Exercício 1

Calcule:

a)

$\log_2\left(\frac{1}{64}\right)$;

$$\log_2\left(\frac{1}{64}\right) = \log_2\left(2^6\right) = 6$$

b)

$\log\left(1000\right)$;

$$\log\left(1000\right) = \log\left(10^2\right) = 2$$

c)

$\ln\left(e^3\right)$;

$$\ln\left(e^3\right) = 3$$

d)

$\ln\left(\sqrt[5]{e}\right)$;

$$\ln\left(\sqrt[5]{e}\right) = \ln\left(e^{\frac{1}{5}}\right) = \frac{1}{5}$$

e)

$\ln\left(e^2\right) + \ln\left(e^{-10}\right)\ln\left(1\right)$;

$$\ln\left(e^2\right) + \ln\left(e^{-10}\right)\ln\left(1\right) = -8$$

f)

$\log_3\left(\frac{\sqrt{27}}{81^8}\right)$;

$$\log_3\left(\frac{\sqrt{27}}{81^8}\right) = \log_3 3^{\frac{3}{2}} - \log_3 3^{32} = \frac{3}{2} - 32 = -\frac{61}{2}$$

g)

$\log_4\left(64\right)$;

$$\log_4\left(64\right) = \log_4\left(4^3\right) = 3$$

h)

$\log_2\left(\sqrt{32}\right)$;

$$\log_2\left(\sqrt{32}\right) = \log_2\left(2^{\frac{5}{2}}\right) = \frac{5}{2}$$

i)

$\log_5(1)$;

$$\log_5(1) = 0$$

Exercício 2

Seja $f(x) = \frac{1+2\ln(x)}{x}$**.**

a)

Determine D_f**.**

$$D_f = \{x \in \mathbb{R} : x > 0 \wedge x \neq 0\} =]0, +\infty[$$

b)

Resolva a inequação $f(x) \geq 0$**.**

$$\frac{1+2\ln(x)}{x} \geq 0$$

x	0		$\frac{1}{\sqrt{e}}$	$+\infty$
$1+2\ln(x)$		$+$	0	$+$
x		$-$	$+$	$+$
$\frac{1+2\ln 2}{x}$		$-$	0	$+$
			Crescente	

C.A.

$$C.S. = [\frac{1}{\sqrt{e}}, +\infty[$$

Exercício 3

Para cada uma das funções seguintes, determine o domínio, o contradomínio e os zeros. Caracterize, caso exista, a função inversa.

a)

$m(x) = 5 - \log(x+5)$;

$$D_m = \{x \in \mathbb{R} : x > -5\} =]-5, +\infty[$$

$$D'_m = \mathbb{R}$$

$$m^{-1} = 10^{-x+5} - 5$$

$$m^{-1} : \mathbb{R} \to]-5, +\infty[$$

$$x \rightarrowtail 10^{-x+5} - 5$$

b)

$g(x) = 3 + \frac{1}{2}\log_7(2x-1)$;

$$D_g = \{x \in \mathbb{R} : x > \frac{1}{2}\} =]\frac{1}{2}, +\infty[$$

$$D'_g = \mathbb{R}$$

$$g^{-1} = \frac{7^{2x-6}+1}{2}$$

$$g^{-1} : \mathbb{R} \to]\frac{1}{2}, +\infty[$$

$$x \rightarrowtail \frac{7^{2x-6}+1}{2}$$

c)

$f(x) = e^{x-3} - 2$;

$$D_f = \mathbb{R}$$

$$D'_f =]-2, +\infty[$$

$$f^{-1} = \ln(x+2) + 3$$

$$f^{-1} :]-2, +\infty[\to \mathbb{R}$$

$$x \rightarrowtail \ln(x+2) + 3$$

Exercício 4

Resolva, em $\mathbb{R}$, cada uma das seguintes condições:

a)

$\ln(x^2-1) = 1$;

$$\ln(x^2-1) = 1 \Leftrightarrow x^2 = 1 + e \Leftrightarrow x = \pm\sqrt{1+e}$$

$$C.S = \{-\sqrt{1+e}, \sqrt{1+e}\}$$

b)

$\log_2(1-2x) > \log 2(x)$;

$$D = \{x \in \mathbb{R} : 1 - 2x > 0 \wedge x > 0\} =]0, \frac{1}{2}[$$

$$x < \frac{1}{3} \wedge D$$

$$D \cap]-\infty, \frac{1}{3}[=]0, \frac{1}{2}[\cap]-\infty, \frac{1}{3}[=]0, \frac{1}{3}[$$

c)

$\log(1-x^2) < 1$.

$$D = \{x \in \mathbb{R} : 1 - x^2 > 0\} =]-1, 1[$$

$$x < -3 \vee x > 3 \wedge D$$

$$D \cap]-\infty, -3[\cup]3, +\infty[=]-1, 1[\cap, -\infty, -3[\cup]3, +\infty[=]-1, 1[$$

Exercício 5

Considere a função real, de variável real, definida por

$$f(x) = 1 - 3^x$$

a)

Calcule $f(0) + f(\log_3 2)$.

$$f(0) + f(\log_3 2) = -1$$

b)

Caracterize, caso exista, a função inversa f^{-1}.

$$D_f = \mathbb{R}$$

$$D'_f =]-\infty, 1[$$

$$f^{-1} = \log_3(-x+1)$$

$$f^{-1} :]-\infty, 1[\to \mathbb{R}[$$

$$x \rightarrowtail \log_3(-x+1)$$