AN ANALYSIS OF DIFFRACTION, BLACK BODIES, AND QUANTUM MECHANICS

G. CHEN
Lab Technician, Data Analyst

L. CHEN
Lab Technician, Data Analyst

E. SHEN
Lab Technician, Data Analyst,

K. WAN
Lab Technician, Data Analyst

U. YOUSAFZAI
Lab Technician, Data Analyst

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ABSTRACT

This lab investigates Planck's constant with LEDs, single slit diffraction and the Uncertainty Principle, and black body radiation. Planck's constant was empirically derived to be $(7 \pm 1) \times 10^{-34}$ Js with a residual value of 5.7% from theory. The single slit diffraction pattern matched the theoretical model with residual values of up to 4.7%, and the Heisenberg Uncertainty Principle was derived. The black body radiation analysis yielded an upper bound for the temperature of the lightbulb filament to be around 4100K, around 24% higher than the expected value.

I INTRODUCTION

Quantum mechanics redefined the model of the microworld and permitted the creation of many new technologies. Thus, empirical investigations on phenomena that demonstrate quantum behaviour are necessary to characterize universal constants. This lab aims to investigate properties of the quantization of light, diffraction, and black body radiation to compare to current theory.

II THEORY

The theory consists of several equations that were necessary to conduct a reliable analysis.

The relationship between energy and frequency is given as:

$$E = hf \qquad [1]$$

(HRW, 2010)

Where:

E = energy of photon (J)
h = Planck's constant (Js)
f = frequency of radiation (Hz)

The relationship between the frequency and wavelength of a light source is given as:

$$c = \lambda f \qquad [2]$$

(HRW, 2010)

Where:

c = speed of light (ms^{-1})
λ = wavelength (m)

The relationship between the potential at which an LED starts emitting photons and their energies is the following:

$$V_a = \frac{E_P}{e} + \frac{\varphi}{c} \qquad [3]$$

(HRW, 2010)

Where:

V_a = activation potential (V)
e = elementary charge (C)
E_p = energy of emitted photons (J)
$\frac{\varphi}{c}$ = energy loss in p-n junction of LED (J)

The de Broglie equation describes the relationship between the wavelength and momentum of a moving particle.

$$h = p\lambda \qquad [4]$$

(HRW, 2010)

Where:

p = momentum of particle (kg ms^{-1})

The Fraunhofer diffraction model describes the distance from a fringe minimum to the central maximum in a single slit diffraction pattern with the following equation:

$$d \sin\theta = m\lambda \qquad [5]$$

(HRW, 2010)

Where:

d = width of slit (m)
θ = deflection angle (rad)
m = the index of the fringe, excluding 0

Wien's Law relating the peak wavelength and temperature is described as:

$$\lambda_{max}T = b \qquad [6]$$

(HRW, 2010)

Where:

λ_{max} = Wavelength of maximum intensity (m)
T = Temperature (K)
b = Wien's constant (mK)

III Method

The LED experiment was performed using green, red, yellow, blue and white color LEDs. A circuit with a voltage divider, a 100 Ω resistor, and a DC power supply was constructed. A voltmeter was used to measure the potential across the LED at the time of activation. Using equations [1], [2], and [3], combined with values of the wavelength of coloured light and the potential at which the LEDs started emitted photons, the Planck constant was derived.

The single-slit diffraction experiment was performed using an IF-HN20 2.0 mW helium neon laser and a PASCO diffraction plate. A Canon EOS 1000D DSLR camera was mounted behind the laser. A meter stick was mounted on the wall below a screen made of sheets of paper. The laser was aimed through the single slit and the diffraction pattern was captured by the camera in a dark room.

The black body experiment was performed with an incandescent lightbulb and a diffraction grating. The resulting spectra was imaged along with a ruler for scale on the DSLR camera at different exposure times and analyzed for the intensity profile of light. The spectra were analyzed to determine the temperature of the filament, assuming a perfect black body model, and compared to the operating temperatures of the lightbulb.

IV Data

The wavelengths of the LEDs were sourced from the manufacturers are listed:

Colour	Wavelength (nm)
Green	525±26
Red	660±33
Yellow	590±30
Blue	470±24
White	450±23

Fig 1. Table of wavelengths of coloured-LEDs. Wavelength of light emitted by LEDs used to determine Planck's constant.

In the single-slit diffraction experiment, the wavelength of the laser was (600±10) nm and the width of the slit was (70.3±.5) µm. The distance from the slit to the wall was measured to be (2.28±.01) m.

In the black body experiment, the distance between the grating and the wall was (33.5±.2) cm. The density of slits on the diffraction grating was given by the manufacturer to be 5360 cm^{-1}.

V Analysis

As mentioned previously, the energy of the emitted photons from an LED is related to the Planck's constant by [1]. In an LED, the free electrons are in the conduction band while the holes are in the valence band. The gap between these two bands is called the band gap. For the electrons to cross this gap, a certain potential must be effected across the diode, also known as the activation potential. This activation potential is related to the energy of the emitted photons by [3]. The term $\frac{\varphi}{c}$ is a constant that relates to the energy loss inside the diode's p-n junction and it was assumed to be constant for all LEDs.

The regressed equation was produced in Excel via the LINEST function for the set of LEDs as shown below:

$$E_{BG} = (7 \pm 1)\mathrm{E} - 34 \cdot \mathrm{f} - (2 \pm 2)\mathrm{E} - 28[7]$$

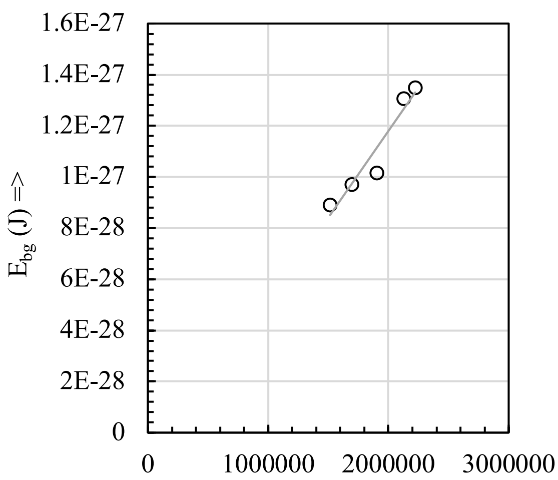

Fig 2. Energy required to cross the band gap of the first set of LEDs vs. the frequency of light emitted. The data collected was from the activation potential of the LEDs and regressed line's slope was used to calculate Planck's constant.

The approximation of Planck's constant using the activation potential method with the five LEDs was found to be $(7 \pm 1)\mathrm{E} - 34$ Js with a residual of 5.7% from the accepted value of $6.621\mathrm{E} - 34$ Js.

The diffraction of light can be comprehended through both classical mechanics and modern quantum mechanics. In classical mechanics, the wave model of light postulates that monochromatic light passing through a slit will undergo diffraction near the edges, thus causing a diffraction pattern to occur once the light travels onto the screen. Particularly, the pattern of the dark fringes can be modeled via [5].

To reconcile the established theory of single slit diffraction with empirical data, the interference pattern constructed with the IF-HN20 laser and the diffraction plate was analyzed.

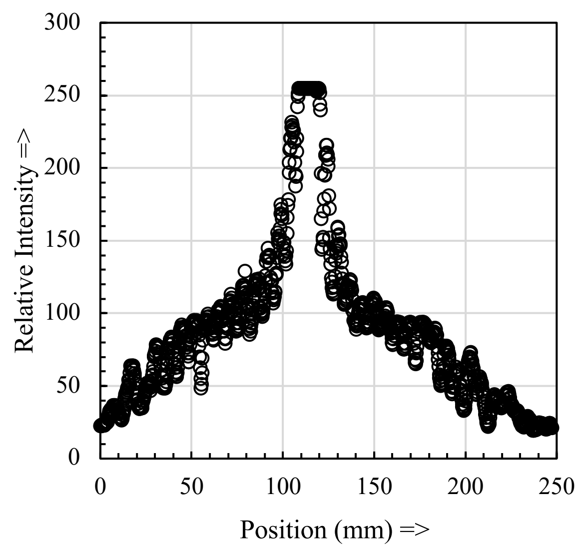

Fig 3. Graph of relative intensity versus position of the interference pattern. The intensity has a maximum value of 255. A central maximum with smaller local maxima and minima can be observed.

The intensity profile of the resulting interference pattern was imaged and analyzed. First, the non-linear sRGB values were converted into linear RGB values to accurately represent the ratio of photons being captured by the camera. The luminance of the interference pattern was determined using a central slit of pixels, and the profile was graphed as shown in Fig 3.

The minima of this profile were analyzed to obtain the position of the dark fringes relative to the central maximum, and the empirical values were compared to the theoretical values determined using [5].

INDEX	THEORETICAL POSITION (MM)	EMPIRICAL POSITION (MM)	RESIDUALS (%)
1	8.5 ± .1	8.1 ± .5	4.7
2	17.1 ± .3	17.1 ± .6	0.0
3	25.6 ± .4	25.0 ± .8	2.3
4	34.1 ± .4	33.1 ± .9	2.9

Fig 4. O-C table comparing the theoretical and empirical locations of dark fringes. The theoretical values are calculated using Fraunhofer approximation. The empirical values were obtained by finding minima.

It was observed that the empirical values closely matched that of the theoretical values, with a residue of less than 5%. This suggests that the wave model of the single slit diffraction phenomenon does match with empirical evidence.

However, a phenomenon that cannot be explained by the classical wave model is the quantum model described by the Theoretical Primer. The quantized behaviour of light in a single slit diffraction pattern can be used to derive, through uncertainty analysis, the Heisenberg Uncertainty Principle.

A singular photon that crosses the slit has its y-position bound between the slits, described with

$$\sigma_y = \pm\frac{d}{2} \qquad [8]$$

where d is the width of the slit. Furthermore, as it is known that the photon has intrinsic wave properties, its path through the slit is not necessarily straight and the y-momentum is uncertain, as described with

$$\sigma_{py} = \pm p\,\Delta\,\Theta \qquad [9]$$

where p is the initial horizontal momentum of the photon, and $\Delta\theta$ describes the spread all the way to the first minimum. Using small angle approximation and the classical model, $\Delta\,\theta = \frac{\lambda}{2\pi d}$. As it is known that photons have energies that are described by frequency, it can be described with de Broglie's theorem. Thus, multiplying the two uncertainties together, it is shown that

$$\sigma_y\sigma_{py} = \frac{h}{4\pi} \qquad [10]$$

which is a crude derivation of Heisenberg's Uncertainty Principle.

For the black body experiment, the camera was calibrated using a colour block. An image of the block was taken under normal lighting conditions and with the lightbulb to reconcile the colour of the spectra. Next, different points on the block were used to map the true intensity to the intensity recorded by the DSLR.

The RGB values of each point in the spectra produced by the diffraction grating were recorded using a Python script and their true relative radiances were effected. The data from the three curves were averaged to obtain the intensity curve of the spectra. Although most algorithms would use a weighted average, the method described was chosen to correct for the human eye's bias to picking green light which is not representative of the true radiance.

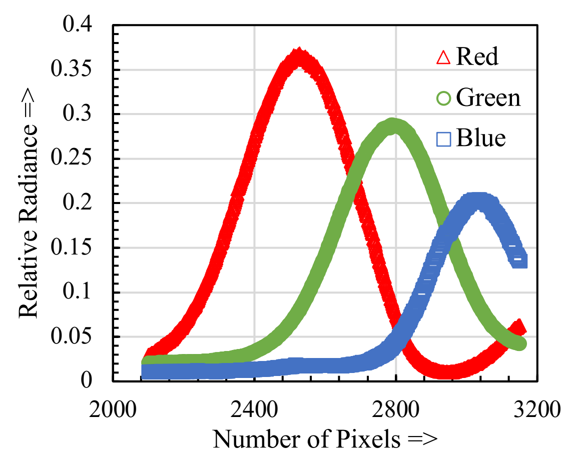

Fig 5. Graph of red, green, and blue intensity per pixel. Each point represents an average of the radiance of the pixels on that column well within the rainbow.

At first, a mapping between RGB values and wavelengths was sought for. Although by converting format to HSL would allow hue to be used as an indication of the wavelength of the light, this process was deemed too inaccurate. Instead, as the deflection angle of a beam of light is a function of its wavelength, the wavelength was determined using this angle with [5]. The total relative intensity was thus graphed against the deflection angle and the wavelength.

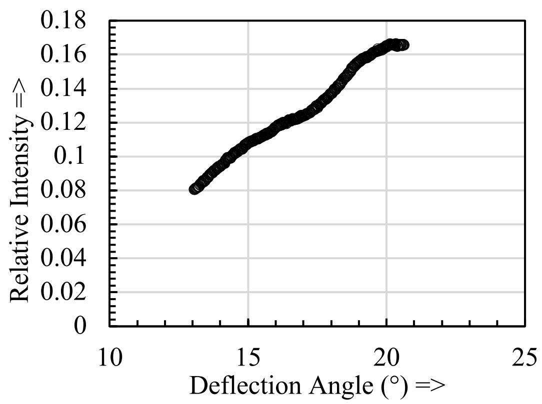

Fig 6. Graph of relative intensity and deflection angle. Note that the intensity increases with the angle.

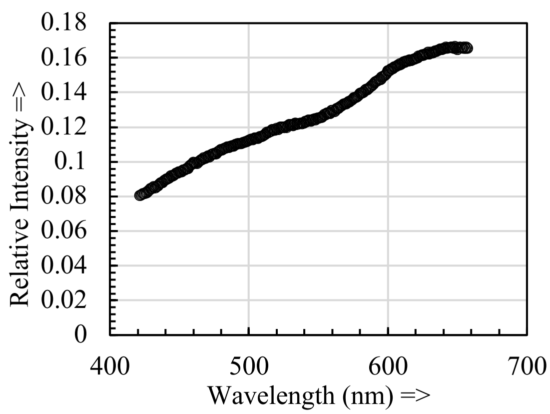

Fig 7. Graph of relative intensity and wavelength. The relative radiance gradually increases with wavelength as described by Wien's law. Note that no peak is observable as it is in the infrared.

Using equation [6], the temperature of the filament could be computed. However, the peak intensity clearly fell within the infrared and could not be picked up by the DSLR. Hence, only maximum observable wavelength for peak intensity is around 697.2 nm. This gives an upper bound of around 4100 K for the temperature of the filament. Sourced values stated that the temperature of the bulb would be roughly 3300 K which agrees with the upper bound.

VI SOURCES OF ERROR

For the experiment to determine Planck's constant, measuring the activation potential through observation was inaccurate. Instead, discharging a capacitor across the LED would have allowed for the observation of the diode's V-i curve, leading to a more accurate value for the activation potential.

The single slit experiment yielded images that lacked clarity, affecting the measurement of the fringe distances. Furthermore, specular lighting from imperfections on the wall caused halos of light to occur that were not part of the direct interference pattern, causing intensity analysis to yield higher values than usual.

Many of the spectra images were overexposed, creating plateaus in the intensity profile, leading to the data collected from those images being unusable.

VII CONCLUSION

The basic principles of quantum mechanics were analyzed in this experiment. Using LED, Planck's constant was experimentally derived to be $(7 \pm 1)\mathrm{E}-34$ Js with a residual of 5.7%. For the single slit diffraction experiment, the theoretical positions were found to be within 4.7% of the measured position, and the Heisenberg Uncertainty Principle was roughly demonstrated. Finally, for the blackbody experiment, an upper bound for the filament's temperature was 4100K.

VIII SOURCES

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