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Calculate double integrals over a rectangle R.

a)

$$\iint_R xy\sqrt{1+x^2+y^2}\,dxdy \qquad R: 0 \le x \le 1, 0 \le y \le 1$$

b)

$$\iint_R \frac{1}{(x+y+1)^3}\mathrm{d}x\mathrm{d}y \qquad R: 0 \le x \le 2, 0 \le y \le 1$$

$$= \int_0^1 \int_0^2 \frac{1}{(x+y+1)^3}\mathrm{d}x\mathrm{d}y$$

$$= \int_0^1 \left[-\frac{1}{2(x+y+1)^2}\right]_0^2 \mathrm{d}y$$

$$= \int_0^1 \left(-\frac{1}{2(3+y)^2} + \frac{1}{2(y+1)^2}\right) \mathrm{d}y$$

$$= -\frac{1}{2}\int_0^1 \frac{1}{(3+y)^2} + \frac{1}{2}\int_0^1 \frac{1}{(y+1)^2}\mathrm{d}y$$

$$= -\frac{1}{2}\left[-\frac{1}{3+y}\right]_0^1 + \frac{1}{2}\left[-\frac{1}{1+y}\right]_0^1$$

$$= -\frac{1}{2}\left[-\frac{1}{4}+\frac{1}{3})\right] + \frac{1}{2}\left[-\frac{1}{2}+1\right] = \frac{5}{24}$$

c)

$$\iint_R x\sin(xy)\mathrm{d}x\mathrm{d}y \qquad R: 0 \le x \le 2, \pi \le y \le 2\pi$$

$$= \int_\pi^{2\pi} \int_0^1 x\sin(xy)\mathrm{d}x\mathrm{d}y$$

$$= \int_0^1 \int_\pi^{2\pi} x\sin(xy)\mathrm{d}y\mathrm{d}x$$

$$= \int_0^1 \left[-\cos(xy)\right]_\pi^{2\pi} \mathrm{d}x$$

$$= \int_0^1 \left(-\cos(2\pi x) + \cos(\pi x)\right)\mathrm{d}x$$

$$= \left[-\frac{1}{2\pi}\sin(2\pi x)\right]_0^1 + \left[\frac{1}{\pi}\sin(\pi x)\right]_0^1 = 0$$

d)

$$\iint_R \left(2x - 3y^2\right) \mathrm{d}x\mathrm{d}y \qquad R: -1 \leq x \leq 1, 0 \leq y \leq 2$$

$$= \int_0^2 \int_{-1}^1 \left(2x - 3y^2\right) \mathrm{d}x\mathrm{d}y$$

$$= \int_0^2 \left[x^2 - 3y^2x\right]_{-1}^1 \mathrm{d}y$$

$$= \int_0^2 -6y^2\mathrm{d}y$$

$$= \left[-2y^3\right]_0^2$$

$$= -16$$