Comprimento de uma arco:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ \mathrm{d}x$$

$$L = \int \mathrm{d}s$$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ se $y = f(x)$, $a \leq x \leq b$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\ \mathrm{d}y$ se $x = h(y)$, $c \leq x \leq d$

$$L = \int \mathrm{d}s$$

1.

Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

$$f'(\ln(\sec x)) = \tan(x)$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)} = \int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^2(x)}\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec x + \tan x}{\sec x + \tan x} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\ \mathrm{d}x$$

$$u = \sec x + \tan x \qquad du = \sec^2 x + \sec x \tan x dx$$

$$u(0) = 1 \qquad u(\frac{\pi}{4}) = \sqrt{2} + 1$$

$$= \int_1^{\sqrt{2}+1} \frac{1}{u}\ \mathrm{d}u$$

$$= [\ln|u|]_1^{\sqrt{2}+1} = \ln(\sqrt{2} + 1)$$

2.

Determine the length of x=$\frac{2}{3}\,(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

$$f'(\frac{2}{3}\,(y-1)^{\frac{3}{2}}) = \sqrt{(y-1)}$$

$$L = \int_1^4 \sqrt{1+(\sqrt{(y-1)})^2}\ \mathrm{d}y = \int_1^4 \sqrt{y}\ \mathrm{d}y$$

$$= \left[\frac{2}{3}y^{\frac{3}{2}}\right]_1^4 = \frac{2}{3}4^{\frac{3}{2}} - \frac{2}{3} = \frac{14}{3}$$

3.

Redo the previous example using the function in the form $y = f(x)$ instead.

4.

Determine the length of $x = \frac{1}{2}y^2$ for $0 \leq x \leq \frac{1}{2}$ .Assume that y is positive.

Comprimento de uma curva:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ \mathrm{d}t = \int_a^b ||\vec{r}'(t)||\ \mathrm{d}t$$

1.

Determine the length of the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ on the interval $0 \leq t \leq 2\pi$.

2.

Determine the arc length function for $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$.

3.

Where on the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ are we after traveling for a distance of $\frac{\pi\sqrt{10}}{3}$?

Área de superfície:

$$A(S) = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\ \mathrm{d}x\,\mathrm{d}y$$

Valores Médios:

$$A = \iint_D \mathrm{d}x\,\mathrm{d}y$$

$$\bar{f} = \frac{\iint_D (x,y)\ \mathrm{d}x\,\mathrm{d}y}{A}$$

$$\bar{f} = \frac{1}{A(R)} \iint_R f(x,y)\ \mathrm{d}A$$

Centro de Massa:

$$M = \iint_R \rho(x,y)\ \mathrm{d}A \qquad M_x = \iint_R y\rho(x,y)\ \mathrm{d}A \qquad M_y = \iint_R x\rho(x,y)\ \mathrm{d}A$$

$$R = (\bar{x}.\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$$\bar{x} = \frac{1}{A} \iint_D x f(x,y)\ \mathrm{d}x\,\mathrm{d}y$$

$$\bar{y} = \frac{1}{A} \iint_D y f(x,y)\ \mathrm{d}x\,\mathrm{d}y$$