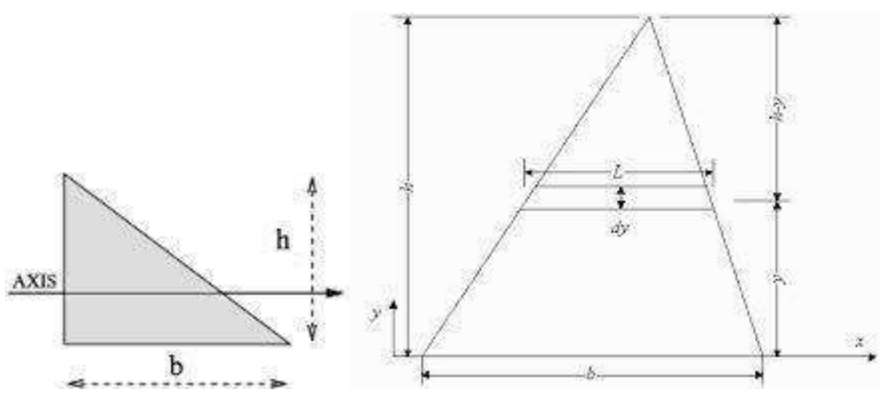


Figure13: Moment of inertia of triangle

From similar triangles; $L/b = (h-y)/h$

$L = b\,(h-y)/h$

Area of Element $dA = L * dy = b\,(h-y)/h\; dy$.

$$I_{xx} = \int y^2 dA = \int_0^h y^2 b\,(h-y)/h\, dy \quad = bh^3/12$$

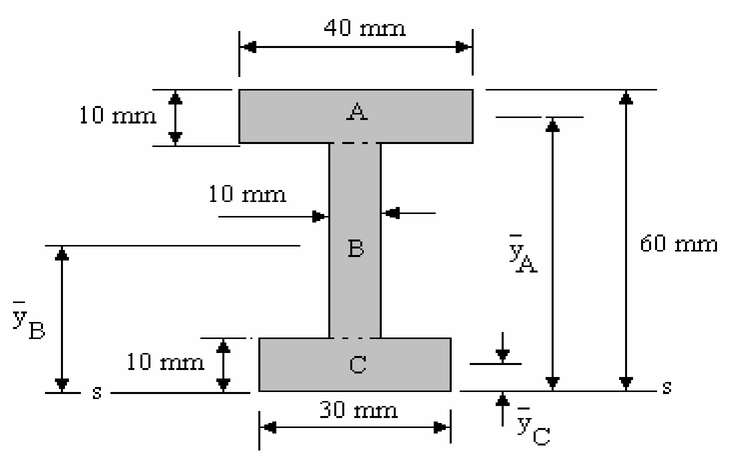
$$I_{xc} = I_{xx} + Ay_c^2 = I_{xx} + (bh/2)(h/3)^2 = bh^3/36$$

$$\text{Similarly } I_{yy} = \int x^2 dA = \int_0^b x^2 h\,(b-x)/b\,dy \qquad = b^3h/12$$

$$I_{yc} = I_{yy} + Ay_c^2 = I_{xx} + (bh/2)(b/3)^2 = b^3h/36$$

MOMENT OF INERTIA:

Calculate the 1st. moment of area for the shape shown about the axis s-s and find the position of the Centroid.



SOLUTION:

The shape is not symmetrical so the centroid is not half way between the top and bottom edges. First determine the distance from the axis s-s to the centre of each part A, B and C. A systematic tabular method is recommended.

Part	Area	Centroid (y)	A*y
A	400	55	22000
B	400	30	12000
C	300	5	1500
Total	1100		35500

The total first moment of area is 35,500 mm^3.

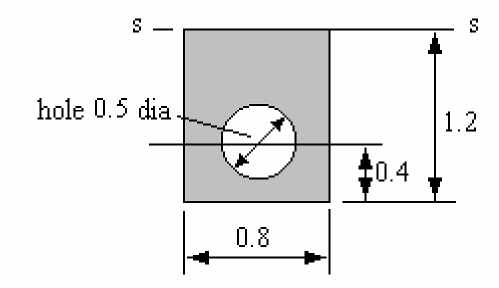
This must also be given by A y' for the whole section hence

Y' = 35 500/1100 = 32.27 mm.

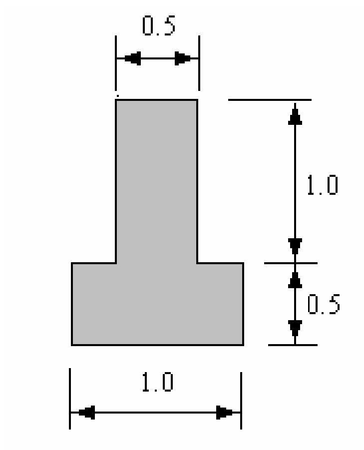
The centroid is 32.77 mm from the bottom edge.

1. Find the distance of the centroid from the axis s – s. All dimensions are in meters.

(0.549 m).



2. Find the distance of the Centroid from the bottom edge. All dimensions are in meters. (0.625 m)



Product of Inertia:

Relative to two rectangular axes, the sum of the products formed by multiplying the mass (or, sometimes, the area) of each element of a figure by the product of the coordinates corresponding to those axis. gure by the product of the coordinates corresponding to those axes.

Product of inertia

The product of inertia of area *A* relative to the indicated *XY* rectangular axes is I_{XY} = ∫ *xy dA* (see illustration). The product of inertia of themass contained in volume *V* relative to the *XY* axes is I_{XY} = ∫ *xyρ dV*—similarly for I_{YZ} and I_{ZX}.

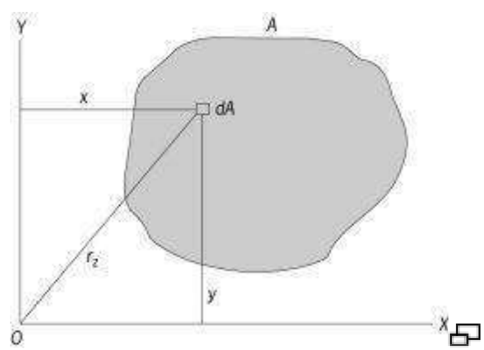


Figure13: Product of Inertia

Product of inertia of an area:

Relative to principal axes of inertia, the product of inertia of a figure is zero. If a figure is mirror symmetrical about a *YZ* plane, I_{ZX} = I_{XY} = 0.*See*

Moment of inertia Principal Moments of Inertia:

One of the major interests in the moment of inertia of area A is determining the orientation of the orthogonal axes passing a pole on the area with maximum or minimum moment of inertia about the axes.

Product of Inertia: Similar to the moment of inertia, a product of inertia can also be obtained from an integral over an area by multiplying the product of the coordinates x and y about the reference coordinate

$$\Delta I_{xy} = xy\Delta A$$

$$\Rightarrow \sum \Delta I_{xy} = \sum xy\Delta A$$

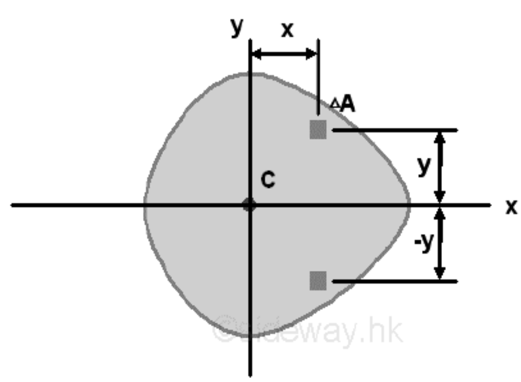
$$\Rightarrow \lim_{\Delta A \to 0} \sum \Delta I_{xy} = \lim_{\Delta A \to 0} \sum xy\Delta A$$

$$\Rightarrow \sum dI_{xy} = \sum xydA$$

$$I_{xy} = \int xydA$$

When considering the second moment of an area as the effect of the first moment acting on the same reference axis, the product moment of an area can be considered as the cross effect of the first moment acting on the orthogonal axis through a origin O at the specified orientation with respect to the area A.

Unlike the moment of inertia, although the elemental area is positive, the product of inertia can be positive, negative, or zero because the value of the coordinates x and y can be positive, negative, or zero. Similar to the first moment of an area about the the axis of symmetry, when one or both of the coordinate axes, x and y are the axis of symmetry of the area A, the integral, the product of inertia I_{xy} about the coordinate axes is zero. For example, a symmetrical area,



Although the area A is not symmetrical about axis y, however since the area is symmetrical about axis x, for any elemental area at a distance y above the axis x, there is always an elemental area below the axis x at the same mirror location of distance -y below the axis x. Therefore the product of inertia of a paired elemental area will cancel out each other and becomes zero, and the integral will reduces to zero also. Imply

$$I_{xy} = \int xydA$$

$$dA = dxdy = dydx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2} \int_{-y}^{y} xydydx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2} \left[\frac{1}{2} xy^2 \right]_{-y}^{y} dx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2} 0dx = 0$$

Since the product of inertia of a symmetrical area about one or two axes of symmetry must be zero, the product of inertia of an area with respect to axes can be used to test the dissymmetry or imbalance of the area about x and y axes because when the product of inertia about x and y axes is not equal to zero, the area is not symmetrical about both x and y axes. But when the product of inertia about x and y axes is equal to zero, the area may be not symmetrical about x and y axes.

Example of Product Moment of Inertia of a Right Angle Triangle

Product Moment of Inertia of a Right Angle Triangle by Double Integration

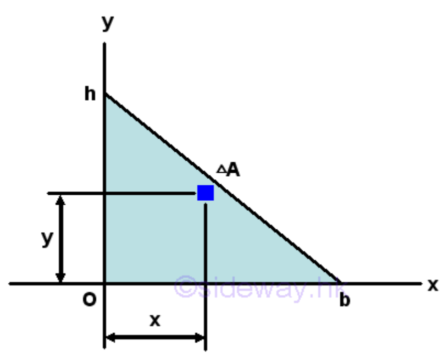


Figure14: Product Moment of Inertia of a Right Angle Triangle

The product moment of an area A of a right angle triangle about the axes xy is

$$\begin{cases} dA = dydx \\ \frac{y}{h} = 1 - \frac{x}{b} \Rightarrow y = h\left(1 - \frac{x}{b}\right) \end{cases}$$

$$I_{xy} = \int xydA$$

$$\Rightarrow I_{xy} = \int_0^b \int_0^y xydydx = \int_0^b \left[\frac{1}{2} xy^2 \right]_0^y dx = \int_0^b \frac{1}{2} xy^2 dx$$

$$\Rightarrow I_{xy} = \int_0^b \frac{1}{2} x \left(h\left(1 - \frac{x}{b}\right) \right)^2 dx = \frac{1}{2} h^2 \int_0^b \left(x - \frac{2x^2}{b} + \frac{x^3}{b^2} \right) dx$$

$$\Rightarrow I_{xy} = \frac{1}{2} h^2 \left[\frac{x^2}{2} - \frac{2x^3}{3b} + \frac{x^4}{4b^2} \right]_0^b = \frac{1}{2} h^2 \left(\frac{b^2}{2} - \frac{2b^3}{3b} + \frac{b^4}{4b^2} \right)$$

$$\Rightarrow I_{xy} = \frac{1}{2} h^2 b^2 \left(\frac{6}{12} - \frac{8}{12} + \frac{3}{12} \right) = \frac{1}{24} h^2 b^2$$

Support Reactions:

If a support prevents translation of a body in a given direction, a force is developed on the body in that direction. The three common types of connections which join a built structure to its foundation are roller, pinned and fixed. A fourth type, not often found in building structures, is known as a simple support. This is often idealized as a frictionless surface. All of these supports can be located anywhere along a structural element. They are found at the ends, at midpoints, or at any other intermediate points. The type of support connection determines the type of load that the support can resist. The support type also has a great effect on the load bearing capacity of each element, and therefore the system.

Roller Supports: Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped at any angle. The resulting reaction force is always a single force that is perpendicular to, and away from, the surface. Roller supports are commonly located at one end of long bridges. This allows the bridge structure to expand and contract with temperature changes. A roller support cannot provide resistance to lateral forces.

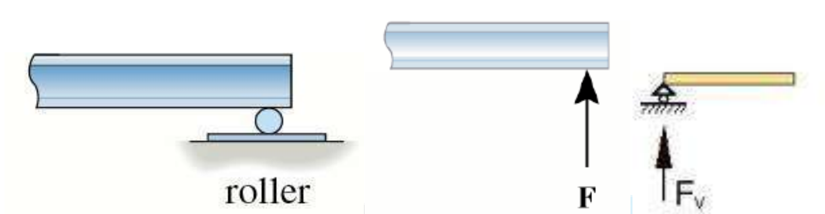


Figure15: Roller Support

The connection point on the bar cannot move downward.

Pinned support: A pinned support can resist both vertical and horizontal forces but not a moment. They allow the rotation, but not to translate in any direction. The knee can be idealized as a connection which allows rotation in only one direction and provides resistance to lateral movement.

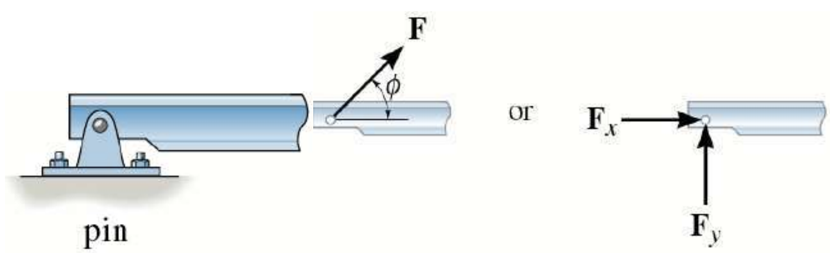


Figure16: Roller Support

The joint cannot move in vertical and horizontal directions.

FIXED SUPPORTS: Fixed support can resist vertical and horizontal forces as well as a moment. Since they restrain both rotation and translation, they are also known as rigid supports. A flagpole set into a concrete base is a good example of this kind of support. The representation of fixed supports always includes two forces (horizontal and vertical) and a moment.

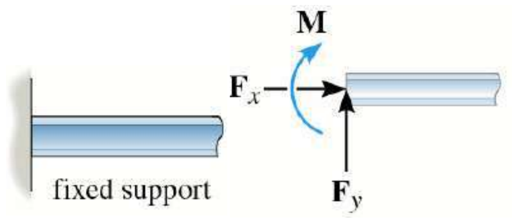


Figure17: Fixed Support

The support prevents translation in vertical and horizontal directions and also rotation, Hence a couple moments is developed on the body in that direction as well.

Types of beams:

Simply supported beam: The beam with one end hinged and the other end roller is called simply supported beam.

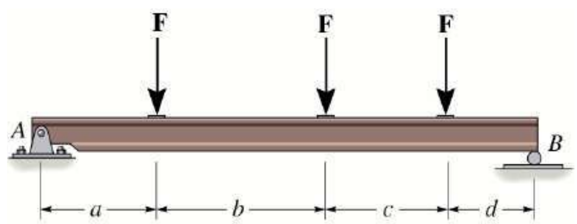


Figure18: Simply Supported Beam

Cantilever beam: The beam with one end fixed and the other end free is called Cantilever beam.

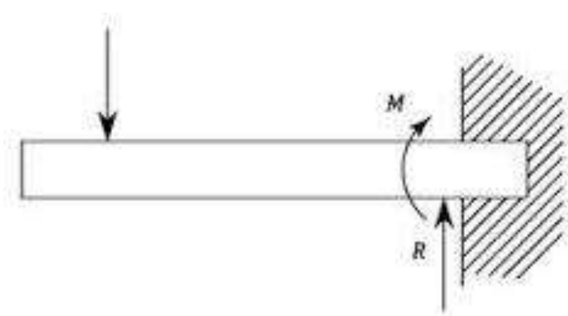


Figure19: Cantilever Beam

Determinate and indeterminate beams: If a beam has equal number of support reactions and static equilibrium conditions, then it is called as statically determinate beam. If a beam has more number of support reactions than the numbers of static equilibrium conditions, then it is called as statically indeterminate beam.

Dynamics

For dynamics as the mathematical analysis of the motion of bodies as a result of impressed forces, see analytical dynamics.

Kinematics is the branch of mechanics that deals with *motion* without regard to forces or energy.

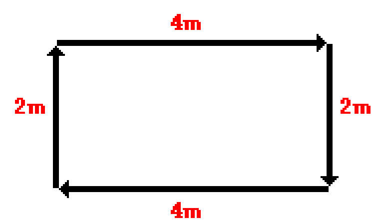
Kinetics is the branch of mechanics that deals with *motion* with regard to forces or energy.

Dynamics is a branch of applied mathematics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to *kinematics*, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.

Distance and displacement are two quantities that may seem to mean the same thing yet have distinctly different definitions and meanings.

- Distance is a scalar quantity that refers to "how much ground an object has covered" during its motion.
- Displacement is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

To test your understanding of this distinction, consider the motion depicted in the diagram below. A man walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North.



Even though the man has walked a total distance of 12 meters, his displacement is 0 meters.

Velocity is a vector quantity that is defined as the rate at which an object changes its position.
Acceleration is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity.
Newton's First Law of Motion:

Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces. They have been expressed in several different ways, over nearly three centuries,[1] and can be summarized as follows.

First law when viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force.

Second law: The vector sum of the external forces F on an object is equal to the mass *m* of that object multiplied by the acceleration vector a of the object: F = *m*a.

Third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

I. Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

II. The relationship between an object's mass *m*, its acceleration a, and the applied force *F* is *F = ma*. Acceleration and force are vectors (as indicated by their symbols being displayed in slant bold font); in this law the direction of the force vector is the same as the direction of the acceleration vector.

III. For every action there is an equal and opposite reaction.

Newton's first law

The first law states that if the net force (the vector sum of all forces acting on an object) is zero, then the velocity of the object is constant.

The first law can be stated mathematically as

$$\Sigma F = 0, dv/dt = 0$$

Consequently,

- An object that is at rest will stay at rest unless an external force acts upon it.
- An object that is in motion will not change its velocity unless an external force acts upon it.

This is known as *uniform motion*. An object *continues* to do whatever it happens to be doing unless a force is exerted upon it. If it is at rest, it continues in a state of rest. If an object is moving, it continues to move without turning or changing its speed. This is evident in space probes that continually move in outer space. Changes in motion must be imposed against the tendency of an object to retain its state of motion. In the absence of net forces, a moving object tends to move along a straight line path indefinitely.

Newton's second law

The second law states that the net force on an object is equal to the rate of change (that is, the *derivative*) of its momentum in an inertial reference frame:

$$\Sigma F = m, dv/dt = ma,$$

Where F is the net force applied, *m* is the mass of the body, and a is the body's acceleration. Thus, the net force applied to a body produces a proportional acceleration. In other words, if a body is accelerating, then there is a force on it.

Consistent with the first law, the time derivative of the momentum is non-zero when the momentum changes direction, even if there is no change in its magnitude; such is the case with uniform circular motion. The relationship also implies the conservation of momentum: when the net force on the body is zero, the momentum of the body is constant. Any net force is equal to the rate of change of the momentum.

Newton's third law

The third law states that all forces between two objects exist in equal magnitude and opposite direction: if one object *A* exerts a force F_A on a second object *B*, then *B* simultaneously exerts a force F_B on *A*, and the two forces are equal and opposite: $F_A = -F_B$. The third law means that all forces are *interactions* between different bodies, and thus that there is no such thing as a unidirectional force or a force that acts on only one body. This law is sometimes referred to as the *action-reaction law*, with F_A called the "action" and F_B the "reaction". The action and the reaction are simultaneous, and it does not matter which is called the

action **and which is called** ***reaction*****; both forces are part of a single interaction, and neither force exists without the other.**

The two forces in Newton's third law are of the same type (e.g., if the road exerts a forward frictional force on an accelerating car's tires, then it is also a frictional force that Newton's third law predicts for the tires pushing backward on the road).

From a conceptual standpoint, Newton's third law is seen when a person walks: they push against the floor, and the floor pushes against the person. Similarly, the tires of a car push against the road while the road pushes back on the tires—the tires and road simultaneously push against each other. In swimming, a person interacts with the water, pushing the water backward, while the water simultaneously pushes the person forward—both the person and the water push against each other. The reaction forces account for the motion in these examples. These forces depend on friction; a person or car on ice, for example, may be unable to exert the action force to produce the needed reaction force.

Newton's law of universal gravitation

Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This is a general physical law derived from empirical observations by what Isaac Newton called induction.

Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them

$$F = (G\, m_1 m_2)/r^2$$

Where:

- ***F*** **is the force between the masses;**
- ***G*** **is the gravitational constant ($6.674\times10^{-11}\ \mathrm{N}\cdot(\mathrm{m/kg})^2$);**
- **m_1 is the first mass;**
- **m_2 is the second mass;**

r **is the distance between the centers of the masses.**

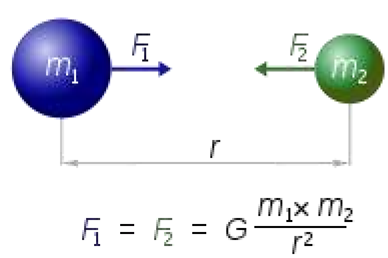


Figure20: Newton's law of universal gravitation

Assuming SI units, F is measured in Newton (N), m_1 and m_2 in kilograms (kg), r in meters (m), and the constant G is approximately equal to 6.674×10^{-11} N m^2 kg^{-2}.

Motion: Motion is a change in position of an object with respect to time. Motion is typically described in terms of displacement, distance (scalar), velocity, acceleration, time and speed. Motion of a body is observed by attaching a frame of reference to an observer and measuring the change in position of the body relative to that frame.

If the position of a body is not changing with the time with respect to a given frame of reference the body is said to be *at rest*, *motionless*, *immobile*, *stationary*, or to have constant (time-invariant) position. An object's motion cannot change unless it is acted upon by a force, as described by Newton's first law. Momentum is a quantity which is used for measuring motion of an object. An object's momentum is directly related to the object's mass and velocity, and the total momentum of all objects in an isolated system (one not affected by external forces) does not change with time, as described by the law of conservation of momentum.

As there is no absolute frame of reference, *absolute motion* cannot be determined. Thus, everything in the universe can be considered to be moving.

Main types of simple motion

There are two types of basic motion: translation and rotation. Translation means motion along a path. Rotation means motion around a fixed axis. An axis is the centre around which something rotates. As we have mentioned before, each type of motion is controlled by a different type of force. Translation is defined by the net force (sum of different forces) acting on an object. Rotation is defined by torque. Torque is a force which causes the rotation of an object.

1. Linear motion or translatory motion is the most basic of all motions. Linear motion is the type of motion in which all parts of an object move in the same direction and each part moves an equal distance. Linear motion is measured by speed and direction. Distance travelled by an object per unit of time is called velocity. The 2 types are-

a) Rectilinear motion -motion of a body in a straight line-- eg: a car moving on the road, a coconut falling down from the tree, a child moving down in a slide.
b) Curvilinear translatory motion-motion of a body along a curved path--eg: Man running in a 400 m race along the circular path.

2. Rotary motion or circular motion is motion in a circle. This type of motion is the starting point of many mechanisms. Example: a spinning wheel. The types are
a) Rotatary motion-- part of the body occupies a particular position at a time. eg: rotation of the earth on its axis, rotation of the blades of a fan, movement of the hands of a clock.
b) Revolution- motion of the whole body in a circle around a central fixed point. eg: movement of the earth around the sun.
3. Reciprocating motion or oscillatory motion is back and forth motion. Example: motion of a swing, movement of the 'bob' of a pendulum in a clock.
4. Periodic motion the bodies occupy a particular position at regular intervals.

eg; position of minute hand in a clock once in every 60 minutes, hour hand in a clock once in every 12 hours., position of planets in their orbits around the sun. non periodic motion-vibratory motion of the drum

5. Irregular or random motion is motion which has no obvious pattern to its movement. Example: a flying bee, movement of football players in the field.

Very often, objects move by complicated motion. Complicated motion can be broken down into simpler types of motion. An example of complicated motion is a flying Frisbee. The movement of a Frisbee consists of a linear motion and a rotary motion.

