Exercício 1

$$(a-b)(a+b)+b(b+2)-2b = a^2-b^2+b^2+2b-2b = a^2$$

Exercício 2

a)

$$\frac{1-2x}{2} \leq x - \frac{x-1}{3} \Leftrightarrow 3-6x \leq 6x-(2x-2) \Leftrightarrow -6x-6x+2x \leq 2-3 \Leftrightarrow$$

$$\Leftrightarrow -10x \leq -1 \Leftrightarrow 10x \geq 1 \Leftrightarrow x \geq \frac{1}{10}$$

$$S = \left[\frac{1}{10}, +\infty\right[$$

b)

$$4x^4 = x^2 \Leftrightarrow x^2(4x^2-1)=0 \Leftrightarrow x^2=0 \vee 4x^2-1=0 \Leftrightarrow x=0 \vee x^2=\frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow x=0 \vee x=\pm\sqrt{\frac{1}{4}} \Leftrightarrow x=0 \vee x=\pm\frac{1}{2}$$

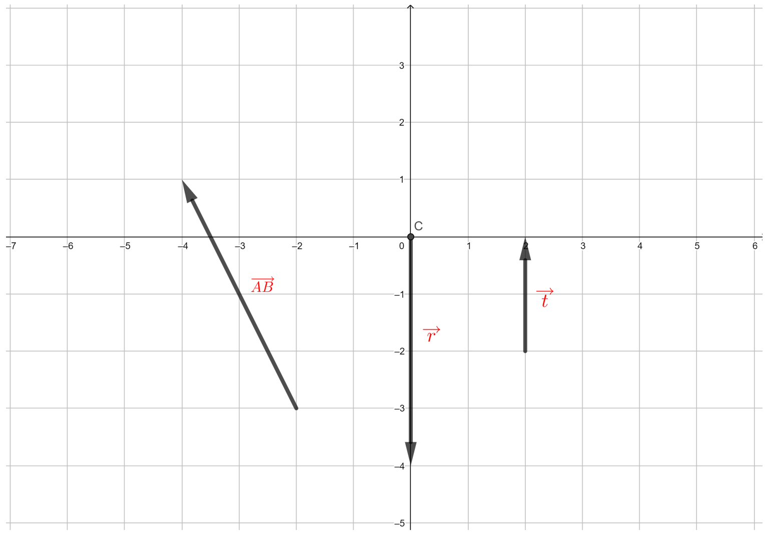
$$S = \left\{\frac{1}{2}, 0, -\frac{1}{2}\right\}$$

c)

$$|3-x|=2 \Leftrightarrow 3-x=2 \vee 3-x=-2 \Leftrightarrow -x=-1 \vee -x=-5 \Leftrightarrow x=1 \vee x=5$$

$$S=\{1,5\}$$

Exercício 3



Exercício 4

$$x^2-2x+y^2+6y=-9 \Leftrightarrow (x-1)^2-1^2+(y+3)^2-3^2=9 \Leftrightarrow (x-1)^2+(y+3)^2=1$$

Centro: $C=(1,-3)$, raio: $r=\sqrt{1}=1$

Exercício 5

a)

Vetor perpendicular a r: $\overrightarrow{v}=(2,-1)$
Declive da reta s: $m=-\dfrac{1}{2}$

$$\begin{aligned} s: & y=-\frac{1}{2}x+b \\ & 1=-\frac{1}{2}*1+b \Leftrightarrow b=\frac{3}{2} \\ s: & y=-\frac{1}{2}x+\frac{3}{2} \end{aligned}$$

b)

$$d_{P,r}=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|2*1-1*1+3|}{\sqrt{2^2+(-1)^2}}=\frac{4}{\sqrt{5}}=\frac{4\sqrt{5}}{5}$$

Exercício 6

$$\begin{aligned} -2\sin x-\sqrt{2}=0 \Leftrightarrow -2\sin x=\sqrt{2} \Leftrightarrow \sin x=-\frac{\sqrt{2}}{2} \\ \Leftrightarrow x=-\frac{\pi}{4}+2k\pi, k\in\mathbb{Z} \vee x=\pi+\frac{\pi}{4}+2k\pi, k\in\mathbb{Z} \\ \Leftrightarrow x=\frac{\pi}{4}+2k\pi, k\in\mathbb{Z} \vee x=\frac{5\pi}{4}+2k\pi, k\in\mathbb{Z} \end{aligned}$$

Exercício 7

$$\frac{\sin x\cos x}{\tan x}=\frac{\sin x\cos x}{\frac{\sin x}{\cos x}}=\frac{\sin x\cos x\cos x}{\sin x}=cos^2x$$

Exercício 8

$$\begin{aligned} -x+1=0 \Leftrightarrow -x=-1 \Leftrightarrow x=1 \\ x^2+1=0 \Leftrightarrow x^2=-1 \rightarrow \text{ impossível} \end{aligned}$$

x	$-\infty$		1		$+\infty$
$-x+1$	$+$	$+$	0	$-$	$-$
x^2+1	$+$	$+$	$+$	$+$	$+$
$\dfrac{-x+1}{x^2+1}$	$+$	$+$	0	$-$	$-$

$S =]-\infty, 1]$

Exercício 9

a)

$$1 + \frac{n+1}{n} = \frac{11}{5} \Leftrightarrow \frac{n+1}{n} = \frac{6}{5} \Leftrightarrow 5(n+1) = 6n \Leftrightarrow 5n+5 = 6n \Leftrightarrow n = 5 \in \mathbb{N}$$

Portanto, $\dfrac{11}{5}$ é um termo da sucessão.

b)

$$\begin{aligned} u_{n+1} - u_n &= 1 + \frac{(n+1)+1}{n+1} - \left(1 + \frac{n+1}{n}\right) = \frac{n+2}{n+1} - \frac{n+1}{n} = \\ &= \frac{n(n+2)}{n(n+1)} - \frac{(n+1)^2}{n(n+1)} = \frac{n^2+2n-(n^2+2n+1)}{n^2+n} \\ &= -\frac{1}{n^2+n} < 0, \forall n \in \mathbb{N} \end{aligned}$$

Portanto, $(u_n)_n$ é estritamente decrescente.

c)

$$\lim_n u_n = \lim\left(1 + \frac{n+1}{n}\right) = \lim\left(1 + 1 + \frac{1}{n}\right) = 2 \in \mathbb{R}$$

Portanto, $(u_n)_n$ é convergente. Além disso, é limitada porque todas as sucessões convergentes são limitadas.

Exercício 10

a)

$$\lim_n \frac{2n-5}{\sqrt{4n^2+1}} = \lim_n \frac{n(2-\frac{5}{n})}{\sqrt{n^2(4+\frac{1}{n^2})}} = \lim_n \frac{2-\frac{5}{n}}{\sqrt{4+\frac{1}{n^2}}} = \frac{2-0}{\sqrt{4+0}} = \frac{2}{2} = 1$$

b)

$$\begin{aligned} \lim_n \left(\frac{n+1}{n-2}\right)^{3n} &= \lim_n \left(\frac{n(1+\frac{1}{n})}{n(1-\frac{2}{n})}\right)^{3n} = \lim_n \frac{\left((1+\frac{1}{n})^n\right)^3}{\left((1-\frac{2}{n})^n\right)^3} = \frac{e^3}{(e^{-2})^3} = \frac{e^3}{e^{-6}} \\ &= e^{3-(-6)} = e^9 \end{aligned}$$

Exercício 11

a) $\lim_{x \to 0^-} f(x) = -2$

b) $\lim_{x \to -2^-} f(x) = -6$

c) $\lim_{x \to +\infty} f(x) = -\infty$

Exercício 12

a)

$$\begin{aligned} &2^{x-1} > 0 \Leftrightarrow -2^{x-1} < 0 \Leftrightarrow 10 - 2^{x-1} < 10 \\ &Df = \mathbb{R},\ D'f =]-\infty, 10[\end{aligned}$$

b)

$$\begin{aligned} Df^{-1} =]-\infty, 10[,\ D'f^{-1} = \mathbb{R} \\ 10 - 2^{x-1} = y \Leftrightarrow 2^{x-1} = -y + 10 \Leftrightarrow x - 1 = \log_2(-y + 10) \\ \Leftrightarrow x = \log_2(-y + 10) + 1 \end{aligned}$$

$$\begin{array}{cccc} f: &]-\infty, 10[& \to & \mathbb{R} \\ & x & \mapsto & \log_2(-x + 10) + 1 \end{array}$$

c)

$$10 - 2^{x-1} = -6 \Leftrightarrow 2^{x-1} = 16 \Leftrightarrow 2^{x-1} = 2^4 \Leftrightarrow x - 1 = 4 \Leftrightarrow x = 5$$

$$S = \{5\}$$

Exercício 13

$$\begin{aligned} f'(x) &= -\frac{4x^3}{4} + 2 * 2x = -x^3 + 4x \\ f'(1) &= -1^3 + 4 * 1 = 3 \\ y &= 3x + b \\ f(1) &= -\frac{1^4}{4} + 2 * 1^2 = -\frac{1}{4} + \frac{8}{4} = \frac{7}{4} \\ \frac{7}{4} &= 3 * 1 + b \Leftrightarrow x = \frac{7}{4} - \frac{12}{4} \Leftrightarrow x = -\frac{5}{4} \\ y &= 3x - \frac{5}{4} \end{aligned}$$