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$$E = \{f : [0, 1] \mapsto \mathbb{R} \text{ cont ua}\}$$

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \quad w(t) = t$$

Calcula a proje  o ortogonal de  $f(t)$  sobre o plano gerado por  $u(t)$  e  $w(t)$ .

*Usando Gram – Schmidt :*

$$v_1 := u$$

$$v_2 := w - \text{proj}_{v_1} w \implies \forall t \in [0, 1] : v_2(t) = w(t) - (\text{proj}_{v_1} w)(t) = t - \frac{\int_0^1 s \, ds}{\int_0^1 1 \, ds} u(t) = t - \frac{1}{2}$$

$$\|v_2\| = \sqrt{\langle v_2, v_2 \rangle} = \sqrt{\int_0^1 (t^2 + \frac{1}{4}) \, dt} = \sqrt{\frac{7}{12}}$$

$$b_1 := 1 \quad b_2 := \frac{v_2}{\|v_2\|} = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$\begin{aligned} \text{proj}_{b_1} f + \text{proj}_{b_2} f &= \langle f, b_1 \rangle b_1 + \langle f, b_2 \rangle b_2 \\ &= \left( \int_0^1 (t^2 - t + 1) \, dt \right) \cdot 1 + \left( \int_0^1 (t^2 - t + 1) \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}} \, dt \right) \cdot \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}} \end{aligned}$$