$$E = \{f: [0,1] \mapsto \mathbb{R} \text{ continua} \}$$

$$\langle f,g \rangle = \int_0^1 f(t)g(t)dt$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \qquad w(t) = t$$

Calcula a projeção ortogonal de f(t) sobre o plano gerado por u(t) e w(t). Usando Gram-Schmidt:

$$v_{1} := u$$

$$v_{2} := w - proj_{v_{1}}w \implies \forall t \in [0, 1] : v_{2}(t) = w(t) - (proj_{v_{1}}w)(t) = t - \frac{\int_{0}^{1} s \, \mathrm{d}s}{\int_{0}^{1} 1 \, \mathrm{d}s} u(t) = t - \frac{1}{2}$$

$$||v_{2}|| = \sqrt{\langle v_{2}, v_{2} \rangle} = \sqrt{\int_{0}^{1} (t^{2} + \frac{1}{4}) \, \mathrm{d}t} = \sqrt{\frac{7}{12}}$$

$$\forall t \in [0, 1] : b_{2}(t) = \frac{v_{2}(t)}{||v_{2}||} = \frac{t - \frac{1}{2}}{||v_{2}||}$$

$$b_{1} := 1 \qquad b_{2}(t) = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$proj_{b_{1}} f + proj_{b_{2}} f = \langle f, b_{1} \rangle b_{1} + \langle f, b_{2} \rangle b_{2}$$