Escola Secundária de Paredes

Proposta de Resolução do TPC 1

Matemática A

12.º Ano de Escolaridade | Turma: J

1. .

1.1. $\lim\left(1+\frac{3}{4n}\right)^{-n}=\left[\lim\left(1+\frac{3}{4n}\right)^{n}\right]^{-1}=\left[\lim\left(1+\frac{\frac{3}{4}}{n}\right)^{n}\right]^{-1}=\left(e^{\frac{3}{4}}\right)^{-1}=e^{-\frac{3}{4}}=\frac{1}{e^{\frac{3}{4}}}=\frac{1}{\sqrt[4]{e^3}}$

1.2. $\lim\left(\frac{3n+2}{3n-2}\right)^{2n}=\left[\lim\left(\frac{3n+2}{3n-2}\right)^{n}\right]^{2}=\left[\lim\left(\frac{3n\left(1+\frac{2}{3n}\right)}{3n\left(1-\frac{2}{3n}\right)}\right)^{n}\right]^{2}=\left[\frac{\lim\left(1+\frac{\frac{2}{3}}{n}\right)^{n}}{\lim\left(1+\frac{-\frac{2}{3}}{n}\right)^{n}}\right]^{2}=$

$=\left(\frac{e^{\frac{2}{3}}}{e^{-\frac{2}{3}}}\right)^{2}=\left(e^{\frac{2}{3}+\frac{2}{3}}\right)^{2}=\left(e^{\frac{4}{3}}\right)^{2}=e^{\frac{8}{3}}=\sqrt[3]{e^8}=\sqrt[3]{e^3\times e^3\times e^2}=e^2\sqrt[3]{e^2}$

1.3. $\lim\left(\frac{2n-3}{3n+4}\right)^{n+1}=\lim\left(\frac{2n\left(1-\frac{3}{2n}\right)}{3n\left(1+\frac{4}{3n}\right)}\right)^{n+1}=\lim\left(\frac{2}{3}\right)^{n+1}\times\frac{\lim\left(1-\frac{3}{2n}\right)^{n+1}}{\lim\left(1+\frac{4}{3n}\right)^{n+1}}=$

$=\lim\left(\frac{2}{3}\right)^{n+1}\times\frac{\lim\left(1-\frac{3}{2n}\right)^{n}\times\lim\left(1-\frac{3}{2n}\right)}{\lim\left(1+\frac{4}{3n}\right)^{n}\times\lim\left(1+\frac{4}{3n}\right)}=$

$=\lim\left(\frac{2}{3}\right)^{n+1}\times\frac{\lim\left(1+\frac{-\frac{3}{2}}{n}\right)^{n}\times\lim\left(1-\frac{3}{2n}\right)}{\lim\left(1+\frac{\frac{4}{3}}{n}\right)^{n}\times\lim\left(1+\frac{4}{3n}\right)}=$

$=0\times\frac{e^{-\frac{3}{2}}\times 1}{e^{\frac{4}{3}}\times 1}=0$

1.4. $\lim\left(\frac{4n+3}{2n+1}\right)^{\frac{n}{2}} = \left[\lim\left(\frac{4n+3}{2n+1}\right)^{n}\right]^{\frac{1}{2}} = \left[\lim\left(\frac{4n\left(1+\frac{3}{4n}\right)}{2n\left(1+\frac{1}{2n}\right)}\right)^{n}\right]^{\frac{1}{2}} = \left[\lim 2^n \times \frac{\lim\left(1+\frac{3}{4n}\right)^n}{\lim\left(1+\frac{1}{2n}\right)^n}\right]^{\frac{1}{2}} =$

$$= \left[\lim 2^n \times \frac{\lim\left(1+\frac{\frac{3}{4}}{n}\right)^n}{\lim\left(1+\frac{\frac{1}{2}}{n}\right)^n}\right]^{\frac{1}{2}} = \left(+\infty \times \frac{e^{\frac{3}{4}}}{e^{\frac{1}{2}}}\right)^{\frac{1}{2}} = +\infty$$

2. $\lim\left(\frac{6n+5}{6n-2}\right)^{2n} = \lim\left(\frac{6n\left(1+\frac{5}{6n}\right)}{6n\left(1-\frac{2}{6n}\right)}\right)^{2n} = \frac{\lim\left(1+\frac{5}{6n}\right)^{2n}}{\lim\left(1-\frac{2}{6n}\right)^{2n}} = \frac{\lim\left(1+\frac{\frac{5}{3}}{2n}\right)^{2n}}{\lim\left(1+\frac{-\frac{2}{3}}{2n}\right)^{2n}} =$

$$= \frac{e^{\frac{5}{3}}}{e^{-\frac{2}{3}}} = e^{\frac{5}{3}+\frac{2}{3}} = e^{\frac{7}{3}}$$

Assim,

$$\lim\left(\frac{6n+5}{6n-2}\right)^{2n} = \frac{e}{e^{-3k-2}} \Leftrightarrow e^{\frac{7}{3}} = \frac{e}{e^{-3k-2}} \Leftrightarrow e^{\frac{7}{3}} = e^{1+3k+2} \Leftrightarrow e^{\frac{7}{3}} = e^{3k+3} \Leftrightarrow 3k+3 = \frac{7}{3} \Leftrightarrow$$

$$\Leftrightarrow 9k+9 = 7 \Leftrightarrow 9k = 7-9 \Leftrightarrow 9k = -2 \Leftrightarrow k = -\frac{2}{9}$$