

4. (2 valores) Determine uma base ortonormada do plano  $P \subset \mathbb{R}^3$  gerado pelos vetores  $(1, 1, 1)$  e  $(0, 1, 2)$ .

$$v_1 = (1, 1, 1)$$

$$u_1 := v_1$$

$$e_1 = \frac{u_1}{\|u_1\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$v_2 = (0, 1, 2)$$

Usando Gram-Schmidt

$$u_2 = v_2 - \text{proj}_{u_1} v_2 = (0, 1, 2) + (-1, -1, -1) = (-1, 0, 1)$$

$$e_2 = \frac{u_2}{\|u_2\|} = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

Basis:

$$\left\{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right\}$$

5. (2 valores) Calcule a matriz  $3 \times 3$  que define, relativamente à base canónica de  $\mathbb{R}^3$ , a projeção ortogonal sobre o plano  $P$  definido no exercício 4.

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{bmatrix}$$