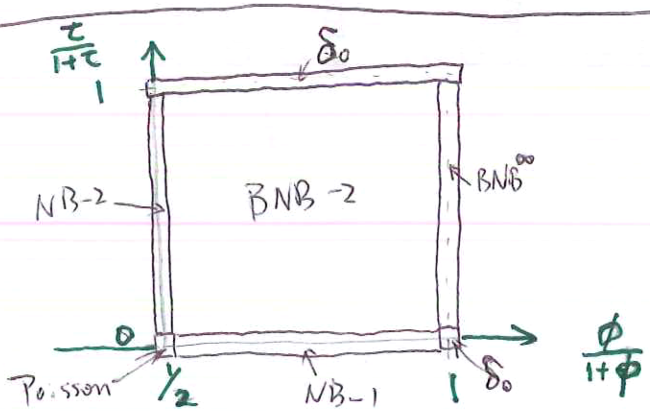
limiting cases of BNB with $\mu > 0$

τ	ϕ	dist'n / pmf	mgf	$V(\mu)$
0	1	Poisson: $\frac{\mu^y}{y!} \cdot \frac{1}{e^{\mu}}$	$e^{\mu(e^t - 1)}$	μ
0	$(1, \infty)$	NB-1: $\frac{\left(\frac{\phi-1}{\phi}\right)^y \Gamma\left(y + \frac{\mu}{\phi-1}\right)}{y!} \cdot \frac{1}{\phi^{\frac{\mu}{\phi-1}} \Gamma\left(\frac{\mu}{\phi-1}\right)}$ $NB\left(r = \frac{\mu}{\phi-1}, p = \frac{1}{\phi}\right)$	$\frac{1}{[\phi - e^t(\phi-1)]^{\frac{\mu}{\phi-1}}}$	$\phi\mu$
0	∞	$I(y=0)$	1	0
$(0, \infty)$	1	NB-2: $\left(\frac{\mu\tau}{1+\mu\tau}\right)^y \frac{\Gamma\left(y+\frac{1}{\tau}\right)}{y!} \frac{1}{(1+\mu\tau)^{1/\tau} \Gamma\left(\frac{1}{\tau}\right)}$ $N\left(r = \frac{1}{\tau}, p = \frac{1}{1+\mu\tau}\right)$	$\frac{1}{\left(1 - (e^t - 1)\mu\tau\right)^{1/\tau}}$	$\mu(1+\tau\mu)$
$(0, \infty)$	$(1, \infty)$	BNB-2: $\frac{\Gamma\left(y+\frac{1}{\tau}\right) \Gamma\left(y + \mu\tau \cdot \frac{\phi}{\phi-1} + \frac{\mu}{\phi-1}\right)}{y! \; \Gamma\left(y + \left(\mu\tau + \frac{1}{\tau} + 2\right)\frac{\phi}{\phi-1} + \frac{\mu-1}{\phi-1}\right)} \cdot$ Const $BNB\left(\alpha = 1 + \frac{\phi+1/\tau}{\phi-1}, \beta = \frac{1}{\tau}, r = \mu\frac{\tau\phi+1}{\phi-1}\right)$	$\nexists$	★ $\phi\mu(1+\tau\mu)$
$(0, \infty)$	∞	BNB-∞: $\frac{\Gamma\left(y+\frac{1}{\tau}\right) \Gamma(y + \mu\tau)}{y! \; \Gamma\left(y + \frac{1}{\tau} + \mu\tau + 2\right)} \cdot \frac{1+\tau}{\tau}\mu(1+\tau\mu)$ $BNB\left(\alpha = 2, \beta = \frac{1}{\tau}, r = \mu\tau\right)$	$\nexists$	∞
∞	1	$I(y=0)$	1	0
∞	$(1, \infty)$	$I(y=0)$	1	0
∞	∞	$I(y=0)$	1	0



~~★ $E(X^3)$ exist when $1 < \phi \le 2$ or ($\phi > 2$ & $0 < \tau < \frac{1}{\phi-2}$)~~

★ For BNB, $E(X^r)$ exist when $1 < \phi \le \frac{r-1}{r-2}$ or $\left(\phi > \frac{r-1}{r-2} \text{ but } 0 < \tau < \frac{1}{(r-2)(\phi-1)-1}\right)$
for $r > 2$