Preparação para exame

12.º Ano de Escolaridade | Turma G-K

TRIGONOMETRIA

1. .

1.1. $\arctan(-1) + f\left[\cos\left(\frac{5\pi}{3}\right)\right] = -\frac{\pi}{4} + \arccos\left[\cos\left(\frac{5\pi}{3}\right)\right] + \frac{\pi}{4} = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

1.2. **Ponto** A

$f(0) = \arccos(0) + \frac{\pi}{4} = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Logo, $A\left(0; \frac{3\pi}{4}\right)$

Ponto B

$f(x) = \frac{7\pi}{12} \Leftrightarrow \arccos(x) + \frac{\pi}{4} = \frac{7\pi}{12} \Leftrightarrow \arccos(x) = \frac{7\pi}{12} - \frac{\pi}{4} \Leftrightarrow \arccos(x) = \frac{4\pi}{12} \Leftrightarrow$

$\Leftrightarrow \arccos(x) = \frac{\pi}{3} \Leftrightarrow x = \cos\left(\frac{\pi}{3}\right) \wedge x \in [-1; 1] \Leftrightarrow x = \frac{1}{2}$

Logo, $B\left(\frac{1}{2}; \frac{7\pi}{12}\right)$

2. $\arccos\left(\frac{\sqrt{3}}{2}\right) - \arcsin(1) + \arctan\left(\tan\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{6} - \frac{\pi}{2} + \arctan(1) = \frac{\pi}{6} - \frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{12}$

3. .

3.1. $f(x) = \arcsin(4x) - \frac{3\pi}{4}$

O domínio da função é $D_f = \{x \in \mathbb{R} : -1 \leq 4x \leq 1\} = \left\{x \in \mathbb{R} : -\frac{1}{4} \leq x \leq \frac{1}{4}\right\} = \left[-\frac{1}{4}; \frac{1}{4}\right]$

3.2. $f(x) = \arcsin(4x) - \frac{3\pi}{4}$

$f\left(\frac{1}{8}\right) = \arcsin\left(4 \times \frac{1}{8}\right) - \frac{3\pi}{4} = \arcsin\left(\frac{1}{2}\right) - \frac{3\pi}{4} = \frac{\pi}{6} - \frac{3\pi}{4} = -\frac{7\pi}{12}$

$f(0) = \arcsin(4 \times 0) - \frac{3\pi}{4} = \arcsin(0) - \frac{3\pi}{4} = 0 - \frac{3\pi}{4} = -\frac{3\pi}{4}.$

Assim, $f\left(\frac{1}{8}\right) - f(0) = -\frac{7\pi}{12} + \frac{3\pi}{4} = \frac{2\pi}{12} = \frac{\pi}{6}$

3.3. $-\frac{\pi}{2} \leq \arcsin(4x) \leq \frac{\pi}{2}, \forall x \in \left[-\frac{1}{4}; \frac{1}{4}\right]$

$\therefore -\frac{\pi}{2} - \frac{3\pi}{4} \leq \arcsin(4x) - \frac{3\pi}{4} \leq \frac{\pi}{2} - \frac{3\pi}{4}, \forall x \in\in \left[-\frac{1}{4}; \frac{1}{4}\right]$

$\therefore -\frac{5\pi}{4} \leq \arcsin(4x) - \frac{3\pi}{4} \leq -\frac{\pi}{4}, \forall x \in \left[-\frac{1}{4}; \frac{1}{4}\right]$

$\therefore -\frac{5\pi}{4} \leq f(x) \leq -\frac{\pi}{4}, \forall x \in \left[-\frac{1}{4}; \frac{1}{4}\right]$

Logo, $D'_f = \left[-\frac{5\pi}{4}; -\frac{\pi}{4}\right]$

3.4. $f(x)=-\frac{3\pi}{4} \Leftrightarrow \arcsin(4x)-\frac{3\pi}{4}=-\frac{3\pi}{4} \Leftrightarrow \arcsin(4x)=\frac{3\pi}{4}-\frac{3\pi}{4} \Leftrightarrow \arcsin(4x)=0 \Leftrightarrow$
$\Leftrightarrow 4x=\sin(0) \Leftrightarrow 4x=0 \Leftrightarrow x=0 \wedge x\in\left[-\frac{1}{4};\frac{1}{4}\right]$
Logo, $x=0$
O conjunto-solução é $C.S.=\{0\}$

4. $\tan\left(\arccos\left(\frac{1}{2}\right)\right)=\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$

Outro processo
Seja $x=\arccos\left(\frac{1}{2}\right)$
Pretende-se $\tan\left(\arccos\left(\frac{1}{2}\right)\right)=\tan(x)$

Ora,
$x=\arccos\left(\frac{1}{2}\right) \Leftrightarrow \cos(x)=\frac{1}{2}$, com $x\in[0;\pi]$
Assim, de $1+\tan^2(x)=\frac{1}{\cos^2(x)}$, vem
$1+\tan^2(x)=\frac{1}{\left(\frac{1}{2}\right)^2} \Leftrightarrow 1+\tan^2(x)=\frac{1}{\frac{1}{4}} \Leftrightarrow 1+\tan^2(x)=4 \Leftrightarrow \tan^2(x)=4-1 \Leftrightarrow$
$\Leftrightarrow \tan^2(x)=3 \Leftrightarrow \tan(x)=\pm\sqrt{3}$, como $x\in[0;\pi]$ e $\cos(x)>0$, tem-se que $x\in 1^{\text{o}}$Q
Logo, $\tan(x)=\sqrt{3}$
Portanto, $\tan\left(\arccos\left(\frac{1}{2}\right)\right)=\sqrt{3}$

5. .

5.1. $\cos(2a)=\cos(a+a)=\cos(a)\cos(a)-\sin(a)\sin(a)=\cos^2(a)-\sin^2(a)$

5.2. $\sin(a-b)=\cos\left(\frac{\pi}{2}-(a-b)\right)=\cos\left[\left(\frac{\pi}{2}-a\right)+b\right]=\cos\left(\frac{\pi}{2}-a\right)\cos(b)-\sin\left(\frac{\pi}{2}-a\right)\sin(b)=$
$=\sin(a)\cos(b)-\cos(a)\sin(b)$

5.3. $\cos(3a)=\cos(a+2a)=\cos(a)\cos(2a)-\sin(a)\sin(2a)=$
$=\cos(a)(\cos^2(a)-\sin^2(a))-\sin(a)2\sin(a)\cos(a)=$
$=\cos(a)(1-\sin^2(a)-\sin^2(a))-2\sin^2(a)\cos(a)=$
$=\cos(a)(1-2\sin^2(a))-2\sin^2(a)\cos(a)=$
$=\cos(a)(1-4\sin^2(a))$
Cálculo auxiliar
$\sin(2a)=\cos\left(\frac{\pi}{2}-2a\right)=\cos\left[\left(\frac{\pi}{2}-a\right)+(-a)\right]=\cos\left(\frac{\pi}{2}-a\right)\cos(-a)-\sin\left(\frac{\pi}{2}-a\right)\sin(-a)=$
$=\sin(a)\cos(a)+\cos(a)\sin(a)=2\sin(a)\cos(a)$

6. $\frac{\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)}{2}-2\sin^2\left(\frac{\pi}{12}\right)+2\cos^2\left(\frac{\pi}{12}\right)=\frac{2\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)}{4}+2\left[\cos^2\left(\frac{\pi}{12}\right)-\sin^2\left(\frac{\pi}{12}\right)\right]=$
$=\frac{\sin\left(2\times\frac{5\pi}{12}\right)}{4}+2\cos\left(2\times\frac{\pi}{12}\right)=\frac{\sin\left(\frac{5\pi}{6}\right)}{4}+2\cos\left(\frac{\pi}{6}\right)=\frac{\frac{1}{2}}{4}+2\times\frac{\sqrt{3}}{2}=\frac{1}{8}+\sqrt{3}=\frac{1+8\sqrt{3}}{8}$

7. Seja $x = \arcsin\left(\frac{5}{13}\right)$

Pretende-se $\cos\left(\frac{\pi}{3} - \arcsin\left(\frac{5}{13}\right)\right) = \cos\left(\frac{\pi}{3} - x\right) = \cos\left(\frac{\pi}{3}\right)\cos(x) + \sin\left(\frac{\pi}{3}\right)\sin(x)$

Ora,

$x = \arcsin\left(\frac{5}{13}\right) \Leftrightarrow \sin(x) = \frac{5}{13}$, com $x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

Assim, de $\sin^2(x) + \cos^2(x) = 1$, vem,

$$\left(\frac{5}{13}\right)^2 + \cos^2(x) = 1 \Leftrightarrow \frac{25}{169} + \cos^2(x) = 1 \Leftrightarrow \cos^2(x) = 1 - \frac{25}{169} \Leftrightarrow \cos^2(x) = \frac{169-25}{169} \Leftrightarrow$$
$$\Leftrightarrow \cos^2(x) = \frac{144}{169} \Leftrightarrow \cos(x) = \pm\sqrt{\frac{144}{169}} \Leftrightarrow \cos(x) = \pm\frac{12}{13},$$

como $x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ e $\sin(x) > 0$, então $x \in 1^{\text{o}}\text{Q}$

Logo, $\cos(x) = \frac{12}{13}$

Portanto,

$$\cos\left(\frac{\pi}{3} - \arcsin\left(\frac{5}{13}\right)\right) = \cos\left(\frac{\pi}{3} - x\right) = \cos\left(\frac{\pi}{3}\right)\cos(x) + \sin\left(\frac{\pi}{3}\right)\sin(x) =$$
$$= \frac{1}{2} \times \frac{12}{13} + \frac{\sqrt{3}}{2} \times \frac{5}{13} = \frac{12 + 5\sqrt{3}}{26}$$

8. $\sin(\alpha) = -\frac{4}{5} \wedge \alpha \in \left]0; \frac{3\pi}{2}\right[$

Assim, de $\sin^2(\alpha) + \cos^2(\alpha) = 1$, vem,

$$\left(-\frac{4}{5}\right)^2 + \cos^2(\alpha) = 1 \Leftrightarrow \frac{16}{25} + \cos^2(\alpha) = 1 \Leftrightarrow \cos^2(\alpha) = 1 - \frac{16}{25} \Leftrightarrow \cos^2(\alpha) = \frac{25-16}{25} \Leftrightarrow$$
$$\Leftrightarrow \cos^2(\alpha) = \frac{9}{25} \Leftrightarrow \cos(\alpha) = \pm\sqrt{\frac{9}{25}} \Leftrightarrow \cos(\alpha) = \pm\frac{3}{5},$$

como $\alpha \in \left]0; \frac{3\pi}{2}\right[$ e $\sin(\alpha) < 0$, então $x \in 3^{\text{o}}\text{Q}$

Logo, $\cos(\alpha) = -\frac{3}{5}$

Portanto,

$$\sin\left(\frac{\pi}{4} - \alpha\right) = \sin\left(\frac{\pi}{4}\right)\cos(\alpha) - \cos\left(\frac{\pi}{4}\right)\sin(\alpha) = \frac{\sqrt{2}}{2} \times \left(-\frac{3}{5}\right) - \frac{\sqrt{2}}{2} \times \left(-\frac{4}{5}\right) =$$
$$= \frac{-3\sqrt{2} + 4\sqrt{2}}{10} = \frac{\sqrt{2}}{10}$$

e

$$\cos\left(\frac{\pi}{6} + \alpha\right) = \cos\left(\frac{\pi}{6}\right)\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha) = \frac{\sqrt{3}}{2} \times \left(-\frac{3}{5}\right) - \frac{1}{2} \times \left(-\frac{4}{5}\right) =$$
$$= \frac{-3\sqrt{3} + 4}{10} = \frac{4 - 3\sqrt{3}}{10}$$

9. .

9.1. $\cos^2\left(\frac{x}{4}\right) - \sin^2\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(2 \times \frac{x}{4}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(\frac{x}{2}\right) = \cos\left(\frac{\pi}{6}\right) \Leftrightarrow$
$\Leftrightarrow \frac{x}{2} = \pm\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow x = \pm\frac{2\pi}{6} + 4k\pi, k \in \mathbb{Z} \Leftrightarrow x = \pm\frac{\pi}{3} + 4k\pi, k \in \mathbb{Z}$

9.2. $\cos(2x) - \cos(x) + 1 = 0 \Leftrightarrow \cos^2(x) - \sin^2(x) - \cos(x) + 1 = 0 \Leftrightarrow$
$\Leftrightarrow \cos^2(x) - \left[1 - \cos^2(x)\right] - \cos(x) + 1 = 0 \Leftrightarrow \cos^2(x) - 1 + \cos^2(x) - \cos(x) + 1 = 0 \Leftrightarrow$
$\Leftrightarrow 2\cos^2(x) - \cos(x) = 0 \Leftrightarrow \cos(x)\left[2\cos(x) - 1\right] = 0 \Leftrightarrow \cos(x) = 0 \vee 2\cos(x) - 1 = 0 \Leftrightarrow$
$\Leftrightarrow \cos(x) = 0 \vee \cos(x) = \frac{1}{2} \Leftrightarrow \cos(x) = \cos\left(\frac{\pi}{2}\right) \vee \cos(x) = \cos\left(\frac{\pi}{3}\right) \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{2} + k\pi \vee x = \pm\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

9.3. $\cos(x) - \sqrt{3}\sin(x) = \sqrt{2} \Leftrightarrow \frac{1}{2}\cos(x) - \frac{\sqrt{3}}{2}\sin(x) = \frac{\sqrt{2}}{2} \Leftrightarrow$
$\Leftrightarrow \cos\left(\frac{\pi}{3}\right)\cos(x) - \sin\left(\frac{\pi}{3}\right)\sin(x) = \frac{\sqrt{2}}{2} \Leftrightarrow \cos\left(\frac{\pi}{3} + x\right) = \cos\left(\frac{\pi}{4}\right) \Leftrightarrow$
$\Leftrightarrow \frac{\pi}{3} + x = \frac{\pi}{4} + 2k\pi \vee \frac{\pi}{3} + x = -\frac{\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{4} - \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{4} - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = -\frac{\pi}{12} + 2k\pi \vee x = -\frac{7\pi}{12} + 2k\pi, k \in \mathbb{Z}$