

Exercício 1

Calcule:

a)

$$\log_2 \left(\frac{1}{64} \right);$$

$$\log_2 \left(\frac{1}{64} \right) = \log_2 (2^6) = 6$$

b)

$$\log (1000);$$

$$\log (1000) = \log (10^3) = 3$$

c)

$$\ln (e^3);$$

$$\ln (e^3) = 3$$

d)

$$\ln (\sqrt[5]{e});$$

$$\ln (\sqrt[5]{e}) = \ln \left(e^{\frac{1}{5}} \right) = \frac{1}{5}$$

e)

$$\ln (e^2) + \ln (e^{-10}) \ln (1);$$

$$\ln (e^2) + \ln (e^{-10}) \ln (1) = -8$$

f)

$$\log_3 \left(\frac{\sqrt{27}}{81^8} \right);$$

$$\log_3 \left(\frac{\sqrt{27}}{81^8} \right) = \log_3 3^{\frac{3}{2}} - \log_3 3^{32} = \frac{3}{2} - 32 = -\frac{61}{2}$$

g)

$$\log_4 (64);$$

$$\log_4 (64) = \log_4 (4^3) = 3$$

h)

$$\log_2 (\sqrt{32});$$

$$\log_2 (\sqrt{32}) = \log_2 \left(2^{\frac{5}{2}} \right) = \frac{5}{2}$$

i)

$$\log_5(1);$$

$$\log_5(1) = 0$$

Exercício 2

Seja $f(x) = \frac{1+2\ln(x)}{x}$.

a)

Determine D_f .

$$D_f = \{x \in \mathbb{R} : x > 0 \wedge x \neq 0\} =]0, +\infty[$$

b)

Resolva a inequação $f(x) \geq 0$.

$$\frac{1+2\ln(x)}{x} \geq 0$$

x	0		$\frac{1}{\sqrt{e}}$	$+\infty$
$1+2\ln(x)$		-	0	+
x		+	+	+
$\frac{1+2\ln 2}{x}$		-	0	+

C.A.

Crescente

$$C.S. = \left[\frac{1}{\sqrt{e}}, +\infty\right[$$

Exercício 3

Para cada uma das funções seguintes, determine o domínio, o contradomínio e os zeros. Caracterize, caso exista, a função inversa.

a)

$$m(x) = 5 - \log(x+5);$$

$$D_m = \{x \in \mathbb{R} : x > -5\} =]-5, +\infty[$$

$$D'_m = \mathbb{R}$$

$$m^{-1} = 10^{-x+5} - 5$$

$$m^{-1} : \mathbb{R} \rightarrow]-5, +\infty[$$

$$x \mapsto 10^{-x+5} - 5$$

b)

$$g(x) = 3 + \frac{1}{2} \log_7(2x - 1);$$

$$D_g = \{x \in \mathbb{R} : x > \frac{1}{2}\} =]\frac{1}{2}, +\infty[$$

$$D'_g = \mathbb{R}$$

$$g^{-1} = \frac{7^{2x-6} + 1}{2}$$

$$g^{-1} : \mathbb{R} \rightarrow]\frac{1}{2}, +\infty[$$

$$x \mapsto \frac{7^{2x-6} + 1}{2}$$

c)

$$f(x) = e^{x-3} - 2;$$

$$D_f = \mathbb{R}$$

$$D'_f =]-2, +\infty[$$

$$f^{-1} = \ln(x + 2) + 3$$

$$f^{-1} :]-2, +\infty[\rightarrow \mathbb{R}$$

$$x \mapsto \ln(x + 2) + 3$$

Exercício 4

Resolva, em \mathbb{R} , cada uma das seguintes condições:

a)

$$\ln(x^2 - 1) = 1;$$

$$D = \{x \in \mathbb{R} : x^2 - 1 > 0\} =]-\infty, -1[\cup]1, +\infty[$$

$$\ln(x^2 - 1) = 1 \Leftrightarrow x^2 - 1 = e \Leftrightarrow x^2 = 1 + e \Leftrightarrow x = \pm\sqrt{1+e}$$

$$C.S = \{-\sqrt{1+e}, \sqrt{1+e}\}$$

b)

$$\log_2(1 - 2x) > \log_2(x);$$

$$D = \{x \in \mathbb{R} : 1 - 2x > 0 \wedge x > 0\} =]0, \frac{1}{2}[$$

$$x < \frac{1}{3} \wedge x \in D$$

$$D \cap]-\infty, \frac{1}{3}[=]0, \frac{1}{2}[\cap]-\infty, \frac{1}{3}[=]0, \frac{1}{3}[$$

c)

$$\log(1 - x^2) < 1.$$

$$D = \{x \in \mathbb{R} : 1 - x^2 > 0\} =]-1, 1[$$

$$\underbrace{x^2 > -9}_{\text{Impossível}} \wedge x \in D$$

$$x \in \mathbb{R} \wedge x \in D$$

$$D \cap \mathbb{R} =]-1, 1[$$

Exercício 5

Considere a função real, de variável real, definida por

$$f(x) = 1 - 3^x$$

a)

Calcule $f(0) + f(\log_3 2)$.

$$f(0) + f(\log_3 2) = -1$$

b)

Caracterize, caso exista, a função inversa f^{-1} .

$$D_f = \mathbb{R}$$

$$D'_f =]-\infty, 1[$$

$$f^{-1} = \log_3(-x + 1)$$

$$f^{-1} :]-\infty, 1[\rightarrow \mathbb{R}[$$

$$x \mapsto \log_3(-x + 1)$$