6. *(1 valor)* Determine uma base ortonormada do plano $P = \{x + y - z = 0\} \subset \mathbb{R}^3$.

Como também se pode determinar que:

$$z = y + y$$

então

$$(x, y, x + y)$$

e daí

$$x(1, 0, 1) + y(0, 1, 1)$$

Podemos determinar os vetores

$$w_1 = v_1 = (1, 0, 1)$$

$$v_2 = (0, 1, 1)$$

$$w_2 = u_2 - \frac{\langle v_2, w_1 \rangle}{||w_1||^2} w_1 = (-\frac{1}{2}, 1, \frac{1}{2})$$

$$\frac{w_2}{||w_2||} = \frac{1}{\frac{\sqrt{6}}{2}}(-\frac{1}{2}, 1, \frac{1}{2}) = \frac{2}{\sqrt{6}}(-\frac{1}{2}, 1, \frac{1}{2}) = (-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$$

$$\frac{w_1}{||w_1||} = \frac{1}{\sqrt{2}}(1, 0, 1) = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$$

$$Span(P) = \{(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})\}$$

Outro método

Sabe-se que um vetor normal deste plano é: $n = (1, 1, -1)$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle -1, -2, 1 \rangle$$

$$w_2 = (-1, -2, 1)$$

$$\frac{w_2}{||w_2||} = \frac{1}{\sqrt{6}}(-1, -2, 1) = (-\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$$

$$Span(P) = \{(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})\}$$

8. *(1 valor)* Calcule a fatorização QR (ou seja, determine uma matriz ortogonal Q e uma matriz triangular superior R tais que $A = QR$) da matriz

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 5 \end{pmatrix}$$

$$u_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$v_1 = u_1 = (1, 1)$$

$$v_2 = u_2 - \text{proj}_{w_1} u_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{||v_1||^2} v_1$$

$$\frac{\langle u_2, v_1 \rangle}{||v_1||^2} v1 = \frac{(1,1) \cdot (-1,5)}{2} (1,1) = (2,2)$$

$$= (-1, 5) - (2, 2) = (-3, 3)$$

$$||v_1|| = \sqrt{2}, ||v_2|| = 3\sqrt{2}$$

$$q_1 = \frac{v_1}{||v_1||} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$q_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$q_2 = \frac{v_2}{||v_2||} = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$q_2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$R = \begin{pmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle \end{pmatrix}$$

$$R = \begin{pmatrix} \sqrt{2} & 2\sqrt{2} \\ 0 & 3\sqrt{2} \end{pmatrix}$$

Find equations for the planes tangent to the surfaces in Exercises 1–4 at the indicated points.

1. $z = x^3 + y^3 - 6xy$; $(1, 2, -3)$
2. $z = (\cos x)(\cos y)$; $(0, \pi/2, 0)$
3. $z = (\cos x)(\sin y)$; $(0, \pi/2, 1)$
4. $z = 1/xy$; $(1, 1, 1)$

1.

$$z = x^3 + y^3 - 6xy$$

$$P = (1, 2, -3)$$

$$z - x^3 - y^3 + 6xy = 0$$

$$\frac{\partial}{\partial x} = -3x^2 + 6y$$

$$\frac{\partial}{\partial x}(1, 2, -3) = 9$$

$$\frac{\partial}{\partial y} = -3y^2 + 6x$$

$$\frac{\partial}{\partial y}(1, 2, -3) = -6$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = -9x + 6y - 6$$

2.

$$z = (\cos x)(\cos y)$$

$$P = (0, \frac{\pi}{2}, 0)$$

$$z - (\cos x)(\cos y) = 0$$

$$\frac{\partial}{\partial x} = (\sin x)(\cos y)$$

$$\frac{\partial}{\partial x}(0, \frac{\pi}{2}, 0) = 0$$

$$\frac{\partial}{\partial y} = (\sin y)(\cos x)$$

$$\frac{\partial}{\partial y}(0, \frac{\pi}{2}, 0) = 1$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = \frac{\pi}{2} - y$$

3.

$$z = (\cos x)(\sin y)$$

$$P = (0, \frac{\pi}{2}, 1)$$

$$z - (\cos x)(\cos y) = 0$$

$$\frac{\partial}{\partial x} = (\sin x)(\sin y)$$

$$\frac{\partial}{\partial x}(0, \frac{\pi}{2}, 1) = 0$$

$$\frac{\partial}{\partial y} = -(\cos x)(\cos y)$$

$$\frac{\partial}{\partial y}(0, \frac{\pi}{2}, 1) = 0$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = 1$$

4.

$$z = \frac{1}{xy}$$

$$P = (1, 1, 1)$$

$$z - \frac{1}{xy} = 0$$

$$\frac{\partial}{\partial x} = \frac{1}{x^2 y}$$

$$\frac{\partial}{\partial x}(1, 1, 1) = 1$$

$$\frac{\partial}{\partial y} = \frac{1}{xy^2}$$

$$\frac{\partial}{\partial y}(1, 1, 1) = 1$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = -x - y + 3$$

Find the equation of the plane tangent to the graph of $z = f(x, y) = x^2 + 2y^3 + 1$ at the points in Exercises 5–8.

5. $(1, 1, 4)$
6. $(-1, -1, 0)$
7. $(0, 0, 1)$
8. $(1, -1, 0)$

$$z = f(x, y) = x^2 + 2y^3 + 1$$

$$z - x^2 - 2y^3 - 1 = 0$$

$$\frac{\partial}{\partial x} = -2x$$

$$\frac{\partial}{\partial y} = -6y^2$$

$$\frac{\partial}{\partial z} = 1$$

5.

$$P = (1, 1, 4)$$

$$\frac{\partial}{\partial x}(1, 1, 4) = -2$$

$$\frac{\partial}{\partial y}(1, 1, 4) = -6$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = -2x + 6y - 4$$

6.

$$P = (-1, -1, 0)$$

$$\frac{\partial}{\partial x}(-1, -1, 0) = 2$$

$$\frac{\partial}{\partial y}(1, 1, 4) = -6$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = -2x + 6y + 4$$

7.

$$P = (0, 0, 1)$$

$$\frac{\partial}{\partial x}(0, 0, 1) = 0$$

$$\frac{\partial}{\partial y}(0, 0, 1) = 0$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = 1$$

8.

$$P = (0, 0, 1)$$

$$\frac{\partial}{\partial x}(1, -1, 0) = -2$$

$$\frac{\partial}{\partial y}(1, -1, 0) = -6$$

$$\frac{\partial}{\partial z} = 1$$

Reta tangente ao plano:

$$z = 2x + 6y + 4$$

Find the equation of the tangent plane of the graph of f at the point $(x_0, y_0, f(x_0, y_0))$ for the functions and points in Exercises 9–12.

9. $f(x, y) = x - y + 2$; $(x_0, y_0) = (1, 1)$.
10. $f(x, y) = x^2 + 4y^2$; $(x_0, y_0) = (2, -1)$.
11. $f(x, y) = xy$; $(x_0, y_0) = (1, 1)$.
12. $f(x, y) = x/(x + y)$; $(x_0, y_0) = (1, 0)$.

9.

$$f(x, y) = x - y + 2$$

;

$$(x_0, y_0) = (1, 1)$$

$$f_x = \frac{\partial f}{\partial x} = 1$$

$$f_x = \frac{\partial f}{\partial x}(1, 1) = 1$$

$$f_y = \frac{\partial f}{\partial y} = -1$$

$$f_y = \frac{\partial f}{\partial y}(1, 1) = -1$$

Reta tangente ao plano:

$$z = x - y + 2$$

13.

$$g(x, z, y) = z - f(x, y) = 0$$

$$n = \frac{\nabla f}{||\nabla f||} = \frac{\langle -1, 1, 1\rangle}{\sqrt{(1)^2 + (-1)^2 + 1}} = \langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$$