Proposta de Resolução da Ficha de Trabalho 3

Matemática A

12.º Ano de Escolaridade | Turma: J

1. .

1.1. Ora,

$$\lim_{x\to 2} f(x) = 3$$

$$\lim_{x\to 2} g(x) = 2$$

Portanto,

$$\lim_{x\to -1}(f+g)(x) = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 3 + 2 = 5$$

1.2. Ora,

$$\lim_{x\to -1} f(x) = 1$$

$$\lim_{x\to -1} g(x) = -2$$

Portanto,

$$\lim_{x\to -1}(g-f)(x) = \lim_{x\to -1} g(x) - \lim_{x\to -1} f(x) = -2 - 1 = -3$$

1.3. Ora,

$$\lim_{x\to 2} f(x) = 3$$

$$\lim_{x\to 2} g(x) = 2$$

Portanto,

$$\lim_{x\to 2}(g\times f)(x) = \lim_{x\to 2} f(x) \times \lim_{x\to 2} g(x) = 3\times 2 = 6$$

1.4. Ora,

$$\lim_{x\to -1} f(x) = 1$$

$$\lim_{x\to -1} g(x) = -2$$

Portanto,

$$\lim_{x\to -1}\left(\frac{g}{f}\right)(x) = \frac{\lim_{x\to -1} g(x)}{\lim_{x\to -1} f(x)} = \frac{-2}{1} = -2$$

2. .

2.1. Ora,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}(2x^2-1) = 2\times 0^2 - 1 = 0 - 1 = -1$$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0}\frac{3-4x}{2-x} = \frac{3-4\times 0}{2-0} = \frac{3-0}{2} = \frac{3}{2}$$

Logo, $\lim_{x\to 0}(f+g)(x) = \lim_{x\to 0} f(x) + \lim_{x\to 0} g(x) = -1 + \frac{3}{2} = \frac{1}{2}$

2.2. Ora,

$$\lim_{x\to 1} f(x) = \lim_{x\to 1}(2x^2-1) = 2\times 1^2 - 1 = 2-1 = 1$$

$$\lim_{x\to 1} g(x) = \lim_{x\to 1} \frac{3-4x}{2-x} = \frac{3-4\times 1}{2-1} = \frac{3-4}{1} = -1$$

Logo, $\lim\limits_{x\to 1}(g-f)(x) = \lim\limits_{x\to 1} g(x) - \lim\limits_{x\to 1} f(x) = -1-1 = -2$

2.3. $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-}(2x^2-1) = 2\times 2^2 - 1 = 8-1 = 7$

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} \frac{3-4x}{2-x} = \frac{3-4\times 2}{2-2^-} = \frac{-5}{0^+} = -\infty$$

Logo, $\lim\limits_{x\to 2^-}(f\times g)(x) = \lim\limits_{x\to 2^-} f(x) \times \lim\limits_{x\to 2^-} g(x) = 7\times(-\infty) = -\infty$

2.4. $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+}(2x^2-1) = 2\times 2^2 - 1 = 8-1 = 7$

$$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \frac{3-4x}{2-x} = \frac{3-4\times 2}{2-2^+} = \frac{-5}{0^-} = +\infty$$

Logo,

$$\lim_{x\to 2^+} \left(\frac{f}{g}\right)(x) = \frac{\lim\limits_{x\to 2^+} f(x)}{\lim\limits_{x\to 2^+} g(x)} = \frac{7}{+\infty} = 0$$

3. .

3.1. $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to 1} \dfrac{(x-1)(x+1)}{x-1} = \lim\limits_{x\to 1}(x+1) = 2$

3.2. $\lim\limits_{x\to 3} \dfrac{2x-6}{x^2-9} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to 3} \dfrac{2(x-3)}{(x-3)(x+3)} = \lim\limits_{x\to 3} \dfrac{2}{x+3} = \dfrac{2}{6} = \dfrac{1}{3}$

3.3. $\lim\limits_{x\to -2} \dfrac{(x+2)^2}{x^2+2x} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to -2} \dfrac{(x+2)^2}{x(x+2)} = \lim\limits_{x\to -2} \dfrac{x+2}{x} = \dfrac{0}{-2} = 0$

3.4. $\lim\limits_{x\to 4^+} \dfrac{8-2x}{(x-4)^2} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to 4^+} \dfrac{-2(x-4)}{(x-4)^2} = \lim\limits_{x\to 4^+} \dfrac{-2}{x-4} = -\dfrac{2}{0^+} = -\infty$

3.5. $\lim\limits_{x\to -5^-} \dfrac{x^2+5x}{x^2+10x+25} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to -5^-} \dfrac{x(x+5)}{(x+5)^2} = \lim\limits_{x\to -5^-} \dfrac{x}{x+5} = \dfrac{-5}{0^-} = +\infty$

3.6. $\lim\limits_{x\to 1} \dfrac{(x-1)^2}{x^3-1} =^{\left(\frac{0}{0}\right)} \lim\limits_{x\to 1} \dfrac{(x-1)^2}{(x-1)(x^2+x+1)} = \lim\limits_{x\to 1} \dfrac{x-1}{x^2+x+1} = \dfrac{0}{3} = 0$

Cálculo auxiliar

Se, 1 é zero de x^3-1

Então,

$x^3 - 1 = (x-1)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|ccc|c} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

Logo, $Q(x) = x^2+x+1$

$x^3 - 1 = (x-1)\times(x^2+x+1)$

3.7. $\lim_{x\to+\infty}(x^2-x) =^{(\infty-\infty)} \lim_{x\to+\infty}\left[x^2\left(1-\frac{x}{x^2}\right)\right] = \lim_{x\to+\infty}(x^2)\times\lim_{x\to+\infty}\left(1-\frac{1}{x}\right) = +\infty\times\left(1-\frac{1}{+\infty}\right) =$

$= +\infty\times(1-0) = +\infty$

3.8. $\lim_{x\to-\infty}(2x^3+x^2+1) =^{(\infty-\infty)} \lim_{x\to-\infty}\left[2x^3\left(1+\frac{x^2}{2x^3}+\frac{1}{2x^3}\right)\right] = \lim_{x\to-\infty}(2x^3)\times\lim_{x\to-\infty}\left(1+\frac{1}{2x}+\frac{1}{2x^3}\right) =$

$= -\infty\times\left(1+\frac{1}{-\infty}+\frac{1}{-\infty}\right) = -\infty\times(1+0+0) = -\infty$

3.9. $\lim_{x\to+\infty}(-3x^4+x+2) =^{(\infty-\infty)} \lim_{x\to+\infty}\left[-3x^4\left(1+\frac{x}{-3x^4}+\frac{2}{-3x^4}\right)\right] = \lim_{x\to+\infty}(-3x^4)\times\lim_{x\to+\infty}\left(1-\frac{1}{3x^3}-\frac{2}{3x^4}\right) =$

$= -\infty\times\left(1-\frac{1}{+\infty}-\frac{1}{+\infty}\right) = -\infty\times(1-0-0) = -\infty$

3.10. $\lim_{x\to-\infty}(4x^4+x^2+x) =^{(\infty-\infty)} \lim_{x\to-\infty}\left[4x^4\left(1+\frac{x^2}{4x^4}+\frac{x}{4x^4}\right)\right] = \lim_{x\to-\infty}(4x^4)\times\lim_{x\to-\infty}\left(1+\frac{1}{4x^2}+\frac{1}{4x^3}\right) =$

$= +\infty\times\left(1+\frac{1}{+\infty}+\frac{1}{-\infty}\right) = +\infty\times(1+0+0) = +\infty$

3.11. $\lim_{x\to+\infty}(\sqrt{x+2}-\sqrt{x+1}) =^{(\infty-\infty)} \lim_{x\to+\infty}\frac{(\sqrt{x+2}-\sqrt{x+1})(\sqrt{x+2}+\sqrt{x+1})}{\sqrt{x+2}+\sqrt{x+1}} = \lim_{x\to+\infty}\frac{(\sqrt{x+2})^2-(\sqrt{x+1})^2}{\sqrt{x+2}+\sqrt{x+1}} =$

$= \lim_{x\to+\infty}\frac{|x+2|-|x+1|}{\sqrt{x+2}+\sqrt{x+1}} = \lim_{x\to+\infty}\frac{x+2-x-1}{\sqrt{x+2}+\sqrt{x+1}} = \lim_{x\to+\infty}\frac{1}{\sqrt{x+2}+\sqrt{x+1}} = \frac{1}{+\infty} = 0$

3.12. $\lim_{x\to+\infty}(\sqrt{x^2+1}-\sqrt{x^2+x+1}) =^{(\infty-\infty)} \lim_{x\to+\infty}\frac{(\sqrt{x^2+1}-\sqrt{x^2+x+1})(\sqrt{x^2+1}+\sqrt{x^2+x+1})}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} =$

$= \lim_{x\to+\infty}\frac{(\sqrt{x^2+1})^2-(\sqrt{x^2+x+1})^2}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} = \lim_{x\to+\infty}\frac{|x^2+1|-|x^2+x+1|}{\sqrt{x+2}+\sqrt{x+1}} = \lim_{x\to+\infty}\frac{x^2+1-x^2-x-1}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} =$

$= \lim_{x\to+\infty}\frac{-x}{\sqrt{x^2+1}+\sqrt{x^2+x+1}} = -\lim_{x\to+\infty}\frac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+\sqrt{x^2\left(1+\frac{x}{x^2}+\frac{1}{x^2}\right)}} =$

$= -\lim_{x\to+\infty}\frac{x}{|x|\sqrt{1+\frac{1}{x^2}}+|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} = -\lim_{x\to+\infty}\frac{x}{|x|\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)} =$

$= -\lim_{x\to+\infty}\frac{x}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)} = -\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} =$

$= -\frac{1}{\sqrt{1+\lim_{x\to+\infty}\frac{1}{x^2}}+\sqrt{1+\lim_{x\to+\infty}\frac{1}{x}+\lim_{x\to+\infty}\frac{1}{x^2}}} = -\frac{1}{\sqrt{1+\frac{1}{+\infty}}+\sqrt{1+\frac{1}{+\infty}+\frac{1}{+\infty}}} =$

$= -\frac{1}{\sqrt{1+0}+\sqrt{1+0+0}} = -\frac{1}{2}$

3.13. $\lim_{x\to+\infty}\frac{2x^2+1}{x^3+3} =^{\left(\frac{\infty}{\infty}\right)} \lim_{x\to+\infty}\frac{x^2\left(2+\frac{1}{x^2}\right)}{x^3\left(1+\frac{3}{x^3}\right)} = \lim_{x\to+\infty}\frac{1}{x}\times\frac{\lim_{x\to+\infty}\left(2+\frac{1}{x^2}\right)}{\lim_{x\to+\infty}\left(1+\frac{3}{x^3}\right)} = \frac{1}{+\infty}\times\frac{2+\frac{1}{+\infty}}{1+\frac{1}{+\infty}} =$

$= 0\times\frac{2+0}{1+0} = 0\times 2 = 0$

3.14. $\lim_{x\to+\infty}\frac{-3x^3-2x-3}{x^2+x-4} =^{\left(\frac{\infty}{\infty}\right)} \lim_{x\to+\infty}\frac{x^3\left(-3-\frac{2x}{x^3}-\frac{3}{x^3}\right)}{x^2\left(1+\frac{x}{x^2}-\frac{4}{x^2}\right)} = \lim_{x\to+\infty}(x)\times\frac{\lim_{x\to+\infty}\left(-3-\frac{2}{x^2}-\frac{3}{x^3}\right)}{\lim_{x\to+\infty}\left(1+\frac{1}{x}-\frac{4}{x^2}\right)} =$

$= +\infty\times\frac{-3-\frac{2}{+\infty}-\frac{3}{+\infty}}{1+\frac{1}{+\infty}-\frac{4}{+\infty}} = +\infty\times\frac{-3-0-0}{1+0-0} = -\infty$

3.15. $\lim_{x\to-\infty}\frac{x^5-1}{x^4+1} \stackrel{(\frac{\infty}{\infty})}{=} \lim_{x\to-\infty}\frac{x^5\left(1-\frac{1}{x^5}\right)}{x^4\left(1+\frac{1}{x^4}\right)} = \lim_{x\to-\infty}(x)\times\frac{\lim_{x\to-\infty}\left(1-\frac{1}{x^5}\right)}{\lim_{x\to-\infty}\left(1+\frac{1}{x^4}\right)} =$

$= -\infty\times\frac{1-\frac{1}{-\infty}}{1+\frac{1}{+\infty}} = -\infty\times\frac{1-0}{1+0} = -\infty$

3.16. $\lim_{x\to-\infty}\frac{2-3x-x^2}{x^4+1} \stackrel{(\frac{\infty}{\infty})}{=} \lim_{x\to-\infty}\frac{x^2\left(-1-\frac{3x}{x^2}+\frac{2}{x^2}\right)}{x^4\left(1+\frac{1}{x^4}\right)} = \lim_{x\to-\infty}\frac{1}{x^2}\times\frac{\lim_{x\to-\infty}\left(-1-\frac{3}{x}+\frac{2}{x^2}\right)}{\lim_{x\to-\infty}\left(1+\frac{1}{x^4}\right)} =$

$= \frac{1}{+\infty}\times\frac{-1-\frac{3}{-\infty}+\frac{2}{+\infty}}{1+\frac{1}{+\infty}} = 0\times\frac{-1-0+0}{1+0} = 0\times(-1) = 0$

3.17. $\lim_{x\to+\infty}\left[\frac{1}{x+3}\times(x^2-9)\right] \stackrel{(0\times\infty)}{=} \lim_{x\to+\infty}\frac{x^2-9}{x+3} \stackrel{(\frac{\infty}{\infty})}{=} \lim_{x\to+\infty}\frac{x^2\left(1-\frac{9}{x^2}\right)}{x\left(1+\frac{3}{x}\right)} =$

$= \lim_{x\to+\infty}(x)\times\frac{\lim_{x\to+\infty}\left(1-\frac{9}{x^2}\right)}{\lim_{x\to+\infty}\left(1+\frac{3}{x}\right)} = +\infty\times\frac{1-\frac{9}{+\infty}}{1+\frac{3}{+\infty}} = +\infty\times\frac{1-0}{1+0} = +\infty$

3.18. $\lim_{x\to+\infty}\left[\frac{-2}{1-x^2}\times(x+1)\right] \stackrel{(0\times\infty)}{=} \lim_{x\to+\infty}\left[\frac{-2x-2}{1-x^2}\right] \stackrel{(\frac{\infty}{\infty})}{=} \lim_{x\to+\infty}\left[\frac{x\left(-2-\frac{2}{x}\right)}{x^2\left(-1+\frac{1}{x^2}\right)}\right] =$

$= \lim_{x\to+\infty}\frac{1}{x}\times\frac{\lim_{x\to+\infty}\left(-2-\frac{2}{x}\right)}{\lim_{x\to+\infty}\left(-1+\frac{1}{x^2}\right)} = \frac{1}{+\infty}\times\frac{-2-\frac{2}{+\infty}}{-1+\frac{1}{+\infty}} = 0\times\frac{-2-0}{-1+0} = 0$

3.19. $\lim_{x\to-2^+}\left[(x^2+4x+4)\times\frac{1}{x+2}\right] \stackrel{(0\times\infty)}{=} \lim_{x\to-2^+}\frac{x^2+4x+4}{x+2} \stackrel{(\frac{0}{0})}{=} \lim_{x\to-2^+}\frac{(x+2)^2}{x+2} =$

$= \lim_{x\to-2^+}(x+2) = 0$

3.20. $\lim_{x\to-1^-}\left[(x+1)\times\frac{1}{x^3+1}\right] \stackrel{(0\times\infty)}{=} \lim_{x\to-1^-}\frac{x+1}{x^3+1} \stackrel{(\frac{0}{0})}{=} \lim_{x\to-1^-}\frac{x+1}{(x+1)(x^2-x+1)} =$

$= \lim_{x\to-1^-}\frac{1}{x^2-x+1} = \frac{1}{3}$

Cálculo auxiliar

Se, -1 é zero de x^3+1

Então,

$x^3+1 = (x+1)\times Q(x)$

Determinemos $Q(x)$, recorrendo, por exemplo, à regra de Ruffini

$$\begin{array}{c|ccc|c} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

Logo, $Q(x) = x^2-x+1$

$x^3+1 = (x+1)\times(x^2-x+1)$