

$\mathbf{L} = \int_R e^{-\frac{x^2}{2}} dx$
 Coordenadas Polares

$$\begin{aligned} x &= r \cos \theta & y &= r \sin \theta & r^2 &= x^2 + y^2 \\ L^2 &= \int_R e^{-\frac{x^2}{2}} dx \cdot \int_R e^{-\frac{y^2}{2}} dy = \iint_{R^2} e^{-\frac{1}{2}(x^2+y^2)} dx \\ J &= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r \cos^2 \theta + r \sin^2 \theta = r \\ L^2 &= \iint_{R^2} e^{-\frac{r^2}{2}} r dr d\theta = \int_0^\infty e^{-\frac{r^2}{2}} r dr \cdot \int_0^{2\pi} d\theta \end{aligned}$$

$$u = -\frac{r^2}{2} \quad -du = r dr$$

$$\begin{aligned} u(\infty) &= -\infty & u(0) &= 0 \\ -\int_0^{-t} e^u du &= \lim_{t \rightarrow \infty} [e^u]_0^{-t} = -(0 - 1) = 1 \\ -\int_0^\infty e^u du \cdot 2\pi &= 1 \cdot 2\pi \end{aligned}$$

Então a integral é $\sqrt{2\pi}$
 $\mathbf{L} = \int_R e^{-x^2} dx$

$$\begin{aligned} L^2 &= \int_R e^{-x^2} dx \cdot \int_R e^{-y^2} dy = \iint_{R^2} e^{-(x^2+y^2)} dx \\ \int_0^\infty e^{-r^2} r dr &= -\frac{1}{2} \lim_{t \rightarrow \infty} [e^{-r^2}]_0^t = -\frac{1}{2}(0 - 1) = \frac{1}{2} \\ L^2 &= \iint_{R^2} e^{-r^2} r dr d\theta = \int_0^\infty e^{-r^2} r dr \cdot \int_0^{2\pi} d\theta = \frac{1}{2} \cdot 2\pi = \pi \end{aligned}$$

Então a integral é $\sqrt{\pi}$

Comprimento de uma arco:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$L = \int ds$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \text{ se } y = f(x), a \leq x \leq b$$

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \text{ se } x = h(y), c \leq y \leq d$$

$$L = \int ds$$

1.

Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

$$f'(\ln(\sec x)) = \tan(x)$$

$$\begin{aligned} L &= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)} \, dx = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)} \, dx = \int_0^{\frac{\pi}{4}} \sec(x) \, dx \\ &= \int_0^{\frac{\pi}{4}} \frac{\sec x + \tan x}{\sec x + \tan x} \sec(x) \, dx \\ &= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx \end{aligned}$$

$$u = \sec x + \tan x \quad du = \sec^2 x + \sec x \tan x \, dx$$

$$u(0) = 1 \quad u\left(\frac{\pi}{4}\right) = \sqrt{2} + 1$$

$$\begin{aligned} &= \int_1^{\sqrt{2}+1} \frac{1}{u} \, du \\ &= [\ln |u|]_1^{\sqrt{2}+1} = \ln(\sqrt{2} + 1) \end{aligned}$$

2.

Determine the length of $x = \frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

$$f'\left(\frac{2}{3}(y-1)^{\frac{3}{2}}\right) = \sqrt{(y-1)}$$

$$\begin{aligned} L &= \int_1^4 \sqrt{1 + (\sqrt{(y-1)})^2} \, dy = \int_1^4 \sqrt{y} \, dy \\ &= \left[\frac{2}{3} y^{\frac{3}{2}} \right]_1^4 = \frac{2}{3} 4^{\frac{3}{2}} - \frac{2}{3} = \frac{14}{3} \end{aligned}$$

3.

Redo the previous example using the function in the form $y = f(x)$ instead.

4.

Determine the length of $x = \frac{1}{2}y^2$ for $0 \leq x \leq \frac{1}{2}$. Assume that y is positive.

Comprimento de uma curva:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b \|\vec{r}'(t)\| dt$$

1.

Determine the length of the curve $\vec{r}(t) = \langle 2t, 3 \sin(2t), 3 \cos(2t) \rangle$ on the interval $0 \leq t \leq 2\pi$.

2.

Determine the arc length function for $\vec{r}(t) = \langle 2t, 3 \sin(2t), 3 \cos(2t) \rangle$.

3.

Where on the curve $\vec{r}(t) = \langle 2t, 3 \sin(2t), 3 \cos(2t) \rangle$ are we after traveling for a distance of $\frac{\pi\sqrt{10}}{3}$?

Área de superfície:

$$A(S) = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} dx dy$$

Valores Médios:

$$\begin{aligned} A &= \iint_D dx dy \\ \bar{f} &= \frac{\iint_D f(x, y) dx dy}{A} \\ \bar{f} &= \frac{1}{A(R)} \iint_R f(x, y) dA \end{aligned}$$

Centro de Massa:

$$M = \iint_R \rho(x, y) dA \quad M_x = \iint_R y \rho(x, y) dA \quad M_y = \iint_R x \rho(x, y) dA$$

$$\bar{R} = (\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M} \right)$$

$$\bar{x} = \frac{1}{A} \iint_D x f(x, y) dx dy$$

$$\bar{y} = \frac{1}{A} \iint_D y f(x, y) \, dx \, dy$$

If $R = \{(x, y) | -1 \leq x \leq 1, -2 \leq y \leq 2\}$, evaluate $\iint_R \sqrt{1-x^2} \, dA$

Método 1:

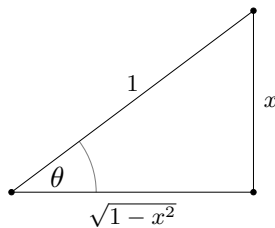
Como $z = \sqrt{1-x^2}$ é metade de um cilindro então:

$$V_{cilindro} = \pi r^2 h$$

$$V = \frac{1}{2} \pi r^2 h = \frac{1}{2} \pi 1 \cdot 4 = 2\pi$$

Método 2:

$$\int_{-2}^2 \int_{-1}^1 \sqrt{1-x^2} \, dx \, dy$$



Substituição trigonométrica:

$$x = \sin \theta \quad dx = \cos \theta \, d\theta$$

$$\cos \theta = \sqrt{1-x^2}$$

Se $x = -1$ então $\theta = -\frac{\pi}{2}$

Se $x = 1$ então $\theta = \frac{\pi}{2}$

$$\int_{-2}^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta \, dy$$

$$\cos 2\theta = \sin^2 \theta - \cos^2 \theta \Leftrightarrow \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\int_{-2}^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (1 + \cos 2\theta) \, d\theta \, dy = \int_{-2}^2 \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dy$$

$$= \left[\frac{\pi}{2} y \right]_{-2}^2 = 2\pi$$