Ficha de Trabalho nº 21
Proposta de resolução

Preparação para exame

12.º Ano de Escolaridade | Turma G-K

Números Complexos

1. .

1.1. $w_1 + 2w_2 = -2 - 3i + 2 \times (1+i) = -2 - 3i + 2 + 2i = -i$

1.2. $w_2 \times w_1 = (-2-3i) \times (1+i) = -2 - 2i - 3i - 3i^2 = -2 - 5i + 3 = 1 - 5i$

1.3. $w_1 \times \overline{w_2} - iw_1 = (-2-3i) \times (1-i) - i(-2-3i) = -2 + 2i - 3i + 3i^2 + 2i + 3i^2 =$
$= -2 - i - 3 + 2i - 3 = -8 + i$

2. .

2.1. $|z_1| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

2.2. $|z_1 + z_2| = |-1 + 4i + 3 + 2i| = |2 + 6i| = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$

2.3. $|z_1 z_2| = |(-1+4i)(3+2i)| = |-3 - 2i + 12i + 8i^2| = |-3 + 10i - 8| = |-11 + 10i| =$
$= \sqrt{(-11)^2 + (10)^2} = \sqrt{221}$

Outro processo

$|z_1 z_2| = |z_1||z_2| = \sqrt{(-1)^2 + 4^2}\sqrt{3^2 + 2^2} = \sqrt{17}\sqrt{13} = \sqrt{221}$

3. .

3.1. $z\overline{z} - iw = |z|^2 - i\left(\frac{1}{2} - \frac{1}{2}i\right) = 2^2 + (-2)^2 - \frac{1}{2}i + \frac{1}{2}i^2 = 8 - \frac{1}{2}i - \frac{1}{2} = \frac{15}{2} - \frac{1}{2}i$

3.2. $z + 2w - Im(z) = i\overline{2a + bi} - R_e(3+3i) \Leftrightarrow 2 - 2i + 2 \times \left(\frac{1}{2} - \frac{1}{2}i\right) - Im(2-2i) = i(2a - bi) - 3 \Leftrightarrow$
$\Leftrightarrow 2 - 2i + 1 - i + 2 = 2ai - bi^2 - 3 \Leftrightarrow 5 - 3i = 2ai + b - 3 \Leftrightarrow 5 - 3i = b - 3 + 2ai \Leftrightarrow$
$\Leftrightarrow b - 3 = 5 \wedge 2a = -3 \Leftrightarrow b = 8 \wedge a = -\frac{3}{2}$

3.3. Seja w' o número complexo inverso de w

$$w' = \frac{1}{w} = \frac{\overline{w}}{|w|^2} = \frac{\frac{1}{2} + \frac{1}{2}i}{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{1}{4} + \frac{1}{4}} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{2}{4}} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} + \frac{\frac{1}{2}i}{\frac{1}{2}} = 1 + i$$

Outro processo

$$w' = \frac{1}{w} = \frac{\overline{w}}{w\overline{w}} = \frac{\frac{1}{2} + \frac{1}{2}i}{\left(\frac{1}{2} - \frac{1}{2}i\right)\left(\frac{1}{2} + \frac{1}{2}i\right)} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{1}{4} + \frac{1}{4}} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{2}{4}} = \frac{\frac{1}{2} + \frac{1}{2}i}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} + \frac{\frac{1}{2}i}{\frac{1}{2}} = 1 + i$$

3.4. $$z_1 = \frac{\overline{z} - 2w}{i(3+3i)} = \frac{2 + 2i - 2\left(\frac{1}{2} - \frac{1}{2}i\right)}{3i + 3i^2} = \frac{2 + 2i - 1 + i}{-3 + 3i} = \frac{1 + 3i}{-3 + 3i} = \frac{(1+3i)(-3-3i)}{(-3+3i)(-3-3i)} =$$
$$= \frac{-3 - 3i - 9i - 9i^2}{(-3)^2 + 3^2} = \frac{-3 - 3i - 9i + 9}{18} = \frac{6 - 12i}{18} = \frac{6}{18} - \frac{12}{18}i = \frac{1}{3} - \frac{2}{3}i$$

4. .

4.1. $w_1 + \overline{w_1} = \sqrt{2} - \sqrt{2}i + \sqrt{2} + \sqrt{2}i = 2\sqrt{2}$
Assim, o inverso de $w_1 + \overline{w_1}$ é $\dfrac{1}{2\sqrt{2}}$

4.2. $w_1 - \overline{w_1} = \sqrt{2} - \sqrt{2}i - (\sqrt{2} + \sqrt{2}i) = -2\sqrt{2}i$
Assim, o inverso de $w_1 - \overline{w_1}$ é $\dfrac{1}{-2\sqrt{2}i} = \dfrac{2\sqrt{2}i}{(-2\sqrt{2}i)(2\sqrt{2}i)} = \dfrac{2\sqrt{2}i}{(2\sqrt{2})^2} = \dfrac{2\sqrt{2}i}{8} = \dfrac{\sqrt{2}}{4}i$

4.3. $w_1\overline{w_1} - \sqrt{2}w_1 = |w_1|^2 - \sqrt{2}(\sqrt{2} - \sqrt{2}i) = \left(\sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2}\right)^2 - 2 + 2i = 4 - 2 + 2i = 2 + 2i$
Assim, o inverso de $w_1\overline{w_1} - \sqrt{2}w_1$ é $\dfrac{1}{2+2i} = \dfrac{2-2i}{(2+2i)(2-2i)} = \dfrac{2-2i}{2^2 + (-2)^2} = \dfrac{2-2i}{8} =$
$= \dfrac{2}{8} - \dfrac{2}{8}i = \dfrac{1}{4} - \dfrac{1}{4}i$

4.4. $w_1\overline{w_1} + \sqrt{2}w_1 = |w_1|^2 + \sqrt{2}(\sqrt{2} - \sqrt{2}i) = \left(\sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2}\right)^2 + 2 - 2i = 4 + 2 - 2i = 6 - 2i$
Assim, o inverso de $w_1\overline{w_1} + \sqrt{2}w_1$ é $\dfrac{1}{6-2i} = \dfrac{6+2i}{(6-2i)(6+2i)} = \dfrac{6+2i}{6^2 + (-2)^2} = \dfrac{6+2i}{40} =$
$= \dfrac{6}{40} + \dfrac{2}{40}i = \dfrac{3}{20} + \dfrac{1}{20}i$

5. $z^2 + 2z + 2 = 0 \Leftrightarrow z = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2 \times 1} \Leftrightarrow z = \dfrac{-2 \pm \sqrt{-4}}{2} \Leftrightarrow z = \dfrac{-2 \pm 2i}{2} \Leftrightarrow$
$\Leftrightarrow z = -1 + i \vee z = -1 - i$

Representação dos afixos $A(-1;1)$ e $B(-1;-1)$ no plano complexo

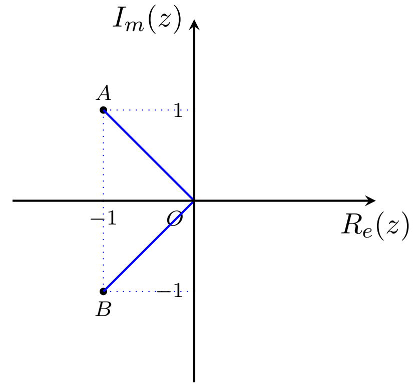

Figura 1

6. .

6.1. $\dfrac{z}{w} = \dfrac{x+yi}{-x-yi} = \dfrac{x+yi}{-(x+yi)} = -1$

Outro processo
$\dfrac{z}{w} = \dfrac{x+yi}{-x-yi} = \dfrac{(x+yi)(-x+yi)}{(-x-yi)(-x+yi)} = \dfrac{-x^2 + (yi)^2}{x^2 - (yi)^2} = \dfrac{-x^2 - y^2}{x^2 + y^2} = \dfrac{-(x^2 + y^2)}{x^2 + y^2} = -1$

6.2. $\frac{iw}{z} = \frac{i(-x-yi)}{x+yi} = \frac{-i(x+yi)}{x+yi} = -i$

O afixo deste número complexo pertence ao semieixo imaginário negativo

Outro processo

$\frac{iw}{z} = \frac{i(-x-yi)}{x+yi} = \frac{-xi-yi^2}{x+yi} = \frac{y-xi}{x+yi} = \frac{(y-xi)(x-yi)}{(x+yi)(x-yi)} = \frac{xy-y^2i-x^2i+xyi^2}{x^2-(yi)^2} =$

$= \frac{xy-y^2i-x^2i-xy}{x^2+y^2} = \frac{-(x^2+y^2)i}{x^2+y^2} = -i$

O afixo deste número complexo pertence ao semieixo imaginário negativo

7. $w = z^2+2\overline{z}-1 = (a+bi)^2+2(a-bi)-1 = a^2+2abi-b^2+2a-2bi-1 = a^2-b^2+2a-1+(-2b+2ab)i$

   Assim, w é um número real se e só se, $-2b+2ab=0$

   $-2b+2ab=0 \Leftrightarrow 2b(a-1)=0 \Leftrightarrow 2b=0 \vee a-1=0 \Leftrightarrow b=0 \vee a=1$

   Portanto,

   Se $a=1 \rightarrow z=1+bi$, com $b \in \mathbb{R}$

   Se $b=0 \rightarrow z=a$, com $a \in \mathbb{R}$