$$E = \{f : [0,1] \mapsto \mathbb{R} \text{ contínua}\}$$
$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt$$
$$f(t) = t^2 - t + 1$$
$$u(t) = 1 \qquad w(t) = t$$

Calcula a projeção ortogonal de f(t) sobre o plano gerado por u(t) e w(t).

Usando Gram-Schmidt:

$$v_1 := u$$
$$v_2 := w - proj_{v_1} w \implies \forall t \in [0,1] : v_2(t) = w(t) - (proj_{v_1} w)(t) = t - \frac{\int_0^1 s \, \mathrm{d}s}{\int_0^1 1 \, \mathrm{d}s} u(t) = t - \frac{1}{2}$$
$$||v_2|| = \sqrt{\langle v_2, v_2\rangle} = \sqrt{\int_0^1 (t^2 + \frac{1}{4}) \, \mathrm{d}t} = \sqrt{\frac{7}{12}}$$
$$\forall t \in [0,1] : b_2(t) = \frac{v_2(t)}{||v_2||} = \frac{t - \frac{1}{2}}{||v_2||}$$
$$b_1 := 1 \qquad b_2(t) = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$
$$proj_{b_1} f + proj_{b_2} f = \langle f, b_1\rangle b_1 + \langle f, b_2\rangle b_2$$
$$= \left(\int_0^1 \left(t^2 - t + 1 \, \mathrm{dt}\right)\right) \cdot 1 + \left(\int_0^1 \left(t^2 - t + 1 \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}} \, \mathrm{dt}\right)\right) \cdot \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$