
$$E = \{f : [0, 1] \mapsto \mathbb{R} \text{ cont ua}\}$$

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \quad w(t) = t$$

Calcula a proje  o ortogonal de $f(t)$ sobre o plano gerado por $u(t)$ e $w(t)$.

Usando Gram-Schmidt:

$$v_1 := u$$

$$v_2 := w - \text{proj}_{v_1} w \implies \forall t \in [0, 1] : v_2(t) = w(t) - (\text{proj}_{v_1} w)(t) = t - \frac{\int_0^1 s \, ds}{\int_0^1 1 \, ds} u(t) = t - \frac{1}{2}$$

$$\|v_2\| = \sqrt{\langle v_2, v_2 \rangle} = \sqrt{\int_0^1 (t^2 + \frac{1}{4}) \, dt} = \sqrt{\frac{7}{12}}$$

$$\forall t \in [0, 1] : b_2(t) = \frac{v_2(t)}{\|v_2\|} = \frac{t - \frac{1}{2}}{\|v_2\|}$$

$$b_1 := 1 \quad b_2(t) = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$\begin{aligned} \text{proj}_{b_1} f + \text{proj}_{b_2} f &= \langle f, b_1 \rangle b_1 + \langle f, b_2 \rangle b_2 \\ &= \left(\int_0^1 (t^2 - t + 1) \, dt \cdot 1 \right) + \left(\int_0^1 \left(t^2 - t + 1 \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}} \, dt \right) \cdot \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}} \right) \end{aligned}$$