Tópicos de Matemática II - 2019/ 2020
2º Teste – Tópicos de resolução

Exercício 1

a) $D_h = \{x \in IR : -3x+5>0\} = \{x \in IR : -3x > -5\} = \left\{x \in IR : x < \frac{5}{3}\right\} = \left]-\infty, \frac{5}{3}\right[$

b) $h(x) = -1 \Leftrightarrow \log_3(-3x+5) = -1 \Leftrightarrow -3x+5 = \frac{1}{3} \wedge x \in D_h \Leftrightarrow -9x+15 = 1 \wedge x \in D_h$

$\Leftrightarrow -9x = -14 \wedge x \in D_h$

$\Leftrightarrow x = \frac{14}{9}$

Resposta: $\left(\frac{14}{9}, -1\right)$

c) $D'_{h^{-1}} = D_h = \left]-\infty, \frac{5}{3}\right[$

$D_{h^{-1}} = D'_h = IR$

$y = h(x) \Leftrightarrow y = \log_3(-3x+5) \Leftrightarrow 3^y = -3x+5 \Leftrightarrow x = \frac{5-3^y}{3}$

$$\begin{aligned} h^{-1} : IR &\to \left]-\infty, \frac{5}{3}\right[\\ x &\mapsto \frac{5-3^x}{3} \end{aligned}$$

Exercício 2

a) $\lim\limits_{x\to-\infty}\left[x^3\left(1+\frac{1}{x}+\frac{5}{x^2}\right)\right] = (-\infty)(1+0+0) = -\infty$

b) $\lim\limits_{x\to1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim\limits_{x\to1}\frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim\limits_{x\to1}\frac{1}{\sqrt{x}+1} = \frac{1}{2}$

Exercício 3

a) $3^x\left(x^2-4\right)=0 \Leftrightarrow 3^x=0 \vee x^2-4=0 \Leftrightarrow x\in\phi \vee x^2=4 \Leftrightarrow x=\pm 2$

b) $5^{x^2+2}=\left(5^2\right)^{-\frac{1}{2}x+2} \Leftrightarrow 5^{x^2+2}=5^{-x+4} \Leftrightarrow x^2+2=-x+4$

$$\Leftrightarrow x^2+x-2=0$$

$$\Leftrightarrow x=\frac{-1\pm\sqrt{9}}{2}$$

$$\Leftrightarrow x=\frac{-1\pm 3}{2}$$

$$\Leftrightarrow x=-2 \vee x=1$$

Exercício 4

a) $\lim\limits_{x\to-\infty} h(x)=\lim\limits_{x\to-\infty}\dfrac{x^3-27}{-x^2+9}=\lim\limits_{x\to-\infty}\dfrac{x^3\left(1-\frac{27}{x^3}\right)}{x^2\left(-1+\frac{9}{x^2}\right)}=\lim\limits_{x\to-\infty}\dfrac{x\left(1-\frac{27}{x^3}\right)}{-1+\frac{9}{x^2}}=\dfrac{(-\infty)(1-0)}{-1+0}=+\infty$

b) $\lim\limits_{x\to3^-} h(x)=\lim\limits_{x\to3^-}\dfrac{x^3-27}{-x^2+9}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to3^-}\dfrac{(x-3)\left(x^2+3x+9\right)}{(x-3)(-x-3)}=\lim\limits_{x\to3^-}\dfrac{x^2+3x+9}{-x-3}=\dfrac{27}{-6}=-\dfrac{9}{2}$ **(*)**

$\lim\limits_{x\to3^+} h(x)=\lim\limits_{x\to3^+}\left(-\dfrac{x^2}{2}\right)=-\dfrac{9}{2}$

Como $\lim\limits_{x\to3^-} h(x)=\lim\limits_{x\to3^+} h(x)=-\dfrac{9}{2}$, então $\lim\limits_{x\to3} h(x)=-\dfrac{9}{2}$.

(*) Cálculos auxiliares:

	1	0	0	-27
3		3	9	27
	1	3	9	0

	-1	0	9
3		-3	-9
	-1	-3	0

Exercício 5

$$x^2+2x=0 \wedge x^2-4\neq 0 \Leftrightarrow x(x+2)=0 \wedge x^2\neq 4 \Leftrightarrow (x=0 \vee x=-2) \wedge x\neq -2 \wedge x\neq 2 \Leftrightarrow x=0$$

Exercício 6

$$f'(x) = -\frac{2}{5}\times 5x^4+2\sqrt{2}\,x = -2x^4+2\sqrt{2}\,x$$

$$f'(\sqrt{2}) = -2(\sqrt{2})^4+2\sqrt{2}\times\sqrt{2} = -2\times\sqrt{16}+2\times 2 = -2\times 4+4 = -4$$

Logo, o declive da reta tangente ao gráfico de *f*, no ponto de abcissa $\sqrt{2}$, é igual a -4

Exercício 7

a) $y' = 2\left(2x^3+2\right)\left(2x^3+2\right)' = \left(4x^3+4\right)\times 6x^2 = 24x^5+24x^2$

b) $y = \dfrac{10-4x}{x}$

$$y' = \frac{(10-4x)'\times x - (10-4x)\times x'}{x^2} = \frac{-4x-10+4x}{x^2} = -\frac{10}{x^2}$$

Exercício 8

$$\log_a\left(b\sqrt{a}\right) = \log_a b+\log_a\sqrt{a} = \frac{1}{3}+\log_a a^{\frac{1}{2}} = \frac{1}{3}+\frac{1}{2} = \frac{2}{6}+\frac{3}{6} = \frac{5}{6}$$