

## 1º Teste – Tópicos de resolução

### Exercício 1

$$C = ]-\infty, -3[ \cup [0, +\infty[.$$

### Exercício 2

$$-2 < -\frac{3}{2} < \pi < \sqrt{16} < \frac{9}{2}.$$

### Exercício 3

$$\begin{aligned} \text{a)} \quad (x^2 + 4)(x + 1) = 0 &\Leftrightarrow x^2 + 4 = 0 \vee x + 1 = 0 \Leftrightarrow x^2 = -4 \vee x = -1 \Leftrightarrow \\ &\Leftrightarrow x \in \emptyset \vee x = -1 \Leftrightarrow x = -1 \quad C.S. = \{-1\}. \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \frac{1-x}{2} - 2 \leq -\frac{x-1}{3} &\Leftrightarrow 3 - 3x - 12 \leq -2x + 2 \Leftrightarrow -3x + 2x \leq 2 - 3 + 12 \Leftrightarrow \\ &\Leftrightarrow -x \leq 11 \Leftrightarrow x \geq -11 \Leftrightarrow x \in [-11, +\infty[ \quad C.S. = [-11, +\infty[. \end{aligned}$$

### Exercício 4

$$\text{a)} \quad 3 - 2 - 5 = 0 \Leftrightarrow -4 = 0 \text{ Proposição falsa} \quad \therefore P \notin r.$$

$$\text{b)} \quad d_{p,r} = \frac{|1 \times 3 + 1 \times (-2) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|-4|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}.$$

### Exercício 5

$$\text{a)} \quad \text{O vetor } \vec{v} = (3, -1) \text{ é perpendicular à reta } p, \text{ logo é um vetor director da reta } t.$$

$$\therefore t: (x, y) = (0, 0) + k(3, -1), k \in \mathbb{R}.$$

$$\text{b)} \quad m = \frac{-3}{-1} = 3 \rightarrow \text{Declive da reta } p = \text{declive da reta } q.$$

$$y = 3x + b$$

$$1 = 3 \times 1 + b \Leftrightarrow b = -2$$

$$\therefore y = 3x - 2.$$

### Exercício 6

$$\text{a)} \quad \overrightarrow{AB} = B - A = (1, -1)$$

$$\|\overrightarrow{AB}\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}.$$

b)  $C: (x + 1)^2 + (y - 3)^2 = 2.$

### Exercício 7

$$x^2 - 6x + y^2 - 2y - 39 = 0 \Leftrightarrow x^2 - 6x + 9 + y^2 - 2y + 1 = 39 + 9 + 1 \Leftrightarrow \\ \Leftrightarrow (x - 3)^2 + (y - 1)^2 = 49$$

Coordenadas do centro:  $(3, 1).$

Raio:  $\sqrt{49} = 7.$

### Exercício 8

a)  $2\cos\theta = 1 \Leftrightarrow \cos\theta = \frac{1}{2} \Leftrightarrow \theta = \frac{\pi}{3} + 2k\pi \vee \theta = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}.$

b)  $\sqrt{2}\sin\theta - \sqrt{2} = 0 \Leftrightarrow \sqrt{2}\sin\theta = \sqrt{2} \Leftrightarrow \sin\theta = \frac{\sqrt{2}}{\sqrt{2}} \Leftrightarrow \sin\theta = 1 \Leftrightarrow \\ \Leftrightarrow \theta = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}.$

### Exercício 9

$$(1 + \tan^2(\theta))(1 - \sin^2\theta) = \frac{1}{\cos^2\theta} \times \cos^2\theta = 1.$$

Cálculo auxiliar:

$$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \cos^2\theta = 1 - \sin^2\theta.$$

