1.

Evaluate the following line integrals.

a)

$\int_C \left(xy + z^3\right)\, \mathrm{d}s$**, where** C **is the part of the helix** $\vec{r}(t) = \langle \cos t, \sin t, t\rangle$ **from** $t = 0$ **to** $t = \pi$

$$\vec{r}'(t) = \langle -\sin t, \cos t, 1\rangle$$

$$||\vec{r}'(t)|| = \sqrt{2}\ \mathrm{d}t$$

$$= \sqrt{2}\int_0^{\pi} = \cos t \sin t + t^3\ \mathrm{d}t$$

$$= \sqrt{2}\left[\frac{1}{2}\sin^2 t + \frac{1}{4}t^4\right]_0^{\pi}$$

$$\frac{\sqrt{2}\pi^4}{4}$$

b)

$\int_C \left(\frac{x}{1+y^2}\right)\, \mathrm{d}s$**, where** C **is given parametrically by** $x = 1 + 2t$**,**$y = t$**,for** $0 \le t \le 1$

$$\mathrm{d}x = 2 \qquad \mathrm{d}y = 1$$

$$\mathrm{d}s = \sqrt{5}\ \mathrm{d}t$$

$$\sqrt{5}\int_0^1 \frac{1}{1+t^2}\ \mathrm{d}t + \sqrt{5}\int_0^1 \frac{2t}{1+t^2}\ \mathrm{d}t$$

$$= \sqrt{5}\left[\arctan t\right]_0^1 + \sqrt{5}\left[\ln 1 + t^2\right]_0^1$$

$$= \sqrt{5}\left(\frac{\pi}{4} + \ln 2\right)$$

2.

Find the mass of a thin wire in the form of $y = \sqrt{9 - x^2}\,(0 \le x \le 3)$ **if the density function is** $f(x, y) = x\sqrt{y}$

$$\int_C f(x, y)\ \mathrm{d}s$$

$$x = 3\cos\theta \qquad y = 3\sin\theta$$

$$\mathrm{d}s = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t = 3\ \mathrm{d}t$$

$$\int_0^{\frac{\pi}{2}} 3\cos\theta\,(3\sin\theta)\,3\ \mathrm{d}t$$

$$= 9\sqrt{3}\int_0^{\frac{\pi}{2}} \cos\theta\,(\sin\theta)\ \mathrm{d}t$$

$$= 6\sqrt{3}\left[(\sin\theta)^{\frac{3}{2}}\right]_0^{\frac{\pi}{2}}$$

$$= 6\sqrt{3}$$