

$$E = \{f : [0, 1] \mapsto \mathbb{R} \text{ contínua}\}$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \quad w(t) = t$$

Calcula a projeção ortogonal de $f(t)$ sobre o plano gerado por $u(t)$ e $w(t)$.

$$\langle f, u \rangle = \int_0^1 f(t)u(t)dt = \int_0^1 t^2 - t + 1dt = \left[\frac{1}{3}t^3 - \frac{1}{2}t^2 + t \right]_0^1 = \frac{5}{6}$$

$$\langle u, u \rangle = \int_0^1 u(t)u(t)dt = [t]_0^1 = 1$$

$$\langle f, w \rangle = \int_0^1 f(t)w(t)dt = \int_0^1 t^3 - t^2 + tdt = \left[\frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2 \right]_0^1 = \frac{5}{12}$$

$$\langle w, w \rangle = \int_0^1 w(t)w(t)dt = \int_0^1 t^2dt = \left[\frac{1}{3}t^3 \right]_0^1 = \frac{1}{3}$$

$$\frac{\langle f(t), u(t) \rangle}{\langle u(t), u(t) \rangle} u(t) = \frac{5}{6}$$

$$\frac{\langle f(t), w(t) \rangle}{\langle w(t), w(t) \rangle} w(t) = \frac{5t}{4}$$

Projeção ortogonal de $f(t)$ sobre o plano gerado de $u(t)=1$ e $w(t)=t$ é:

$$\frac{5}{6} + \frac{5t}{4}$$