L $= \int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x$

Coordenadas Polares

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r^2 = x^2 + y^2$$

$$L^2 = \int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x \cdot \int_R e^{-\frac{y^2}{2}}\ \mathrm{d}y = \iint_{R^2} e^{-\frac{1}{2}(x^2+y^2)}\ \mathrm{d}x$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

$$L^2 = \iint_{R^2} e^{-\frac{r^2}{2}} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-\frac{r^2}{2}} r\ \mathrm{d}r \cdot \int_0^{2\pi} \mathrm{d}\theta$$

$$u = -\frac{r^2}{2} \qquad -du = r\ \mathrm{d}r$$

$$u(\infty) = -\infty \qquad u(0) = 0$$

$$-\int_0^{-t} e^u\ \mathrm{d}u = \lim_{t\to\infty} [e^u]_0^{-t} = -(0-1) = 1$$

$$-\int_0^\infty e^u\ \mathrm{d}u \cdot 2\pi = 1 \cdot 2\pi$$

Então a integral é $\sqrt{2\pi}$

L $= \int_R e^{-x^2}\ \mathrm{d}x$

$$L^2 = \int_R e^{-x^2}\ \mathrm{d}x \cdot \int_R e^{-y^2}\ \mathrm{d}y = \iint_{R^2} e^{-(x^2+y^2)}\ \mathrm{d}x$$

$$\int_0^\infty e^{-r^2} r\ \mathrm{d}r = -\frac{1}{2}\lim_{t\to\infty}\left[e^{-r^2}\right]_0^t = -\frac{1}{2}(0-1) = \frac{1}{2}$$

$$L^2 = \iint_{R^2} e^{-r^2} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-r^2} r\ \mathrm{d}r \cdot \int_0^{2\pi} \mathrm{d}\theta = \frac{1}{2}\cdot 2\pi = \pi$$

Então a integral é $\sqrt{\pi}$

Comprimento de uma arco:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ \mathrm{d}x$$

$$L = \int \mathrm{d}s$$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ se $y = f(x)$, $a \leq x \leq b$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\ \mathrm{d}y$ se $x = h(y)$, $c \leq x \leq d$

$$L = \int \mathrm{d}s$$

1.

Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

$$f'(\ln(\sec x)) = \tan(x)$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)} = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)}\ \mathrm{d}x = \int_0^{\frac{\pi}{4}} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec x + \tan x}{\sec x + \tan x} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\ \mathrm{d}x$$

$$u = \sec x + \tan x \qquad du = \sec^2 x + \sec x \tan x dx$$

$$u(0) = 1 \qquad u(\frac{\pi}{4}) = \sqrt{2} + 1$$

$$= \int_1^{\sqrt{2}+1} \frac{1}{u}\ \mathrm{d}u$$

$$= [\ln|u|]_1^{\sqrt{2}+1} = \ln(\sqrt{2} + 1)$$

2.

Determine the length of x=$\frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

$$f'(\frac{2}{3}(y-1)^{\frac{3}{2}}) = \sqrt{(y-1)}$$

$$L = \int_1^4 \sqrt{1 + (\sqrt{(y-1)})^2}\ \mathrm{d}y = \int_1^4 \sqrt{y}\ \mathrm{d}y$$

$$= \left[\frac{2}{3} y^{\frac{3}{2}}\right]_1^4 = \frac{2}{3} 4^{\frac{3}{2}} - \frac{2}{3} = \frac{14}{3}$$

3.

Redo the previous example using the function in the form $y = f(x)$ instead.

4.

Determine the length of $x = \frac{1}{2}y^2$ for $0 \leq x \leq \frac{1}{2}$ .Assume that y is positive.

Comprimento de uma curva:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, \mathrm{d}t = \int_a^b ||\vec{r}'(t)|| \, \mathrm{d}t$$

1.

Determine the length of the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ on the interval $0 \leq t \leq 2\pi$.

$$\vec{r}'(t) = \langle 2, 6\cos(2t), -6\sin(2t)\rangle$$

$$||\vec{r}'(t)|| = \sqrt{2^2 + (6\cos(2t))^2 + (-6\sin(2t))^2}$$

$$= 2\sqrt{10}$$

$$\int_0^{2\pi} ||\vec{r}'(t)|| \, \mathrm{d}t = \int_0^{2\pi} 2\sqrt{10} \, \mathrm{d}t = 4\pi\sqrt{10}$$

2.

Determine the arc length function for $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$.

$$||\vec{r}'(t)|| = 2\sqrt{10}$$

$$s(t) = \int_0^t ||\vec{r}'(u)|| \, \mathrm{d}u$$

$$s(t) = \int_0^t 2\sqrt{10} \, \mathrm{d}u = \left[2\sqrt{10}u\right]_0^t 2\sqrt{10}t$$

$$t = \frac{s}{2\sqrt{10}}$$

Parametrizar:

$$\vec{r}(t(s)) = \langle \frac{s}{\sqrt{10}}, 3\sin(\frac{s}{\sqrt{10}}), 3\cos(\frac{s}{\sqrt{10}})\rangle$$

3.

Where on the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ are we after traveling for a distance of $\frac{\pi\sqrt{10}}{3}$?

$$\vec{r}(t(\frac{\pi\sqrt{10}}{3})) = \langle \frac{\pi}{3}, 3\sin(\frac{\pi}{3}), 3\cos(\frac{\pi}{3})\rangle$$
$$= \langle \frac{\pi}{3}, \frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$$

Practice Problems

For problems 1&2 determine the length of the vector function on the given interval.

1.

$\vec{r}(t) = (3-4t)\hat{\mathbf{i}} + 6t\hat{\mathbf{j}} - (9+2t)\hat{\mathbf{k}}$ from $-6 \le t \le 8$.

$$||\vec{r}'(t)|| = \sqrt{16+36+4} = 2\sqrt{14}$$

$$= \int_{-6}^{8} ||\vec{r}'(t)||\ \mathrm{d}t = \int_{-6}^{8} 2\sqrt{14}\ \mathrm{d}t$$

$$2\sqrt{14}\,[t]_{-6}^{8} = 28\sqrt{14}$$

2.

$\vec{r}(t) = \langle \frac{1}{3}t^3, 4t, \sqrt{2}t^2\rangle$ from $0 \le t \le 2$.

$$||\vec{r}'(t)|| = \sqrt{t^4+16+8t^2} = \sqrt{(t^2+4)^2} = t^2+4$$

$$= \int_{0}^{2} ||\vec{r}'(t)||\ \mathrm{d}t = \int_{0}^{2} t^2+4\ \mathrm{d}t$$

$$= \left[\frac{1}{3}t^3 + 4t\right]_0^2 = \frac{32}{3}$$

or problems 3&4 find the arc length function for the given vector function.

3.

$\vec{r}(t) = \langle t^2, 2t^3, 1-t^3\rangle$

$$||\vec{r}'(t)|| = \sqrt{4t^2+45t^4} = t\sqrt{4+45t^2}$$

$$s(t) = \int_0^t u\sqrt{4+45u^2}\ \mathrm{d}u = \frac{1}{135}\left[\left(4+45u^2\right)^{\frac{3}{2}}\right]_0^t$$

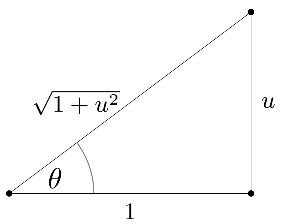
$$= \frac{1}{135}\left[\left(4+45t^2\right)^{\frac{3}{2}} - 8\right]$$

4.

$\vec{r}(t) = \langle 4t, -2t, \sqrt{5}t^2 \rangle$

$$||\vec{r}'(t)|| = \sqrt{20+20t^2} = 2\sqrt{5}\sqrt{1+t^2}$$

$$s(t) = 2\sqrt{5}\int_0^t \sqrt{1+u^2}\ \mathrm{d}u$$



$$u = \tan(\theta) \qquad \mathrm{d}u = \sec^2(\theta)\ \mathrm{d}\theta$$

$$\sqrt{1+(\tan\theta)^2} = |\sec^2\theta| = \sec\theta$$

$$0 = \tan\theta \qquad \tan^{-1}(t) = \theta$$

$$s(t) = 2\sqrt{5}\int_0^{\tan^{-1}(t)} \sec^3(\theta)\ \mathrm{d}\theta$$

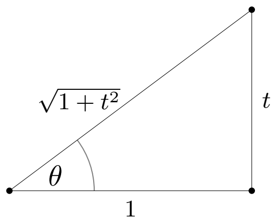
$$f = \sec(\theta) \qquad f' = \sec(\theta)\tan(\theta)$$

$$g' = \sec^2(\theta)\ \mathrm{d}\theta \qquad g = \tan(\theta)$$

$$2\sqrt{5}\left([\sec(\theta)\tan(\theta)]_0^{tan^{-1}(t)} - \int_0^{tan^{-1}(t)} \sec(\theta)\tan^2(\theta) \right)$$

$$2\sqrt{5}\left(\sec(tan^{-1}(t))\tan(tan^{-1}(t)) - \int_0^{tan^{-1}(t)} \sec^3(\theta)\ \mathrm{d}\theta + \int_0^{tan^{-1}(t)} \sec(\theta)\ \mathrm{d}\theta \right)$$

$$\sqrt{5}\left(\sec\left(tan^{-1}(t)\right)\tan\left(tan^{-1}(t)\right) + ln|\sec\left(tan^{-1}(t)\right) + \tan\left(tan^{-1}(t)\right)|\right)$$



$$\theta = \tan^{-1}(t) \qquad \sec(\theta) = \sqrt{1+t^2} \qquad \sec\left(tan^{-1}(t)\right) = \sec\theta = \sqrt{1+t^2}$$

$$s(t) = \sqrt{5}\left(t\sqrt{1+t^2} + ln|\sqrt{1+t^2} + t|\right)$$

5.

$$t = \sqrt{\frac{(135s+8)^{\frac{2}{3}} - 4}{45}}$$

$$t(20) = \sqrt{\frac{(2708)^{\frac{2}{3}} - 4}{45}} = 2.05633$$

$$r(t(\vec{20})) = \langle 4.22849, 17.39035, -7.69518\rangle$$

For problems 1&2 determine the length of the vector function on the given interval.

1.

$\vec{r}(t) = 4\cos(2t)\hat{\mathbf{i}} + 3t\hat{\mathbf{j}} - 4\sin(2t)\hat{\mathbf{k}}$ from $0 \le t \le 3\pi$.

$$||\vec{r}'(t)|| = \sqrt{(-8\sin 2t)^2 + 9 + (-8\cos 2t)^2} = \sqrt{73}$$

$$= \int_0^{3\pi} ||\vec{r}'(t)||\ \mathrm{d}t = \int_0^{3\pi} \sqrt{73}\ \mathrm{d}t$$

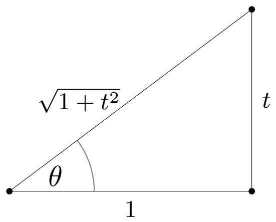
$$\sqrt{73}\,[t]_0^{3\pi} = 3\pi\sqrt{73}$$

2.

$\vec{r}(t) = \langle 9 - 2t, 4 + 2t, \sqrt{2}t^2 \rangle$ from $0 \leq t \leq 1$.

$$||\vec{r}'(t)|| = \sqrt{(-2)^2 + 2^2 + (2\sqrt{2}t)^2} = 2\sqrt{2}\sqrt{t^2 + 1}$$

$$= \int_0^1 ||\vec{r}'(t)|| \text{ d}t = \int_0^1 2\sqrt{2}\sqrt{t^2 + 1} \text{ d}t$$



$$t = \tan(\theta) \qquad \text{d}t = \sec^2(\theta) \text{ d}\theta$$

$$\sqrt{1 + (\tan\theta)^2} = |\sec^2\theta| = \sec\theta$$

$$0 = \tan\theta \qquad 1 = \tan\theta = \frac{\pi}{4}$$

$$= 2\sqrt{2}\int_0^{\frac{\pi}{4}} \sec^3\theta \text{ d}\theta$$

$$= 2\sqrt{2}\left([\sec(\theta)\tan(\theta)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sec^3(\theta) \text{ d}\theta + \int_0^{\frac{\pi}{4}} \sec(\theta) \text{ d}\theta\right) = 3.25$$

3.

$\vec{r}(t) = 2t\vec{i} + \frac{1}{2}t^2\vec{j} + \ln(t^2)\vec{k}, \sqrt{2}t^2$ from $1 \leq t \leq 3$.

$$||\vec{r}'(t)|| = \sqrt{t^4 + 16 + 8t^2} = \sqrt{(t^2 + 4)^2} = t^2 + 4$$

$$= \int_0^2 ||\vec{r}'(t)|| \text{ d}t = \int_0^2 t^2 + 4 \text{ d}t$$

$$= \left[\frac{1}{3}t^3 + 4t\right]_0^2 = \frac{32}{3}$$

The curvature measures how fast a curve is changing direction at a given point.

$$k = \left\| \frac{\mathrm{d}\vec{T}}{\mathrm{d}s} \right\|$$

Alternative:

$$k = \frac{\left\|\vec{T}'(t)\right\|}{\|\vec{r}'(t)\|} \qquad k = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$$

If $y = f(x)$:

$$\vec{r}(x) = x\vec{i} + f(x)\vec{j}$$

Curvature formula:

$$k = \frac{|f''(x)|}{\left(1 + [f'(x)]^2\right)^{\frac{3}{2}}}$$

Determine the curvature for $\vec{r}(t) = \langle t, 3\sin(t), 3\cos(t)\rangle$.

$$\vec{r}'(t) = \langle 1, 3\cos(t), -3\sin(t)\rangle$$

$$||\vec{r}'(t)|| = \sqrt{10}$$

$$\vec{T}(t) = \langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\cos(t), -\frac{3}{\sqrt{10}}\sin(t)\rangle$$

$$\vec{T}'(t) = \langle 0, -\frac{3}{\sqrt{10}}\sin(t), -\frac{3}{\sqrt{10}}\cos(t)\rangle$$

$$||\vec{T}'(t)|| = \frac{3}{\sqrt{10}}$$

$$k = \frac{\left\|\vec{T}'(t)\right\|}{\|\vec{r}'(t)\|} = \frac{\frac{3}{\sqrt{10}}}{\sqrt{10}} = \frac{3}{10}$$

Determine the curvature of $\vec{r}(t) = t^2\vec{i} + t\vec{k}$

$$\vec{r}'(t) = 2t\vec{i} + \vec{k}$$

$$||\vec{r}'(t)|| = \sqrt{4t^2 + 1}$$

$$\vec{r}''(t) = 2\vec{i}$$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t & 0 & 1 \\ 2 & 0 & 0 \end{vmatrix} = 2\vec{j}$$

$$\left\|\vec{r}'(t) \times \vec{r}''(t)\right\| = 2$$

$$k = \frac{2}{(4t^2+1)^{\frac{3}{2}}}$$

Find the curvature for each the following vector functions.

1.

$\vec{r}(t) = \langle \cos(2t), -\sin(2t), 4t \rangle$

$$\vec{r}'(t) = \langle -2\sin(2t), -2\cos(2t), 4 \rangle$$

$$||\vec{r}'(t)|| = \sqrt{20} = 2\sqrt{5}$$

$$\vec{T}(t) = \langle -\frac{1}{\sqrt{5}} \sin(2t), -\frac{1}{\sqrt{5}} \cos(2t), \frac{2}{\sqrt{5}} \rangle$$

$$\vec{T}'(t) = \langle -\frac{2}{\sqrt{5}} \cos(2t), \frac{2}{\sqrt{5}} \sin(2t), 0 \rangle$$

$$||\vec{T}'(t)|| = \sqrt{\frac{4}{\sqrt{5}} \cos^2(2t) + \frac{4}{\sqrt{5}} \sin^2(2t)} = \frac{2}{\sqrt{5}}$$

$$k = \frac{\left\|\vec{T}'(t)\right\|}{\|\vec{r}'(t)\|} = \frac{\frac{2}{\sqrt{5}}}{2\sqrt{5}} = \frac{1}{5}$$

2.

$\vec{r}(t) = \langle 4t, -t^2, 2t^3 \rangle$

$$\vec{r}'(t) = \langle 4, -2t, 6t^2 \rangle$$

$$||\vec{r}'(t)|| = \sqrt{16 + 4t^2 + 36t^4}$$

$$\vec{r}''(t) = \langle 0, -2, 12t \rangle$$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2t & 6t^2 \\ 0 & -2 & 12t \end{vmatrix} = -12t^2\vec{i} - 48t\vec{j} - 8\vec{k}$$

$$\left\|\vec{r}'(t) \times \vec{r}''(t)\right\| = \sqrt{144t^4 + 2304t^2 + 64}$$

$$k = \frac{\sqrt{144t^4 + 2304t^2 + 64}}{(16 + 4t^2 + 36t^4)^{\frac{3}{2}}}$$

1.

$\vec{r}(t) = \langle 5t, 1 - 2t, 4t^{\frac{3}{2}} \rangle$

$$\vec{r}'(t) = \langle 5, -2, 6t^{\frac{1}{2}} \rangle$$

$$||\vec{r}'(t)|| = \sqrt{29 + 36t}$$

$$\vec{r}''(t) = \langle 0, 0, 3t^{-\frac{1}{2}} \rangle$$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -2 & 6t^{\frac{1}{2}} \\ 0 & 0 & 3t^{-\frac{1}{2}} \end{vmatrix} = -6t^{-\frac{1}{2}}\vec{i} - 15t^{-\frac{1}{2}}\vec{j}$$

$$\left\|\vec{r}'(t) \times \vec{r}''(t)\right\| = \sqrt{261}t^{-\frac{1}{2}}$$

$$k = \frac{\sqrt{261}t^{-\frac{1}{2}}}{(29 + 36t)^{\frac{3}{2}}}$$

2.

$\vec{r}(t) = \langle 6, e^{-5t}, 3te^{-5t} \rangle$

$$\vec{r}'(t) = \langle 5, -2, 6t^{\frac{1}{2}} \rangle$$

$$||\vec{r}'(t)|| = \sqrt{29 + 36t}$$

$$\vec{r}''(t) = \langle 0, 0, 3t^{-\frac{1}{2}} \rangle$$

$$\vec{r}'(t)\times\vec{r}''(t)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & -2 & 6t^{\frac{1}{2}}\\ 0 & 0 & 3t^{-\frac{1}{2}}\end{vmatrix}=-6t^{-\frac{1}{2}}\vec{i}-15t^{-\frac{1}{2}}\vec{j}$$

$$\left\|\vec{r}'(t)\times\vec{r}''(t)\right\|=\sqrt{261}t^{-\frac{1}{2}}$$

$$k=\frac{\sqrt{261}t^{-\frac{1}{2}}}{(29+36t)^{\frac{3}{2}}}$$

3.

$\vec{r}(t)=\langle\cos(\omega t),t,\sin(\omega t)\rangle$

$$\vec{r}'(t)=\langle 5,-2,6t^{\frac{1}{2}}\rangle$$

$$||\vec{r}'(t)||=\sqrt{29+36t}$$

$$\vec{r}''(t)=\langle 0,0,3t^{-\frac{1}{2}}\rangle$$

$$\vec{r}'(t)\times\vec{r}''(t)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & -2 & 6t^{\frac{1}{2}}\\ 0 & 0 & 3t^{-\frac{1}{2}}\end{vmatrix}=-6t^{-\frac{1}{2}}\vec{i}-15t^{-\frac{1}{2}}\vec{j}$$

$$\left\|\vec{r}'(t)\times\vec{r}''(t)\right\|=\sqrt{261}t^{-\frac{1}{2}}$$

$$k=\frac{\sqrt{261}t^{-\frac{1}{2}}}{(29+36t)^{\frac{3}{2}}}$$

Área de superfície:

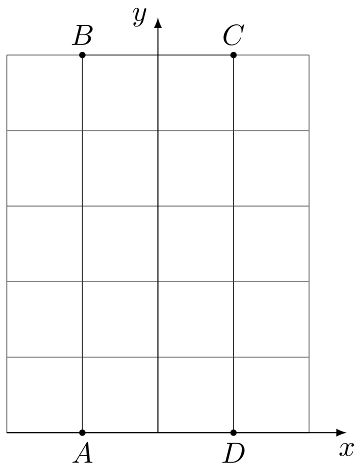
$$A(S)=\iint_D\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\,\mathrm{d}x\,\mathrm{d}y$$

Valores Médios:

$$A=\iint_D \mathrm{d}x\,\mathrm{d}y$$

$$\bar{f}=\frac{\iint_D(x,y)\,\mathrm{d}x\,\mathrm{d}y}{A}$$

$$\bar{f}=\frac{1}{A(R)}\iint_R f(x,y)\,\mathrm{d}A$$



$$A(R) = 2 \cdot 5 = 10$$

$$\bar{f} = \frac{1}{10} \int_{-1}^{1} \int_{0}^{5} x^2 y \; \mathrm{d}y \, \mathrm{d}x$$

$$= \frac{1}{10} \int_{-1}^{1} \left[x^2 \frac{y^2}{2} \right]_0^5 \mathrm{d}x = \frac{1}{10} \int_{-1}^{1} \frac{25}{2} x^2 \; \mathrm{d}x = \frac{25}{20} \left[\frac{x^3}{3} \right]_{-1}^{1} = \frac{5}{6}$$

Centro de Massa:

$$M = \iint_R \rho(x,y) \; \mathrm{d}A \qquad M_x = \iint_R y\rho(x,y) \; \mathrm{d}A \qquad M_y = \iint_R x\rho(x,y) \; \mathrm{d}A$$

$$R = (\bar{x}.\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M} \right)$$

$$\bar{x} = \frac{1}{A} \iint_D x f(x,y) \; \mathrm{d}x \, \mathrm{d}y$$

$$\bar{y} = \frac{1}{A} \iint_D y f(x,y) \; \mathrm{d}x \, \mathrm{d}y$$

Find the volume of the solid that is bounded about by $f(x,y) = y \sin(xy)$ and below $R = [1,2] \times [0,\pi]$

$$V = \int_0^{\pi} \int_1^2 y \sin(xy) \; \mathrm{d}x \, \mathrm{d}y$$

$$= \int_0^{\pi} \left[-\frac{y}{y} \cos(xy) \right]_1^2 \mathrm{d}y = \int_0^{\pi} -\cos(2y) + \cos(y) \; \mathrm{d}y$$

$$= \left[-\frac{1}{2} \sin(2y) + \sin(y) \right]_0^{\pi} = 0$$

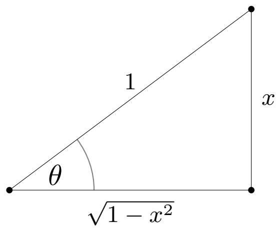
If $R = \{(x,y) | -1 \le x \le 1, -2 \le y \le 2\}$, evaluate $\iint_R \sqrt{1-x^2}\ \mathrm{d}A$
Método 1:
Como $z = \sqrt{1-x^2}$ é metade de um cilindro então:

$$V_{cilindro} = \pi r^2 h$$

$$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi 1 \cdot 4 = 2\pi$$

Método 2:

$$\int_{-2}^{2}\int_{-1}^{1} \sqrt{1-x^2}\ \mathrm{d}x\,\mathrm{d}y$$



Substituição trigonométrica:

$$x = \sin\theta \qquad \mathrm{d}x = \cos\theta\ \mathrm{d}\theta$$

$$\cos\theta = \sqrt{1-x^2}$$

Se $x = -1$ então $\theta = -\frac{\pi}{2}$
Se $x = 1$ então $\theta = \frac{\pi}{2}$

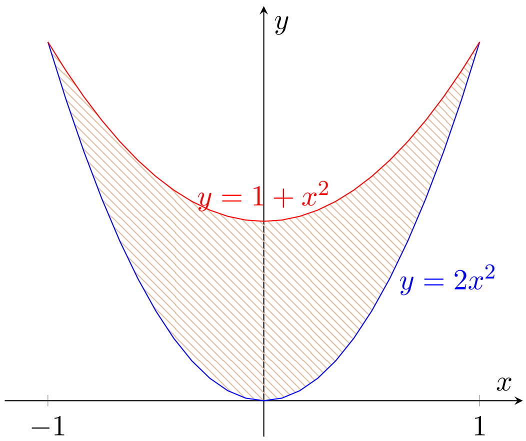
$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta\,\mathrm{d}y$$

$$\cos 2\theta = \sin^2\theta - \cos^2\theta \Leftrightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\left(1+\cos 2\theta\right)\ \mathrm{d}\theta\,\mathrm{d}y = \int_{-2}^{2} \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\ \mathrm{d}y$$

$$= \left[\frac{\pi}{2}y\right]_{-2}^{2} = 2\pi$$

Evaluate $\iint_D (x+2y)\ \mathrm{d}A$ where D is the region bounded by $y = 2x^2$ and $y = 1 + x^2$



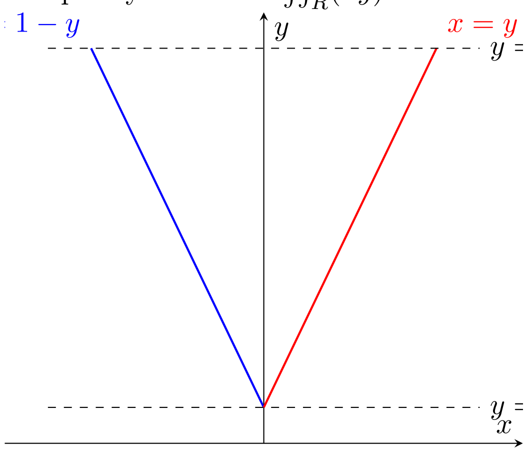
$$\int_{-1}^{1}\int_{2x^2}^{1+x^2}(x+2y)\,\mathrm{d}y\,\mathrm{d}x = \int_{-1}^{1}\left[xy+y^2\right]_{2x^2}^{1+x^2}\,\mathrm{d}x$$

$$= \int_{-1}^{1}\left[(x(1+x^2)+(1+x^2)^2)-(x(2x^2)+(2x^2)^2)\right]\,\mathrm{d}x$$

$$= \int_{-1}^{1}(-x^3-3x^4+x+2x^2+1)\,\mathrm{d}x = \left[-\frac{1}{4}x^4-\frac{3}{5}x^5+\frac{1}{2}x^2+\frac{2}{3}x^3+x\right]_{-1}^{1}$$

$$= \left[(-\frac{1}{4}-\frac{3}{5}+\frac{1}{2}+\frac{2}{3}+1)-(-\frac{1}{4}+\frac{3}{5}+\frac{1}{2}-\frac{2}{3}-1)\right] = \frac{32}{15}$$

Setup only! Evaluate $\iint_R (xy)\,\mathrm{d}A$ where R is the region bounded by $y=-x+1, y=x+1$ and $y=3$



Horizontal fixamos o x

$$\int_{1}^{3}\int_{1-y}^{y-1}(xy)\,\mathrm{d}x\,\mathrm{d}y$$

Find the volume of the solid that lies under $z=xy$ and and about D where D is the region bounded by

$$\int_{-2}^{4}\int_{\frac{1}{2}y^3-3}^{y+1}(xy)\ \mathrm{d}x\,\mathrm{d}y$$

Calculate $\iiint_R = (x+y+2z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\ R: x^2+z^2=4,y=2,y=3.$
Coordenadas cilíndricas:

$$x = r\cos(\theta) \qquad z = r\sin\theta$$

$$\int_2^3\int_0^{2\pi}\int_0^2 (r\cos(\theta)+y+2r\sin(\theta))\,r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3\int_0^{2\pi}\int_0^2 (r^2\cos(\theta)+yr+2r^2\sin(\theta))\ \mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3\int_0^{2\pi}\left[(\frac{1}{3}r^3\cos(\theta)+\frac{1}{2}yr^2+\frac{2}{3}r^3\sin(\theta))\right]_0^2\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3\int_0^{2\pi}(\frac{8}{3}\cos(\theta)+2y+\frac{16}{3}\sin(\theta))\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3\left[(\frac{8}{3}\sin(\theta)+2y\theta-\frac{16}{3}\cos(\theta))\right]_0^{2\pi}\ \mathrm{d}y$$

$$= \left[2\pi y^2\right]_2^3 = 18\pi - 8\pi = 10\pi$$

Calculate the volume of the region $\iiint_R = (x+y+2z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\ R: x^2+z^2=4,y=2,y=3.$

$$\int_2^3\int_0^{2\pi}\int_0^2 r\ \mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3\int_0^{2\pi}\left[\frac{1}{2}r^2\right]_0^2\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_2^3 [2\theta]_0^{2\pi}\ \mathrm{d}y$$

$$= [4\pi y]_2^3 = 12\pi - 8\pi = 4\pi$$

Calculate $\iiint_R = y\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\ R: x^2+y^2=3,z=-1,z=2.$
Coordenadas cilíndricas:

$$y = r\sin\theta$$

$$\int_{-1}^{2}\int_0^{2\pi}\int_0^{\sqrt{3}} r\sin(\theta)\,r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} r^2 \sin(\theta)\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \left[\frac{1}{3} r^3 \sin(\theta)\right]_0^{\sqrt{3}} \mathrm{d}\theta\, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \sqrt{3} \sin(\theta)\, \mathrm{d}\theta\, \mathrm{d}z$$

$$= [0]_{-1}^{2} = 0$$

Calculate the volume of the region $\iiint_R = y\ \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z,\ R : x^2 + y^2 = 3, z = -1, z = 2.$

$$\int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} 1\, r\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} r\ \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \left[\frac{1}{2} r^2\right]_0^{\sqrt{3}} \mathrm{d}\theta\, \mathrm{d}z$$

$$= \int_{-1}^{2} \left[\frac{3}{2}\theta\right]_0^{2\pi} \mathrm{d}z$$

$$= [3\pi z]_{-1}^{2} = 9\pi$$

Line integral with respect to arc length.
Parametric equations:

$$x = h(t) \qquad y = g(t) \qquad a \leq t \leq b$$

Parametric equations as a vector function:

$$\vec{r}(t) = h(t)\vec{i} + g(t)\vec{j} \qquad a \leq t \leq b$$

The curve is called smooth if $\vec{r}(t)$ is continuous and $\vec{r}(t) \neq 0$

$$L = \int_a^b \mathrm{d}s \qquad \mathrm{d}s = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$

$$\int_C f(x, y)\ \mathrm{d}s = \int_a^b f(h(t), g(t)) \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$

$$\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} = ||\vec{r}(t)||$$

Curve	Parametric Equations	
	Counter-Clockwise	Clockwise
$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (Ellipse)	$x=a\cos(t)$ $y=b\sin(t)$ $0\le t\le 2\pi$	$x=a\cos(t)$ $y=-b\sin(t)$ $0\le t\le 2\pi$
	Counter-Clockwise	Clockwise
$x^2+y^2=r^2$ (Circle)	$x=r\cos(t)$ $y=r\sin(t)$ $0\le t\le 2\pi$	$x=r\cos(t)$ $y=-r\sin(t)$ $0\le t\le 2\pi$
$y=f(x)$	$x=t$ $y=f(t)$	
$x=g(y)$	$x=g(t)$ $y=t$	
Line Segment From (x_0,y_0,z_0) to (x_1,y_1,z_1)	$\vec{r}(t)=(1-t)\langle x_0,y_0,z_0\rangle+t\langle x_1,y_1,z_1\rangle\ ,\ 0\le t\le 1$ or $x=(1-t)\,x_0+t\,x_1$ $y=(1-t)\,y_0+t\,y_1\quad,\quad 0\le t\le 1$ $z=(1-t)\,z_0+t\,z_1$	

Evaluate $\int_C xy^4\ \mathrm{d}s$ where C is the right half of the circle, $x^2+y^2=16$ traced out in a counter clockwise direction.

$$x=4\cos(t)\qquad y=4\sin(t)\qquad -\frac{\pi}{2}\le t\le\frac{\pi}{2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-4\sin(t)\qquad \frac{\mathrm{d}y}{\mathrm{d}t}=4\cos(t)$$

$$\mathrm{d}s=\sqrt{(-4\sin(t))^2+(4\cos(t))^2}\ \mathrm{d}t=4\ \mathrm{d}t$$

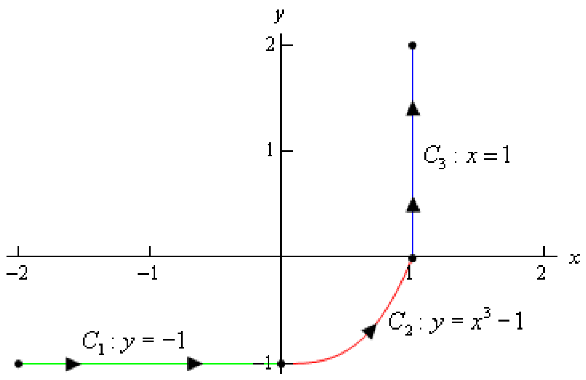
$$\int_C xy^4\ \mathrm{d}s=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(4\cos(t))\,(4\sin(t))^4\,4\ \mathrm{d}t$$

$$=4096\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos(t))\,(\sin(t))^4\ \mathrm{d}t$$

$$=\frac{4096}{5}\left[(\sin(t))^5\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{4096}{5}[2]=\frac{8192}{5}$$

Evaluate $\int_C 4x^3 \; \mathrm{d}s$ where C is the curve shown below.



$$C_1 : x = t \qquad y = -1 \qquad -2 \le t \le 0$$

$$C_1 : \; \mathrm{d}x = 1 \qquad \mathrm{d}y = 0 \qquad \mathrm{d}s = \sqrt{1} \; \mathrm{d}t = \; \mathrm{d}t$$

$$C_2 : x = t \qquad y = t^3 - 1 \qquad 0 \le t \le 1$$

$$C_2 : \; \mathrm{d}x = 1 \qquad \mathrm{d}y = 3t^2 \qquad \mathrm{d}s = \sqrt{1 + 9t^4} \; \mathrm{d}t$$

$$C_3 : x = 1 \qquad y = t \qquad 0 \le t \le 2$$

$$C_3 : \; \mathrm{d}x = 0 \qquad \mathrm{d}y = 1 \qquad \mathrm{d}s = \sqrt{1} \; \mathrm{d}t = \; \mathrm{d}t$$

$$\int_{C_1} 4x^3 \; \mathrm{d}s = \int_{-2}^{0} 4t^3 \; \mathrm{d}t$$

$$\int_{C_2} 4x^3 \; \mathrm{d}s = \int_0^1 4t^3 \sqrt{1 + 9t^4} \; \mathrm{d}t = \frac{2}{27} \left[(1 + 9t^4)^{\frac{3}{2}} \right]_0^1 = \frac{2}{27} \left[10^{\frac{3}{2}} - 1 \right] = 2.268354$$

$$\int_{C_3} 4x^3 \; \mathrm{d}s = \int_0^2 4(1)^3 \; \mathrm{d}t = [4t]_0^2 = 8$$

Evaluate $\int_C 4x^3 \; \mathrm{d}s$ where C is the line segment from $(-2, -1)$ to $(1, 2)$.

$$\vec{r}(t) = (1 - t)\langle -2, -1 \rangle + t\langle 1, 2 \rangle$$

$$= \langle 3t - 2, 3t - 1 \rangle$$

$$x = 3t - 2 \qquad y = 3t - 1$$

$$||\vec{r}(t)|| = \sqrt{18} = 3\sqrt{2}\ \mathrm{d}t$$

$$12\sqrt{2}\int_0^1 (3t-2)^3\ \mathrm{d}t = \sqrt{2}\left[(3t-2)^4\right]_0^1 = -15\sqrt{2}$$

Evaluate $\int_C 4x^3\ \mathrm{d}s$ where C is the line segment from $(1,2)$ to $(-2,-1)$.

$$\vec{r}(t) = (1-t)\langle 1,2\rangle + t\langle -2,-1\rangle$$

$$= \langle -3t+1, -3t - +2\rangle$$

$$x = -3t + 1 \qquad y = -3t + 2$$

$$||\vec{r}(t)|| = \sqrt{18} = 3\sqrt{2}\ \mathrm{d}t$$

$$12\sqrt{2}\int_0^1 (-3t+1)^3\ \mathrm{d}t = -\sqrt{2}\left[(-3t+1)^4\right]_0^1 = -15\sqrt{2}$$