Find the derivative matrices in Exercises 5–8 and evaluate at the given points.

5. $\partial(x, y)/\partial(u, v)$; $x = u \sin v$, $y = e^{uv}$; at $(0, 1)$.

$$x = u\sin(v) \qquad y = e^{uv}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} \sin(v) & u\cos(v) \\ ve^{uv} & ue^{uv} \end{pmatrix} = \begin{pmatrix} \sin 1 & 0 \\ 1 & 0 \end{pmatrix}$$

6. $\partial(x, y, z)/\partial(r, \theta, \phi)$; where $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$; at $(2, \pi/3, \pi/4)$.

$$x = r\sin\phi\cos\theta \qquad y = r\sin\phi\sin\theta \qquad x = r\cos\phi$$

$$\frac{\partial(x,y,z)}{\partial(r,\phi,\theta)} = \begin{pmatrix} \sin\phi\cos\theta & -r\sin\phi\sin\theta & r\cos\phi\cos\theta \\ \sin\phi\sin\theta & r\sin\phi\cos\theta & r\cos\phi\sin\theta \\ \cos\phi & 0 & -r\sin\phi \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{2} \\ \frac{\sqrt{2}}{2} & 0 & -\sqrt{2} \end{pmatrix}$$

Compute $\partial z/\partial x$ and $\partial z/\partial y$ in Exercises 21–24 using matrix multiplication and by direct substitution.

21. $z = u^2 + v^2$; $u = 2x + 7$, $v = 3x + y + 7$.
22. $z = u^2 + 3uv - v^2$; $u = \sin x$, $v = -\cos x + \cos y$.
23. $z = \sin u \cos v$; $u = 3x^2 - 2y$, $v = x - 3y$.
24. $z = u/v^2$; $u = x + y$, $v = xy$.

21.

$$z = u^2 + v^2 \qquad u = 2x + 7 \qquad v = 3x + y + 7$$

$$\begin{pmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{pmatrix} \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 4u & 6u + 2v \end{pmatrix} = \begin{pmatrix} 8x + 28 & 12x + 42 + 6x + 2y + 14 \end{pmatrix}$$