Proposta de Teste Global nº 1

Grupo I

1. C

P	P	P	P	P	P
4	1	1	4	3	2

$4 \times 1 \times 1 \times {}^{4}A_3 = 96$

P	P	P	P	I	I
4	1	1	4	5	4

$4 \times 1 \times 1 \times 4 \times {}^{5}A_2 \times {}^{3}C_1 = 960$

escolher posição para nº par

I	I	I	I	I	I
5	1	1	4	3	2

$5 \times 1 \times 1 \times {}^{4}A_3 = 120$

I	I	I	I	P	P
5	1	1	4	5	4

$5 \times 1 \times 1 \times 4 \times {}^{5}A_2 \times {}^{3}C_1 = 1200$

escolher posição ímpar

$96 + 960 + 120 + 1200 = 2376$

2. D

$${}^{n-2}C_{n-12} + 2 \times {}^{n-2}C_{n-11} + {}^{n-2}C_8$$

$$= {}^{n-2}C_{n-12} + {}^{n-2}C_{n-11} + {}^{n-2}C_{n-11} + {}^{n-2}C_{n-2-8}$$

$$= {}^{n-1}C_{n-11} + {}^{n-2}C_{n-11} + {}^{n-2}C_{n-10}$$

Regra da simetria:

$${}^{n}C_p = {}^{n}C_{n-p} \Rightarrow {}^{n-2}C_8 = {}^{n-2}C_{n-2-8}$$

$$= {}^{n-1}C_{n-11} + {}^{n-1}C_{n-10}$$

Regra de Stiefel:

$${}^{n}C_p + {}^{n}C_{p+1} = {}^{n+1}C_{p+1} \Rightarrow {}^{n-2}C_{\underbrace{n-12}_{p}} + {}^{n-2}C_{\underbrace{n-11}_{p}} = {}^{n-1}C_{n-11}$$

$$= {}^{n}C_{n-10}$$

$\downarrow$

16º elemento $\quad {}^{n}C_{15}$

$n - 10 = 15 \Leftrightarrow n = 25$

3. C

Os termos do desenvolvimento de $\left(\frac{x^3}{m}+\frac{m}{x}\right)^m$ são:

$${}^mC_p \cdot \left(\frac{x^3}{m}\right)^{m-p} \cdot \left(\frac{m}{x}\right)^p = {}^mC_p \cdot \frac{x^{3m-3p}}{m^{m-p}} \cdot \frac{m^p}{x^p} =$$

$$= {}^mC_p \cdot x^{3m-3p-p} \cdot m^{p-(m-p)}$$

$$= {}^mC_p \cdot x^{\overbrace{3m-4p}^{2}} \cdot m^{\overbrace{2p-m}^{4}}$$

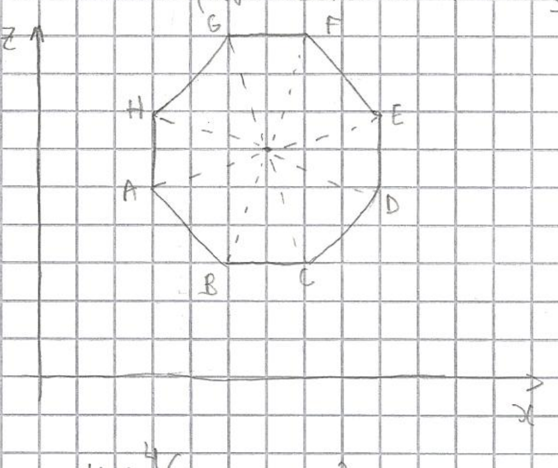
Logo $\begin{cases} 3m-4p=2 \\ 2p-m=4 \end{cases} \Leftrightarrow \begin{cases} 3(2p-4)-4p=2 \\ 2p-4=m \end{cases} \Leftrightarrow$

$$\begin{cases} 6p-12-4p=2 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} 2p=14 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} p=7 \\ 2\times 7-4=m \end{cases} \Leftrightarrow \begin{cases} p=7 \\ m=10 \end{cases}$$

4. B

casos possíveis: ${}^{10}C_3$

casos favoráveis: $4 \times {}^4C_3$



Há 4 planos, que contêm vértices do sólido, $\perp$ a xOz:

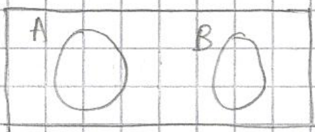
- QHD (contém os vértices Q, H, D, P)
- QGC (contém os vértices Q G C P)
- QAE (contém os vértices QAEP)
- QBF (contém os vértices QBFP)

$$p = \frac{4\times {}^4C_3}{{}^{10}C_3} = \frac{2}{15}$$

5. A

A e B são incompatíveis $\Leftrightarrow A \cap B = \{\} \Rightarrow P(A \cap B) = 0 \Rightarrow$

$\Rightarrow P(A \cup B) = P(A) + P(B)$



$A \cap (A \cup B) = A$

$P(A \mid (A \cup B)) = 0{,}25 \Leftrightarrow \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = 0{,}25$

$\Leftrightarrow \dfrac{P(A)}{P(\overline{A})} = 0{,}25 \Leftrightarrow P(A) = 0{,}25\,(1 - P(A))$

$\Leftrightarrow P(A) = 0{,}25 - 0{,}25\,P(A)$

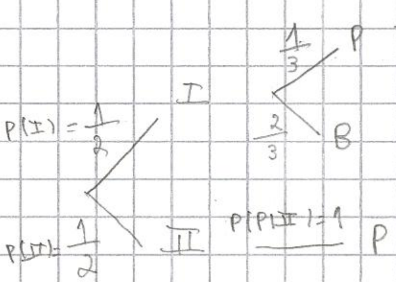
$\Leftrightarrow P(A) + 0{,}25\,P(A) = 0{,}25$

$\Leftrightarrow 1{,}25\,P(A) = 0{,}25$

$\Leftrightarrow P(A) = \dfrac{0{,}25}{1{,}25} = 0{,}2$

6. A

Caixa I < P BB   Caixa II – P P



$\rightarrow P(I \cap P) = P(I) \times P(P \mid I)$

$P(I \mid P) = \dfrac{P(I \cap P)}{P(P)} =$

$= \dfrac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{2}}$

$= \dfrac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{2}_{(3)}} = \dfrac{\frac{1}{\cancel{6}}}{\frac{4}{\cancel{6}}} = \dfrac{1}{4}$

ou através da tabela

$\cap$	I	II	p.m
B	$\frac{1}{3}$	0	$\frac{1}{3}$
P	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$
p.m	$\frac{1}{2}$	$\frac{1}{2}$	1

$$P(P|I) = \frac{1}{3} \Leftrightarrow \frac{P(P \cap I)}{P(I)} = \frac{1}{3}$$

$$\Leftrightarrow P(P \cap I) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$P(I|P) = \frac{P(I \cap P)}{P(P)} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{3}{12} = \frac{1}{4}$$

7. B

$$\underbrace{P(X=2 \mid 2 \leq X \leq 3)} = \frac{3}{4}$$

$$\frac{P(X=2 \cap 2 \leq X \leq 3)}{P(2 \leq X \leq 3)} = \frac{P(X=2)}{P(X=2)+P(X=3)} = \frac{a}{a+a^2}$$

$$\therefore \frac{a}{a+a^2} = \frac{3}{4} \Leftrightarrow 4a = 3a + 3a^2$$

$$\Leftrightarrow 0 = 3a^2 - a$$

$$\Leftrightarrow a(3a-1) = 0$$

$$\Leftrightarrow a = 0 \vee 3a - 1 = 0$$

$$\Leftrightarrow a = 0 \vee a = \frac{1}{3}$$

$$\Leftrightarrow a = 0 \vee \boxed{a = \frac{1}{3}}$$

$$b + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + b = 1 \Leftrightarrow 2b = 1 - \frac{1}{3} - \frac{1}{9}$$

$$\Leftrightarrow 2b = \frac{5}{9} \Leftrightarrow \boxed{b = \frac{5}{18}}$$