4. *(2 valores)* Determine uma base ortonormada do plano $P \subset \mathbb{R}^3$ gerado pelos vetores $(1,1,1)$ e $(0,1,2)$.

$$v_1 = (1,1,1)$$
$$u_1 := v_1$$
$$e_1 = \frac{u_1}{||u_1||} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$
$$v_2 = (0,1,2)$$

Usando Gram-Schmidt

$$u_2 = v_2 - proj_{u_1} v_2 = (0,1,2) + (-1,-1,-1) = (-1,0,1)$$
$$e_2 = \frac{u_2}{||u_2||} = (-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$$

Basis:
$$\{(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})\}$$

5. *(2 valores)* Calcule a matriz 3×3 que define, relativamente à base canónica de $\mathbb{R}^3$, a projeção ortogonal sobre o plano P definido no exercício 4.

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{bmatrix}$$

9. *(2 valores)* Calcule valores e vetores próprios da matriz

$$A = \begin{pmatrix} 1 & i \\ -i & 0 \end{pmatrix}$$

Os valores próprios são $\frac{1+\sqrt{5}}{2}$ e $\frac{1-\sqrt{5}}{2}$

$$\begin{bmatrix} \frac{1-\sqrt{5}}{2} & i \\ -i & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1+\sqrt{5}}{2} & i \\ -i & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix}$$