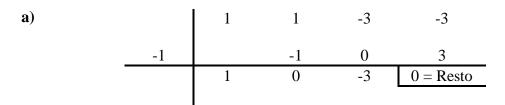




Tópicos de Matemática II - 2017/ 2018 2º Teste - Tópicos de resolução

Exercício 1



Logo:
$$p(x) = [x-(-1)](x^2-3) = (x+1)(x^2-3)$$

b)

$$\lim_{x \to -\infty} \frac{p(x)}{-x^2 + 1} = \lim_{x \to -\infty} \frac{x^3 \left(1 + \frac{1}{x} - \frac{3}{x^2} - \frac{3}{x^3}\right)}{x^2 \left(-1 + \frac{1}{x^2}\right)} = \lim_{x \to -\infty} \frac{x \left(1 + \frac{1}{x} - \frac{3}{x^2} - \frac{3}{x^3}\right)}{-1 + \frac{1}{x^2}} = \lim_{x \to -\infty} \frac{(-\infty)(1 + 0 - 0 - 0)}{-1 + 0} = +\infty$$

ii)
$$\lim_{x \to -1} \frac{p(x)}{x^2 - 1} = \lim_{x \to -1} \frac{(x+1)(x^2 - 3)}{(x+1)(x-1)} = \lim_{x \to -1} \frac{x^2 - 3}{x - 1} = \frac{1 - 3}{-1 - 1} = 1$$

Exercício 2

Cálculos auxiliares:

•
$$1-x^2=0 \Leftrightarrow x^2=1 \Leftrightarrow x=\pm 1$$

•
$$x^2 - x = 0 \Leftrightarrow x(x-1) = 0 \Leftrightarrow x = 0 \lor x = 1$$

x	- 8	-1		0		1	+∞
$1-x^2$	-	0	+	+	+	0	-
x^2-x	+	+	+	0	_	0	+
$\frac{1-x^2}{x^2-x}$	-	0	+	s.s.	_	s.s.	_

$$C.S. = [-1,0[$$

Exercício 3

$$\mathbf{a)} \ D_f = IR$$

$$e^{2x+1} > 0 \Leftrightarrow -e^{2x+1} < 0 \Leftrightarrow 10 - e^{2x+1} < 10$$

$$D'_f = \left] -\infty, 10 \right[$$

b)
$$D_{f^{-1}} = D'_f =]-\infty, 10[$$

$$D'_{f^{-1}} = D_f = IR$$

$$y = f(x) \iff y = 10 - e^{2x+1} \iff e^{2x+1} = 10 - y \iff 2x+1 = \ln(10 - y) \iff x = \frac{\ln(10 - y) - 1}{2}$$

$$f^{-1}:]-\infty,10[$$
 \rightarrow IR

$$x \mapsto \frac{\ln(10-x)-1}{2}$$

c)
$$10 - e^{2x+1} \ge 0 \Leftrightarrow -e^{2x+1} \ge -10 \Leftrightarrow e^{2x+1} \le 10 \Leftrightarrow 2x+1 \le \ln 10 \Leftrightarrow x \le \frac{-1 + \ln 10}{2}$$

$$C.S. = \left] -\infty, \frac{-1 + \ln 10}{2} \right]$$

Exercício 4

- **a**) 2
- **b**) 1
- **c)** 1
- $\mathbf{d}) \infty$

Exercício 5

a)
$$4^{x}(x^{2}-x)=0 \Leftrightarrow 4^{x}=0 \lor x^{2}-x=0 \Leftrightarrow x \in \phi \lor x(x-1)=0 \Leftrightarrow x=0 \lor x=1$$

C.S. = $\{0,1\}$

b)
$$\log_3(4x-3)=2 \Leftrightarrow 4x-3=3^2 \land x > \frac{3}{4} \Leftrightarrow 4x=12 \land x > \frac{3}{4} \Leftrightarrow x=3$$

$$C.S. = {3}$$

Exercício 6

a)
$$y' = 6x^2 + 10x$$

b)
$$y' = \frac{2e^{2x}(x+1)-e^{2x}}{(x+1)^2} = \frac{2xe^{2x}+2e^{2x}-e^{2x}}{(x+1)^2} = \frac{2xe^{2x}+e^{2x}}{(x+1)^2}$$

b)
$$y' = \frac{1}{x}(x+1) + \ln x = \frac{x+1+x\ln x}{x}$$

Exercício 7

$$\log_c \left(\frac{c}{\sqrt[3]{b^2}}\right) = \log_c c - \log_c b^{\frac{2}{3}} = 1 - \frac{2}{3}\log_c b = 1 - \frac{2}{3} \times 2 = 1 - \frac{4}{3} = -\frac{1}{3}$$

Outra possível resolução:

$$\log_c b = 2 \Leftrightarrow b = c^2$$
. Então:

$$\log_c \left(\frac{c}{\sqrt[3]{b^2}}\right) = \log_c \left(\frac{c}{\sqrt[3]{c^4}}\right) = \log_c \left(\frac{c}{\sqrt[4]{\frac{4}{3}}}\right) = \log_c \left(c^{-\frac{1}{3}}\right) = -\frac{1}{3}$$