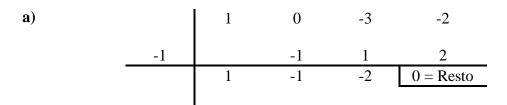




Tópicos de Matemática II - 2016/ 2017 2º Teste - Tópicos de resolução

Exercício 1



Logo:
$$p(x) = [x-(-1)](x^2-x-2) = (x+1)(x^2-x-2)$$

b) Cálculo auxiliar:
$$x^2 - x - 2 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{2} \Leftrightarrow x = -1 \lor x = 2$$

X	$-\infty$	-1		0		2	$+\infty$
x+1	_	0	+	+	+	+	+
$x^2 - x - 2$	+	0	_	_	_	0	+
X	_	_	l	0	+	+	+
$\frac{p(x)}{x}$	+	0	+	s.s.	_	0	+

C.S. =
$$\{-1\} \cup]0,2]$$

Exercício 2

$$D = \{x \in IR: x - 4 > 0 \land 10 - x > 0\} = \{x \in IR: x > 4 \land x < 10\} =]4,10[$$

$$\log(x-4) \ge \log(10-x)$$

$$\Leftrightarrow x-4 \ge 10-x \land x \in]4,10[$$

$$\Leftrightarrow$$
 2 $x \ge 14 \land x \in]4,10[$

$$\Leftrightarrow x \ge 7 \land x \in]4,10[$$

$$\Leftrightarrow x \in [7,10[$$

Exercício 3

$$D_g = \{x \in IR: 2x + 5 > 0\} = \left[-\frac{5}{2}, +\infty \right] = D'_{g^{-1}}$$

$$D'_g = IR = D_{g^{-1}}$$

$$y = g(x) \iff y - 1 = \log(2x + 5) \iff 2x + 5 = 10^{y - 1} \iff x = \frac{10^{y - 1} - 5}{2}$$

$$g^{-1}: IR \longrightarrow \left[-\frac{5}{2}, +\infty \right[$$

$$x \longmapsto \frac{10^{x-1} - 5}{2}$$

Exercício 4

a)
$$2-e^x = -5 \Leftrightarrow e^x = 7 \Leftrightarrow x = \ln 7$$

Resposta: $(\ln 7, -5)$

b)
$$e^x > 0 \Leftrightarrow -e^x < 0 \Leftrightarrow 2 - e^x < 2$$

$$D_f' =]-\infty, 2[$$

c)
$$f'(x) = (2-e^x)' = 0-e^x = -e^x$$

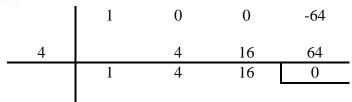
O declive da reta tangente ao gráfico de f no ponto de abcissa zero é igual a:

$$f'(0) = -e^0 = -1$$

Exercício 5

a)
$$\lim_{x \to -\infty} \frac{x^3 \left(1 - \frac{64}{x^3}\right)}{x \left(1 - \frac{4}{x}\right)} = \lim_{x \to -\infty} \frac{x^2 \left(1 - \frac{64}{x^3}\right)}{1 - \frac{4}{x}} = \frac{(+\infty)(1 - 0)}{1 - 0} = +\infty$$

b) Cálculo auxiliar:



$$\lim_{x \to 4^{-}} \frac{(x-4)(x^2+4x+16)}{x-4} = \lim_{x \to 4^{-}} (x^2+4x+16) = 16+16+16=48$$

$$\lim_{x \to 4^+} \frac{\left(\sqrt{x} - 2\right)\left(\sqrt{x} + 2\right)}{\left(x - 4\right)\left(\sqrt{x} + 2\right)} = \lim_{x \to 4^+} \frac{x - 4}{\left(x - 4\right)\left(\sqrt{x} + 2\right)} = \lim_{x \to 4^+} \frac{1}{\sqrt{x} + 2} = \frac{1}{4}$$

 $\lim_{x \to 4^{-}} f(x) \neq \lim_{x \to 4^{+}} f(x)$. Logo, não existe $\lim_{x \to 4} f(x)$.

Exercício 6

a)
$$y' = 3(4x+1)^2(4x+1)' = 3(4x+1)^2 \times 4 = 12(4x+1)^2$$

b)
$$y' = (2x-1)'(x^3-3)+(2x-1)(x^3-3)' = 2(x^3-3)+(2x-1)(3x^2) = 8x^3-3x^2-6$$

Exercício 7

$$\log_2\left(\frac{a^5}{8}\right) = \log_2 a^5 - \log_2 8 = 5\log_2 a - \log_2 2^3 = 5 \times \frac{1}{5} - 3 = -2$$

Outra resolução:

$$\log_2 a = \frac{1}{5} \Leftrightarrow a = 2^{1/5}$$
. Então:

$$\log_2\left(\frac{a^5}{8}\right) = \log_2\left(\frac{\left(2^{1/5}\right)^5}{8}\right) = \log_2\left(\frac{2}{8}\right) = \log_2\left(\frac{1}{4}\right) = \log_22^{-2} = -2$$