Quiz Assignment 4

Solutions

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1. It is always possible to copy the state of one qubit into another qubit. [**1 point**]

a) True b) False

Answer: The No cloning theorem forbids the copying of a quantum state. Therefore, the answer is, b) **false**.

2. A quantum gate is a physical object and the qubit state changes when it interacts with this object. [**1 point**]

a) True b) False

Answer: As explained in the lectures, the gates are transformations that are usually applied as signals controlled by a classical device. The answer is, b) **false**.

3. A classical computer is required in order to use a quantum computer. [**1 point**]

a) True b) False

Answer: For the reason described in the previous question and for purposes of state preparation and measurement, a classical computer is requires. The answer is, a) **true**.

4. The quantum oracle of an n-bit Boolean function requires a quantum computer with exactly n-qubits. **[1 point]**

a) True   b) False

Answer: As defined in the lectures, the oracle for an n-bit Boolean function requires n-qubits for the input $\mathbf{x}$ and one more for the output $F(\mathbf{x})$. Therefore, atleast $(n+1)$-qubits are needed. The answer is, b) **false**.

5. The operation performed on the two-qubit system in the quantum circuit shown in (Figure 1) is: **[2 points]**

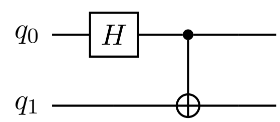

Figure 1: Circuit for Question 5

a) $(H \otimes I) \circ \text{CNOT}$ b) $(H \otimes H) \circ \text{CNOT}$

c) $\text{CNOT} \circ (H \otimes I)$ d) $\text{CNOT} \circ (I \otimes H)$

Answer: By the convention defined in the lectures, In the first stage the operation $(H \otimes I)$ is applied and the CNOT is applied in the second stage. The answer is c) $\text{CNOT} \circ (H \otimes I)$.

6. The operation performed on the two-qubit system in the quantum circuit shown in (Figure 2) is: **[2 points]**

a) $\text{CNOT}_0^1 \circ (H \otimes Z) \circ \text{CNOT}_0^1$ b) $\text{CNOT}_1^0 \circ (H \otimes Z) \circ \text{CNOT}_0^1$

c) $\text{CNOT}_0^1 \circ (H \otimes Z) \circ \text{CNOT}_1^0$ d) $\text{CNOT}_1^0 \circ (H \otimes Z)$

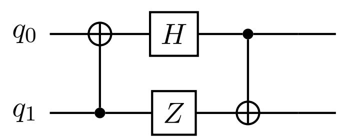

Figure 2: Circuit for Question 6

Answer: By the conventions defined in the lectures, In the first stage the operation CNOT_0^1 is applied, in the second stage, $(H \otimes Z)$ is applied and finally, CNOT_1^0 is applied in the third stage. The answer is b) $\mathrm{CNOT}_1^0 \circ (H \otimes Z) \circ \mathrm{CNOT}_0^1$.

7. The operation performed by the gates in the shaded area of the quantum circuit in (Figure 3) is: [**1 points**]

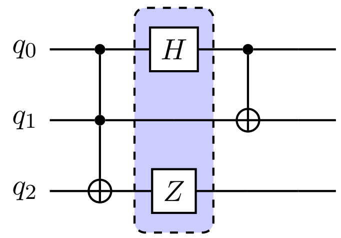

Figure 3: Circuit for Question 7

a) $(Z \otimes I \otimes H)$

b) $(H \otimes Z)$

c) $(H \otimes I \otimes Z)$

d) $(H \otimes Z \otimes I)$

Answer: The answer is c) $(H \otimes I \otimes Z)$.

8. The resultant state for the quantum circuit shown in (Figure 4) is: [**3 points**]

a) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

b) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

c) $\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle)$

d) $\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$

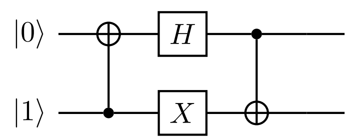

Figure 4: Circuit for Question 8

Answer: The total transformation is, $\text{CNOT}_1^0 \circ (H \otimes X) \circ \text{CNOT}_0^1$. The qubits are numbered q_0 and q_1 and the state $|q_0\rangle |q_1\rangle$, same as the previous question. The initial state is $|01\rangle$, an element of the two-qubit computational basis, the result can be evaluated using the actions as follows.

$$|01\rangle \xrightarrow{\text{CNOT}_0^1} |11\rangle \xrightarrow{H \otimes X} |-\rangle |0\rangle \equiv \frac{1}{\sqrt{2}} (|00\rangle - |10\rangle)$$
$$\frac{1}{\sqrt{2}} (|00\rangle - |10\rangle) \xrightarrow{\text{CNOT}_1^0} \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

The answer is b) $\frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$.

9. The resultant state for the quantum circuit shown in (Figure 5) is: [**4 points**]

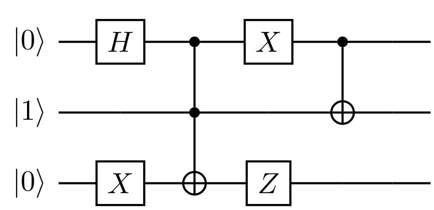

Figure 5: Circuit for Question 9

a) $\frac{1}{\sqrt{2}} (|000\rangle - |111\rangle)$

b) $\frac{1}{\sqrt{2}} (|010\rangle - |101\rangle)$

c) $\frac{1}{\sqrt{2}} (|011\rangle - |101\rangle)$

d) $\frac{1}{\sqrt{2}} (|100\rangle - |111\rangle)$

Answer: The operation, $\left(\text{CNOT}_1^0 \otimes I\right) \circ (X \otimes I \otimes Z) \circ \text{CCX} \circ (H \otimes I \otimes X)$ is applied by this circuit. The qubits are numbered q_0, q_1 and q_2 and the state $|q_0\rangle\,|q_1\rangle\,|q_2\rangle$, similar to the previous questions. The initial state is $|010\rangle$, an element of the three-qubit computational basis, the result can be evaluated using the actions as follows.

$$|010\rangle \xrightarrow{H\otimes I\otimes X} |+\rangle\,|1\rangle\,|1\rangle \equiv \frac{1}{\sqrt{2}}\left(|011\rangle + |111\rangle\right) \xrightarrow{\text{CCX}} \frac{1}{\sqrt{2}}\left(|011\rangle + |110\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|011\rangle + |110\rangle\right) \xrightarrow{X\otimes I\otimes Z} \frac{1}{\sqrt{2}}\left(-|111\rangle + |010\rangle\right) \xrightarrow{\text{CNOT}_1^0\otimes I} \frac{1}{\sqrt{2}}\left(-|101\rangle + |010\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(-|101\rangle + |010\rangle\right) \equiv \frac{1}{\sqrt{2}}\left(|010\rangle - |101\rangle\right)$$

The answer is b) $\frac{1}{\sqrt{2}}\left(|010\rangle - |101\rangle\right)$.

10. The Boolean function represented by the oracle given in (Figure 6), where $x_0, x_1 \in \{0,1\}$ and $+$ denotes the XOR operation. More than one option may be correct: **[4 points]**

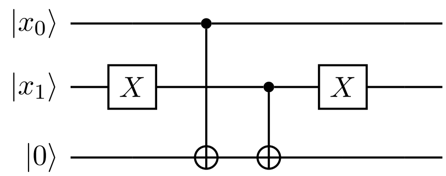

Figure 6: Circuit for Question 10

a) $x_0 + \overline{x_1}$

b) $\overline{x_0 + x_1}$

c) $\overline{x_0} + \overline{x_1}$

d) $1 + x_0 + x_1$

Answer: By noting that $X\,|x\rangle = |\overline{x}\rangle$, where $x \in \{0,1\}$, the effect of the circuit can be evaluated by stages as follows: (the target qubit in state $|0\rangle$ is labelled q_2)

$$|x_0 x_1\rangle\,|0\rangle \xrightarrow{I\otimes X\otimes I} |x_0\overline{x_1}\rangle\,|0\rangle \xrightarrow{\text{CNOT}_2^0} |x_0\overline{x_1}\rangle\,|x_0\rangle$$
$$|x_0\overline{x_1}\rangle\,|x_0\rangle \xrightarrow{I\otimes\text{CNOT}_2^1} |x_0\overline{x_1}\rangle\,|x_0 + \overline{x_1}\rangle$$

The target qubit is in the state $|x_0 + \overline{x_1}\rangle$, therefore $F(\mathbf{x}) = x_0 + \overline{x_1}$.

It should be noted that since $\overline{x_1} = 1 + x_1$, which implies $F(\mathbf{x}) = 1 + x_0 + x_1$ is also correct and by the same logic, $1 + (x_0 + x_1) = \overline{x_0 + x_1}$.

This means $F(\mathbf{x}) = \overline{x_0 + x_1}$ is also correct.

The correct answers are a),b) and d).