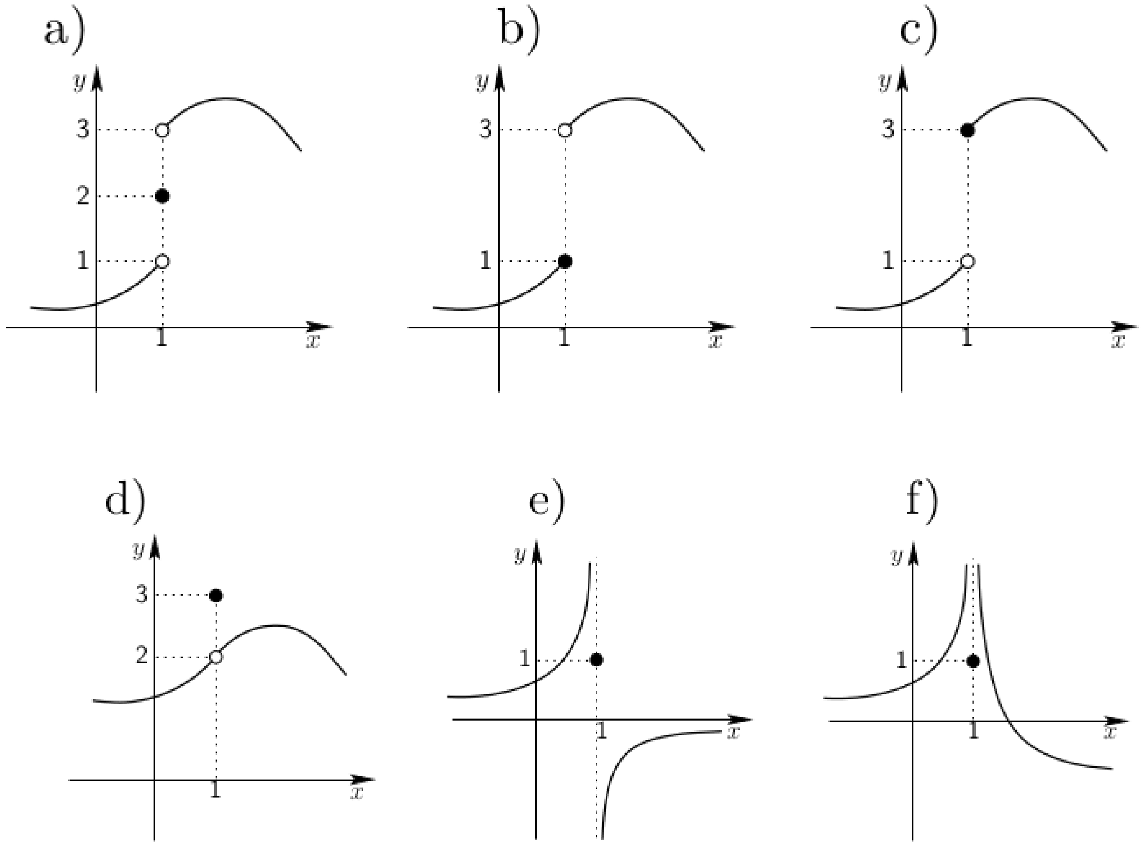
Exercício 1

Para cada uma das alíneas seguintes, indique:



i) $\lim\limits_{x \to 1^-} f(x)$;

- a) 1;
- b) 1;
- c) 1;
- d) 2;
- e) $+\infty$;
- f) $+\infty$;

ii) $\lim\limits_{x \to 1^+} f(x)$;

- a) 3;
- b) 3;
- c) 3;

d) 2;
e) $-\infty$;
f) $+\infty$;

iii) $f(1)$.

a) 2;
b) 1;
c) 3;
d) 3;
e) 1;
f) 1;

Exercício 2

Sendo a função h definida, em $\mathbb{R}$,por

$$h(x) = \begin{cases} 2x,\text{se } x \geq 3 \\ x^2 - 3 \text{ se } x < 3 \end{cases}$$

Calcule

$\lim\limits_{x\to 5} h(x)$;

$$\lim_{x\to 5} h(x) = \lim_{x\to 5} 2x = 10$$

$\lim\limits_{x\to -\infty} h(x)$;

$$\lim_{x\to -\infty} h(x) = \lim_{x\to -\infty} x^2 - 3 = +\infty$$

$\lim\limits_{x\to 3^-} h(x)$;

$$\lim_{x\to 3^-} h(x) = \lim_{x\to 3^-} x^2 - 3 = 6$$

$\lim\limits_{x\to 3^+} h(x)$;

$$\lim_{x\to 3^+} h(x) = \lim_{x\to 3^+} 2x = 6$$

Diga se existe $\lim\limits_{x\to 3} h(x)$.

Existe limite pois só existe um limite.

Exercício 3

Calcule, se existirem, os seguintes limites:

a)

$\lim\limits_{x\to 3^-} \frac{x^2}{x-3}$

$$\lim_{x\to 3^-} \frac{x^2}{x-3} = \frac{9}{0^-} = -\infty$$

b)

$\lim_{x \to -1^+} \frac{4x-3}{x+1}$

$$\lim_{x \to -1^+} \frac{4x-3}{x+1} = \frac{-7}{0^+} = -\infty$$

c)

$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x^2}\right)$

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x^2}\right) = \lim_{x \to 0^+} \frac{x-1}{x^2} = \frac{-1}{0^+} = -\infty$$

d)

$\lim_{x \to 0} \frac{5x^3+8x^2}{3x^4-16x^2}$

$$\lim_{x \to 0} \frac{5x^3+8x^2}{3x^4-16x^2} = \lim_{x \to 0} \frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)} = -\frac{1}{2}$$

e)

$\lim_{x \to 2} \frac{x^3-5x^2+8x-4}{x^3-3x^2+4}$

C.A.

$x^3 - 5x^2 + 8x - 4$

$$\begin{array}{c|cccc} & 1 & -5 & 8 & -4 \\ 2 & & 2 & -6 & 4 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{c|ccc} & 1 & -3 & 2 \\ 2 & & 2 & -2 \\ \hline & 1 & -1 & 0 \end{array}$$

$x^3 - 5x^2 + 8x - 4 = (x-2)^2 (x-1)$

$x^3 - 3x^2 + 4 = (x-2)^2 (x+1)$

$$\begin{array}{c|cccc} & 1 & -3 & 0 & 4 \\ 2 & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$\begin{array}{c|ccc} & 1 & -1 & -2 \\ 2 & & 2 & 2 \\ \hline & 1 & 1 & 0 \end{array}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)^2}(x-1)}{\cancel{(x-2)^2}(x+1)} = \frac{1}{3}$$

f)

$\lim_{x\to-\infty} \frac{2x^2+5x}{3x+2-4x^2}$

$$\lim_{x\to-\infty} \frac{2x^2+5x}{3x+2-4x^2} = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(2+\cancelto{0}{\frac{5}{x}}\right)}{\cancel{x^2}\left(\cancelto{0}{\frac{3}{x}}+\cancelto{0}{\frac{2}{x^2}}-4\right)} = -\frac{1}{2}$$

g)

$\lim_{x\to+\infty} \frac{x^2}{x^3+9}$

$$\lim_{x\to+\infty} \frac{x^2}{x^3+9} = \lim_{x\to+\infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(x+\cancelto{0}{\frac{9}{x^2}}\right)} = \frac{1}{+\infty} = 0$$

h)

$\lim_{x\to+\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$

$$\lim_{x\to+\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) = \lim_{x\to+\infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(x+\cancelto{0}{\frac{9}{x^2}}\right)} = \frac{1}{+\infty} = 0$$

i)

$\lim_{x\to-\infty} e^{-2x}$

$$\lim_{x\to-\infty} e^{-2x} = e^{+\infty} = +\infty$$

j)

$\lim_{x\to-\infty} \frac{2^x}{3^x}$

$$\lim_{x\to-\infty} \frac{2^x}{3^x} = \lim_{x\to-\infty} \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-\infty} = +\infty$$