Tópicos de Matemática II - 2015/ 2016
2º Teste – Tópicos de resolução

Exercício 1

a)

	1	-3	1	0	4
2		2	-2	-2	-4
	1	-1	-1	-2	0=Resto
2		2	2	2	
	1	1	1	0=Resto	

Logo: $p(x)=(x-2)(x-2)(x^2+x+1)=(x-2)^2(x^2+x+1)$

b) Cálculo auxiliar: $x^2+x+1=0 \Leftrightarrow x=\frac{-1\pm\sqrt{1-4}}{2} \Leftrightarrow x\in\phi$

x	$-\infty$	2	$+\infty$
$(x-2)^2$	+	0	+
x^2+x+1	+	+	+
$p(x)$	+	0	+

C.S. = $]-\infty,2[\cup]2,+\infty[$

Exercício 2

$$y=f(x)\Leftrightarrow y-7=2e^{-3x}\Leftrightarrow e^{-3x}=\frac{y-7}{2}\Leftrightarrow -3x=\ln\left(\frac{y-7}{2}\right)\Leftrightarrow x=-\frac{1}{3}\ln\left(\frac{y-7}{2}\right)$$

$f^{-1}(x)=-\frac{1}{3}\ln\left(\frac{x-7}{2}\right)$; $D_{f^{-1}}=D'_f=]7,+\infty[$; $D'_{f^{-1}}=D_f=IR$

$$\begin{array}{ccc} f^{-1}:]7,+\infty[& \rightarrow & IR \\ x & \mapsto & -\frac{1}{3}\ln\left(\frac{x-7}{2}\right) \end{array}$$

Exercício 3

a) Cálculo auxiliar:

	1	0	0	0	-1
1		1	1	1	1
	1	1	1	1	0

$$\lim_{x\to 1}\frac{(x-1)\left(x^3+x^2+x+1\right)}{(x-1)} = \lim_{x\to 1}\left(x^3+x^2+x+1\right) = 1+1+1+1=4$$

b) $\lim_{x\to 9}\frac{\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)}{(x-9)\left(\sqrt{x}+3\right)} = \lim_{x\to 9}\frac{\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)}{(x-9)\left(\sqrt{x}+3\right)} = \lim_{x\to 9}\frac{x-9}{(x-9)\left(\sqrt{x}+3\right)} = \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$

Exercício 4

a) $3^{-x-4} = \left(3^3\right)^{3x+5} \Leftrightarrow 3^{-x-4} = 3^{9x+15} \Leftrightarrow -x-4 = 9x+15 \Leftrightarrow -10x = 19 \Leftrightarrow x = -\frac{19}{10}$

b) $D = \{x\in IR : 2x > 0\} =]0,+\infty[$

$$\log(2x) \le \log 7 \Leftrightarrow 2x \le 7 \wedge x \in]0,+\infty[\Leftrightarrow x \le \frac{7}{2} \wedge x \in]0,+\infty[\Leftrightarrow x \in \left]0,\frac{7}{2}\right]$$

c) $D = \{x\in IR : 3-x > 0\} =]-\infty,3[$

$$\ln(3-x) \ge -1 \Leftrightarrow 3-x \ge e^{-1} \wedge x \in]-\infty,3[\Leftrightarrow -x \ge \frac{1}{e} - 3 \wedge x \in]-\infty,3[$$

$$\Leftrightarrow x \le 3-\frac{1}{e} \wedge x \in]-\infty,3[\Leftrightarrow x \in \left]-\infty,3-\frac{1}{e}\right]$$

Exercício 5

a) $\lim\limits_{x\to-\infty}\dfrac{x^2\left(2+\dfrac{6}{x}-\dfrac{20}{x^2}\right)}{x^2\left(\dfrac{25}{x^2}-1\right)} = \lim\limits_{x\to-\infty}\dfrac{2+\dfrac{6}{x}-\dfrac{20}{x^2}}{\dfrac{25}{x^2}-1} = \dfrac{2+0-0}{0-1} = -2$

b) $\lim\limits_{x\to0^-}\dfrac{2x^2+6x-20}{25-x^2} = \dfrac{0+0-20}{25-0} = -\dfrac{20}{25} = -\dfrac{4}{5}$

$\lim\limits_{x\to0^+}\dfrac{2}{x+1} = \dfrac{2}{0+1} = 2$

$\lim\limits_{x\to0^-}h(x) \neq \lim\limits_{x\to0^+}h(x)$. Logo, não existe $\lim\limits_{x\to0}h(x)$**.**

Exercício 6

Cálculo auxiliares:

$x^2-5x+6=0$

$\Leftrightarrow x=\dfrac{5\pm\sqrt{25-24}}{2}$

$\Leftrightarrow x=2 \vee x=3$

$2x^2-7x+3=0$

$\Leftrightarrow x=\dfrac{7\pm\sqrt{49-24}}{4}$

$\Leftrightarrow x=\dfrac{1}{2} \vee x=3$

$\dfrac{x^2-5x+6}{2x^2-7x+3}=0 \Leftrightarrow x^2-5x+6=0 \wedge 2x^2-7x+3\neq0 \Leftrightarrow (x=2\vee x=3) \wedge x\neq\dfrac{1}{2} \wedge x\neq3 \Leftrightarrow x=2$

Exercício 7

a) $+\infty$ **b)** $-\infty$ **c)** $+\infty$ **d)** 1

Exercício 8

a) $y'=2(3x^2-x)(3x^2-x)' = (6x^2-2x)(6x-1) = 36x^3-18x^2+2x$

b) $y'=\dfrac{(x-1)'(x^2+1)-(x-1)(x^2+1)'}{(x^2+1)^2} = \dfrac{1(x^2+1)-(x-1)2x}{(x^2+1)^2} = \dfrac{-x^2+2x+1}{(x^2+1)^2}$

Exercício 9

$\log_n\left(\log_n\sqrt[n]{n}\right)=\log_n\left(\log_n n^{1/n}\right)=\log_n\left(\dfrac{1}{n}\right)=\log_n n^{-1}=-1$