Proposta de Teste N.º 3

Grupo II

Ex 1.

1.1. Se $u = y^2$, então $E = \dfrac{\sqrt[3]{y^2 \cdot \sqrt[4]{(y^2 \cdot y)^2}}}{y\sqrt{y^2 \cdot y}} \times \sqrt[3]{y \cdot y^2}$

$$= \frac{\sqrt[3]{y^2 \cdot \sqrt{y^3}} \times \sqrt[3]{y^4}}{y^2\sqrt{y}} = \frac{\sqrt[3]{\sqrt{y^4 \times y^3}} \cdot \cancel{y}\sqrt[3]{y}}{y^{\cancel{2}}\sqrt{y}}$$

$$= \frac{\sqrt[6]{y^7} \times \sqrt[3]{y}}{\cancel{y}\sqrt{y}} = \frac{\cancel{y}\sqrt[6]{y} \times \sqrt[3 \times 2]{y^{2 \times 2}}}{\cancel{y}\sqrt[\times 3]{y^{\times 3}}} = \sqrt[6]{\frac{y \cdot y^2}{y^3}} = \sqrt[6]{\frac{y^3}{y^3}} =$$

$$= \sqrt[6]{1} = 1$$

1.2. $E = \dfrac{\sqrt[3]{uy}}{y}$

$$E = \frac{\sqrt[3]{u\sqrt[4]{(uy)^2}}}{y\sqrt{uy}} \times \sqrt[3]{uy^2} = \frac{\sqrt[3]{\sqrt{uy \cdot u^2}} \times \sqrt[3 \times 2]{u^2 y^{2 \times 2}}}{\sqrt[\times 3]{u^3 y^{3 \times 3}}}$$

$$= \sqrt[6]{\frac{\cancel{u^3} \cdot y \cdot u^2 y^4}{\cancel{u^3} \cdot y^9}} = \sqrt[6:2]{\frac{u^{2:2}}{y^{4:2}}} = \sqrt[3]{\frac{u}{y^2}} = \frac{\sqrt[3]{u}}{\sqrt[3]{y^2}} \times \frac{\sqrt[3]{y}}{\sqrt[3]{y}}$$

$$= \frac{\sqrt[3]{uy}}{y}$$

1.3. $u = 3$; $y = 9$ $\rightarrow$ $E = \dfrac{\sqrt[3]{27}}{9} = \dfrac{3}{9} = \dfrac{1}{3}$

$$\frac{6\sqrt{E} + 1}{\sqrt{3} + 2} = \frac{6 \cdot \sqrt{1/3} + 1}{\sqrt{3} + 2} = \frac{6 \cdot \frac{1}{\sqrt{3}} + 1}{\sqrt{3} + 2} = \frac{6 \cdot \frac{\sqrt{3}}{3} + 1}{\sqrt{3} + 2}$$

$$= \frac{2\sqrt{3} + 1}{\sqrt{3} + 2} \times \frac{\sqrt{3} - 2}{\sqrt{3} - 2} = \frac{2 \times 3 - 4\sqrt{3} + \sqrt{3} - 2}{(\sqrt{3})^2 - 2^2} = \frac{6 - 3\sqrt{3} - 2}{3 - 4} =$$

$$= \frac{4 - 3\sqrt{3}}{-1} = \cancel{4} - 4 + 3\sqrt{3}$$

Ex 2

2.1. $\begin{cases} B(1) = 6 \\ B(-3) = -2 \end{cases} \Leftrightarrow \begin{cases} b \times 1^3 + (2b+a) \times 1^2 + (2a-b) \times 1 + a = 6 \\ b \times (-3)^3 + (2b+a) \times (-3)^2 + (2a-b) \times (-3) + a = -2 \end{cases}$

$\Leftrightarrow \begin{cases} \cancel{b} + 2b + a + 2a - \cancel{b} + a = 6 \\ -27b + 18b + 9a - 6a + 3b + a = -2 \end{cases}$

$\Leftrightarrow \begin{cases} 4a + 2b = 6 \\ -6b + 4a = -2 \end{cases} \underset{:2}{\Leftrightarrow} \begin{cases} 2a + b = 3 \\ -3b + 2a = -2 \end{cases} \Leftrightarrow \begin{cases} b = 3 - 2a \\ -3(3 - 2a) + 2a = -1 \end{cases}$

$\Leftrightarrow \begin{cases} \text{———} \\ -9 + 6a + 2a = -1 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ 8a = 8 \end{cases} \Leftrightarrow \begin{cases} b = 3 - 2 \times 1 \\ a = 1 \end{cases} \Leftrightarrow \begin{cases} b = 1 \\ a = 1 \end{cases}$

$\therefore \; a = b = 1$

Logo, $B(x) = x^3 + 3x^2 + x + 1$

2.2. $B(x) \geq 5x + 1$

$\Leftrightarrow x^3 + 3x^2 + x + \cancel{1} \geq 5x + \cancel{1} \Leftrightarrow x^3 + 3x^2 - 4x \geq 0$

$\Leftrightarrow x(x^2 + 3x - 4) \Leftrightarrow x(x+4)(x-1) \geq 0$

C Aux:

$x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-4)}}{2 \times 1} \qquad x = \dfrac{-3 \pm \sqrt{25}}{2} \qquad x = -4 \vee x = 1$

$\therefore \; x^2 + 3x - 4 = (x+4)(x-1)$

	$-\infty$	-4		0		1	$+\infty$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+4$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
Produto	$-$	0	$+$	0	$-$	0	$+$

$C.S. = [-4, 0] \cup [1, +\infty[$

Ex 3.

3.1. Tem-se que:

$P(x) = (x^3 - 2x + 1)(3x + 1) + (-20x^2 - 21x + 23) =$

$= 3x^4 + x^3 - 6x^2 - 2x + 3x + 1 - 20x^2 - 21x + 23 =$

$= 3x^4 + x^3 - 26x^2 - 20x + 24.$

	3	1	-26	-20	24
-2	↓	-6	10	32	-24
	3	-5	-16	12	0
-2	↓	-6	22	-12	
	3	-11	6	0	
-2	↓	-6	34		
	3	-17	~~40~~ 70		

→ -2 é raiz de P

∴ -2 é raiz de multiplicidade 2 de P

3.2. $P(x) = (x+2)^2 (3x^2 - 11x + 6)$

C Aux:

$x = \frac{11 \pm \sqrt{(-11)^2 - 4 \times 3 \times 6}}{2 \times 3} \Leftrightarrow x = \frac{11 \pm \sqrt{49}}{6} \Leftrightarrow x = \frac{4}{6} \vee x = \frac{18}{6}$

$\Leftrightarrow x = \frac{2}{3} \vee x = 3$

∴ zeros de $p = \left\{-2, \frac{2}{3}, 3\right\}$

não esquecer

Logo, $P(x) = 3 (x+2)^2 \left(x - \frac{2}{3}\right)(x-3)$

3.3. $3(x+2)^3 (x-3)^2 \left(x - \frac{2}{3}\right) \leq 0$

	$-\infty$		-2		$2/3$		3	$+\infty$
$3(x+2)^3$	−		0	+	+	+	+	+
$(x-3)^2$	+		+	+	+	+	0	+
$(x-2/3)$	−		−	−	0	+	+	+
Produto	+		0	−	0	+	0	+

C.S. = $[-2; 2/3] \cup \{3\}$

Ex4

4.1. $2x^2 + 2y^2 - 4x + 16y - 16 = 0$

$\underset{:2}{\Leftrightarrow} x^2 + y^2 - 2x + 8y - 8 = 0$

$\Leftrightarrow x^2 - 2x + 1^2 + y^2 + 8y + 4^2 = 8 + 1^2 + 4^2$

$\Leftrightarrow (x-1)^2 + (y+4)^2 = 25$

$\therefore$ Centro $(1, -4)$, Raio $= \sqrt{25} = 5$

4.2. i) $A \in Ox \Rightarrow A(x, 0)$, $x > 0$

$A \in$ circunferência : $(x-1)^2 + (0+4)^2 = 25 \Leftrightarrow$

$\Leftrightarrow x - 1 = \pm\sqrt{9} \Leftrightarrow x - 1 = 3 \lor x - 1 = -3$

$\Leftrightarrow x = 4 \lor x = -2$

$\therefore A(4, 0)$

ii) C tem a mesma abcissa que A. Logo $C(4, 4)$ e $C \in$ circunferência :

$(4-1)^2 + (y+4)^2 = 25 \Leftrightarrow (y+4)^2 = 25 - 9 \Leftrightarrow$

$\Leftrightarrow y + 4 = \pm\sqrt{16} \Leftrightarrow y + 4 = -4 \lor y + 4 = 4 \Leftrightarrow$

$\Leftrightarrow y = -8 \lor y = 0$

↳ ordenada do A

$\therefore C(4, -8)$

iii) B tem abcissa -3 e pertence à circunferência :

$(-3-1)^2 + (y+4)^2 = 25 \Leftrightarrow (y+4)^2 = 25 - 16$

$\Leftrightarrow y + 4 = \pm\sqrt{9} \Leftrightarrow y + 4 = -3 \lor y + 4 = 3 \Leftrightarrow$

$\Leftrightarrow y = -7 \lor y = -1$

A ordenada B é maior que a ordenada do centro da circunferência $(1, -4)$

$\therefore B(-3, -1)$

4.3. $B(-3, -1)$, $C(4, -8)$

$(x+3)^2 + (y+1)^2 = (x-4) + (y+8)^2$

$\Leftrightarrow x^2 + 6x + 9 + y^2 + y + 2y + 1 = x^2 - 8x + 16 + y^2 + 16y + 64$

$\Leftrightarrow -14y = -14x + 70 \Leftrightarrow y = x - 5$ //

4.4. $P\ (2y+2\ ;\ y)$

$d(P,C) = 9 \iff \left(\sqrt{(2y+2-4)^2 + (y+8)^2}\right)^2 = 9^2$

$\iff (2y-2)^2 + (y+8)^2 = 81$

$\iff 4y^2 - 8y + 4 + y^2 + 16y + 64 = 81$

$\iff 5y^2 + 8y - 13 = 0$

$\iff y = \dfrac{-8 \pm \sqrt{8^2 - 4\times 5\times(-13)}}{2\times 5}$

$\iff y = \dfrac{-8 \pm \sqrt{324}}{10}$

$\iff y = \dfrac{-8-18}{10} \ \vee\ y = \dfrac{-8+18}{10}$

$\iff y = -\dfrac{13}{5} \ \vee\ y = 1$

$\therefore P\ (2\times 1 + 2\ ;\ 1) = (4, 1)$

$P\left(2\times\left(-\dfrac{13}{5}\right) + 2,\ -\dfrac{13}{5}\right) = \left(-\dfrac{16}{5},\ -\dfrac{13}{5}\right)$

4.5. $s \parallel Oy$ e $A(4,0) \in s \Rightarrow s: x = 4$

$t \parallel Ox$ e $B(-3,-1) \in t \Rightarrow t: y = -1$

$R: y = x$

$(x-1)^2 + (y+4)^2 \leq 25 \wedge x \leq 4 \wedge y \leq 1 \wedge y \leq x$

Ex 5

5.1. a) $d(P, F_1) + d(P, F_2) = 2a$

$\left(5, \dfrac{4\sqrt{6}}{7}\right) \in$ Elipse :

$(5k)^2 + 49\times\left(\dfrac{4\sqrt{6}}{7}\right)^2 = 196 \iff 25k^2 + \cancel{49}\times\dfrac{4^2\times 6}{\cancel{49}} = 196$

$\iff 25k^2 = 196 - 16\times 6 \iff 25k^2 = 100 \iff k^2 = 4$

Logo, $(kx)^2 + 49y^2 = 196$

$\Leftrightarrow k^2 \cdot x^2 + 49y^2 = 196$

$\Leftrightarrow 4x^2 + 49y^2 = 196$

$\Leftrightarrow \dfrac{4x^2}{196} + \dfrac{49y^2}{196} = 1$

$\Leftrightarrow \dfrac{x^2}{49} + \dfrac{y^2}{4} = 1$

$\therefore a^2 = 49 \Rightarrow a = 7$ eixo maior é $2 \times 7 = 14$

$b^2 = 4 \Rightarrow b = 2$ $\therefore d(P, F_1) + d(P, F_2) = 14$ //

b) i) eixo menor $= 2 \times 2 = 4$

ii) $a^2 = b^2 + c^2 \Leftrightarrow$

$\Leftrightarrow 49 = 4 + c^2 \Leftrightarrow$

$\Leftrightarrow c^2 = 45 \Leftrightarrow$

$\Leftrightarrow c = \pm\sqrt{45}$

$\Leftrightarrow c = \pm 3\sqrt{5}$

$$\begin{array}{r|l} 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$\therefore F_1(3\sqrt{5}, 0)$ e $F_2(-3\sqrt{5}, 0)$

5.2. $B(3\sqrt{5}, y) \in$ Elipse

$\dfrac{(3\sqrt{5})^2}{49} + \dfrac{y^2}{4} = 1 \Leftrightarrow \dfrac{y^2}{4} = 1 - \dfrac{9 \times 5}{49} \Leftrightarrow$

$\Leftrightarrow y^2 = \dfrac{4 \times 4}{49} \Leftrightarrow y = \pm\sqrt{\dfrac{16}{49}} \Leftrightarrow y = \pm\dfrac{4}{7}$ $\therefore B\left(3\sqrt{5}, \dfrac{4}{7}\right)$

Logo $\overline{AB} = 2 \times \dfrac{4}{7} = \dfrac{8}{7}$

$\therefore A_{[ABCD]} = \overline{AD} \times \overline{AB} = 6\sqrt{5} \times \dfrac{8}{7} = \dfrac{48\sqrt{5}}{7}$ //