

1.

Determine as seguintes integrais:

1)

$$\begin{aligned}& \int_0^{\sqrt{\frac{\pi}{2}}} x \sin x^2 dx \\&= \frac{1}{2} \int_0^{\sqrt{\frac{\pi}{2}}} 2x \sin x^2 dx \\&= \frac{1}{2} (-\cos x^2) \Big|_0^{\sqrt{\frac{\pi}{2}}} \\&= \frac{1}{2} (-\cos (\sqrt{\frac{\pi}{2}})^2 + \cos 0) \\&= \frac{1}{2}\end{aligned}$$

2)

$$\begin{aligned}& \int_0^{\pi} (x+2) \cos x dx \\& f' = \cos x \quad f = \sin x \\& g = x+2 \quad g' = 1 \\& (x+2) \sin x \Big|_0^{\pi} - \int_0^{\pi} \sin x dx \\& (\pi+2) \sin \pi - 2 \sin 0 + \cos x \Big|_0^{\pi} \\&= -2\end{aligned}$$

3)

$$\begin{aligned}& \int_1^2 x 2^x dx \\& f' = 2^x \quad f = \frac{2^x}{\ln 2} \\& g = x \quad g' = 1 \\& \frac{2^x}{\ln 2} x \Big|_1^2 - \int_1^2 \frac{2^x}{\ln 2} \\&= \frac{8}{\ln 2} - \frac{2}{\ln 2} - \frac{1}{\ln 2} \frac{2^x}{\ln 2} \Big|_1^2\end{aligned}$$

$$\begin{aligned}
&= \frac{6}{\ln 2} - \frac{2}{\ln^2 2} \\
&= \frac{6 \ln 2 - 2}{\ln^2 2}
\end{aligned}$$

4)

$$\begin{aligned}
&\int_0^1 \frac{e^x}{\sqrt{e^x + 1}} dx \\
&2(e^x + 1)^{\frac{1}{2}} \Big|_0^1 \\
&2((e + 1)^{\frac{1}{2}} - (2)^{\frac{1}{2}})
\end{aligned}$$

2.

a)

Calcule $\int_0^{\frac{\pi}{2}} e^x \sin(x) dx$:

$$\begin{aligned}
&\int_0^{\frac{\pi}{2}} e^x \sin(x) dx \\
&\begin{aligned} f' &= \sin(x) & f &= -\cos(x) \\ g &= e^x & g' &= e^x \end{aligned} \\
&= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = -\cos(x)e^x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos(x)e^x dx \\
&\begin{aligned} f' &= \cos(x) & f &= \sin(x) \\ g &= e^x & g' &= e^x \end{aligned} \\
&= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = 1 + e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin(x)e^x dx \\
&= \int_0^{\frac{\pi}{2}} e^x \sin(x) dx = \frac{1}{2}(1 + e^{\frac{\pi}{2}})
\end{aligned}$$

3.

Usando uma substituição, calcule os seguintes integrais

1)

$$\begin{aligned}
& \int_{-1}^1 e^{\arcsin(x)} dx \\
& \sin(x) : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1] \\
& x = \sin(t) \quad dx = \cos(t) dt \\
& = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\arcsin(\sin(t))} \cos(t) dt \\
& = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt \\
& f' = \cos(t) \quad f = \sin(t) \\
& g = e^t \quad g' = e^t \\
& = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt = (\sin(t)e^t) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \sin(t) dt \\
& f' = \sin(t) \quad f = -\cos(t) \\
& g = e^t \quad g' = e^t \\
& = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt = e^{\frac{\pi}{2}} - e^{-\frac{\pi}{2}} - [(-\cos(t)e^t) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -e^t \cos(t) dt] \\
& = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt = e^{\frac{\pi}{2}} - e^{-\frac{\pi}{2}} + (\cos(t)e^t) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt \\
& = 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t) dt = e^{\frac{\pi}{2}} - e^{-\frac{\pi}{2}}
\end{aligned}$$

2)

3)

$$\begin{aligned}
& \int_0^{\frac{3}{2}} 2^{\sqrt{2x+1}} dx \\
& u = \sqrt{2x+1} \\
& \frac{1}{2}(u^2 - 1) = x \\
& dx = u du \\
& \int_0^2 2^u u du
\end{aligned}$$

$$\begin{aligned}
 f' &= 2^u & f &= \frac{2^u}{\ln(2)} \\
 g &= u & g' &= 1 \\
 \frac{2^u}{\ln(2)} \cdot u \Big|_1^2 - \int_0^2 \frac{2^u}{\ln(2)} du \\
 \frac{2^u}{\ln(2)} \cdot u \Big|_1^2 - \frac{1}{\ln(2)} \int_0^2 2^u du \\
 \frac{2^u}{\ln(2)} \cdot u \Big|_1^2 - \frac{1}{\ln^2(2)} 2^u \Big|_1^2
 \end{aligned}$$

4)

$$\begin{aligned}
 \int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}} dx \\
 x &= \sin(t) \\
 dx &= \cos(t) dt \\
 \arcsin\left(\frac{\sqrt{2}}{2}\right) &= \arcsin(\sin(t)) \\
 t &= \frac{\pi}{4} \\
 \arcsin(0) &= \arcsin(\sin(t)) \\
 t &= 0 \\
 \int_0^{\frac{\pi}{4}} \frac{\sin^2(t)}{\cancel{\cos(t)}} \cancel{\cos(t)} dt \\
 \int_0^{\frac{\pi}{4}} \frac{1 - \cos(2t)}{2} dt \\
 \int_0^{\frac{\pi}{4}} \frac{1}{2} - \frac{\cos(2t)}{2} dt \\
 \int_0^{\frac{\pi}{4}} \frac{1}{2} dt - \frac{1}{2} \int_0^{\frac{\pi}{4}} \cos(2t) dt \\
 \frac{1}{2} t \Big|_0^{\frac{\pi}{4}} - \frac{1}{4} \sin(2t) \Big|_0^{\frac{\pi}{4}} \\
 \frac{1}{2} \cdot \frac{\pi}{4} - \frac{\pi}{4} \cdot \sin\left(2 \cdot \frac{\pi}{4}\right) \\
 \frac{\pi - 2}{8}
 \end{aligned}$$