

1.

Determine as seguintes primitivas:

1)

$$\begin{aligned} & \int (x^2 - 4x + \frac{5}{x}) dx \\ &= \frac{x^3}{3} - \frac{4x^2}{2} - 5 \ln |x| + C \end{aligned}$$

2)

$$\begin{aligned} & \int \frac{2x+1}{x^2+x+3} dx \\ &= \ln |x^2+x+3| + C \end{aligned}$$

3)

$$\begin{aligned} & \int \frac{3}{2x-1} dx \\ &= \frac{3}{2} \int \frac{2}{2x-1} dx \\ &= \frac{3}{2} \ln |2x-1| + C \end{aligned}$$

4)

$$\begin{aligned} & \int \frac{1}{x} \cos(\ln x) dx \\ &= \sin(\ln x) + C \end{aligned}$$

5)

$$\begin{aligned} & \int \frac{\sqrt{1+2\ln x}}{x} dx \\ &= \int \frac{1}{x} (1+2\ln x)^{\frac{1}{2}} dx \\ &= \frac{1}{2} \int \frac{2}{x} (1+2\ln x)^{\frac{1}{2}} dx \\ &= \frac{1}{2} \frac{(1+2\ln x)^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{(1+2\ln x)^{\frac{3}{2}}}{3} + C \end{aligned}$$

6)

$$\begin{aligned}\int \sin x \cos^4 x dx \\ = -\frac{1}{5} \cos^5 x\end{aligned}$$

2.

Recorrendo à primitivação por partes, determine as seguintes primitivas:

1)

$$\begin{aligned}\int x \sin 2x dx \\ f' = \sin 2x \quad f = -\frac{1}{2} \cos 2x \\ g = x \quad g' = 1 \\ = -\frac{x}{2} \cos 2x - \int \cos 2x dx \\ = -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x dx \\ = -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + C\end{aligned}$$

2)

$$\begin{aligned}\int (2x^2 - 1)e^x dx \\ f' = e^x \quad f = e^x \\ g = 2x^2 - 1 \quad g' = 4x \\ = (2x^2 - 1)e^x - \int e^x 4x dx \\ = (2x^2 - 1)e^x - 4 \int e^x x dx \\ f' = e^x \quad f = e^x \\ g = x \quad g' = 1 \\ = (2x^2 - 1)e^x - 4e^x x + 4e^x \\ = e^x(2x^2 - 4x + 3) + C\end{aligned}$$

3)

$$\begin{aligned}
 & \int \arctan x dx \\
 & \int 1 \cdot \arctan x dx \\
 & f' = 1 \quad f = x \\
 & g = \arctan x \quad g' = \frac{1}{x^2 + 1} \\
 & = x \arctan x - \int \frac{x}{x^2 + 1} dx \\
 & = x \arctan x - \frac{1}{2} \int \frac{2x}{x^2 + 1} dx \\
 & = x \arctan x - \frac{1}{2} \ln |x^2 + 1| + C
 \end{aligned}$$

3.

Recorde que  $\cos^2 x = \frac{\cos 2x + 1}{2}$  e determine  $\int \cos^2 x dx$ .

$$\begin{aligned}
 \int \cos^2 x dx &= \int \frac{\cos 2x + 1}{2} dx \\
 &= \frac{1}{2} \left( \frac{1}{2} \sin 2x + x \right) \\
 &= \frac{1}{4} \sin 2x + \frac{x}{2} + C
 \end{aligned}$$

1.

Determine as seguintes primitivas:

1)

$$\begin{aligned}
 & \int \ln x dx \\
 & \int 1 \cdot \ln x dx \\
 & f' = 1 \quad f = x \\
 & g = \ln x \quad g' = \frac{1}{x} \\
 & = x \ln x - \int 1 dx \\
 & = x \ln x - x \\
 & = x(\ln x - 1) + C
 \end{aligned}$$

2)

$$\begin{aligned} & \int \frac{e^{\arctan x}}{1+x^2} dx \\ &= \int \frac{1}{1+x^2} e^{\arctan x} dx \\ &= e^{\arctan x} + C \end{aligned}$$

3)

$$\begin{aligned} & \int \frac{-3}{x \ln^3 x} dx \\ &= -3 \int \frac{1}{x} (\ln^{-3} x) dx \\ &= \frac{3}{2} (\ln^{-2} x) \\ &= \frac{3}{2 \ln^2 x} + C \end{aligned}$$

4)

$$\begin{aligned} & \int -3x^2 \cos x dx \\ &= -3 \int x^2 \cos x dx \\ & \quad \begin{array}{ll} f' = \cos x & f = \sin x \\ g = x^2 & g' = 2x \end{array} \\ &= -3(x^2 \sin x - \int \sin x 2x dx) \\ &= -3(x^2 \sin x - 2 \int \sin x x dx) \\ & \quad \begin{array}{ll} f' = \sin x & f = -\cos x \\ g = x & g' = 1 \end{array} \\ &= -3(x^2 \sin x - 2(-x \cos x + \sin x)) \\ &= -3(x^2 \sin x + 2x \cos x - 2 \sin x) \\ &= -3x^2 \sin x - 6x \cos x + 6 \sin x + C \end{aligned}$$

5)

$$\begin{aligned}
 & \int \frac{\sin x}{\sqrt{1 + \cos x}} dx \\
 &= - \int -\sin x (1 + \cos x)^{-\frac{1}{2}} dx \\
 &= - \int -\sin x \frac{(1 + \cos x)^{\frac{1}{2}}}{\frac{1}{2}} \\
 &= -2(1 + \cos x)^{\frac{1}{2}} \\
 &= -2\sqrt{1 + \cos x} + C
 \end{aligned}$$

6)

$$\begin{aligned}
 & \int \arcsin x dx \\
 & \int 1 \cdot \arcsin x dx \\
 & f' = 1 \quad f = x \\
 & g = \arcsin x \quad g' = \frac{1}{\sqrt{1 - x^2}} \\
 &= x \arcsin x - \int \frac{x}{\sqrt{1 - x^2}} dx \\
 &= x \arcsin x - \int x(1 - x^2)^{-\frac{1}{2}} dx \\
 &= x \arcsin x + \frac{1}{2} \int -2x(1 - x^2)^{-\frac{1}{2}} dx \\
 &= x \arcsin x + \frac{1}{2} \left( \frac{(1 - x^2)^{\frac{1}{2}}}{\frac{1}{2}} \right) \\
 &= x \arcsin x + \sqrt{1 - x^2}
 \end{aligned}$$