Tópicos de Matemática II - 2017/ 2018
2º Teste – Tópicos de resolução

Exercício 1

a)

$$
\begin{array}{c|cccc}
 & 1 & 1 & -3 & -3 \\
-1 & & -1 & 0 & 3 \\
\hline
 & 1 & 0 & -3 & \boxed{0 = \text{Resto}}
\end{array}
$$

Logo: $p(x) = [x-(-1)](x^2-3) = (x+1)(x^2-3)$

b)

i) $\lim\limits_{x\to-\infty} \dfrac{p(x)}{-x^2+1} = \lim\limits_{x\to-\infty} \dfrac{x^3\left(1+\frac{1}{x}-\frac{3}{x^2}-\frac{3}{x^3}\right)}{x^2\left(-1+\frac{1}{x^2}\right)} = \lim\limits_{x\to-\infty} \dfrac{x\left(1+\frac{1}{x}-\frac{3}{x^2}-\frac{3}{x^3}\right)}{-1+\frac{1}{x^2}} = \lim\limits_{x\to-\infty} \dfrac{(-\infty)(1+0-0-0)}{-1+0} = +\infty$

ii) $\lim\limits_{x\to-1} \dfrac{p(x)}{x^2-1} = \lim\limits_{x\to-1} \dfrac{(x+1)(x^2-3)}{(x+1)(x-1)} = \lim\limits_{x\to-1} \dfrac{x^2-3}{x-1} = \dfrac{1-3}{-1-1} = 1$

Exercício 2

Cálculos auxiliares:

- $1-x^2=0 \Leftrightarrow x^2=1 \Leftrightarrow x=\pm 1$
- $x^2-x=0 \Leftrightarrow x(x-1)=0 \Leftrightarrow x=0 \vee x=1$

x	$-\infty$	-1		0		1	$+\infty$
$1-x^2$	$-$	0	$+$	$+$	$+$	0	$-$
x^2-x	$+$	$+$	$+$	0	$-$	0	$+$
$\dfrac{1-x^2}{x^2-x}$	$-$	0	$+$	s.s.	$-$	s.s.	$-$

C.S. = $[-1, 0[$

Exercício 3

a) $D_f = IR$

$e^{2x+1} > 0 \Leftrightarrow -e^{2x+1} < 0 \Leftrightarrow 10 - e^{2x+1} < 10$

$D'_f = \left]-\infty, 10\right[$

b) $D_{f^{-1}} = D'_f = \left]-\infty, 10\right[$

$D'_{f^{-1}} = D_f = IR$

$y = f(x) \Leftrightarrow y = 10 - e^{2x+1} \Leftrightarrow e^{2x+1} = 10 - y \Leftrightarrow 2x+1 = \ln(10-y) \Leftrightarrow x = \dfrac{\ln(10-y)-1}{2}$

$$\begin{array}{cccc} f^{-1}: \left]-\infty, 10\right[& \to & IR \\ x & \mapsto & \dfrac{\ln(10-x)-1}{2} \end{array}$$

c) $10 - e^{2x+1} \geq 0 \Leftrightarrow -e^{2x+1} \geq -10 \Leftrightarrow e^{2x+1} \leq 10 \Leftrightarrow 2x+1 \leq \ln 10 \Leftrightarrow x \leq \dfrac{-1+\ln 10}{2}$

C.S. = $\left]-\infty, \dfrac{-1+\ln 10}{2}\right]$

Exercício 4

a) 2 **b)** 1 **c)** 1 **d)** $-\infty$

Exercício 5

a) $4^x(x^2 - x) = 0 \Leftrightarrow 4^x = 0 \vee x^2 - x = 0 \Leftrightarrow x \in \phi \vee x(x-1) = 0 \Leftrightarrow x = 0 \vee x = 1$

C.S. = $\{0, 1\}$

b) $\log_3(4x-3) = 2 \Leftrightarrow 4x - 3 = 3^2 \wedge x > \dfrac{3}{4} \Leftrightarrow 4x = 12 \wedge x > \dfrac{3}{4} \Leftrightarrow x = 3$

C.S. = $\{3\}$

Exercício 6

a) $y' = 6x^2 + 10\ x$

b) $y' = \dfrac{2e^{2x}(x+1) - e^{2x}}{(x+1)^2} = \dfrac{2xe^{2x} + 2e^{2x} - e^{2x}}{(x+1)^2} = \dfrac{2xe^{2x} + e^{2x}}{(x+1)^2}$

b) $y' = \dfrac{1}{x}(x+1) + \ln x = \dfrac{x + 1 + x\ln x}{x}$

Exercício 7

$$\log_c\left(\frac{c}{\sqrt[3]{b^2}}\right) = \log_c c - \log_c b^{\frac{2}{3}} = 1 - \frac{2}{3}\log_c b = 1 - \frac{2}{3}\times 2 = 1 - \frac{4}{3} = -\frac{1}{3}$$

Outra possível resolução:

$\log_c b = 2 \Leftrightarrow b = c^2$. Então:

$$\log_c\left(\frac{c}{\sqrt[3]{b^2}}\right) = \log_c\left(\frac{c}{\sqrt[3]{c^4}}\right) = \log_c\left(\frac{c}{c^{\frac{4}{3}}}\right) = \log_c\left(c^{-\frac{1}{3}}\right) = -\frac{1}{3}$$