Quantum Computing Course

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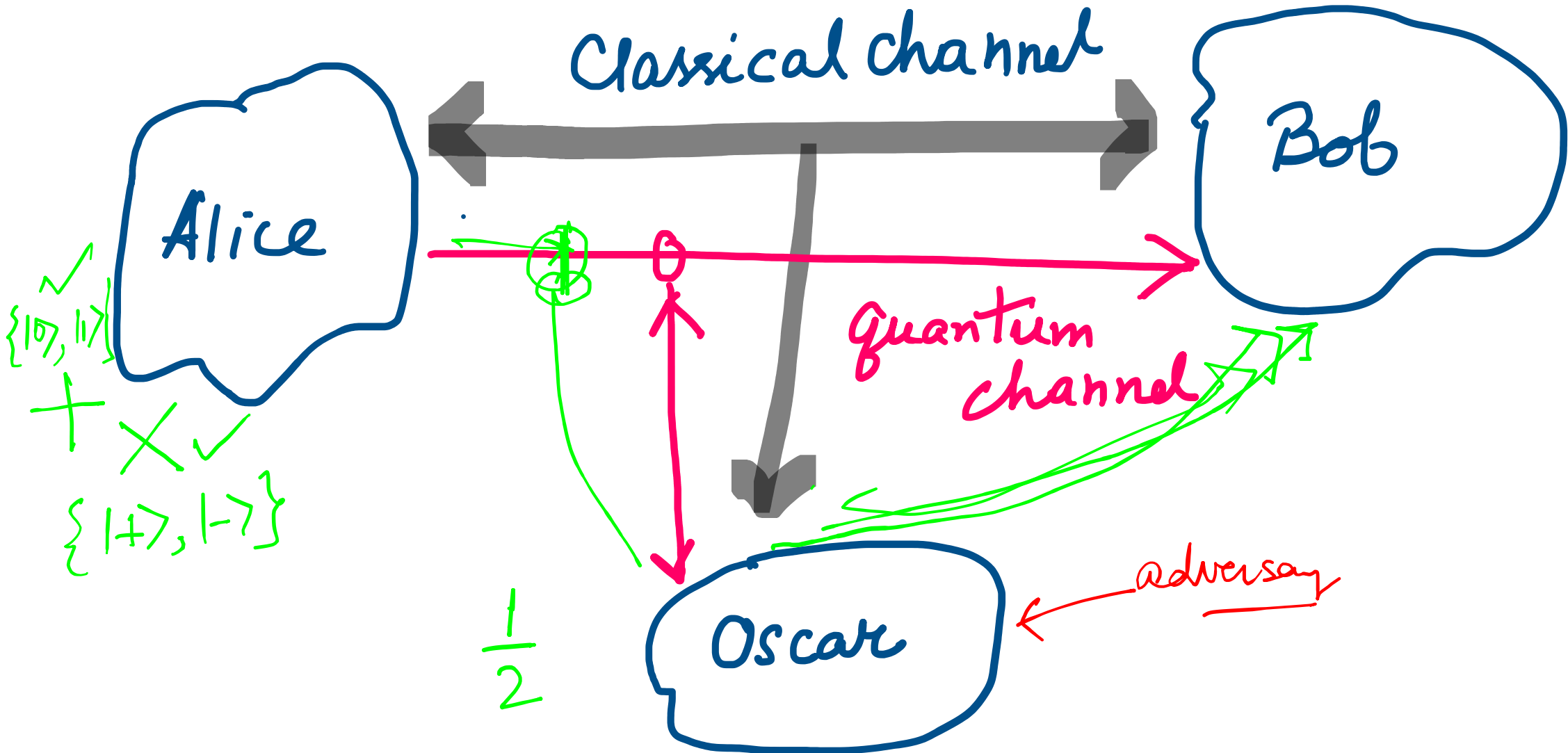
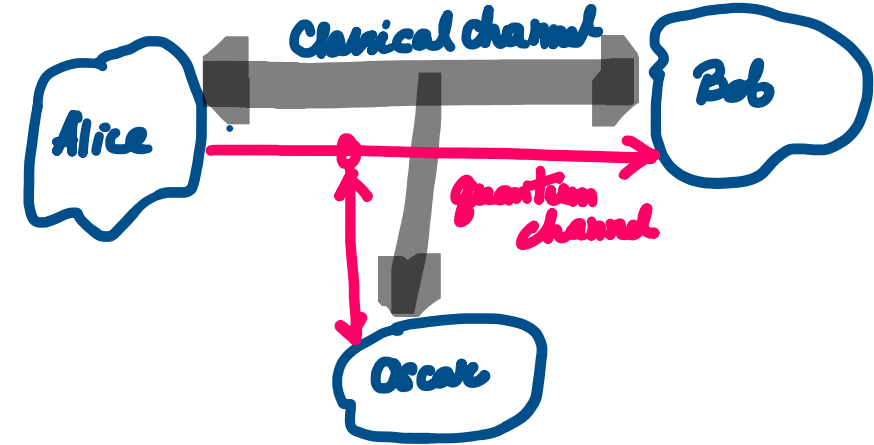Indian Institute of Technology Roorkee

Module 4

Lecture 2: Quantum Key Distribution

- BB84 in the presence of an eavesdropper
- Key distillation

A quantum key distribution protocol: BB84

BB84

- Encoding using the standard basis

  - $0 \rightarrow |0\rangle$
  - $1 \rightarrow |1\rangle$

- Encoding using the Hadamard basis

  - $0 \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
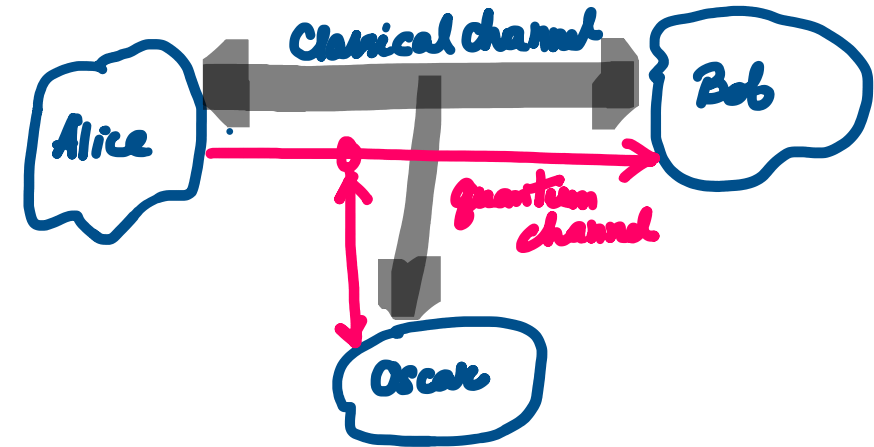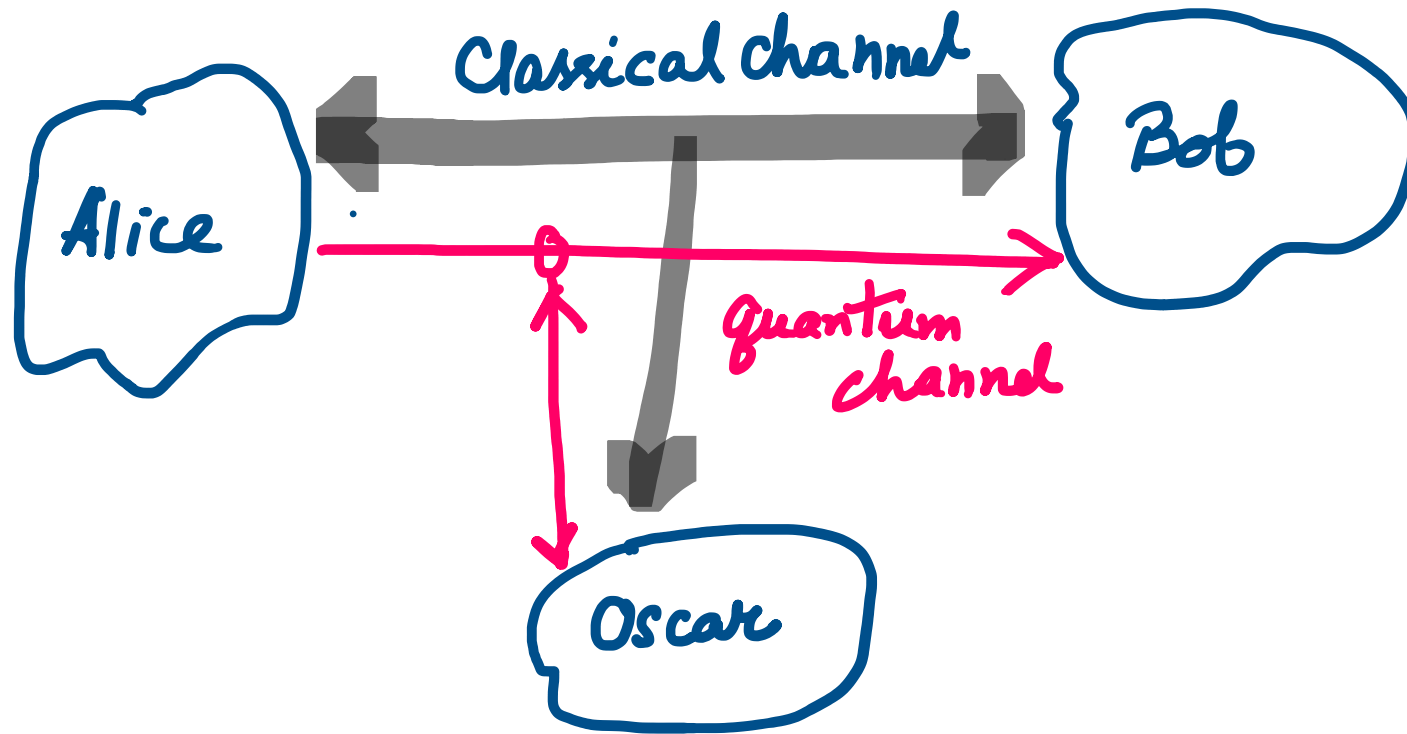 - $1 \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

BB84

- Encoding using the standard basis
    - $0 \rightarrow |0\rangle$
    - $1 \rightarrow |1\rangle$
- Encoding using the Hadamard basis
    - $0 \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
    - $1 \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

- Alice uses quantum or classical means to generate a random sequence of classical bit values
- Alice then randomly encodes each bit of this sequence in the polarization state of a photon by randomly choosing for each bit one of the following two agreed-upon bases in which to encode it.
- Bob measures the state of each photon he receives by randomly picking either basis.
- Over the classical channel Alice and Bob check that Bob has received a photon for every one Alice has sent.
- Then Alice and Bob tell each other the bases they used for encoding and decoding each bit.

- Encoding using the standard basis
  - $0 \to |0\rangle$
  - $1 \to |1\rangle$
- Encoding using the Hadamard basis
  - $0 \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
  - $1 \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Key sifting
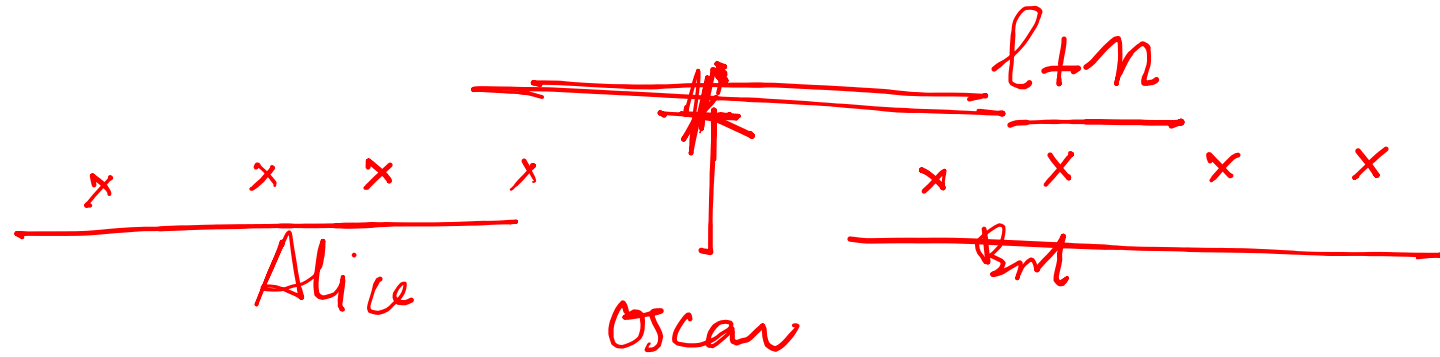
$\ell + n$

Alice

Oscar

Bob

- After sending a long stream of key elements, Alice tells Bob which encoding rule she chose for each key element, and Bob is able to discard all the wrong elements.

$\ell + n$ is the number key elements for which the bases match.

- This part of the protocol is said to be sifting.

- It is assumed that all communications required to do the sifting are made over the classical authenticated channel. So the sifting process is not influenced by Oscar.

Alice and Bob use a classical authenticated channel:

Detecting eavesdropping

Handwritten annotations (red): {0⟩, |1⟩} → {|0⟩, |1⟩} ... |+⟩, |−⟩ ... |+⟩ ... |0⟩ ... $\frac{1}{2}$ ... $\frac{1}{4}$ Probability that oscar

Handwritten annotations (green): |0⟩ ... gou undetected = 3/4 ... |+⟩, |−⟩ → $\frac{1}{4}$ ... {|+⟩, |−⟩}
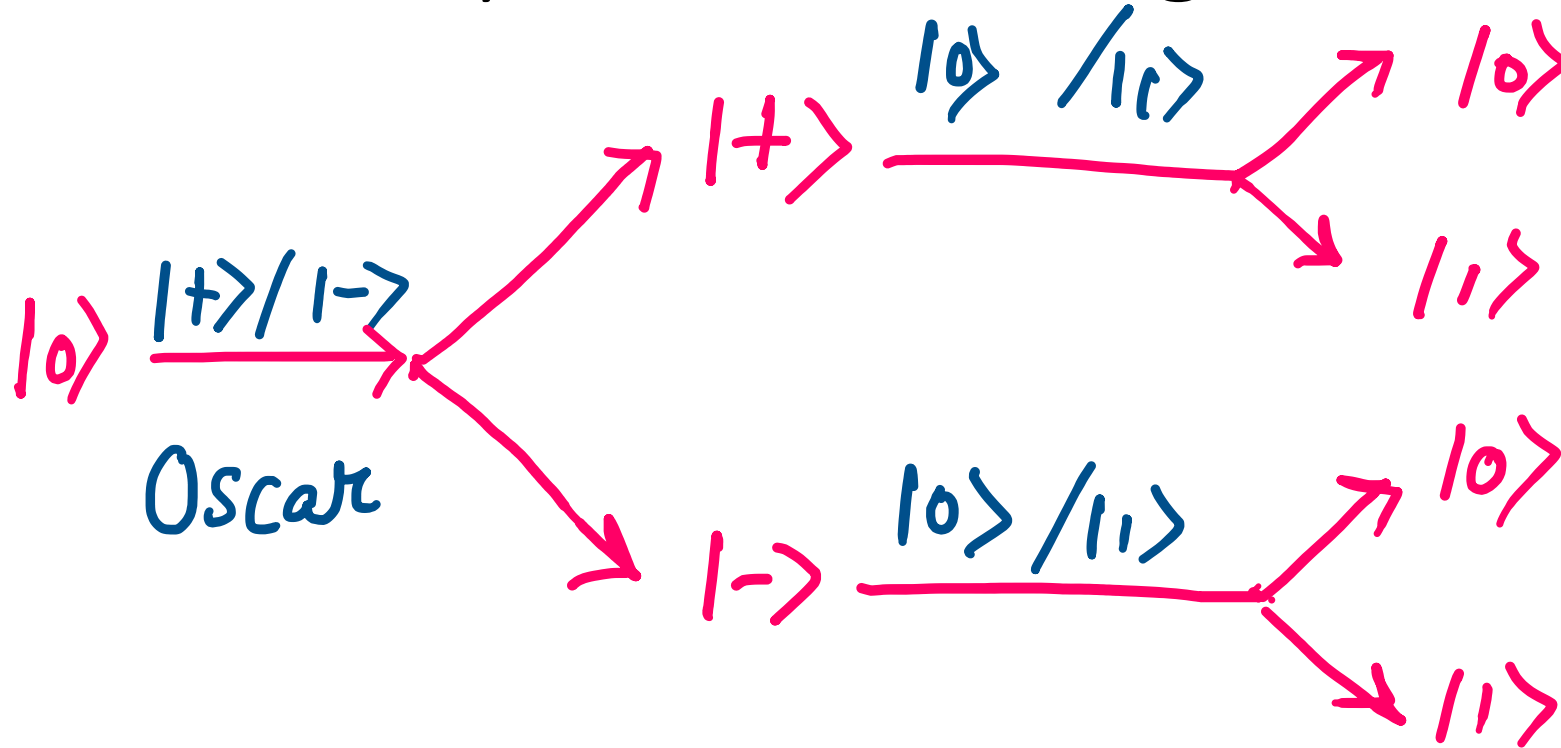
- Suppose, by the way of attack, Oscar intercept a photon, measures it, and sends the photon resulting from the measurement to Bob.

  - Oscar has probability of $\frac{1}{2}$ of measuring with the correct basis.

  - When he does, he does not disturb the state and goes unnoticed.

  - When he measures in the wrong basis, he sends the wrong state to Bob.

  - With the wrong state, Bob will basically measure a random bit, which has a probability $\frac{1}{2}$ of matching Alice's bit and probability $\frac{1}{2}$ of being wrong.

Handwritten annotations (red): Probability that Alice & Bob will detect em.

Probability of introducing error

$|0\rangle$ $\xrightarrow{|+\rangle/|-\rangle}$ Oscar

$|+\rangle$ $\xrightarrow{|0\rangle/|1\rangle}$ $|0\rangle$, $|1\rangle$

$|-\rangle$ $\xrightarrow{|0\rangle/|1\rangle}$ $|0\rangle$, $|1\rangle$

- So Oscar has a probability $\frac{1}{4}$ of introducing an error between Alice and Bob's bits.

Detecting eavesdropping

$\ell + n \longrightarrow$ randomly chosen

- Alice and Bob discloses a part of the sifted key.

    - A given protocol might specify that after a transmission of $\ell + n$ key elements numbered from 0 to $\ell + n - 1$, Alice randomly chooses n indices and communicates to Bob.

    - Alice and Bob then reveal the corresponding n key elements to count the number of errors.

    - Any error means that there was some eavesdropping.

- The absence of error gives some statistical confidence on the fact that there was no eavesdropping.

Sure !

Distilling a secret key

$$\ell = \boxed{\frac{k'}{n}}$$

(handwritten, top right) $n = $ no. of disclosed key bits

- Alice and Bob count the number of errors in the disclosed key elements to obtain and estimate of the expected error factor e.

- From this they statistically estimate that Oscar knows no more than I_E bits on the ℓ key elements. *(handwritten)* They estimate a number not the position.

- Using the public classical authenticated channel Alice and Bob can still try to make a fully secret key. This is called key distillation.

Distilling a secret key

- Reconciliation

  $I_E + |M|$

  - Alice and Bob discloses $|M|$ number of parity check bits.

  - Therefore Oscar has information on $I_E + |M|$ bits.

- Privacy amplification

  $\ell - I_E - |M|$

  - Oscar has no information on $\ell - I_E - |M|$ bits.

  - This is done by a function $f: \{0, 1\}^\ell \to \{0, 1\}^m$ that spreads the ignorance of Oscar over the entire output.

$f: \{0, 1\}^\ell \longrightarrow \{0, 1\}^m$

m

x_1	x_2	$x_1 \oplus x_2$
0	0	0
0	1	1
1	0	1
1	1	0