L $= \int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x$
Coordenadas Polares

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r^2 = x^2 + y^2$$

$$L^2 = \int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x \cdot \int_R e^{-\frac{y^2}{2}}\ \mathrm{d}y = \iint_{R^2} e^{-\frac{1}{2}(x^2+y^2)}\ \mathrm{d}x$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

$$L^2 = \iint_{R^2} e^{-\frac{r^2}{2}} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-\frac{r^2}{2}} r\ \mathrm{d}r \cdot \int_0^{2\pi} \mathrm{d}\theta$$

$$u = -\frac{r^2}{2} \qquad -du = r\ \mathrm{d}r$$

$$u(\infty) = -\infty \qquad u(0) = 0$$

$$-\int_0^{-t} e^u\ \mathrm{d}u = \lim_{t\to\infty} [e^u]_0^{-t} = -(0-1) = 1$$

$$-\int_0^\infty e^u\ \mathrm{d}u \cdot 2\pi = 1 \cdot 2\pi$$

Então a integral é $\sqrt{2\pi}$
L $= \int_R e^{-x^2}\ \mathrm{d}x$

$$L^2 = \int_R e^{-x^2}\ \mathrm{d}x \cdot \int_R e^{-y^2}\ \mathrm{d}y = \iint_{R^2} e^{-(x^2+y^2)}\ \mathrm{d}x$$

$$\int_0^\infty e^{-r^2} r\ \mathrm{d}r = -\frac{1}{2}\lim_{t\to\infty}\left[e^{-r^2}\right]_0^t = -\frac{1}{2}(0-1) = \frac{1}{2}$$

$$L^2 = \iint_{R^2} e^{-r^2} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-r^2} r\ \mathrm{d}r \cdot \int_0^{2\pi} \mathrm{d}\theta = \frac{1}{2}\cdot 2\pi = \pi$$

Então a integral é $\sqrt{\pi}$
Comprimento de uma arco:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ \mathrm{d}x$$

$$L = \int \mathrm{d}s$$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ se $y = f(x)$, $a \le x \le b$
$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\ \mathrm{d}y$ se $x = h(y)$, $c \le x \le d$

$$L = \int \mathrm{d}s$$

1.

Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

$$f'(\ln(\sec x)) = \tan(x)$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)} = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)}\ \mathrm{d}x = \int_0^{\frac{\pi}{4}} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec x + \tan x}{\sec x + \tan x} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\ \mathrm{d}x$$

$$u = \sec x + \tan x \qquad du = \sec^2 x + \sec x \tan x dx$$

$$u(0) = 1 \qquad u(\frac{\pi}{4}) = \sqrt{2} + 1$$

$$= \int_1^{\sqrt{2}+1} \frac{1}{u}\ \mathrm{d}u$$

$$= [\ln|u|]_1^{\sqrt{2}+1} = \ln(\sqrt{2}+1)$$

2.

Determine the length of x=$\frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

$$f'(\frac{2}{3}(y-1)^{\frac{3}{2}}) = \sqrt{(y-1)}$$

$$L = \int_1^4 \sqrt{1 + (\sqrt{(y-1)})^2}\ \mathrm{d}y = \int_1^4 \sqrt{y}\ \mathrm{d}y$$

$$= \left[\frac{2}{3} y^{\frac{3}{2}}\right]_1^4 = \frac{2}{3} 4^{\frac{3}{2}} - \frac{2}{3} = \frac{14}{3}$$

3.

Redo the previous example using the function in the form $y = f(x)$ instead.

4.

Determine the length of $x = \frac{1}{2}y^2$ for $0 \leq x \leq \frac{1}{2}$ .Assume that y is positive.

Comprimento de uma curva:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, \mathrm{d}t = \int_a^b ||\vec{r}'(t)||\, \mathrm{d}t$$

1.

Determine the length of the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ on the interval $0 \leq t \leq 2\pi$.

2.

Determine the arc length function for $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$.

3.

Where on the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ are we after traveling for a distance of $\frac{\pi\sqrt{10}}{3}$?

Área de superfície:

$$A(S) = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, \mathrm{d}x\, \mathrm{d}y$$

Valores Médios:

$$A = \iint_D \mathrm{d}x\, \mathrm{d}y$$

$$\bar{f} = \frac{\iint_D (x,y)\, \mathrm{d}x\, \mathrm{d}y}{A}$$

$$\bar{f} = \frac{1}{A(R)} \iint_R f(x,y)\, \mathrm{d}A$$

Centro de Massa:

$$M = \iint_R \rho(x,y)\, \mathrm{d}A \qquad M_x = \iint_R y\rho(x,y)\, \mathrm{d}A \qquad M_y = \iint_R x\rho(x,y)\, \mathrm{d}A$$

$$R = (\bar{x}.\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$$\bar{x} = \frac{1}{A} \iint_D x f(x,y)\, \mathrm{d}x\, \mathrm{d}y$$

$$\bar{y} = \frac{1}{A} \iint_D y f(x,y)\ \mathrm{d}x\,\mathrm{d}y$$

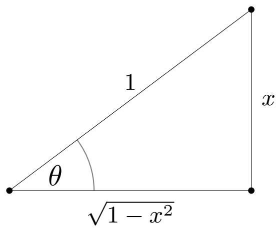
If $R = \{(x,y) | -1 \leq x \leq 1, -2 \leq y \leq 2\}$, evaluate $\iint_R \sqrt{1-x^2}\ \mathrm{d}A$
Método 1:
Como $z = \sqrt{1-x^2}$ é metade de um cilindro então:

$$V_{cilindro} = \pi r^2 h$$

$$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi 1 \cdot 4 = 2\pi$$

Método 2:

$$\int_{-2}^{2}\int_{-1}^{1} \sqrt{1-x^2}\ \mathrm{d}x\,\mathrm{d}y$$



Substituição trigonométrica:

$$x = \sin\theta \qquad \mathrm{d}x = \cos\theta\ \mathrm{d}\theta$$

$$\cos\theta = \sqrt{1-x^2}$$

Se $x = -1$ então $\theta = -\frac{\pi}{2}$
Se $x = 1$ então $\theta = \frac{\pi}{2}$

$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta\,\mathrm{d}y$$

$$\cos 2\theta = \sin^2\theta - \cos^2\theta \Leftrightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\left(1+\cos 2\theta\right)\ \mathrm{d}\theta\,\mathrm{d}y = \int_{-2}^{2} \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\ \mathrm{d}y$$

$$= \left[\frac{\pi}{2}y\right]_{-2}^{2} = 2\pi$$