1. .

1.1. $f'(x) = (3^{-2x+1} + e^x)' = (3^{-2x+1})' + (e^x)' = (-2x+1)' \times \ln(3) \times 3^{-2x+1} + e^x =$
$= -2 \times \ln(3) \times 3^{-2x+1} + e^x = -2\ln(3)3^{-2x+1} + e^x$

1.2. $f'(x) = (10^{2x} \times \log(2x))' = (10^{2x})' \times \log(2x) + (\log(2x))' \times 10^{2x} =$
$= (2x)' \times \ln(10) \times 10^{2x} \times \log(2x) + \dfrac{(2x)'}{\ln(10) \times 2x} \times 10^{2x} =$
$= 2 \times \ln(10) \times 10^{2x} \times \log(2x) + \dfrac{2}{\ln(10) \times 2x} \times 10^{2x} = 10^{2x}\left(2\ln(10)\log(2x) + \dfrac{1}{\ln(10)x}\right)$

1.3. $$f'(x) = \left[\frac{\log_2\left(\frac{x+2}{x+1}\right)}{x+1}\right]' = \frac{\left[\log_2\left(\frac{x+2}{x+1}\right)\right]' \times (x+1) - \log_2\left(\frac{x+2}{x+1}\right) \times (x+1)'}{(x+1)^2} =$$

$$= \frac{\dfrac{\left(\frac{x+2}{x+1}\right)'}{\ln(2) \times \frac{x+2}{x+1}} \times (x+1) - \log_2\left(\frac{x+2}{x+1}\right) \times 1}{(x+1)^2} =$$

$$= \frac{\dfrac{\frac{-1}{(x+1)^2}}{\ln(2) \times \frac{x+2}{x+1}} \times (x+1) - \log_2\left(\frac{x+2}{x+1}\right) \times 1}{(x+1)^2}$$

$$= \frac{-\dfrac{x+1}{\ln(2)(x+2)(x+1)^2} \times (x+1) - \log_2\left(\frac{x+2}{x+1}\right)}{(x+1)^2} =$$

$$= \frac{-\dfrac{(x+1)^2}{\ln(2)(x+2)(x+1)^2} - \log_2\left(\frac{x+2}{x+1}\right)}{(x+1)^2} =$$

$$= \frac{-\dfrac{1}{\ln(2)(x+2)} - \log_2\left(\frac{x+2}{x+1}\right)}{(x+1)^2}$$

**Cálculo auxiliar:**

$$\left(\frac{x+2}{x+1}\right)' = \frac{(x+2)' \times (x+1) - (x+2) \times (x+1)'}{(x+1)^2} = \frac{1 \times (x+1) - (x+2) \times 1}{(x+1)^2} =$$
$$= \frac{x+1-x-2}{(x+1)^2} = -\frac{1}{(x+1)^2}$$

2. Determinemos a abcissa do ponto de tangência

   $f(x) = -3 \Leftrightarrow \left(\frac{1}{2}\right)^{x-2} - 5 = -3 \Leftrightarrow 2^{-x+2} = 2 \Leftrightarrow -x + 2 = 1 \Leftrightarrow x = 1$
   Ponto de tangência: $T(1; -3)$
   Determinemos o declive da reta tangente

   $f'(x) = \left[\left(\frac{1}{2}\right)^{x-2} - 5\right]' = \left[\left(\frac{1}{2}\right)^{x-2}\right]' - 5' = (x-2)' \times \ln\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^{x-2} - 0 =$
   $= 1 \times \ln\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^{x-2} = (\ln(1) - \ln(2)) \times \left(\frac{1}{2}\right)^{x-2} = (0 - \ln(2)) \times \left(\frac{1}{2}\right)^{x-2} = -\ln(2)\left(\frac{1}{2}\right)^{x-2}$
   Assim,

   $m_t = f'(1) = -\ln(2) \times \left(\frac{1}{2}\right)^{1-2} = -\ln(2) \times \left(\frac{1}{2}\right)^{-1} = -\ln(2) \times 2 = -2\ln(2)$

   Logo,

   $t : y = -2\ln(2)x + b$, como a reta "passa"em $T(1; -3)$, vem,

   $-3 = -2\ln(2) \times 1 + b \Leftrightarrow b = -3 + 2\ln(2)$

   Portanto,

   $t : y = -2\ln(2)x - 3 + 2\ln(2)$ $\twoheadrightarrow$equação reduzida da reta tangente pedida