Ficha de Trabalho nº 1
Proposta de resolução

Preparação para exame

12.º Ano de Escolaridade | Turmas G - K

1. $A(4;7)$, $B(0;7)$, $C(0;10)$ e $D(8;10)$,

   1.1. $\overline{AB}=4$; $\overline{CD}=8$; $\overline{BC}=10-7=3$;
   $A_{[ABCD]}=\dfrac{\overline{CD}+\overline{AB}}{2}\times\overline{BC}=\dfrac{8+4}{2}\times 3=18u.a.$

   1.2. Seja $P(x;y)$ um ponto genérico da mediatriz do segmento de reta $[AD]$

   $M\left(\dfrac{4+8}{2};\dfrac{7+10}{2}\right)$
   $M\left(6;\dfrac{17}{2}\right)$
   $\overrightarrow{AD}=D-A=(8-4;10-7)=(4;3)$
   $\overrightarrow{MP}=P-M=\left(x-6;y-\dfrac{17}{2}\right)$
   $\overrightarrow{AD}\cdot\overrightarrow{MP}=0\Leftrightarrow(4;3)\cdot\left(x-6;y-\dfrac{17}{2}\right)=0\Leftrightarrow 4(x-6)+3\left(y-\dfrac{17}{2}\right)=0\Leftrightarrow$
   $\Leftrightarrow 4x-24+3y-\dfrac{51}{2}=0\Leftrightarrow 8x+6y-48-51=0\Leftrightarrow 8x+6y-99=0\Leftrightarrow y=-\dfrac{4}{3}x+\dfrac{33}{2}$

   1.3. $\overline{AD}=\sqrt{(8-4)^2+(10-7)^2}=\sqrt{25}=5$
   Círculo:
   $(x-4)^2+(y-7)^2\leq 5^2$
   $(x-4)^2+(y-7)^2\leq 25$
   Semiplanos:
   $0\leq x\leq 4$; $y\leq 7$
   Condição que define a região sombreada:$(x-4)^2+(y-7)^2\leq 25\wedge 0\leq x\leq 4\wedge y\leq 7$

2. $B(6;3)$; $C(3;0)$; $D(0;-3)$

   2.1. Seja $P(x;y)$ um ponto genérico da mediatriz do segmento de reta $[BC]$

   $M\left(\dfrac{6+3}{2};\dfrac{3+0}{2}\right)$
   $M\left(\dfrac{9}{2};\dfrac{3}{2}\right)$
   $\overrightarrow{BC}=C-D=(3-6;0-3)=(-3;-3)$
   $\overrightarrow{MP}=P-M=\left(x-\dfrac{9}{2};y-\dfrac{3}{2}\right)$
   $\overrightarrow{BC}\cdot\overrightarrow{MP}=0\Leftrightarrow(-3;-3)\cdot\left(x-\dfrac{9}{2};y-\dfrac{3}{2}\right)=0\Leftrightarrow -3\left(x-\dfrac{9}{2}\right)-3\left(y-\dfrac{3}{2}\right)=0\Leftrightarrow$
   $\Leftrightarrow -3x+\dfrac{27}{2}-3y+\dfrac{9}{2}=0\Leftrightarrow -6x-6y+27+9=0\Leftrightarrow 6x+6y-36=0\Leftrightarrow y=-x+6$

2.2. Círculo:
$(x-0)^2+(y-0)^2 \leq 3^2$
$x^2+y^2 \leq 9$

Equação da reta BC
Determinemos o declive da reta: $m_{BC} = \dfrac{3-0}{6-3} = 1$
$BC: y = x + b$, como a reta interseta o eixo Oy no ponto D, logo a ordenada na origem é -3
Assim, $BC: y = x - 3$
Condição que define a região sombreada: $x^2+y^2 \leq 9 \wedge y \leq x-3$

2.3. Área do círculo: $A_{circulo} = \pi \times 3^2 = 9\pi u.a.$
Área do trapézio $[ABCO]$: $A_{[ABCO]} = \dfrac{\overline{AB}+\overline{OC}}{2} \times \overline{OA} = \dfrac{6+3}{2} \times 3 = \dfrac{27}{2} u.a.$
Assim, $A_{tracejada} = A_{[ABCO]} - \dfrac{1}{4}A_{circulo} = \dfrac{27}{2} - \dfrac{9}{4}\pi \approx 6.43 u.a.$

3. 3.1. $A \cap B = \{2;4\}$
 3.2. $A \cup C = \{1,2;3;4;5;7;9\}$
 3.3. $B \cap C = \{\}$, logo, $A \cap (B \cap C) = \{\}$
 3.4. $B \cap (A \cup C) = \{2;4\}$
 3.5. $C \cup (A \cap B) = \{1,2;3;4;5;7;9\}$
 3.6. $B \cup (A \cup C) = \{1,2;3;4;5;6;7;8;9;10\}$

4. .

4.1. $A \cap B = \{x : 3x-1 \geq 5 \wedge |x-2| < 3\}$
$B \cup C = \{x : |x-2| < 3 \vee 1 - |-x-1| < 5\}$
$\overline{A} = \{x : 3x-1 < 5\}$
$\overline{B} = \{x : |x-2| \geq 3\}$

4.2. $3x-1 \geq 5 \Leftrightarrow 3x \geq 6 \Leftrightarrow x \geq 2 \rightarrow A = [2;+\infty[$
$|x-2| < 3 \Leftrightarrow x-2 > -3 \wedge x-2 < 3 \Leftrightarrow x > -1 \wedge x < 5 \rightarrow B =]-1;5[$
$1 - |-x-1| < 5 \Leftrightarrow -|-x-1| < 4 \Leftrightarrow |-x-1| > -4 \rightarrow$ condição universal em $\mathbb{R}$
Logo, $C = \mathbb{R}$

4.2.1. $A \cap B = [2;5[$

4.2.2. $A \cup C = \mathbb{R}$

4.2.3. $A \setminus B = A \cap \overline{B} = [5;+\infty[$
$\overline{B} =]-\infty;-1] \cup [5;+\infty[$

4.2.4. $\overline{A} \setminus \overline{C} = \overline{A} \cap \overline{\overline{C}} = \overline{A} \cap C =]-\infty;2[$
$\overline{A} =]-\infty;2[$

4.2.5. $\overline{B} \setminus A = \overline{B} \cap \overline{A} =]-\infty;-1]$

4.2.6. $\overline{A \cap B} = \overline{A} \cup \overline{B} =]-\infty;2[\cup [5;+\infty[$
Ou
$\overline{A \cap B} = \overline{[2;5[} =]-\infty;2[\cup [5;+\infty[$

4.2.7. $\overline{B \cup \overline{C}} = \overline{B} \cap C =]-\infty;-1] \cup [5;+\infty[$

5. 5.1. $A \cap \left(B \cap \overline{A}\right) = (A \cap \overline{A}) \cap B = \{\} \cap B = \{\}$

   5.2. $(A \cap B) \cup \left(B \cap \overline{A}\right) = B \cap (A \cup \overline{A}) = B \cap U = B$

   5.3. $\left[A \cap \overline{B \cap \overline{A}}\right] \cup \overline{A} = \left[A \cap (\overline{B} \cup \overline{\overline{A}})\right] \cup \overline{A} = \left[A \cap (\overline{B} \cup A)\right] \cup \overline{A} = \left[(A \cap \overline{B}) \cup (A \cap A)\right] \cup \overline{A} =$
   $= \left[(A \cap \overline{B}) \cup A\right] \cup \overline{A} = (A \cap \overline{B}) \cup A \cup \overline{A} = (A \cap \overline{B}) \cup (A \cup \overline{A}) = (A \cap \overline{B}) \cup U = U$