Proposta de Resolução da Ficha de Trabalho 8

Matemática A

12.º Ano de Escolaridade | Turma: J

1. .

   1.1. $-1 \leq \cos\left(x - \frac{\pi}{4}\right) \leq 1, \forall x \in \mathbb{R}$

   $\therefore -1 \times (-2) \geq -2\cos\left(x - \frac{\pi}{4}\right) \geq 1 \times (-2), \forall x \in \mathbb{R}$

   $\therefore 2 \geq -2\cos\left(x - \frac{\pi}{4}\right) \geq -2, \forall x \in \mathbb{R}$

   $\therefore -2 \leq -2\cos\left(x - \frac{\pi}{4}\right) \leq 2, \forall x \in \mathbb{R}$

   $\therefore 1 - 2 \leq 1 - 2\cos\left(x - \frac{\pi}{4}\right) \leq 1 + 2, \forall x \in \mathbb{R}$

   $\therefore -1 \leq f(x) \leq 3, \forall x \in D_f$

   Logo, $D'_f = [-1; 3]$

   Resposta: (C)

   1.2. Seja τ o período positivo mínimo da função f

   $f(x + \tau) = f(x)$

   $\therefore 1 - 2\cos\left(x + \tau - \frac{\pi}{4}\right) = 1 - 2\cos\left(x - \frac{\pi}{4}\right)$

   $\therefore -2\cos\left(x - \frac{\pi}{4} + \tau\right) = -2\cos\left(x - \frac{\pi}{4}\right)$

   $\therefore \cos\left(x - \frac{\pi}{4} + \tau\right) = \cos\left(x - \frac{\pi}{4}\right)$

   Atendendo que a função cosseno é periódica de período positivo mínimo 2π rad, resulta,

   $\tau = 2\pi$ rad

   Logo, o período positivo mínimo da função f é 2π rad

   1.3. Resolvendo a equação $f(x) = 0$, vem,

   $f(x) = 0 \Leftrightarrow 1 - 2\cos\left(x - \frac{\pi}{4}\right) = 0 \Leftrightarrow$

   $\Leftrightarrow -2\cos\left(x - \frac{\pi}{4}\right) = -1 \Leftrightarrow$

   $\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow$

   $\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right) \Leftrightarrow$

   $\Leftrightarrow x - \frac{\pi}{4} = \frac{\pi}{3} + k2\pi \vee x - \frac{\pi}{4} = -\frac{\pi}{3} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

   $\Leftrightarrow x = \frac{\pi}{3} + \frac{\pi}{4} + k2\pi \vee x = -\frac{\pi}{3} + \frac{\pi}{4} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

   $\Leftrightarrow x = \frac{4\pi}{12} + \frac{3\pi}{12} + k2\pi \vee x = -\frac{4\pi}{12} + \frac{3\pi}{12} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{7\pi}{12} + k2\pi \vee x = -\frac{\pi}{12} + k2\pi, k \in \mathbb{Z}$

Atribuindo valores a k, resulta,

Se $k = 0 \rightarrow$

$x = 0 \vee x = \frac{7\pi}{12} \vee x = -\frac{\pi}{12}$

Se $k = 1 \rightarrow$

$x = \frac{7\pi}{12} + 2\pi \vee x = -\frac{\pi}{12} + 2\pi$

$\therefore x = \frac{31\pi}{12} \vee x = -\frac{21\pi}{12}$

Se $k = -1 \rightarrow$

$x = \frac{7\pi}{12} - 2\pi \vee x = -\frac{\pi}{12} - 2\pi$

$\therefore x = -\frac{17\pi}{12} \vee x = -\frac{25\pi}{12}$

Concluindo,

$C.S. = \left\{-\frac{\pi}{12}; 0; \frac{7\pi}{12}\right\}$

2. Sabe-se que $\frac{5\pi}{12} = \frac{2\pi}{3} - \frac{\pi}{4}$

$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \times \left(-\frac{1}{2}\right) =$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$

3. .

3.1. $\frac{1}{2}\sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right) = \frac{1}{2} \times \frac{1}{2} \times 2\sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right) = \frac{1}{4}\sin\left(2 \times \frac{3\pi}{8}\right) = \frac{1}{4}\sin\left(\frac{3\pi}{4}\right) =$

$= \frac{1}{4} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{8}$

3.2. $\sin^4\left(\frac{\pi}{12}\right) - \cos^4\left(\frac{\pi}{12}\right) = \left[\sin^2\left(\frac{\pi}{12}\right)\right]^2 - \left[\cos^2\left(\frac{\pi}{12}\right)\right]^2 = \left[\sin^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{\pi}{12}\right)\right]\left[\sin^2\left(\frac{\pi}{12}\right) - \cos^2\left(\frac{\pi}{12}\right)\right] =$
$= 1 \times \left[\sin^2\left(\frac{\pi}{12}\right) - \cos^2\left(\frac{\pi}{12}\right)\right] = -\left[\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)\right] = -\cos\left(2 \times \frac{\pi}{12}\right) =$

$= -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

4. .

4.1. $2\cos^2(2x) - 2\sin^2(2x) = -\sqrt{2} \Leftrightarrow$

$\Leftrightarrow \cos^2(2x) - \sin^2(2x) = -\frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(2 \times 2x) = -\frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(4x) = -\frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(4x) = \cos\left(\frac{3\pi}{4}\right) \Leftrightarrow$

$\Leftrightarrow 4x = \pm\frac{3\pi}{4} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \pm\frac{3\pi}{16} + k\frac{\pi}{2}, k \in \mathbb{Z}$

$C.S. = \left\{\pm\frac{3\pi}{16} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

4.2. $\sqrt{3}\sin(2x) - \cos(2x) = \sqrt{3} \Leftrightarrow$

$\Leftrightarrow \frac{\sqrt{3}}{2}\sin(2x) - \frac{1}{2}\cos(2x) = \frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \sin\left(\frac{\pi}{3}\right)\sin(2x) - \cos\left(\frac{\pi}{3}\right)\cos(2x) = \frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi}{3}\right)\cos(2x) - \sin\left(\frac{\pi}{3}\right)\sin(2x) = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi}{3} + 2x\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi}{3} + 2x\right) = \cos\left(\frac{5\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \frac{\pi}{3} + 2x = \frac{5\pi}{6} + k2\pi \vee \frac{\pi}{3} + 2x = -\frac{5\pi}{6} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{5\pi}{6} - \frac{\pi}{3} + k2\pi \vee 2x = -\frac{5\pi}{6} - \frac{\pi}{3} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{5\pi}{6} - \frac{2\pi}{6} + k2\pi \vee 2x = -\frac{5\pi}{6} - \frac{2\pi}{6} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = -\frac{3\pi}{6} + k2\pi \vee 2x = -\frac{7\pi}{6} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\frac{3\pi}{12} + k\pi \vee x = -\frac{7\pi}{12} + k\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{4} + k\pi \vee x = -\frac{7\pi}{12} + k\pi, k \in \mathbb{Z}$

$C.S. = \left\{-\frac{\pi}{4} + k\pi; -\frac{7\pi}{12} + k\pi, k \in \mathbb{Z}\right\}$

5. .

5.1. $g(x) = \sin\left(4x + \frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right) + \cos\left(4x + \frac{\pi}{4}\right)\sin\left(x - \frac{\pi}{4}\right) = \sin\left(4x + \frac{\pi}{4} + x - \frac{\pi}{4}\right) = \sin(5x)$

5.2. $\lim\limits_{x\to 0}\frac{e^2 - e^{x+2}}{g(2x)} = \lim\limits_{x\to 0}\frac{e^2 - e^{x+2}}{\sin(10x)} = -\lim\limits_{x\to 0}\frac{e^{x+2} - e^2}{\sin(10x)} = -\lim\limits_{x\to 0}\frac{e^2\left(e^x - 1\right)}{\sin(10x)} = -e^2\lim\limits_{x\to 0}\frac{e^x - 1}{\sin(10x)} =$

$= -e^2\lim\limits_{x\to 0}\frac{\frac{e^x - 1}{x}}{\frac{\sin(10x)}{x}} = -e^2 \times \frac{\lim\limits_{x\to 0}\frac{e^x - 1}{x}}{\lim\limits_{10x\to 0}\frac{\sin(10x)}{10x} \times 10} = -e^2 \times \frac{1}{1\times 10} = -\frac{e^2}{10}$

Nota: aplicaram-se os limites notáveis: $\lim\limits_{x\to 0}\frac{e^x - 1}{x} = 1$ e $\lim\limits_{x\to 0}\frac{\sin(x)}{x} = 1$

5.3. Determinemos a função derivada de g

$g'(x) = [\sin(5x)]' = (5x)'\cos(5x) = 5\cos(5x)$

O declive da reta tangente t é $m_t = g'\left(\frac{\pi}{15}\right) = 5\cos\left(\frac{5\pi}{15}\right) = 5\cos\left(\frac{\pi}{3}\right) = 5 \times \frac{1}{2} = \frac{5}{2}$

Assim,

$t: y = \frac{5}{2}x + b, b \in \mathbb{R}$

Por outro lado,

$g\left(\frac{\pi}{15}\right) = \sin\left(\frac{5\pi}{15}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Portanto, o ponto de tangência é $A\left(\frac{\pi}{15}; \frac{\sqrt{3}}{2}\right)$

Assim,

$\frac{\sqrt{3}}{2} = \frac{5}{2} \times \frac{\pi}{15} + b \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{\pi}{6} + b \Leftrightarrow b = \frac{3\sqrt{3}}{6} - \frac{\pi}{6} \Leftrightarrow b = \frac{3\sqrt{3} - \pi}{6}$

Concluindo, a equação reduzida da reta tangente ao gráfico da função g no ponto de abcissa $\frac{\pi}{15}$, é,

$t: y = \frac{5}{2}x + \frac{3\sqrt{3} - \pi}{6}$

6. .

6.1. $h(x) = 1 - \frac{\sin\left(\frac{x}{2}\right)\left[\cos^2\left(\frac{x}{4}\right) - \sin^2\left(\frac{x}{4}\right)\right]}{\frac{1}{2}\tan(x)} =$

$= 1 - \frac{\sin\left(\frac{x}{2}\right)\cos\left(2 \times \frac{x}{4}\right)}{\frac{1}{2}\tan(x)} = 1 - \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\tan(x)} = 1 - \frac{\sin\left(2 \times \frac{x}{2}\right)}{\tan(x)} = 1 - \frac{\sin(x)}{\tan(x)} =$

$= 1 - \frac{\sin(x)}{\frac{\sin(x)}{\cos(x)}} = 1 - \cos(x)$

6.2. Calculemos a função primeira derivada de i

$i'(x) = [1 - \cos(4x)]' = 1' - [\cos(4x)]' = 0 + (4x)' \times \sin(4x) = 4\sin(4x)$

Calculemos os zeros de $i'(x)$, no intervalo $\left]0; \frac{\pi}{2}\right[$

$i'(x) = 0 \Leftrightarrow 4\sin(4x) = 0 \Leftrightarrow \sin(4x) = 0 \Leftrightarrow \sin(4x) = \sin(0) \Leftrightarrow 4x = 0 + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = k\frac{\pi}{4}, k \in \mathbb{Z}$

Atribuindo valores a k, tem-se,

Se $k = 0 \mapsto x = 0 \notin \left]0; \frac{\pi}{2}\right[$

Se $k = 1 \mapsto x = \frac{\pi}{4} \in \left]0; \frac{\pi}{2}\right[$

Se $k = 2 \mapsto x = \frac{\pi}{2} \notin \left]0; \frac{\pi}{2}\right[$

Se $k = -1 \mapsto x = -\frac{\pi}{4} \notin \left]0; \frac{\pi}{2}\right[$

Portanto, $x = \frac{\pi}{4}$

Sinal de $i'(x)$, no intervalo $\left]0; \frac{\pi}{2}\right[$

$i'(x) > 0 \Leftrightarrow 4\sin(4x) > 0 \Leftrightarrow \sin(4x) > 0 \Leftrightarrow 0 < x < \frac{\pi}{4}$

$i'(x) < 0 \Leftrightarrow 4\sin(4x) < 0 \Leftrightarrow \sin(4x) < 0 \Leftrightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$

Elaborando um quadro de sinal de $i'(x)$

x	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$i'(x)$	$n.d$	$+$	0	$-$	$n.d$
$i(x)$	$n.d$	$\nearrow$	2	$\searrow$	$n.d$

$i\left(\frac{\pi}{4}\right) = 1 - \cos\left(4 \times \frac{\pi}{4}\right) = 1 - \cos(\pi) = 1 - (-1) = 2$

A função i é crescente no intervalo $\left]0; \frac{\pi}{4}\right[$, é decrescente no intervalo $\left]\frac{\pi}{4}; \frac{\pi}{2}\right[$, e atinge o valor máximo 2, para $x = \frac{\pi}{4}$

6.3. $\lim\limits_{x\to 0} \frac{h(x)}{x\sin(2x)} = \lim\limits_{x\to 0} \frac{1-\cos(x)}{x\sin(2x)} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to 0} \frac{(1-\cos(x))\,(1+\cos(x))}{x\,(1+\cos(x))\sin(2x)} = \lim\limits_{x\to 0} \frac{1-\cos^2(x)}{x\,(1+\cos(x))\sin(2x)} =$

$= \lim\limits_{x\to 0} \frac{\sin^2(x)}{x\,(1+\cos(x))\,2\sin(x)\cos(x)} = \frac{1}{2}\lim\limits_{x\to 0} \frac{\sin(x)}{x\,(1+\cos(x))\cos(x)} =$

$= \frac{1}{2} \times \lim\limits_{x\to 0} \frac{\sin(x)}{x} \times \lim\limits_{x\to 0} \frac{1}{(1+\cos(x))\cos(x)} = \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$

7. $\tan(2\alpha) = \frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{2\sin(\alpha)\cos(\alpha)}{\cos^2(\alpha) - \sin^2(\alpha)} = \frac{\frac{2\sin(\alpha)\cos(\alpha)}{\cos^2(\alpha)}}{\frac{\cos^2(\alpha)}{\cos^2(\alpha)} - \frac{\sin^2(\alpha)}{\cos^2(\alpha)}} = \frac{\frac{2\sin(\alpha)}{\cos(\alpha)}}{1 - \left[\frac{\sin(\alpha)}{\cos(\alpha)}\right]^2} = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$

8. $\left[\cos\left(\frac{\alpha}{4}\right) - \sin\left(\frac{\alpha}{4}\right)\right]\left[\cos\left(\frac{\alpha}{4}\right) + \sin\left(\frac{\alpha}{4}\right)\right] = \cos^2\left(\frac{\alpha}{4}\right) - \sin^2\left(\frac{\alpha}{4}\right) = 1 - \sin^2\left(\frac{\alpha}{4}\right) - \sin^2\left(\frac{\alpha}{4}\right) =$

$= 1 - 2\sin^2\left(\frac{\alpha}{4}\right)$

9. $\cos(2x) - \sin(x) = 1 \Leftrightarrow \cos^2(x) - \sin^2(x) - \sin(x) = 1 \Leftrightarrow$

$\Leftrightarrow 1 - \sin^2(x) - \sin^2(x) - \sin(x) = 1 \Leftrightarrow$

$\Leftrightarrow -2\sin^2(x) - \sin(x) = 0 \Leftrightarrow$

$\Leftrightarrow 2\sin^2(x) + \sin(x) = 0 \Leftrightarrow$

$\Leftrightarrow \sin(x)\,[2\sin(x) + 1] = 0 \Leftrightarrow$

$\Leftrightarrow \sin(x) = 0 \vee 2\sin(x) + 1 = 0 \Leftrightarrow$

$\Leftrightarrow \sin(x) = 0 \vee \sin(x) = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \sin(x) = \sin(0) \vee \sin(x) = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow x = 0 + k\pi \vee x = -\frac{\pi}{6} + k2\pi \vee x = \pi - \left(-\frac{\pi}{6}\right) + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = k\pi \vee x = -\frac{\pi}{6} + k2\pi \vee x = \pi + \frac{\pi}{6} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = k\pi \vee x = -\frac{\pi}{6} + k2\pi \vee x = \frac{7\pi}{6} + k2\pi, k \in \mathbb{Z} \Leftrightarrow$

$C.S. = \left\{k\pi; -\frac{\pi}{6} + k2\pi; \frac{7\pi}{6} + k2\pi, k \in \mathbb{Z}\right\}$

10. $2 \in D_f$

A função f é contínua em $x = 2$, se existir $\lim\limits_{x \to 2} f(x)$, ou seja,

se $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

Ora,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{e^{x-2} - 1}{-5x^2 + 25x - 30} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \to 2^-} \frac{e^{x-2} - 1}{(x-2)(-5x+15)} =$$
$$= \lim_{x \to 2^-} \frac{e^{x-2} - 1}{x - 2} \times \lim_{x \to 2^-} \frac{1}{-5x + 15} =$$
$$= \lim_{y \to 0^-} \frac{e^y - 1}{y} \times \frac{1}{5} = 1 \times \frac{1}{5} = \frac{1}{5}$$

Cálculos auxiliares

Fez-se a mudança de variável

$y = x - 2 \Leftrightarrow x = y + 2$

Se $x \mapsto 2^-$, então, $y \mapsto 0^-$

Aplicou-se o limite notável: $\lim\limits_{x \to 0} \frac{e^x - 1}{x} = 1$

$-5x^2 + 25x - 30 = (x - 2) \times Q(x)$

Pela regra de Ruffini, vem,

	-5	25	-30
2		-10	30
	-5	15	0

$Q(x) = -5x + 15$

Logo,

$-5x^2 + 25x - 30 = (x - 2)(-5x + 15)$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{\sin(x-2)}{x^2 + x - 6} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \to 2^+} \frac{\sin(x-2)}{(x-2)(x+3)} = \lim_{x \to 2^+} \frac{\sin(x-2)}{x-2} \times \lim_{x \to 2^+} \frac{1}{x+3} =$$
$$= \lim_{y \to 0^+} \frac{\sin(y)}{y} \times \frac{1}{5} = 1 \times \frac{1}{5} = \frac{1}{5}$$

Cálculos auxiliares

Fez-se a mudança de variável

$y = x - 2 \Leftrightarrow x = y + 2$

Se $x \mapsto 2^+$, então, $y \mapsto 0^+$

Aplicou-se o limite notável: $\lim\limits_{x \to 0} \frac{\sin(x)}{x} = 1$

$x^2 + x - 6 = (x - 2) \times Q(x)$

Pela regra de Ruffini, vem,

	1	1	-6
2		2	6
	1	3	0

$Q(x) = x + 3$

Logo,

$x^2 + x - 6 = (x - 2)(x + 3)$

$f(2) = \ln\left(\frac{k+1}{2}\right)$

Ora, a função f é contínua em $x = 2$, se, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

Então, deverá ter-se,

$\ln\left(\frac{k+1}{2}\right) = \frac{1}{5} \Leftrightarrow \frac{k+1}{2} = e^{\frac{1}{5}} \Leftrightarrow \frac{k+1}{2} = \sqrt[5]{e} \Leftrightarrow k+1 = 2\sqrt[5]{e} \Leftrightarrow k = 2\sqrt[5]{e} - 1$

Portanto, a função f é contínua em $x = 2$, se $k = 2\sqrt[5]{3} - 1$

11. .

11.1. O ponto A tem coordenadas $(\cos(x); \sin(x))$, com $\cos(x) > 0$ e $\sin(x) < 0$

Assim,

$\overline{AD} = 2 \times |\sin(x)| = -2\sin(x)$

$\overline{BC} = 2 \times |\tan(x)| = -2\tan(x)$

Medida de comprimento da altura do trapézio: $h = 1 - |\cos(x)| = 1 - \cos(x)$

Assim, a área do trapézio $[ABCD]$, é dada, em função de x, por

$A(x) = \frac{\overline{AD} + \overline{BC}}{2} \times h = \frac{-2\sin(x) - 2\tan(x)}{2} \times (1 - \cos(x)) = (-\sin(x) - \tan(x)) \times (1 - \cos(x)) =$

$= -\sin(x) + \sin(x)\cos(x) - \tan(x) + \tan(x)\cos(x) = -\sin(x) + \frac{1}{2}\sin(2x) - \tan(x) + \sin(x) =$

$= -\tan(x) + \frac{1}{2}\sin(2x)$, com $x \in \left]\frac{3\pi}{2}; 2\pi\right[$

11.2. Para certo $\alpha \in \left]\frac{3\pi}{2}; 2\pi\right[$, sabe-se que $\tan(\pi - \alpha) = \frac{3}{5}$

Então, $\tan(\pi - \alpha) = \frac{3}{5} \Leftrightarrow -\tan(\alpha) = \frac{3}{5} \Leftrightarrow \tan(\alpha) = -\frac{3}{5}$

Ora, de $1 + \tan^2(\alpha) = \frac{1}{\cos^2(\alpha)}$, vem,

$1 + \left(-\frac{3}{5}\right)^2 = \frac{1}{\cos^2(\alpha)} \Leftrightarrow 1 + \frac{9}{25} = \frac{1}{\cos^2(\alpha)} \Leftrightarrow \frac{34}{25} = \frac{1}{\cos^2(\alpha)} \Leftrightarrow \cos^2(\alpha) = \frac{25}{34} \Leftrightarrow$

$\Leftrightarrow \cos(\alpha) = \pm\sqrt{\frac{25}{34}} \Leftrightarrow \cos(\alpha) = \pm\frac{5}{\sqrt{34}} \Leftrightarrow \cos(\alpha) = \pm\frac{5\sqrt{34}}{34}$, e como $\cos(\alpha) > 0$, vem, $\cos(\alpha) = \frac{5\sqrt{34}}{34}$

De $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$, ou seja, de $\sin(\alpha) = \tan(\alpha) \times \cos(\alpha)$, resulta,

$\sin(\alpha) = -\frac{3}{5} \times \frac{5\sqrt{34}}{34} = -\frac{3\sqrt{34}}{34}$

Assim, a área do trapézio é igual a

$f(\alpha) = -\tan(\alpha) + \frac{1}{2}\sin(2\alpha) = -\tan(\alpha) + \frac{1}{2} \times 2\sin(\alpha)\cos(\alpha) = -\tan(\alpha) + \sin(\alpha)\cos(\alpha) =$

$= \frac{3}{5} - \frac{3\sqrt{34}}{34} \times \frac{5\sqrt{34}}{34} = \frac{3}{5} - \frac{15 \times 34}{34 \times 34} = \frac{3}{5} - \frac{15}{34} = \frac{27}{170}\ u.a.$