Tópicos de Matemática II - 2018/ 2019
2º Teste – Tópicos de resolução

Exercício 1

a) $D_f = \mathbb{R}$

$2^{x+2} > 0, \forall x \in \mathbb{R} \Leftrightarrow -2^{x+2} < 0, \forall x \in \mathbb{R} \Leftrightarrow 1 - 2^{x+2} < 1, \forall x \in \mathbb{R} \Leftrightarrow$
$\Leftrightarrow f(x) < 1, \forall x \in \mathbb{R}$

$\therefore D'_f = \,]-\infty, 1[$

b) $y = 1 - 2^{x+2} \Leftrightarrow 2^{x+2} = 1 - y \Leftrightarrow x + 2 = \log_2(1-y) \Leftrightarrow$
$\Leftrightarrow x = -2 + \log_2(1-y)$

$f^{-1}(x) = -2 + \log_2(1-x)$

$$\therefore\ f^{-1}: \begin{array}{l}]-\infty, 1[\longrightarrow \mathbb{R} \\ \quad x \quad\ \longrightarrow -2 + \log_2(1-x) \end{array}$$

c) $f(x) \geq -15 \Leftrightarrow 1 - 2^{x+2} \geq -15 \Leftrightarrow -2^{x+2} \geq -16 \Leftrightarrow 2^{x+2} \leq 16 \Leftrightarrow$

$\Leftrightarrow 2^{x+2} \leq 2^4 \Leftrightarrow x + 2 \leq 4 \Leftrightarrow x \leq 2$

$C.S. = \,]-\infty, 2]$

Exercício 2

$\frac{-x^2+9}{-x-1} \geq 0 \Leftrightarrow x \in [-3, -1[\,\cup\, [3, +\infty[$

$C.S. = [-3, -1[\,\cup\, [3, +\infty[$

<u>Cálculos auxiliares:</u>

$-x^2 + 9 = 0 \Leftrightarrow -x^2 = -9 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm\sqrt{9} \Leftrightarrow x = \pm 3$
$-x - 1 = 0 \Leftrightarrow -x = 1 \Leftrightarrow x = -1$

x	-∞	-3		-1		3	+∞
$-x^2+9$	-	0	+	+	+	0	-
$-x-1$	+	+	+	0	-	-	-
$\frac{-x^2+9}{-x-1}$	-	0	+	S.S	-	0	+

Exercício 3

a) $\lim\limits_{x\to+\infty}\left(\sqrt{x+2}-\sqrt{x}\right) = \lim\limits_{x\to+\infty}\frac{(\sqrt{x+2}-\sqrt{x})(\sqrt{x+2}+\sqrt{x})}{(\sqrt{x+2}+\sqrt{x})} = \lim\limits_{x\to+\infty}\frac{x+2-x}{(\sqrt{x+2}+\sqrt{x})} =$

$= \lim\limits_{x\to+\infty}\frac{2}{(\sqrt{x+2}+\sqrt{x})} = \frac{2}{+\infty} = 0$

b) $\lim\limits_{x\to+\infty}\left(\frac{1}{x^2+2}\cdot(x+1)\right) = \lim\limits_{x\to+\infty}\frac{x+1}{x^2+2} = \lim\limits_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{2}{x^2}\right)} = \lim\limits_{x\to+\infty}\frac{1+\frac{1}{x}}{x\left(1+\frac{2}{x^2}\right)} = \frac{1}{+\infty} =$
$= 0$

Exercício 4

a) $5^{-x-1} = 25^{2x+3} \Leftrightarrow 5^{-x-1} = (5^2)^{2x+3} \Leftrightarrow 5^{-x-1} = 5^{4x+6} \Leftrightarrow$
$\Leftrightarrow -x-1 = 4x+6 \Leftrightarrow x = -\frac{7}{5}$ $\qquad C.S. = \left\{-\frac{7}{5}\right\}$

b) $D = \{x \in \mathbb{R}: x > 0 \wedge 2x+5 > 0\} = \left\{x \in \mathbb{R}: x > 0 \wedge x > -\frac{5}{2}\right\} = \mathbb{R}^+$

$log(x) > log(2x+3) \Leftrightarrow x > 2x+3 \wedge x \in \mathbb{R}^+ \Leftrightarrow -x > 5 \wedge x \in \mathbb{R}^+ \Leftrightarrow$
$\Leftrightarrow -x > 5 \wedge x \in \mathbb{R}^+ \Leftrightarrow\Leftrightarrow x > 2x+3 \wedge x \in \mathbb{R}^+ \Leftrightarrow -x > 5 \wedge x \in \mathbb{R}^+ \Leftrightarrow$
$\Leftrightarrow x < -5 \wedge x \in \mathbb{R}^+ \Leftrightarrow x \in \{\ \ \}$ Condição impossível. $C.S. = \{\ \ \}$

c) $D = \{x \in \mathbb{R}: x-1 > 0\} =]1,+\infty[$

$ln(x-1) = 2 \Leftrightarrow x-1 = e^2 \wedge x \in]1,+\infty[\Leftrightarrow x = e^2+1 \wedge x \in]1,+\infty[\Leftrightarrow$
$\Leftrightarrow x = e^2+1$ $\qquad C.S. = \{e^2+1\}$

Exercício 5

$\lim\limits_{x\to1^-} h(x) = \lim\limits_{x\to1^+}\frac{3-2x}{x^2+3} = \frac{3-2}{1+3} = \frac{1}{4}$

$\lim\limits_{x\to1^-} h(x) = \lim\limits_{x\to1^+}\frac{x^3-x^2}{x^2+2x-3} = \lim\limits_{x\to1^+}\frac{x^2(x-1)}{(x-1)(x+3)} = \lim\limits_{x\to1^+}\frac{x^2}{x+3} = \frac{1}{4}$
$\therefore \lim\limits_{x\to1} h(x) = \frac{1}{4}$ porque $\lim\limits_{x\to1^-} h(x) = \lim\limits_{x\to1^+} h(x) = \frac{1}{4}$.

<u>Cálculo auxiliar:</u>

$x^2+2x-3 = 0 \Leftrightarrow x = \frac{-2\pm\sqrt{4+12}}{2} \Leftrightarrow x = \frac{-2\pm\sqrt{16}}{2} \Leftrightarrow x = \frac{-2\pm4}{2} \Leftrightarrow x = -3 \vee x = 1$

Exercício 6

$\frac{x^2-3x+2}{x^2-1}=0 \Leftrightarrow (x=1 \vee x=2) \wedge (x \neq -1 \wedge x \neq 1) \Leftrightarrow x=2$

$C.S.=\{2\}$

Cálculos auxiliares:

$x^2-3x+2=0 \Leftrightarrow x=\frac{3\pm\sqrt{9-8}}{2} \Leftrightarrow x=\frac{3\pm\sqrt{1}}{2} \Leftrightarrow x=\frac{3\pm 1}{2} \Leftrightarrow \Leftrightarrow x=1 \vee x=2$
$x^2-1=0 \Leftrightarrow x^2=1 \Leftrightarrow x=\pm\sqrt{1} \Leftrightarrow x=\pm 1$

Exercício7

- **a)** $+\infty$
- **b)** -6
- **c)** $\frac{2}{0^-}=-\infty$
- **d)** -4

Exercício 8

a) $y'=4ln(x)+\frac{4x}{x}=4ln(x)+4$

b) $y'=2e^{2x}\times 5x+e^{2x}\times 5=5e^{2x}(2x+1)$

Exercício 9

$f'(x)=-x^2+2x$
$f'(1)=1$
$f(1)=-\frac{1}{3}+1-7=-\frac{19}{3}$
$y=x+b$
$-\frac{19}{3}=1+b \Leftrightarrow b=-\frac{22}{3}$

Resposta: $y=x-\frac{22}{3}$.