$$E = \{f : [0,1] \mapsto \mathbb{R} \text{ contínua}\}$$

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \qquad w(t) = t$$

Calcula a projeção ortogonal de f(t) sobre o plano gerado por u(t) e w(t).

$$Usando\, Gram - Schmidt :$$

$$v_1 := u$$

$$v_2 := w - proj_{v_1} w \implies \forall t \in [0,1] : v_2(t) = w(t) - (proj_{v_1} w)(t) = t - \frac{\int_0^1 s\,\mathrm{d}s}{\int_0^1 1\,\mathrm{d}s} u(t) = t - \frac{1}{2}$$

$$||v_2|| = \sqrt{\langle v_2, v_2\rangle} = \sqrt{\int_0^1 (t^2 + \frac{1}{4})\,\mathrm{d}t} = \sqrt{\frac{7}{12}}$$

$$b_1 := 1 \qquad b_2 := \frac{v_2(t)}{||v_2||} = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$proj_{b_1} f + proj_{b_1} f = \langle f, b_1\rangle b_1 + \langle f, b_2\rangle b_2$$

$$= (\int_0^1 (t^2 - t + 1)\,\mathrm{dt} \cdot 1 + (\int_0^1 (t^2 - t + 1 \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}\,\mathrm{dt}) \cdot \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$