

Find the derivative matrices in Exercises 5–8 and evaluate at the given points.

5. $\partial(x, y)/\partial(u, v)$; $x = u \sin v$, $y = e^{uv}$; at $(0, 1)$.

$$x = u \sin(v) \quad y = e^{uv}$$

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{pmatrix} \sin(v) & u \cos(v) \\ ve^{uv} & ue^{uv} \end{pmatrix} = \begin{pmatrix} \sin 1 & 0 \\ 1 & 0 \end{pmatrix}$$

6. $\partial(x, y, z)/\partial(r, \theta, \phi)$; where $x = r \sin \phi \cos \theta$,
 $y = r \sin \phi \sin \theta$, $z = r \cos \phi$; at $(2, \pi/3, \pi/4)$.

$$x = r \sin \phi \cos \theta \quad y = r \sin \phi \sin \theta \quad z = r \cos \phi$$

$$\frac{\partial(x, y, z)}{\partial(r, \phi, \theta)} = \begin{pmatrix} \sin \phi \cos \theta & -r \sin \phi \sin \theta & r \cos \phi \cos \theta \\ \sin \phi \sin \theta & r \sin \phi \cos \theta & r \cos \phi \sin \theta \\ \cos \phi & 0 & -r \sin \phi \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{2} \\ \frac{\sqrt{2}}{2} & 0 & -\sqrt{2} \end{pmatrix}$$

Compute $\partial z/\partial x$ and $\partial z/\partial y$ in Exercises 21–24 using matrix multiplication and by direct substitution.

21. $z = u^2 + v^2$; $u = 2x + 7$, $v = 3x + y + 7$.

22. $z = u^2 + 3uv - v^2$; $u = \sin x$,
 $v = -\cos x + \cos y$.

23. $z = \sin u \cos v$; $u = 3x^2 - 2y$, $v = x - 3y$.

24. $z = u/v^2$; $u = x + y$, $v = xy$.

21.

$$z = u^2 + v^2 \quad u = 2x + 7 \quad v = 3x + y + 7$$

$$\begin{pmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{pmatrix} \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = (2u \quad 2v) \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

$$= (4u + 6v \quad 0u + 2v) = (26x + 6y + 70 \quad 6x + 2y + 14)$$

22.

$$z = u^2 + 3uv - v^2 \quad u = \sin x \quad v = -\cos x + \cos y$$

$$\begin{aligned} \left(\frac{\partial z}{\partial x} \quad \frac{\partial z}{\partial y} \right) &= \left(\frac{\partial z}{\partial u} \quad \frac{\partial z}{\partial v} \right) \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = (2u + 3v \quad 3u - 2v) \begin{pmatrix} \cos x & 0 \\ \cos x & -\sin y \end{pmatrix} \\ &= ((2u + 3v) \cos x + \cos x(3u - 2v) \quad 0 - \sin y(3u - 2v)) \\ &= (5 \sin x \cos x + 5 \cos^2 x + \cos y \cos x \quad -3 \sin^2 y) \end{aligned}$$