Escola Secundária de Paredes

Proposta de Resolução do 1º Teste de Avaliação

Matemática A

12.º Ano de Escolaridade | outubro de 2020

Turma: 12ºJ

1. .

1.1. $\lim_{x\to+\infty}\left[\frac{3}{2x^2+x+1}\times(x^2+2x+2)\right] =^{(0\times\infty)} \lim_{x\to+\infty}\frac{3x^2+6x+6}{2x^2+x+1} =^{\left(\frac{\infty}{\infty}\right)} \lim_{x\to+\infty}\frac{x^2\left(3+\frac{6x}{x^2}+\frac{6}{x^2}\right)}{x^2\left(2+\frac{x}{x^2}+\frac{1}{x^2}\right)} =$

$$= \frac{\lim_{x\to+\infty}\left(3+\frac{6}{x}+\frac{6}{x^2}\right)}{\lim_{x\to+\infty}\left(2+\frac{1}{x}+\frac{1}{x^2}\right)} = \frac{3+0+0}{2+0+0} = \frac{3}{2}$$

1.2. $\lim_{x\to-5}\frac{x^3+8x^2+5x-50}{x^2+5x} =^{\left(\frac{0}{0}\right)} \lim_{x\to-5}\frac{(x+5)(x^2+3x-10)}{x(x+5)} = \lim_{x\to-5}\frac{x^2+3x-10}{x} = 0$

Cálculos auxiliares

$x^3+8x^2+5x-50=(x+5)\times Q(x)$

Pela regra de Ruffini, tem-se,

	1	8	5	-50
-5		-5	-15	50
	1	3	-10	0

Logo, $Q(x)=x^2+3x-10$

2. Ora,

$$A(x)=\overline{OB}\times\overline{AB}=x\times h(x)=x\times\frac{2}{x-1}=\frac{2x}{x-1}$$

Resposta:

Versão 1: (B)

Versão 2: (D)

3. Ora,

$$a_n = \frac{3n+5}{n+1} = \frac{3n+3+2}{n+1} = \frac{3n+3}{n+1} + \frac{2}{n+1} = \frac{3(n+1)}{n+1} + \frac{2}{n+1} = 3 + \frac{2}{n+1}$$

Logo,

$a_n > 3, \forall n \in \mathbb{N}$

$\lim(a_n) = \lim\left(3 + \frac{2}{n+1}\right) = 3^+$

Assim,

$\lim f(a_n) = 2$

Resposta:

Versão 1: (C)

Versão 2: (B)

4. Ora,

$-1 \in D_g$

$$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} \frac{(x+1)^2}{x^2+x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to -1^-} \frac{(x+1)^2}{x(x+1)} = \lim_{x\to -1^-} \frac{x+1}{x} = 0$$

$$\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} \frac{x^2-1}{x+1} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to -1^+} \frac{(x-1)(x+1)}{x+1} = \lim_{x\to -1^+}(x-1) = -2$$

$g(-1) = 0$

Como, $\lim\limits_{x\to -1^-} g(x) \neq \lim\limits_{x\to -1^+} g(x)$

Logo, Não existe $\lim\limits_{x\to -1} g(x)$

Resposta:

Versão 1: (D)

Versão 2: (A)

5. .

5.1. $$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{-x-2}{3x^2+6x} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to -\infty} \frac{x\left(-1-\frac{2}{x}\right)}{x^2\left(3+\frac{6x}{x^2}\right)} = \lim_{x\to -\infty} \frac{x}{x^2} \times \frac{\lim\limits_{x\to -\infty}\left(-1-\frac{2}{x}\right)}{\lim\limits_{x\to -\infty}\left(3+\frac{6}{x}\right)} =$$

$$= \lim_{x\to -\infty} \frac{1}{x} \times \frac{-1-\frac{2}{-\infty}}{3+\frac{6}{-\infty}} = \frac{1}{-\infty} \times \frac{-1+0}{3-0} = 0$$

5.2. $-2 \in D_f$

A função f é contínua em $x=-2$, se existir $\lim\limits_{x\to -2} f(x)$, ou seja,

se $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^+} f(x) = f(-2)$

Ora,

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{-x-2}{3x^2+6x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to -2^-} \frac{-(x+2)}{3x(x+2)} = \lim_{x\to -2^-} \frac{-1}{3x} = \frac{1}{6}$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{\sqrt{x+11}-3}{(x+2)(x+3)} = \lim_{x\to -2^+} \frac{(\sqrt{x+11}-3)(\sqrt{x+11}+3)}{(x+2)(x+3)(\sqrt{x+11}+3)} =$$

$$= \lim_{x\to -2^+} \frac{(\sqrt{x+11})^2-3^2}{(x+2)(x+3)(\sqrt{x+11}+3)} = \lim_{x\to -2^+} \frac{|x+11|-9}{(x+2)(x+3)(\sqrt{x+11}+3)} =$$

$$= \lim_{x\to -2^+} \frac{x+11-9}{(x+2)(x+3)(\sqrt{x+11}+3)} = \lim_{x\to -2^+} \frac{x+2}{(x+2)(x+3)(\sqrt{x+11}+3)} =$$

$$= \lim_{x\to -2^+} \frac{1}{(x+3)(\sqrt{x+11}+3)} = \frac{1}{(\sqrt{-2+9}+3)} = \frac{1}{6}$$

$f(-2) = 2k+5$

Ora, a função f é contínua em $x=-2$, se, $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^+} f(x) = f(-2)$

Então, deverá ter-se,

$$2k+5 = \frac{1}{6} \Leftrightarrow 2k = \frac{1}{6} - 5 \Leftrightarrow 2k = -\frac{29}{6} \Leftrightarrow k = -\frac{29}{12}$$

Portanto, a função f é contínua em $x=-2$, se $k = -\dfrac{29}{12}$

6. .

6.1. $f(x) = g(x) \Leftrightarrow \dfrac{x+1}{x^2-4} = \dfrac{x}{x+2} \Leftrightarrow \dfrac{x+1}{x^2-4} - \dfrac{x}{x+2} = 0 \Leftrightarrow \dfrac{x+1}{(x-2)(x+2)} - \dfrac{x}{x+2} = 0 \Leftrightarrow$

$$\Leftrightarrow \frac{x+1-x(x-2)}{(x-2)(x+2)} = 0 \Leftrightarrow \frac{x+1-x^2+2x}{(x-2)(x+2)} = 0 \Leftrightarrow \frac{-x^2+3x+1}{(x-2)(x+2)} = 0 \Leftrightarrow$$

$$\Leftrightarrow -x^2+3x+1 = 0 \wedge (x-2)(x+2) \neq 0 \Leftrightarrow x = \frac{-3 \pm \sqrt{3^2 - 4\times(-1)\times 1}}{2\times(-1)} \wedge x \neq -2 \wedge x \neq 2 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{-3\pm\sqrt{13}}{-2} \wedge x \neq -2 \wedge x \neq 2 \Leftrightarrow \left(x = \frac{-3-\sqrt{13}}{-2} \vee x = \frac{-3+\sqrt{13}}{-2}\right) \wedge x \neq -2 \wedge x \neq 2 \Leftrightarrow$$

$$\Leftrightarrow \left(x = \frac{3+\sqrt{13}}{2} \vee x = \frac{3-\sqrt{13}}{2}\right) \wedge x \neq -2 \wedge x \neq 2 \Leftrightarrow x = \frac{3+\sqrt{13}}{2} \vee x = \frac{3-\sqrt{13}}{2}$$

Cálculo auxiliar

$(x-2)(x+2) = 0 \Leftrightarrow x-2 = 0 \vee x+2 = 0 \Leftrightarrow x = 2 \vee x = -2$

$$C.S. = \left\{\frac{3-\sqrt{13}}{2}; \frac{3+\sqrt{13}}{2}\right\}$$

6.2. $f(x) \leq \dfrac{x+1}{2-x} \Leftrightarrow \dfrac{x+1}{x^2-4} \leq \dfrac{x+1}{2-x} \Leftrightarrow \dfrac{x+1}{x^2-4} - \dfrac{x+1}{2-x} \leq 0 \Leftrightarrow \dfrac{x+1}{(x-2)(x+2)} + \dfrac{x+1}{x-2} \leq 0 \Leftrightarrow$

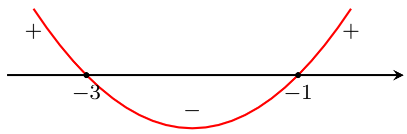
$\Leftrightarrow \dfrac{x+1+(x+1)(x+2)}{(x-2)(x+2)} \leq 0 \Leftrightarrow \dfrac{(x+1)(x+3)}{(x-2)(x+2)} \leq 0$

$\twoheadrightarrow$ **Numerador**

Zeros: $(x+1)(x+3) = 0 \Leftrightarrow x+1 = 0 \vee x+3 = 0 \Leftrightarrow x = -1 \vee x = -3$

Sinal:

Como se trata de uma função quadrática, estuda-se o sinal pela representação gráfica da parábola



Assim,

$(x+1)(x+3) < 0 \Leftrightarrow -3 < x < -1$

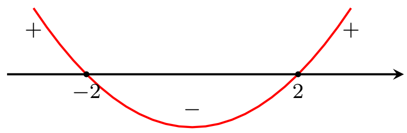
$(x+1)(x+3) > 0 \Leftrightarrow x < -3 \vee x > -1$

$\twoheadrightarrow$ **Denominador**

Zeros: $(x-2)(x+2) = 0 \Leftrightarrow x-2 = 0 \vee x+2 = 0 \Leftrightarrow x = 2 \vee x = -2$

Sinal:

Como se trata de uma função quadrática, estuda-se o sinal pela representação gráfica da parábola



Assim,

$(x-2)(x+2) < 0 \Leftrightarrow -2 < x < 2$

$(x-2)(x+2) > 0 \Leftrightarrow x < -2 \vee x > 2$

Tabela de sinais

x	$-\infty$	-3		-2		-1		2	$+\infty$
$(x+1)(x+3)$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$(x-2)(x+2)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
$\dfrac{(x+1)(x+3)}{(x-2)(x+2)}$	$+$	0	$-$	$s.s.$	$+$	0	$-$	$s.s.$	$+$

Concluindo:

$C.S. = [-3; -2[\cup [-1; 2[$

7. Ora,

$D_h = \left\{x \in \mathbb{R} : x^3 - 3x - 2 \neq 0\right\}$

Sabe-se que 2 anula o polinómio $x^3 - 3x - 2$

Então,

$x^3 - 3x - 2 = (x-2)Q(x)$

Pela regra de Ruffini, vem,

$$\begin{array}{c|ccc|c} & 1 & 0 & -3 & -2 \\ 2 & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

Assim,

$Q(x) = x^2 + 2x + 1 = (x+1)^2$

Logo,

$x^3 - 3x - 2 = (x-2)(x^2+2x+1) = (x-2)(x+1)^2$

Portanto,

$x^3 + 2x - 2 = 0 \Leftrightarrow (x-2)(x+1)^2 = 0 \Leftrightarrow x - 2 = 0 \vee (x+1)^2 = 0 \Leftrightarrow x = 2 \vee x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = 2 \vee x = -1$

Concluindo,

$D_h = \left\{x \in \mathbb{R} : x^3 + 2x - 2 \neq 0\right\} = \mathbb{R} \setminus \{-1; 2\}$

8. $\lim\limits_{x \to -5^-} \dfrac{5x + x^2}{(x+5)^2} =^{\left(\frac{0}{0}\right)} \lim\limits_{x \to -5^-} \dfrac{x(5+x)}{(x+5)^2} = \lim\limits_{x \to -5^-} \dfrac{x}{x+5} = \dfrac{-5}{0^-} = +\infty$

Resposta:

Versão 1: (C)

Versão 2: (B)

9. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+3} + \sqrt{x^2+2}}{x+1} =^{\left(\frac{\infty}{\infty}\right)} \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2\left(1 + \dfrac{3}{x^2}\right)} + \sqrt{x^2\left(1 + \dfrac{2}{x^2}\right)}}{x+1} =$

$= \lim\limits_{x \to -\infty} \dfrac{|x|\sqrt{1 + \dfrac{3}{x^2}} + |x|\sqrt{1 + \dfrac{2}{x^2}}}{x+1} = \lim\limits_{x \to -\infty} \dfrac{-x\sqrt{1 + \dfrac{3}{x^2}} - x\sqrt{1 + \dfrac{2}{x^2}}}{x+1} =$

$= \lim\limits_{x \to -\infty} \dfrac{-x\left(\sqrt{1 + \dfrac{3}{x^2}} + \sqrt{1 + \dfrac{2}{x^2}}\right)}{x\left(1 + \dfrac{1}{x}\right)} = -\dfrac{\sqrt{1 + \lim\limits_{x \to -\infty} \dfrac{3}{x^2}} + \sqrt{1 + \lim\limits_{x \to -\infty} \dfrac{2}{x^2}}}{1 + \lim\limits_{x \to -\infty} \dfrac{1}{x}} = -\dfrac{1+1}{1+0} = -2$