# Proposta de Resolução da Ficha de Trabalho 3

**Matemática A**

**12.º Ano de Escolaridade** | Turma: $B + C + H$

1. .

1.1. Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = 0$

Assim,

$$\lim f(x_n) = \lim \frac{4x_n - 2}{x_n - 4} = \frac{4\lim(x_n) - 2}{\lim(x_n) - 4} = \frac{0-2}{0-4} = \frac{1}{2}$$

Portanto,

$$\lim_{x\to 0} f(x) = \frac{1}{2}$$

1.2. Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = 4^+$

Assim,

$$\lim f(x_n) = \lim \frac{4x_n - 2}{x_n - 4} = \frac{4\lim(x_n) - 2}{\lim(x_n) - 4} = \frac{14}{0^+} = +\infty$$

Portanto,

$$\lim_{x\to 4^+} f(x) = +\infty$$

1.3. Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = -1$

Assim,

$$\lim f(x_n) = \lim \frac{4x_n - 2}{x_n - 4} = \frac{4\lim(x_n) - 2}{\lim(x_n) - 4} = \frac{-6}{-5} = \frac{6}{5}$$

Portanto,

$$\lim_{x\to -1} f(x) = \frac{6}{5}$$

1.4. Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = 4^-$

Assim,

$$\lim f(x_n) = \lim \frac{4x_n - 2}{x_n - 4} = \frac{4\lim(x_n) - 2}{\lim(x_n) - 4} = \frac{14}{0^-} = -\infty$$

Portanto,

$$\lim_{x\to 4^-} f(x) = -\infty$$

2. .

2.1. Seja, $f$, a função racional, definida por $f(x) = \dfrac{3}{x-1}$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = +\infty$

Assim,

$\lim f(x_n) = \lim \dfrac{3}{x_n - 1} = \dfrac{3}{\lim(x_n) - 1} = \dfrac{3}{+\infty} = 0$

Portanto, $\lim\limits_{x \to +\infty} f(x) = 0$

Ou seja,

$\lim\limits_{x \to +\infty} \dfrac{3}{x-1} = 0$

2.2. Seja, $f$, a função racional, definida por $f(x) = \dfrac{4}{2x^2 - 1}$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = +\infty$

Assim,

$\lim f(x_n) = \lim \dfrac{4}{2(x_n)^2 - 1} = \dfrac{4}{2(\lim(x_n))^2 - 1} = \dfrac{4}{+\infty} = 0$

Portanto, $\lim\limits_{x \to +\infty} f(x) = 0$

Ou seja,

$\lim\limits_{x \to +\infty} \dfrac{4}{2x^2 - 1} = 0$

2.3. Seja, $f$, a função racional, definida por $f(x) = \left(5 + \dfrac{1}{2+x}\right)$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = +\infty$

Assim,

$\lim f(x_n) = \lim \left(5 + \dfrac{1}{2 + x_n}\right) = 5 + \dfrac{1}{2 + \lim(x_n)} = 5 + \dfrac{1}{+\infty} = 5 + 0 = 5$

Portanto, $\lim\limits_{x \to +\infty} f(x) = 5$

Ou seja,

$\lim\limits_{x \to +\infty} \left(5 + \dfrac{1}{2+x}\right) = 5$

2.4. Seja, $f$, a função racional, definida por $f(x) = \dfrac{1}{3 - 2x}$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = -\infty$

Assim,

$\lim f(x_n) = \lim \dfrac{1}{3 - 2x_n} = \dfrac{1}{3 - 2\lim(x_n)} = \dfrac{1}{+\infty} = 0$

Portanto, $\lim\limits_{x \to -\infty} f(x) = 0$

Ou seja,

$\lim\limits_{x \to -\infty} \dfrac{1}{3 - 2x} = 0$

2.5. Seja, $f$, a função racional, definida por $f(x) = \dfrac{2}{3 - x^2}$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = -\infty$

Assim,

$\lim f(x_n) = \lim \dfrac{2}{3 - (x_n)^2} = \dfrac{2}{3 - (\lim(x_n))^2} = \dfrac{2}{-\infty} = 0$

Portanto, $\lim_{x\to-\infty} f(x) = 0$

Ou seja,

$$\lim_{x\to-\infty} \frac{2}{3-x^2} = 0$$

2.6. Seja, $f$, a função racional, definida por $f(x) = \left(-3 - \frac{2}{4x+1}\right)$

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = -\infty$

Assim,

$$\lim f(x_n) = \lim\left(-3 - \frac{2}{4x_n+1}\right) = -3 - \frac{2}{4\lim(x_n)+1} = -3 - \frac{2}{-\infty} = -3+0 = -3$$

Portanto, $\lim_{x\to-\infty} f(x) = -3$

Ou seja,

$$\lim_{x\to-\infty} \left(-3 - \frac{2}{4x+1}\right) = -3$$

3. .

Seja, $(x_n)$, uma sucessão de valores pertencente ao domínio de $f$, e tal que $\lim(x_n) = +\infty$

Assim,

$$\lim f(x_n) = \lim\left(3 + \frac{1}{x_n^2+x_n+1}\right) = 3 + \frac{1}{(\lim(x_n))^2+\lim(x_n)+1} = 3 + \frac{1}{+\infty} = 3+0 = 3$$

Portanto, $\lim_{x\to+\infty} f(x) = 3$

4. .

4.1. Sabe-se que , $-2 \in D_f$

Assim, existe $\lim_{x\to-2} g(x)$, se, $\lim_{x\to-2^-} f(x) = \lim_{x\to-2^+} f(x) = f(-2)$

Ora, como,

$$\lim_{x\to-2^-} f(x) = 4$$

$$\lim_{x\to-2^+} f(x) = -2$$

Então, não existe $\lim_{x\to-2} f(x)$

4.2. Sabe-se que , $-2 \notin D_f$

Assim, existe $\lim_{x\to-2} f(x)$, se, $\lim_{x\to-2^-} f(x) = \lim_{x\to-2^+} f(x)$

Ora, como,

$$\lim_{x\to-2^-} f(x) = -2$$

$$\lim_{x\to-2^+} f(x) = -2$$

Então, existe $\lim_{x\to-2} f(x) = -2$

5. .

5.1. Ora,

$$\lim_{x\to 1} f(x) = \lim_{x\to 1}\left(1+\frac{1}{x^2+1}\right) = 1+\frac{1}{2} = \frac{3}{2}$$

$$\lim_{x\to 1} g(x) = \lim_{x\to 1}\frac{x}{x+1} = \frac{1}{2}$$

Logo, $\lim\limits_{x\to 1}(f+g)(x) = \lim\limits_{x\to 1} f(x) + \lim\limits_{x\to 1} g(x) = \frac{3}{2}+\frac{1}{2} = \frac{4}{2} = 2$

5.2. Ora,

$$\lim_{x\to -2} f(x) = \lim_{x\to -2}\left(1+\frac{1}{x^2+1}\right) = 1+\frac{1}{5} = \frac{6}{5}$$

$$\lim_{x\to -2} g(x) = \lim_{x\to -2}\frac{x}{x+1} = \frac{-2}{-1} = 2$$

Logo, $\lim\limits_{x\to -2}(g-f)(x) = \lim\limits_{x\to -2} g(x) - \lim\limits_{x\to -2} f(x) = 2-\frac{6}{5} = \frac{4}{5}$

5.3. $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+}\left(1+\frac{1}{x^2+1}\right) = 1+\frac{1}{2} = \frac{3}{2}$

$$\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+}\frac{x}{x+1} = \frac{-1}{0^+} = -\infty$$

Logo, $\lim\limits_{x\to -1^+}(f\times g)(x) = \lim\limits_{x\to -1^+} f(x) \times \lim\limits_{x\to -1^+} g(x) = \frac{3}{2}\times(-\infty) = -\infty$

5.4. $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-}\left(1+\frac{1}{x^2+1}\right) = 1+\frac{1}{2} = \frac{3}{2}$

$$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-}\frac{x}{x+1} = \frac{-1}{0^-} = +\infty$$

Logo,

$$\lim_{x\to -1^-}\left(\frac{f}{g}\right)(x) = \frac{\lim\limits_{x\to -1^-} f(x)}{\lim\limits_{x\to -1^-} g(x)} = \frac{\frac{3}{2}}{+\infty} = 0$$

6. .

6.1. .

$$\lim_{x\to -1}\frac{x^3+1}{x^3+2x^2-3x-4} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{(x+1)(x^2+x-4)} = \lim_{x\to -1}\frac{x^2-x+1}{x^2+x-4} = \frac{3}{-4} = -\frac{3}{4}$$

**Cálculo auxiliar**

Se, $-1$ é zero de $x^3+1$

Então,

$x^3+1 = (x+1)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{r|rrr|r} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

Logo, $Q(x) = x^2-x+1$

$x^3+1 = (x+1)\times(x^2-x+1)$

Se $-1$ é zero de $x^3+2x^2-3x-4$

Então,

$x^3+2x^2-3x-4=(x+1)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{r|rrr|r} & 1 & 2 & -3 & -4 \\ & & & & \\ -1 & & -1 & -1 & 4 \\ \hline & 1 & 1 & -4 & 0 \end{array}$$

Logo, $Q(x)=x^2+x-4$

$x^3+2x^2-3x-4=(x+1)\times(x^2+x-4)$

6.2. $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{x^3+1}{x^3+2x^2-3x-4}=^{\left(\frac{\infty}{\infty}\right)}\lim\limits_{x\to+\infty}\dfrac{x^3\left(1+\dfrac{1}{x^3}\right)}{x^3\left(1+\dfrac{2x^2}{x^3}-\dfrac{3x}{x^3}-\dfrac{4}{x^3}\right)}=\dfrac{\lim\limits_{x\to+\infty}\left(1+\dfrac{1}{x^3}\right)}{\lim\limits_{x\to+\infty}\left(1+\dfrac{2}{x}-\dfrac{3}{x^2}-\dfrac{4}{x^3}\right)}=$

$=\dfrac{1+0}{1+0-0-0}=1$

7. .

7.1. $\lim\limits_{x\to-2}\dfrac{2x+x^2}{x+2}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to-2}\dfrac{x(x+2)}{x+2}=\lim\limits_{x\to-2}(x)=-2$

7.2. $\lim\limits_{x\to-1}\dfrac{2x+2}{1-x^2}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to-1}\dfrac{2(x+1)}{(1-x)(1+x)}=\lim\limits_{x\to-1}\dfrac{2}{1-x}=\dfrac{2}{2}=1$

7.3. $\lim\limits_{x\to3^+}\dfrac{x^2-9}{(x-3)^2}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to3^+}\dfrac{(x-3)(x+3)}{(x-3)^2}=\lim\limits_{x\to3^+}\dfrac{x+3}{x-3}=\dfrac{6}{0^+}=+\infty$

7.4. $\lim\limits_{x\to-5^-}\dfrac{3x+15}{x^2+10x+25}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to-5^-}\dfrac{3(x+5)}{(x+5)^2}=\lim\limits_{x\to-5^-}\dfrac{3}{x+5}=\dfrac{3}{0^-}=-\infty$

7.5. $\lim\limits_{x\to\sqrt{2}^+}\dfrac{x^2-2}{x^2-2\sqrt{2}x+2}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to\sqrt{2}^+}\dfrac{(x-\sqrt{2})(x+\sqrt{2})}{(x-\sqrt{2})^2}=\lim\limits_{x\to\sqrt{2}^+}\dfrac{x+\sqrt{2}}{x-\sqrt{2}}=\dfrac{2\sqrt{2}}{0^+}=+\infty$

7.6. $\lim\limits_{x\to1}\dfrac{2x^2-x-1}{x^3-1}=^{\left(\frac{0}{0}\right)}\lim\limits_{x\to1}\dfrac{(x-1)(2x+1)}{(x-1)(x^2+x+1)}=\lim\limits_{x\to1}\dfrac{2x+1}{x^2+x+1}=\dfrac{3}{3}=1$

**Cálculo auxiliar**

- Se 1 é zero de $x^3-1$

  Então,

  $x^3-1=(x-1)\times Q(x)$

  Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

  $$\begin{array}{r|rrr|r} & 1 & 0 & 0 & -1 \\ & & & & \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

  Logo, $Q(x)=x^2+x+1$

  $x^3-1=(x-1)\times(x^2+x+1)$

- Se 1 é zero de $2x^2-x-1$

  Então,

$2x^2 - x - 1 = (x-1) \times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|cc|c} & 2 & -1 & -1 \\ & & & \\ 1 & & 2 & 1 \\ \hline & 2 & 1 & 0 \end{array}$$

Logo, $Q(x) = 2x + 1$

$2x^2 - x - 1 = (x-1) \times (2x+1)$

7.7. $\lim\limits_{x\to-\infty}(x^4+x+1) =^{(\infty-\infty)} \lim\limits_{x\to-\infty}\left[x^4\left(1-\frac{x}{x^4}+\frac{1}{x^4}\right)\right] = \lim\limits_{x\to-\infty}(x^4) \times \lim\limits_{x\to-\infty}\left(1-\frac{1}{x^3}+\frac{1}{x^4}\right) =$

$= +\infty \times \left(1 - \frac{1}{-\infty} + \frac{1}{+\infty}\right) = +\infty \times (1+0+0) = +\infty$

7.8. $\lim\limits_{x\to-\infty}(x^3+3x^2+x) =^{(\infty-\infty)} \lim\limits_{x\to-\infty}\left[x^3\left(1+\frac{3x^2}{x^3}+\frac{x}{x^3}\right)\right] = \lim\limits_{x\to-\infty}(x^3) \times \lim\limits_{x\to-\infty}\left(1+\frac{3}{x}+\frac{1}{x^2}\right) =$

$= -\infty \times \left(1 + \frac{3}{-\infty} + \frac{1}{+\infty}\right) = -\infty \times (1-0+0) = -\infty$

7.9. $\lim\limits_{x\to+\infty}(2x^2-2x+4) =^{(\infty-\infty)} \lim\limits_{x\to+\infty}\left[x^2\left(1-\frac{2x}{x^2}+\frac{4}{x^2}\right)\right] = \lim\limits_{x\to+\infty}(x^2) \times \lim\limits_{x\to+\infty}\left(1-\frac{2}{x}+\frac{4}{x^2}\right) =$

$= +\infty \times \left(1 - \frac{1}{+\infty} + \frac{4}{+\infty}\right) = +\infty \times (1-0+0) = +\infty$

7.10. $\lim\limits_{x\to+\infty}(-x^4-x^2+x-5) =^{(\infty-\infty)} \lim\limits_{x\to+\infty}\left[x^4\left(-1-\frac{x^2}{x^4}+\frac{x}{x^4}-\frac{5}{x^4}\right)\right] = \lim\limits_{x\to+\infty}(x^4) \times \lim\limits_{x\to+\infty}\left(-1-\frac{1}{x^2}+\frac{1}{x^3}-\frac{5}{x^4}\right) =$

$= +\infty \times \left(-1 - \frac{1}{+\infty} + \frac{1}{+\infty} - \frac{1}{+\infty}\right) = +\infty \times (-1-0+0-0) = -\infty$

7.11. $\lim\limits_{x\to-\infty}\frac{2x^3+x+4}{x^2-x+1} =^{(\frac{\infty}{\infty})} \lim\limits_{x\to-\infty}\frac{x^3\left(2+\frac{x}{x^3}+\frac{4}{x^3}\right)}{x^2\left(1-\frac{x}{x^2}+\frac{1}{x^2}\right)} = \lim\limits_{x\to-\infty}(x) \times \frac{\lim\limits_{x\to-\infty}\left(2+\frac{1}{x^2}+\frac{4}{x^3}\right)}{\lim\limits_{x\to-\infty}\left(1-\frac{1}{x}+\frac{1}{x^2}\right)} =$

$= -\infty \times \frac{2+\frac{1}{+\infty}+\frac{4}{-\infty}}{1-\frac{1}{-\infty}+\frac{1}{+\infty}} = -\infty \times \frac{2+0-0}{1+0+0} = -\infty$

7.12. $\lim\limits_{x\to-\infty}\frac{x^4-x+1}{x^4+3} =^{(\frac{\infty}{\infty})} \lim\limits_{x\to-\infty}\frac{x^4\left(1-\frac{x}{x^4}+\frac{1}{x^4}\right)}{x^4\left(1+\frac{3}{x^4}\right)} = \frac{\lim\limits_{x\to-\infty}\left(1-\frac{1}{x^3}+\frac{1}{x^4}\right)}{\lim\limits_{x\to-\infty}\left(1+\frac{3}{x^4}\right)} =$

$= \frac{1-\frac{1}{-\infty}+\frac{1}{+\infty}}{1+\frac{3}{+\infty}} = \frac{1+0+0}{1+0} = 1$

7.13. $\lim\limits_{x\to+\infty}\frac{4x+1}{x^3+2x-5} =^{(\frac{\infty}{\infty})} \lim\limits_{x\to+\infty}\frac{x\left(4+\frac{1}{x}\right)}{x^3\left(1+\frac{2x}{x^3}-\frac{5}{x^3}\right)} = \lim\limits_{x\to+\infty}\frac{1}{x^2} \times \frac{\lim\limits_{x\to+\infty}\left(4+\frac{1}{x}\right)}{\lim\limits_{x\to+\infty}\left(1+\frac{2}{x^2}-\frac{5}{x^3}\right)} =$

$= \frac{1}{+\infty} \times \frac{4+\frac{1}{+\infty}}{1+\frac{2}{+\infty}+\frac{5}{+\infty}} = 0 \times \frac{4+0}{1+0+0} = 0 \times 4 = 0$

7.14. $\lim\limits_{x\to-\infty}\left[\frac{2}{x+5}\times(x^2-x+1)\right] =^{(0\times\infty)} \lim\limits_{x\to-\infty}\frac{2x^2-2x+2}{x+5} =^{(\frac{\infty}{\infty})} \lim\limits_{x\to-\infty}\frac{x^2\left(2-\frac{2x}{x^2}+\frac{2}{x^2}\right)}{x\left(1+\frac{5}{x}\right)} =$

$$= \lim_{x\to-\infty}(x) \times \frac{\lim_{x\to-\infty}\left(2-\frac{2}{x}+\frac{2}{x^2}\right)}{\lim_{x\to-\infty}\left(1+\frac{5}{x}\right)} = -\infty \times \frac{2-\frac{2}{-\infty}+\frac{2}{+\infty}}{1+\frac{5}{-\infty}} = -\infty \times \frac{2+0+0}{1-0} = -\infty$$

7.15. $\lim_{x\to-5^-}\left[\frac{1}{25-x^2}\times(x+5)\right] =^{(0\times\infty)} \lim_{x\to-5^-}\frac{x+5}{25-x^2} =^{\left(\frac{0}{0}\right)} \lim_{x\to-5^-}\frac{x+5}{-(x^2-25)} =$

$= -\lim_{x\to-5^-}\frac{x+5}{(x-5)(x+5)} = -\lim_{x\to-5^-}\frac{1}{x-5} = \frac{1}{10}$

7.16. $\lim_{x\to3^+}\left[(2x-6)\times\frac{1}{x^2-6x+9}\right] =^{(0\times\infty)} \lim_{x\to3^+}\frac{2x-6}{x^2-6x+9} =^{\left(\frac{0}{0}\right)} \lim_{x\to3^+}\frac{2(x-3)}{(x-3)^2} =$

$= \lim_{x\to3^+}\frac{2}{x-3} = \frac{2}{0^+} = +\infty$

7.17. $\lim_{x\to2^-}\left[(-x^2+x+2)\times\frac{1}{-3x^3+6x^2+x-2}\right] =^{(0\times\infty)} \lim_{x\to2^-}\frac{-x^2+x+2}{-3x^3+6x^2+x-2} =^{\left(\frac{0}{0}\right)} \lim_{x\to2^-}\frac{(x-2)(-x-1)}{(x-2)(-3x^2+1)} =$

$= \lim_{x\to2^-}\frac{-x-1}{-3x^2+1} = \frac{-3}{-11} = \frac{3}{11}$

**Cálculo auxiliar**

- Se 2 é zero de $-x^2+x+2$

  Então,

  $-x^2+x+2 = (x-2)\times Q(x)$

  Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

  $$\begin{array}{c|cc|c} & -1 & 1 & 2 \\ 2 & & -2 & -2 \\ \hline & -1 & -1 & 0 \end{array}$$

  Logo, $Q(x) = -x-1$

  $-x^2+x+2 = (x-2)\times(-x-1)$

- Se 2 é zero de $-3x^3+6x^2+x-2$

  Então,

  $-3x^3+6x^2+x-2 = (x-2)\times Q(x)$

  Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

  $$\begin{array}{c|ccc|c} & -3 & 6 & 1 & -2 \\ 2 & & -6 & 0 & 2 \\ \hline & -3 & 0 & 1 & 0 \end{array}$$

  Logo, $Q(x) = -3x^2+1$

  $-3x^3+6x^2+x-2 = (x-2)\times(-3x^2+1)$

7.18. $\lim_{x\to+\infty}(\sqrt{2x+4}-\sqrt{2x+2}) =^{(\infty-\infty)} \lim_{x\to+\infty}\frac{(\sqrt{2x+4}-\sqrt{2x+2})(\sqrt{2x+4}+\sqrt{2x+2})}{\sqrt{2x+4}+\sqrt{2x+2}} =$

$= \lim_{x\to+\infty}\frac{(\sqrt{2x+4})^2-(\sqrt{2x+2})^2}{\sqrt{2x+4}+\sqrt{2x+2}} = \lim_{x\to+\infty}\frac{|2x+4|-|2x+2|}{\sqrt{2x+4}+\sqrt{2x+2}} = \lim_{x\to+\infty}\frac{2x+4-2x-2}{\sqrt{2x+4}+\sqrt{2x+2}} =$

$= \lim_{x\to+\infty}\frac{2}{\sqrt{2x+4}+\sqrt{2x+2}} = \frac{2}{+\infty} = 0$

7.19. $\lim_{x\to+\infty}(\sqrt{x^2+2x+3}-\sqrt{x^2+x+2}) =^{(\infty-\infty)}$

$$= \lim_{x\to+\infty}\frac{(\sqrt{x^2+2x+3}-\sqrt{x^2+x+2})(\sqrt{x^2+2x+3}+\sqrt{x^2+x+2})}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} =$$

$$= \lim_{x\to+\infty}\frac{(\sqrt{x^2+2x+3})^2-(\sqrt{x^2+x+2})^2}{\sqrt{x^2+2x+3}+\sqrt{x^2+x+2}} = \lim_{x\to+\infty}\frac{|x^2+2x+3|-|x^2+x+2|}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} =$$

$$= \lim_{x\to+\infty}\frac{x^2+2x+3-x^2-x-2}{\sqrt{x^2+2x+3}+\sqrt{x^2+x+2}} = \lim_{x\to+\infty}\frac{x+1}{\sqrt{x^2+2x+3}+\sqrt{x^2+x+2}} =^{(\frac{\infty}{\infty})}$$

$$= \lim_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\right)}{\sqrt{x^2\left(1+\frac{2x}{x^2}+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{x}{x^2}+\frac{2}{x^2}\right)}} = \lim_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\right)}{|x|\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+|x|\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}} =$$

$$\lim_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\right)}{|x|\left(\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}\right)} = \lim_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}\right)} =$$

$$= \lim_{x\to+\infty}\frac{1+\frac{1}{x}}{\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}} = \frac{1+\lim_{x\to+\infty}\frac{1}{x}}{\sqrt{1+\lim_{x\to+\infty}\frac{2}{x}+\lim_{x\to+\infty}\frac{3}{x^2}}+\sqrt{1+\lim_{x\to+\infty}\frac{1}{x}+\lim_{x\to+\infty}\frac{2}{x^2}}} =$$

$$\frac{1+\frac{1}{+\infty}}{\sqrt{1+\frac{2}{+\infty}+\frac{3}{+\infty}}+\sqrt{1+\frac{1}{+\infty}+\frac{2}{+\infty}}} = \frac{1}{\sqrt{1+0+0}+\sqrt{1+0+0}} = \frac{1}{2}$$

8. $2 \in D_f$

Para existir $\lim_{x\to 2} f(x)$, deve ter-se,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

Ora,

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\frac{x^2-6x+8}{x-2} = \lim_{x\to 2^-}\frac{(x-2)(x-4)}{x-2} = \lim_{x\to 2^-}(x-4) = -2$$

**Cálculo auxiliar**

Se 2 é zero de $x^2-6x+8$

Então,

$x^2-6x+8 = (x-2)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|cc|c} & 1 & -6 & 8 \\ 2 & & 2 & -8 \\ \hline & 1 & -4 & 0 \end{array}$$

Logo, $Q(x) = x-4$

$x^2-6x+8 = (x-2)\times(x-4)$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{-4x^2+4x+8}{3x^2-6x} = \lim_{x\to 2^+} \frac{(x-2)(-4x-4)}{3x(x-2)} = \lim_{x\to 2^+} \frac{-4x-4}{3x} = -2$$

**Cálculo auxiliar**

Se 2 é zero de $-4x^2+4x+8$

Então,

$-4x^2+4x+8 = (x-2)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|cc|c} & -4 & 4 & 8 \\ & & & \\ 2 & & -8 & -8 \\ \hline & -4 & -4 & 0 \end{array}$$

Logo, $Q(x) = -4x-4$

$-4x^2+4x+8 = (x-2)\times(-4x-4)$

$f(2) = -2$

Ora, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

Logo, existe $\lim_{x\to 2} f(x)$, e o seu valor é $-2$

9. $1 \in D_g$

   Para existir $\lim_{x\to 1} g(x)$, deve ter-se,

   $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = g(1)$

   Ora,

   $$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} \frac{-3x^2+5x-2}{2x-2} = \lim_{x\to 1^-} \frac{(x-1)(-3x+2)}{2(x-1)} = \lim_{x\to 1^-} \frac{-3x+2}{2} = -\frac{1}{2}$$

   **Cálculos auxiliares**

   Se 1 é zero de $-3x^2+5x-2$

   Então,

   $-3x^2+5x-2 = (x-1)\times Q(x)$

   Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

   $$\begin{array}{c|cc|c} & -3 & 5 & -2 \\ & & & \\ 1 & & -3 & 2 \\ \hline & -3 & 2 & 0 \end{array}$$

   Logo, $Q(x) = -3x+2$

   $-3x^2+5x-2 = (x-2)\times(-3x+2)$

$$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} \frac{\sqrt{2x+2}-2}{x^2-x} = \lim_{x\to 1^+} \frac{(\sqrt{2x+2}-2)(\sqrt{2x+2}+2)}{x(x-1)(\sqrt{2x+2}+2)} = \lim_{x\to 1^+} \frac{(\sqrt{2x+2})^2-2^2}{x(x-1)(\sqrt{2x+2}+2)} =$$

$$= \lim_{x\to 1^+} \frac{|2x+2|-4}{x(x-1)(\sqrt{2x+2}+2)} = \lim_{x\to 1^+} \frac{2x+2-4}{x(x-1)(\sqrt{2x+2}+2)} = \lim_{x\to 1^+} \frac{2(x-1)}{x(x-1)(\sqrt{2x+2}+2)} =$$

$$= \lim_{x\to 1^+} \frac{2}{x(\sqrt{2x+2}+2)} = \frac{2}{4} = \frac{1}{2}$$

$g(1) = -\dfrac{1}{2}$

Ora, $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} g(x)$

Logo, não existe $\lim_{x\to 1} g(x)$

10. $-2 \in D_h$

Para existir $\lim_{x\to -2} h(x)$, deve ter-se,

$$\lim_{x\to -2^-} h(x) = \lim_{x\to -2^+} h(x) = h(-2)$$

Ora,

$$\lim_{x\to -2^-} h(x) = \lim_{x\to -2^-} \frac{x^2-4}{-7x^2-12x+4} = \lim_{x\to -2^-} \frac{(x+2)(x-2)}{(x+2)(-7x+2)} = \lim_{x\to -2^-} \frac{x-2}{-7x+2} = -\frac{1}{4}$$

**Cálculos auxiliares**

Se $-2$ é zero de $-7x^2-12x+4$

Então,

$-7x^2-12x+4 = (x+2)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|cc|c} & -7 & -12 & 4 \\ & & & \\ -2 & & 14 & -4 \\ \hline & -7 & 2 & 0 \end{array}$$

Logo, $Q(x) = -7x+2$

$-7x^2-12x+4 = (x+2)\times(-7x+2)$

$$\lim_{x\to -2^+} h(x) = \lim_{x\to -2^+} \frac{2-\sqrt{x+6}}{x^2+5x+6} = \lim_{x\to -2^+} \frac{(2-\sqrt{x+6})(2+\sqrt{x+6})}{(x+3)(x+2)(\sqrt{x+6}+2)} = \lim_{x\to -2^+} \frac{2^-(\sqrt{x+6})^2}{(x+3)(x+2)(\sqrt{x+6}+2)} =$$

$$= \lim_{x\to -2^+} \frac{4-|x+6|}{(x+3)(x+2)(\sqrt{x+6}+2)} = \lim_{x\to -2^+} \frac{4-x-6}{(x+3)(x+2)(\sqrt{x+6}+2)} = \lim_{x\to -2^+} \frac{-(x+2)}{(x+3)(x+2)(\sqrt{x+6}+2)} =$$

$$= \lim_{x\to -2^+} \frac{-1}{(x+3)(\sqrt{x+6}+2)} = \frac{-1}{1\times 4} = -\frac{1}{4}$$

**Cálculos auxiliares**

Se $-2$ é zero de $x^2+5x+6$

Então,

$x^2+5x+6 = (x+2)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|cc|c} & 1 & 5 & 6 \\ & & & \\ -2 & & -2 & -6 \\ \hline & 1 & 3 & 0 \end{array}$$

Logo, $Q(x) = x + 1$

$x^2 + 5x + 6 = (x + 2) \times (x + 1)$

$h(-2) = 2 - 3k$

Ora, existe $\lim\limits_{x \to -2} h(x)$, se, $\lim\limits_{x \to -2^-} h(x) = \lim\limits_{x \to -2^+} h(x) = h(-2)$

Então, deverá ter-se,

$2 - 3k = -\dfrac{1}{4} \Leftrightarrow 8 - 12k = -1 \Leftrightarrow 12k = 8 + 1 \Leftrightarrow 12k = 9 \Leftrightarrow k = \dfrac{9}{12} \Leftrightarrow k = \dfrac{3}{4}$

Portanto, existe $\lim\limits_{x \to -2} h(x)$, se $k = \dfrac{3}{4}$