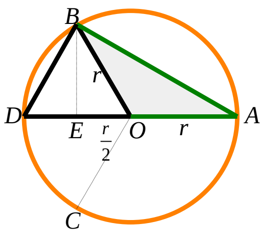
Ficha de Trabalho n.º 3 – Matemática A – 10.º Ano

Radicais e Potências de Expoente Racional

Algumas Resoluções

"Conhece a Matemática e dominarás o Mundo."
Galileu Galilei

8. Considere-se a seguinte figura:



Tem-se que:

- D é o ponto diametralmente oposto a A e E é o ponto médio do segmento de recta $[OD]$

- A amplitude dos arcos AB, BC e CA é 120°, pelo que a amplitude do arco BD é 60°. Logo, o triângulo $[OBD]$ é equilátero.

A área do triângulo $[OAB]$ é dada por $\frac{\overline{OA}\times\overline{DE}}{2}$. Seja $\overline{OA}=\overline{OB}=r$, pelo que $\overline{OE}=\frac{r}{2}$.

Assim, $\overline{DE}^2+\left(\frac{r}{2}\right)^2=r^2 \Leftrightarrow \overline{DE}^2=r^2-\frac{r^2}{4} \Leftrightarrow \overline{DE}^2=\frac{3r^2}{4} \underset{\overline{DE}>0}{\Rightarrow} \overline{DE}=\sqrt{\frac{3r^2}{4}} \underset{r>0}{=} \frac{\sqrt{3}r}{2}$. Portanto:

$$A_{[OAB]}=\frac{\overline{OA}\times\overline{DE}}{2}=\frac{r\times\frac{\sqrt{3}r}{2}}{2}=\frac{\sqrt{3}r^2}{4}\underset{i)}{=}\frac{\sqrt{3}\left(\frac{1}{\pi}\right)^2}{4}=\frac{\frac{\sqrt{3}}{\pi^2}}{4}=\frac{\sqrt{3}}{4\pi^2}$$

i) $P_{circunferência}=2 \Leftrightarrow 2\pi r=2 \Leftrightarrow r=\frac{2}{2\pi} \Leftrightarrow r=\frac{1}{\pi}$

Resposta: B

9.3. $$\frac{\sqrt[4]{4}}{\sqrt[3]{18}}=\frac{\sqrt[4]{2^2}}{\sqrt[3]{2\times3^2}}=\frac{\sqrt{2}}{\sqrt[3]{2\times3^2}}\times\frac{\sqrt[3]{2^2\times3}}{\sqrt[3]{2^2\times3}}=\frac{\sqrt{2}\times\sqrt[3]{2^2\times3}}{\sqrt[3]{2^3\times3^3}}=\frac{\sqrt[6]{2^3}\times\sqrt[6]{\left(2^2\times3\right)^2}}{\sqrt[3]{(2\times3)^3}}=\frac{\sqrt[6]{2^3}\times\sqrt[6]{2^4\times3^2}}{\sqrt[3]{6^3}}=$$

$$=\frac{\sqrt[6]{2^3\times2^4\times3^2}}{6}=\frac{\sqrt[6]{2^7\times3^2}}{6}=\frac{2\sqrt[6]{2\times3^2}}{6}=\frac{\sqrt[6]{18}}{3}$$

9.5. $\frac{\sqrt[4]{3}}{\sqrt{18}+\sqrt{8}}=\frac{\sqrt[4]{3}}{3\sqrt{2}+2\sqrt{2}}=\frac{\sqrt[4]{3}}{5\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt[4]{3}\times\sqrt[4]{2^2}}{5\left(\sqrt{2}\right)^2}=\frac{\sqrt[4]{3\times4}}{5\times2}=\frac{\sqrt[4]{12}}{10}$

Também se pode multiplicar e dividir pelo conjugado:

$$\frac{\sqrt[4]{3}}{\sqrt{18}+\sqrt{8}}\times\frac{\sqrt{18}-\sqrt{8}}{\sqrt{18}-\sqrt{8}}=\frac{\sqrt[4]{3}\times\sqrt{18}-\sqrt[4]{3}\times\sqrt{18}}{\left(\sqrt{18}\right)^2-\left(\sqrt{8}\right)^2}=\frac{\sqrt[4]{3}\times3\sqrt{2}-\sqrt[4]{3}\times2\sqrt{2}}{18-8}=\frac{3\sqrt[4]{3}\times\sqrt[4]{2^2}-2\sqrt[4]{3}\times\sqrt[4]{2^2}}{10}=$$

$$=\frac{3\sqrt[4]{3\times2^2}-2\sqrt[4]{3\times2^2}}{10}=\frac{3\sqrt[4]{12}-2\sqrt[4]{12}}{10}=\frac{\sqrt[4]{12}}{10}$$

9.6. $\frac{\sqrt{2}}{\sqrt[4]{2}-1}=\frac{\sqrt{2}}{\sqrt[4]{2}-1}\times\frac{\sqrt[4]{2}+1}{\sqrt[4]{2}+1}=\frac{\sqrt{2}\left(\sqrt[4]{2}+1\right)}{\left(\sqrt[4]{2}\right)^2-1^2}=\frac{\sqrt{2}\times\sqrt[4]{2}+\sqrt{2}}{\sqrt[4]{2^2}-1}=\frac{\sqrt[4]{2^2}\times\sqrt[4]{2}+\sqrt{2}}{\sqrt{2}-1}=\frac{\sqrt[4]{2^3}+\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=$

$$=\frac{\sqrt[4]{2^3}\times\sqrt{2}+\sqrt[4]{2^3}\times1+\sqrt{2}\times\sqrt{2}+\sqrt{2}}{\left(\sqrt{2}\right)^2-1^2}=\frac{\sqrt[4]{2^3}\times\sqrt[4]{2^2}+\sqrt[4]{8}+2+\sqrt{2}}{2-1}=\frac{\sqrt[4]{2^3\times2^2}+\sqrt[4]{8}+\sqrt{2}+2}{1}=$$

$$=\sqrt[4]{2^5}+\sqrt[4]{8}+\sqrt{2}+2=2\sqrt[4]{2}+\sqrt[4]{8}+\sqrt{2}+2$$

9.7. $\frac{a}{\sqrt{a}-a}=\frac{a}{\sqrt{a}-a}\times\frac{\sqrt{a}+a}{\sqrt{a}+a}=\frac{a\left(\sqrt{a}+a\right)}{\left(\sqrt{a}\right)^2-a^2}=\frac{a\left(\sqrt{a}+a\right)}{a-a^2}=\frac{\cancel{a}\left(\sqrt{a}+a\right)}{\cancel{a}\left(1-a\right)}=\frac{a+\sqrt{a}}{1-a}$

9.10. $\frac{1}{\sqrt{3-\sqrt{8}}}=\frac{1}{\sqrt{3-\sqrt{8}}}\times\frac{\sqrt{3-\sqrt{8}}}{\sqrt{3-\sqrt{8}}}=\frac{\sqrt{3-2\sqrt{2}}}{3-\sqrt{8}}\underset{i)}{=}\frac{\sqrt{\left(1-\sqrt{2}\right)^2}}{3-\sqrt{8}}\underset{ii)}{=}\frac{\sqrt{2}-1}{3-\sqrt{8}}=\frac{\sqrt{2}-1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=$

$$=\frac{3\sqrt{2}+\sqrt{16}-3-\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2}=\frac{3\sqrt{2}+4-3-2\sqrt{2}}{9-8}=\frac{\sqrt{2}+1}{1}=1+\sqrt{2}$$

i) $3-2\sqrt{2}=1^2-2\times1\times\sqrt{2}+\left(\sqrt{2}\right)^2=\left(1-\sqrt{2}\right)^2$

ii) Tem-se que $\sqrt{x^2}=|x|$; $|x|=x$ se $x\geq0$ e $|x|=-x$ se $x<0$.

Logo, como $1-\sqrt{2}<0$ vem que $\sqrt{\left(1-\sqrt{2}\right)^2}=\left|1-\sqrt{2}\right|=-\left(1-\sqrt{2}\right)=\sqrt{2}-1$.

10.3. $\sqrt[3]{108}+\left(\frac{1}{4}\right)^{\frac{1}{6}}+\sqrt[6]{2}\times\left(\left(\frac{1}{2}\right)^{-\frac{3}{2}}\right)^{\frac{1}{3}}=\sqrt[3]{2^2\times3^3}+\sqrt[6]{\left(\frac{1}{2}\right)^2}+\sqrt[6]{2}\times\sqrt[3]{\left(\frac{1}{2}\right)^{-\frac{3}{2}}}=3\sqrt[3]{2^2}+\sqrt[3]{\frac{1}{2}}+\sqrt[6]{2}\times\sqrt[3]{\sqrt{\left(\frac{1}{2}\right)^{-3}}}=$

$$=3\sqrt[3]{4}+\frac{1}{\sqrt[3]{2}}+\sqrt[6]{2}\times\sqrt[6]{2^3}=3\sqrt[3]{4}+\frac{1}{\sqrt[3]{2}}\times\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}+\sqrt[6]{2^4}=3\sqrt[3]{4}+\frac{\sqrt[3]{4}}{\sqrt[3]{2^3}}+\sqrt[3]{2^2}$$

$$=3\sqrt[3]{4}+\frac{\sqrt[3]{4}}{2}+\sqrt[3]{4}=\frac{6\sqrt[3]{4}+\sqrt[3]{4}+2\sqrt[3]{4}}{2}=\frac{9\sqrt[3]{4}}{2}$$

11.3. Se $x=2y$, então:

$$\frac{\sqrt{2y\times y}}{\sqrt{2y}-\sqrt{y}}=\frac{\sqrt{2y^2}}{\sqrt{2y}-\sqrt{y}}\times\frac{\sqrt{2y}+\sqrt{y}}{\sqrt{2y}+\sqrt{y}}\underset{y>0}{=}\frac{y\sqrt{2}\left(\sqrt{2y}+\sqrt{y}\right)}{\left(\sqrt{2y}\right)^2-\left(\sqrt{y}\right)^2}=\frac{y\sqrt{2}\left(\sqrt{2}\times\sqrt{y}+\sqrt{y}\right)}{2y-y}=$$

$$=\frac{\cancel{y}\left(2\sqrt{y}+\sqrt{2}\times\sqrt{y}\right)}{\cancel{y}}=\left(2+\sqrt{2}\right)\times\sqrt{y}$$