An Analysis of DC Transients and Semiconductors

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Abstract

In this investigation, the properties of various fundamental components in DC circuits were analyzed. For resistor-capacitor circuits, it was found that the percent error ranged between 2% and 11% between the theoretical time constants and the observed charging and discharging time constants. For the PN junction diode, the function $\left((1.9 \pm .2)E - 10\right)e^{(22.7 \pm .3)V}$ was found to model the i-V curve. Finally, for an NPN transistor, the amplification curves are modelled as a piece-wise function of two linear fits, and the gain was measured to values of 158±2, 154±3, 150±2, and 151±2 at different currents

I Introduction

Resistors, capacitors, diodes and transistors are some of the most fundamental electrical components used to build modern electronics such as radios and integrated circuits. As such, it is of great importance to understand the function, operation, and reliability of these circuit elements. Using basic equipment and theory, the properties of these components were tested in this lab.

II Theory

The relationship between resistance, current, and potential is given by Ohm's Law:

$$V = IR \qquad [1]$$

(C&J, 2001)

Where:
- I = Current through a circuit
- R = Total resistance of circuit
- V = Potential through a circuit

For a circuit consisting of only a resistor and a capacitor, the capacitor will discharge its stored energy through the resistor. The voltage across the capacitor can be modelled with the following linear differential equation:

$$C\frac{dV}{dt} + \frac{V}{R} = 0 \qquad [2]$$

(C&J, 2001)

Where:
- V = Voltage across capacitor
- C = Capacitance of capacitor
- R = Resistance of resistor

[2] can be used to derive the following equation for voltage as a function of time:

$$V(t) = V_{CC} + (V_0 - V_{CC})e^{-\frac{t}{RC}} \qquad [3]$$

(HRW, 2010)

Where:
- V_{CC} = Voltage of the power supply
- V_0 = Voltage at t=0

The Shockley diode equation was used to produce the i-V characteristic of the diode.

$$I = I_s\left(e^{\frac{V_D}{nV_T}} - 1\right) \qquad [4]$$

(HRW, 2010)

Where:

I = Diode current
I_s = Reverse bias saturation current
V_D = Voltage across diode
V_T = Thermal voltage
n = Ideality factor

The gain of a transistor when it is active is given as

$$I_C = \beta I_B \qquad [5]$$

(HRW, 2010)

Where:

I_B = Base current
I_C = Collector current
β = Gain of the transistor

III Method

For the RC circuit, four resistors of resistance 680 Ω, 1 kΩ, 2.2 kΩ and 4.7 kΩ along with four capacitors of 1, 10, 47 and 100 μF were used to create 16 distinct circuits. An oscilloscope (Tektronix TDS2002B) was used to measure the charging and discharging of the capacitor. These values for the resistors and capacitors were chosen because it would result in a time constant of roughly 0.03 seconds. The time constant was made a few times smaller than the wavelength of the function generator (BK Precision 4011A) in order to ensure that there was adequate time for observing the charging and discharging of the capacitor.

For PN diode analysis, a 1N4128 diode was connected in series with a 100 Ω resistor. The circuit was powered by the function generator in a sawtooth wave setting to test various positive and negative voltages. The voltage across the resistor was measured by the oscilloscope to obtain the voltage and current across the diode.

For transistor analysis, a circuit was built using a 2N4124 NPN transistor, a potentiometer, and a 9-volt battery.

The following are diagrams of circuits that were affected for each experiment:

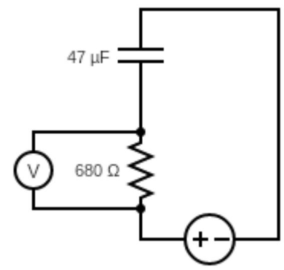

Fig 1. Circuit Diagram used in Resistor-Capacitor Experiment. A function generator is used to provide DC current and the resistor is connected to CH1 of the oscilloscope.

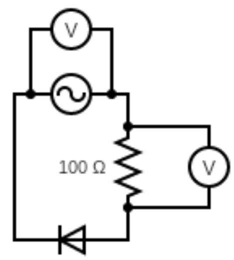

Fig 2. Circuit Diagram used in PN Diode Experiment. The top voltmeter is connected to CH2 on the oscilloscope while the voltmeter on the right is connected to CH1.

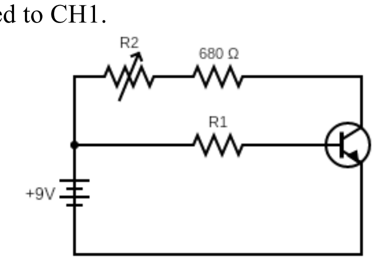

Fig 3. Circuit Diagram used in NPN transistor Experiment. In this circuit, point voltages with respect to ground are measured both before and after the 680Ω resistor.

IV Data

According to the manufacturer, the resistors and the capacitors have an uncertainty of 1% and 20% of their values respectively.

The battery used in the transistor experiment was measured to have a voltage of (9.34±.08) V.

V Analysis

For the PN diode experiment, equation [4] was used to regress the current across the diode as a function of voltage. As the reverse bias saturation current is almost negligible when considering currents in the forward region, the equation can be approximated to remove the second term. This yields:

$$i(V) = \big((1.9 \pm .2)E - 10\big)e^{(22.7\,\pm\,.3)V} \quad [6]$$

From datasheets for the 1N4128 diode, the reverse leakage current is (1±.5)E-8 A and the breakdown voltage is (45.6±.05) V. The maximum reverse voltage the diode was subjected to was 5.12 V which is well below the breakdown voltage. The leakage current is so small that it couldn't be detected accurately. With the oscilloscope measuring to a precision of 4E-2 V with a resistor of 100 Ω, the smallest discernable current using equation [1] is 4E-4 A, much larger than the leakage current. This limit could be improved by using a much larger resistor.

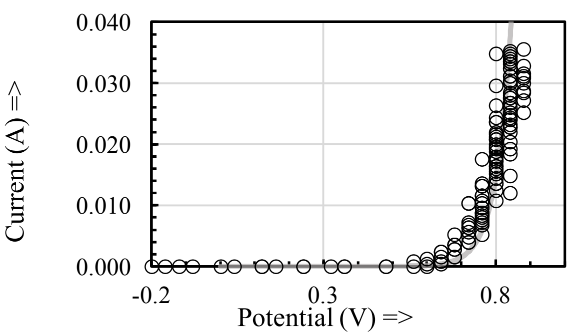

Fig 4. Graph of i vs V in PN Diode Experiment. Overlaid is the regression curve. The negative potentials resulted in a near-constant line around zero. Theoretically, the leakage current would be negative but it couldn't be detected in this experiment.

The thermal voltage represents the flow of electric current across a diode as a function of temperature. It can be computed as

$$V_T = \frac{kT}{q} \quad [7]$$

Where:

$k =$ Boltzmann's Constant
$q =$ Elementary Charge
$T =$ Temperature

If room temperature is taken to be 23°C and constants obtained from HRW are used, the thermal voltage can be computed to be (25.5±.01) mV.

The ideality factor dictates how closely a diode follows the ideal diode equation. An ideality factor of $n = 1$ represents an ideal diode. According to the manufacturer, the 1N4128 has an ideality factor of $n = 1.62$. Using these values, the O-C residual table was computed.

Index	Voltage (V)	Observed Current (I_o) (A)	Theoretical Current (I_t) (A)	Residual (%)
1	0.4	(1.7±.1) E-6	(1.60±.03) E-6	5.9
2	0.6	(1.6±.1) E-4	(2.03±.05) E-4	26.9
3	0.8	(2.2±.1) E-2	(2.57±.04) E-2	16.8

Fig 5. O-C table of current at various potentials. Lower potentials were omitted as the values obtained were highly inaccurate due to the limited number of significant digits from the oscilloscope measurements.

For RC circuits, equation [3] was used to regress the potential difference of the capacitor as a function of time. 16 RC circuits were created from the 4 resistors and capacitors and their charging and discharging curves were measured. Only 6 of the RC circuit configurations are shown below.

Index	Resistor (Ω)	Capacitor (μF)	Theoretical Time Constant (s)
1	680±7	47±9.4	0.031±.006
2	2200±22	47±9.4	0.10±.02
3	4700±47	47±9.4	0.22±.04
4	2200±22	10±2	0.022±.004
5	4700±47	10±2	0.047±.009
6	2200±22	1.0±.2	0.0022±.0004

Fig 6. table of resistance, capacitance, and resulting time constant of RC circuits. Resistor and capacitor pairs were chosen to ensure a short time constant that would allow the capacitor to fully charge or discharge within one cycle of the function generator.

Index	Discharging Time Constant (s)	Charging Time Constant (s)
1	0.0353±.0002	0.0353±.0002
2	0.105±.003	0.101±.003
3	0.23±.03	0.21±.02
4	0.02438±.00009	0.02458±.00009
5	0.0524±.0004	0.0521±.0003
6	0.00241±.00001	0.00246±.00001

Fig 7. table of charging and discharging times of RC circuit. The charging and discharging time constants should be the same.

The residuals for the discharging time constant are all no greater than 10.3% (index 5 in Figure 6 & 7), and for charging, it is bounded by 10.5%. These residuals confirm that components used in the RC circuits functioned according to theory.

For the RC circuit with a 680 Ω resistor and a 47 μF capacitor, the following equation was generated for the discharge curve.

$$V(t) = (-0.07 \pm .01) + (9.34 \pm .02)e^{-(28.3\pm.2)t} \quad [8]$$

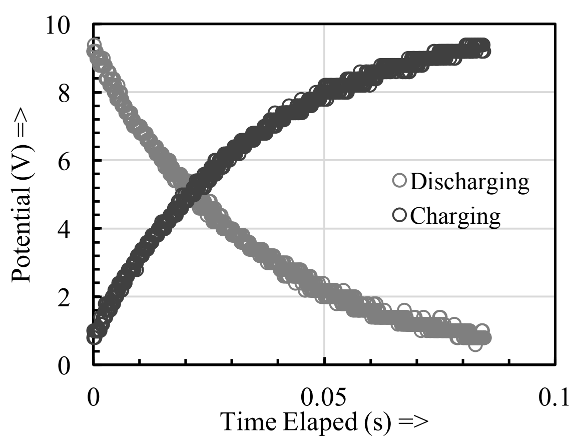

Fig 8. Graph of charging and discharging curves for an RC circuit as a function of time. The capacitor had a capacitance of 47 μF and the resistor had a resistance of 680 Ω.

The equation for the discharging curve in Figure 8 is [8] while the equation for the charging curve is given by the following:

$$V(t) = (10.20 \pm .02) - (9.35 \pm .02)e^{-(28.3\pm.2)t} \quad [9]$$

As can be seen from Figure 8, the charging and discharging curves are symmetrical, confirming the known theory. This results from the underlying symmetry of a capacitor in which two identical conductive plates surround a dielectric material. The time constants for the discharging and charging curves of Figure 8 were both (0.0353±.0002). These deviate approximately 9% from the theoretical value of (0.031±.006). This deviation can be attributed to the oscilloscope's lack of precision which is further discussed in the Sources of Error section of this paper.

For the transistor experiment, the base current must first be effected. To this end, [1] is used to along with the fact that V_{BE} of a transistor is a constant 0.7 V (*University of Pittsburgh*) to effect the base current to the following:

Index	R_1 (kΩ)	I_B (μA)
1	220±2	39.3±.5
2	470±5	18.4±.3
3	680±7	12.7±.2
4	1000±10	8.6±.1

Fig 9. Base Currents. The base currents for each base resistor are calculated and listed here

For each of the above values, I_B, the V_{CE} and I_C values were obtained over time, the following graph was produced with this data:

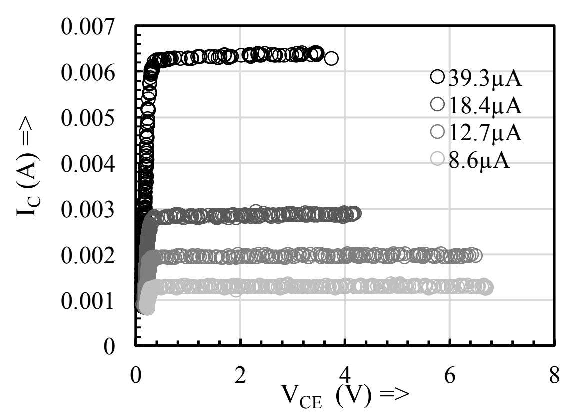

Fig 10. Amplification Curves. This graph shows the relationship between I_C and V_{CE} in both saturation and active modes.

According to the manufacturer, the transistor used for analysis has a saturation voltage for V_{CE} of 0.3V. Thus, for voltages of under 0.3V, the transistor is in saturation mode. Here, the transistor is in forward bias in both directions, so I_C increases rapidly with V_{CE}. Applying a linear fit to the part of the function with an I_C-intercept of 0, we get an equation of the following form for some constant k:

$$I_C = kV_{CE} \tag{10}$$

Using the above equation, the following table of values was effected:

Index	I_B (μA)	k (AV^{-1})
1	39.3	(1.59±.03)E-2
2	18.4	(8.66±.09)E-3
3	12.7	(6.03±.05)E-3
4	8.6	(4.43±.02)E-3

Fig 11. V_{CE} to I_C Ratios. The values of k for [10] as found with a linear fit are listed for each value of I_B.

It was noted that k seems to be directly correlated to I_B which could be used to modify [10]. However, insufficient data was obtained to either accept or reject this hypothesis. During active mode, collector-base junction is now reverse biased. Here, [5] becomes a good approximation for the value of I_C. It was observed that I_C increased slightly to an increase in V_{CE}. To characterize this, a linear fit was effected and the following values were obtained.

Index	I_B (μA)	m (AV^{-1})	b (A)
1	39.3	(5.2±.4)E-5	(6.229±.009)E-3
2	18.4	(1.3±.1)E-5	(2.828±.003)E-3
3	12.7	(6.1±.8)E-6	(1.905±.003)E-3
4	8.6	(5±5)E-7	(1.301±.002)E-3

Fig 12. Active Mode Linear Fit. A linear fit was regresssed for each I_B for the transistor during active mode. m represents the slope and b is the y-intercept of the fit.

Besides the I_B = 8.6μA case, it is seen that I_C does increase slightly with V_{CE}. However, this change is extremely small as an increase of 3V only corresponds to an maximal increase of 2.5%. This confirms that I_C and V_{CE} are effectively independent and matches our expectations. Since this is the case, the value of b in Figure 12 is the value for I_C when the transistor is active. Using [5], the β values are found to be 158±2, 154±3, 150±2, and 151±2 for the four respective I_B.

According to *ON Semiconductors, 2010*, a transistor with I_C between 1mA and 10mA can have their β differ by 25%, which our obtained values clearly fall within. Furthermore, the transistor used has a β range of 50 to 150 of which the obtained values fall within.

VI Sources of Error

The capacitors used for the RC circuits had an uncertainty of 20% which likely propagated into the values in the circuit analysis. Use of more reliable capacitors could have reduced the residuals for the time constants. The was not precise enough as it depended on the pixels of the display, greatly affecting the data obtained from the diode circuit. For the transistor analysis, it was impossible to reach 0 V_{CCE} using the current setup. A voltage divider could be used to divide V_{CCE} arbitrarily and therefore make it reach 0.

VII Conclusion

The properties of fundamental electronics used in modern circuits were analyzed in this lab and it was found that the experimental data conforms closely to the theoretically modelled data.

VIII Sources

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