Quantum Computing Circuits

Introduction to Quantum Computing

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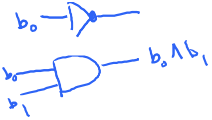
Outline

Representing Quantum transformations

- Consider the figure shown below:

the 'H' gate is applied on a qubit in initial state $|0\rangle$



flow

$$|0\rangle \;—\boxed{H}—\; |+\rangle$$

Figure 1: The Hadamard transformation

Initial state | Gate operation | Resultant state

- This figure shows a qubit that is initially in the $|0\rangle$ state is transformed into the $|+\rangle$ by the H transformation.

- This picture is very similar to classical circuits where the transformation H is treated as the quantum equivalent of the classical logic gate.

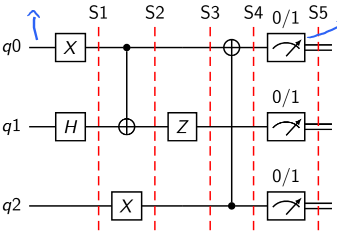
- This picture is referred to as a quantum circuit. This is the model used to describe computation in the IBMQ experience. It should be noted that the above diagram is an open circuit.

Conventions for drawing circuits

- The following conventions are used while drawing quantum circuits:
  - The wires (horizontal single lines) represent the qubits and may be labelled.
  - Classical data/bits are represented by double lines
  - The operations (gates) on these qubits are drawn on the wire.
  - The operations are performed from the left to the right one at a time.
  - The stages of the operations maybe separated with vertical lines, shown below in red.
  - Control qubits are represented by solid circles. Target qubits are represented by '$\oplus$'.

"wire"

qubits q_0, q_1 & q_2 store all the information about the transformations applied



measurement -> control operation

$\oplus$ -> X gate

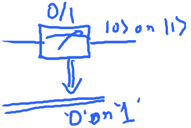


Figure 2: A labelled 3-qubit circuit with stages identified

The order of operations

- As can be seen from the previous circuits, a stage is an interval where an operation (gate) is performed on at least one qubit.
- The operations performed in the previous circuit maybe described as follows:
  - At S1: The X-gate is applied on $q0$; H-gate is applied to $q1$; No gate is applied on $q2$.
  - At S2: The $CNOT_1^0$ is applied, $q0$ controls the transformation on $q1$; X-gate is applied on $q2$.
  - At S3: Z-gate is applied to $q1$; No gates are applied on $q0$ and $q2$.
  - At S4: The $CNOT_0^2$ is applied, $q2$ controls the transformation on $q0$; No gate is applied on $q2$.
  - At S5: All the qubits are measured in the computational basis and the classicall result is transmitted.
- The circuit therefore depicts the order in which transformation are performed on the qubits.

A Classical Computer

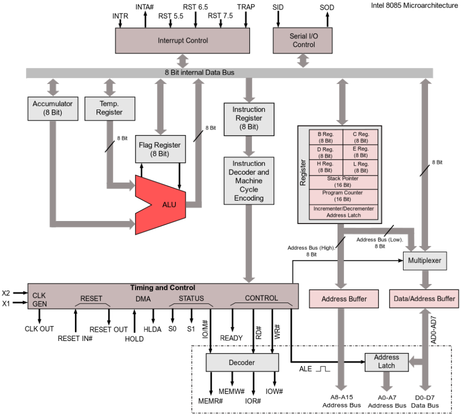


Figure 3: The Intel 8085 Microarchitecture[1]

[1] Appaloosa (https://commons.wikimedia.org/wiki/File:Intel_8085_arch.svg), “Intel 8085 arch”, https://creativecommons.org/licenses/by-sa/3.0/legalcode

Classical Computing

- The information stored and processed in the CPU is in the form of classical bits.
- The data is stored in the registers of the memory. Similarly the state of the CPU is also stored in the various registers.
- The processing/computation is done by the Timing and Control unit, Arithmetic and Logic unit (ALU), etc.
- The information stored in the memory is transferred to the CPU through the data buses (the dark grey arrows).
- The data signal carried through the bus depends on the information stored in the register (0/1). The data can therefore be replicated from one register to another.

Replicating qubit states: is it possible?

- **Let us assume** that there exists a two-qubit unitary transformation U that can replicate any qubit state.
- Considering a qubit in the state $|\psi\rangle$ and a second qubit in state $\underline{|0\rangle}$ and U can be defined as follows:

initialization

$$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle \rightarrow \text{Is this possible?}$$

  the qubit state $|\psi\rangle$ is replicated in the second qubit.
- If there is another state $|\phi\rangle$ that can be replicated then,

$$U|\phi\rangle|0\rangle = |\phi\rangle|\phi\rangle$$

- Therefore for a normalised linear combination of $|\psi\rangle$ and $|\phi\rangle$, for the state, $|\chi\rangle = a|\psi\rangle + b|\phi\rangle$ U is expected to act as shown below:

$\rightarrow \langle\chi|\chi\rangle = 1$

$$\begin{aligned} U|\chi\rangle|0\rangle &= |\chi\rangle|\chi\rangle \\ &= (a|\psi\rangle + b|\phi\rangle) \otimes (a|\psi\rangle + b|\phi\rangle) \\ &= aa|\psi\rangle|\psi\rangle + ab|\psi\rangle|\phi\rangle + ba|\phi\rangle|\psi\rangle + bb|\phi\rangle|\phi\rangle \end{aligned}$$

Replicating qubit states: No Clones

- However, U is also a linear operator, therefore

$a|\psi\rangle + b|\phi\rangle$

$$
\begin{aligned}
U|\chi\rangle|0\rangle &= aU|\psi\rangle|0\rangle + bU|\phi\rangle|0\rangle \\
&= a|\psi\rangle|\psi\rangle + b|\phi\rangle|\phi\rangle
\end{aligned}
$$

- This is a contradiction and therefore the original assumption is wrong. This means U cannot act on $|\chi\rangle|0\rangle$ is the expected manner.

- It is impossible to replicate (or clone) an arbitrary, unknown quantum state.

- The above statement is referred to as the No cloning theorem.

Quantum Computing without clones

- The state of a quantum computer is given by the state of the qubits. Therefore, the qubits are the memory of a quantum computer.
- Since it is impossible to replicate (copy/clone) the state of a qubit. Quantum computation cannot be performed in the same sense as classical computation.
- This also prevents basic read/write operations in the classical sense on a quantum computer.
- All computations are therefore performed on the initial set of qubits that are also the memory.
- Quantum computation is therefore in-memory computation where the operations are performed directly on the memory of the computer.

Circuits as a physical representation

- Understanding that computation being represented here is in-memory, it is possible to reinterpret the circuit as follows.
  - Every stage denotes a different time interval, this is the reason why the operations (gates) are being performed in a definite order.
  - This also implies that a separate device needs to keep track of the time and decide when to apply the gates.
  - This device is usually a classical computer.
  - The circuit is therefore interpreted/complied on a classical computer which will in turn apply appropriate gate transformations on the quantum computer.

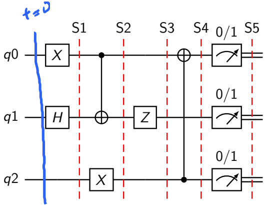


Figure 4: The example circuit

transformations are 8 x 8 unitary matrices

From circuits to matrices

- If a circuit contains n qubits, the overall transformation will correspond to an n-qubit operation.
- The operator corresponding to each stage can be evaluated by taking the tensor product of the independent operators, denoted by '$\otimes$'.
- If no operation is being performed on a qubit in a particular stage, one takes the tensor product with the 2×2 identity matrix
- Across multiple stages, the operator matrices are evaluated by matrix multiplication, denoted by '$\circ$'.
- These rules will be applied to the example circuit to get the resultant operation.

The “example” operation

gate operations are mostly transmitted as signals that interact with the qubit

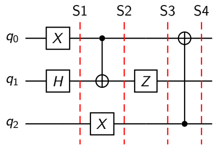


Figure 5: The example circuit with measurement removed

- It is assumed that the circuit state is represented in the *big-endian* notation: $|q_0\rangle\,|q_1\rangle\,|q_2\rangle$.
  - At S1: $X \otimes H \otimes I$.
  - At S2: $CNOT_1^0 \otimes X$.
  - At S3: $I \otimes Z \otimes I$.
  - At S4: $CNOT_0^2$, this operator is being represented as a 3-qubit operation.

$$\left(S_3\right)\left(S_2\right)\left(S_1\right)\left(\quad\right)^{8\times 1}$$

- The resultant operation is therefore given by (note that the matrix order is reversed):

$$\overset{S_4}{CNOT_0^2} \circ \overset{S_3}{(I \otimes Z \otimes I)} \circ \overset{S_2}{\left(CNOT_1^0 \otimes X\right)} \circ \overset{S_1}{(X \otimes H \otimes I)}\,|q_0\rangle\,|q_1\rangle\,|q_2\rangle$$

General Multiqubit States

- Consider a system of n-qubits each of which are in the $|0\rangle$ state.
- The n-qubit state is defined as

$$|0_n\rangle \equiv |0\rangle_1 \otimes |0\rangle_2 \otimes \cdots \otimes |0\rangle_{n-1} \otimes |0\rangle_n$$

- Applying the X gate to all these qubits yields:

$$X_1 \otimes X_2 \otimes \cdots \otimes X_{n-1} \otimes X_n |0_n\rangle = |1\rangle_1 \otimes |1\rangle_2 \otimes \cdots \otimes |1\rangle_{n-1} \otimes |1\rangle_n$$

  this state is represented as $|1_n\rangle$
- Similarly applying H gate to all these qubits yields:

$$H_1 \otimes H_2 \otimes \cdots \otimes H_{n-1} \otimes H_n |0_n\rangle = |+\rangle_1 \otimes |+\rangle_2 \otimes \cdots \otimes |+\rangle_{n-1} \otimes |+\rangle_n$$

  this state is represented as an equal weight superposition of all states corresponding to n-bit classical strings x:

$$\sum_{x \in \{0,1\}^n} \frac{1}{2^{\frac{n}{2}}} |x\rangle$$

The Qubit register

- The quantum circuits are for the previous operation are given as follow:
- The n-qubit state is defined as

Figure 6: X gates on each qubit of $|0_n\rangle$

and,

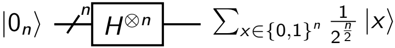


Figure 7: H gates on each qubit of $|0_n\rangle$