1.

Determine as seguintes integrais:

1)

$$\int_0^{\sqrt{\frac{\pi}{2}}} x \sin x^2 dx$$

$$= \frac{1}{2}\int_0^{\sqrt{\frac{\pi}{2}}} 2x \sin x^2 dx$$

$$= \frac{1}{2}(-\cos x^2)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}(-\cos(\sqrt{\frac{\pi}{2}})^2 + \cos 0)$$

$$= \frac{1}{2}$$

2)

$$\int_0^{\pi} (x+2)\cos x dx$$

$$f' = \cos x \qquad f = \sin x$$

$$g = x+2 \qquad g' = 1$$

$$(x+2)\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x dx$$

$$(\pi+2)\sin\pi - 2\sin 0 + \cos x\Big|_0^{\pi}$$

$$= -2$$

3)

$$\int_1^2 x 2^x dx$$

$$f' = 2^x \qquad f = \frac{2^x}{\ln 2}$$

$$g = x \qquad g' = 1$$

$$\frac{2^x}{\ln 2}x\Big|_1^2 - \int_1^2 \frac{2^x}{\ln 2}$$

$$= \frac{8}{\ln 2} - \frac{2}{\ln 2} - \frac{1}{\ln 2}\frac{2^x}{\ln 2}\Big|_1^2$$

$$= \frac{6}{\ln 2} - \frac{2}{\ln^2 2}$$
$$= \frac{6\ln 2 - 2}{\ln^2 2}$$

4)

$$\int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx$$
$$2(e^x+1)^{\frac{1}{2}})\Big|_0^1$$
$$2((e+1)^{\frac{1}{2}} - (2)^{\frac{1}{2}})$$

2.

a)

Calcule $\int_0^{\frac{\pi}{2}} e^x \sin(x) dx$:

$$\int_0^{\frac{\pi}{2}} e^x sin(x) dx$$
$$f' = \sin(x) \qquad f = -\cos(x)$$
$$g = e^x \qquad g' = e^x$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = -\cos(x) e^x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos(x) e^x dx$$
$$f' = \cos(x) \qquad f = \sin(x)$$
$$g = e^x \qquad g' = e^x$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = 1 + e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin(x) e^x dx$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = \frac{1}{2}(1 + e^{\frac{\pi}{2}})$$

3.

Usando uma substituição, calcule os seguintes integrais

1)

$$\int_{-1}^{1} e^{\arcsin(x)} dx$$

$$\sin(x) : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$$

$$x = \sin(t) \qquad dx = \cos(t)dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\arcsin(\sin(t))} \cos(t)dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt$$

$$f' = \cos(t) \qquad f = \sin(t)$$

$$g = e^t \qquad g' = e^t$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt = (\sin(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \sin(t)dt$$

$$f' = \sin(t) \qquad f = -\cos(t)$$

$$g = e^t \qquad g' = e^t$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}} - [(-\cos(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -e^{t} \cos(t)dt]$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}} + (\cos(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt$$

$$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{t} \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}}$$

2)

3)

$$\int_{0}^{\frac{3}{2}} 2^{\sqrt{2x+1}} dx$$

$$u = \sqrt{2x+1}$$

$$\frac{1}{2}(u^2 - 1) = x$$

$$dx = udu$$

$$\int_{-\frac{1}{2}}^{\frac{5}{8}} 2^{u} udu$$

$$f' = 2^u \qquad f = \frac{2^u}{\ln(2)}$$

$$g = u \qquad g' = 1$$

$$\frac{2^u}{\ln(2)} \cdot u - \int_{-\frac{1}{2}}^{\frac{5}{8}} \frac{2^u}{\ln(2)} du$$

$$\frac{2^u}{\ln(2)} \cdot u\Big|_{-\frac{1}{2}}^{\frac{5}{8}} - \frac{1}{\ln(2)} \int_{-\frac{1}{2}}^{\frac{5}{8}} 2^u du$$

$$\frac{2^u}{\ln(2)} \cdot u\Big|_{-\frac{1}{2}}^{\frac{5}{8}} - \frac{1}{\ln^2(2)} 2^u\Big|_{-\frac{1}{2}}^{\frac{5}{8}}$$

4)

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$$

$$x = \sin(t)$$

$$dx = \cos(t) dt$$

$$\arcsin(\frac{\sqrt{2}}{2}) = \arcsin(\sin(t))$$

$$t = \frac{\pi}{4}$$

$$\arcsin(0) = \arcsin(\sin(t))$$

$$t = 0$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin^2(t)}{\cancel{\cos(t)}} \cancel{\cos(t)} dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1-\cos(2t)}{2} dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} - \frac{\cos(2t)}{2} dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} dt - \frac{1}{2} \int_0^{\frac{\pi}{4}} \cos(2t) dt$$

$$\frac{1}{2} t\Big|_0^{\frac{\pi}{4}} - \frac{1}{4} \sin(2t)\Big|_0^{\frac{\pi}{4}}$$

$$\frac{1}{2} \cdot \frac{\pi}{4} - \frac{\pi}{4} \cdot \sin(\cancel{2} \cdot \frac{\pi}{\cancel{4}})$$

$$\frac{\pi - 2}{8}$$