1.

Determine as seguintes integrais:

1)

$$\int_0^{\sqrt{\frac{\pi}{2}}} x \sin x^2 dx$$

$$= \frac{1}{2} \int_0^{\sqrt{\frac{\pi}{2}}} 2x \sin x^2 dx$$

$$= \frac{1}{2}(-\cos x^2)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}(-\cos(\sqrt{\frac{\pi}{2}})^2 + \cos 0)$$

$$= \frac{1}{2}$$

2)

$$\int_0^{\pi} (x+2) \cos x dx$$

$$f' = \cos x \qquad f = \sin x$$

$$g = x + 2 \qquad g' = 1$$

$$(x+2)\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x dx$$

$$(\pi + 2)\sin \pi - 2\sin 0 + \cos x\Big|_0^{\pi}$$

$$= -2$$

3)

$$\int_1^2 x 2^x dx$$

$$f' = 2^x \qquad f = \frac{2^x}{\ln 2}$$

$$g = x \qquad g' = 1$$

$$\frac{2^x}{\ln 2} x\Big|_1^2 - \int_1^2 \frac{2^x}{\ln 2}$$

$$= \frac{8}{\ln 2} - \frac{2}{\ln 2} - \frac{1}{\ln 2}\frac{2^x}{\ln 2}\Big|_1^2$$

$$= \frac{6}{\ln 2} - \frac{2}{\ln^2 2}$$
$$= \frac{6\ln 2 - 2}{\ln^2 2}$$

4)

$$\int_0^1 \frac{e^x}{\sqrt{e^x+1}}dx$$
$$2(e^x+1)^{\frac{1}{2}})\Big|_0^1$$
$$2((e+1)^{\frac{1}{2}} - (2)^{\frac{1}{2}})$$

2.

a)

Calcule $\int_0^{\frac{\pi}{2}} e^x \sin(x)dx$:

$$\int_0^{\frac{\pi}{2}} e^x sin(x)dx$$
$$f' = \sin(x) \qquad f = -\cos(x)$$
$$g = e^x \qquad g' = e^x$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x)dx = -\cos(x)e^x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos(x)e^x dx$$
$$f' = \cos(x) \qquad f = \sin(x)$$
$$g = e^x \qquad g' = e^x$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x)dx = 1 + e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin(x)e^x dx$$
$$= \int_0^{\frac{\pi}{2}} e^x \sin(x)dx = \frac{1}{2}(1 + e^{\frac{\pi}{2}})$$

b)

Determine todas as primitivas de $f(x) = e^x \cos(x)$.

$$\int e^x \cos(x)dx$$
$$f' = \cos(x) \qquad f = \sin(x)$$
$$g = e^x \qquad g' = e^x$$

$$\int e^x \cos(x)dx = \sin(x)e^x - \int \sin(x)e^x$$

$$f' = \sin(x) \qquad f = -\cos(x)$$

$$g = e^x \qquad g' = e^x$$

$$\int e^x \cos(x)dx = \sin(x)e^x + \cos(x)e^x + \int \cos(x)e^x$$

$$2\int e^x \cos(x)dx = \sin(x)e^x + \cos(x)e^x$$

$$\int e^x \cos(x)dx = \frac{1}{2}(\sin(x)e^x + \cos(x)e^x)$$

3.

Usando uma substituição, calcule os seguintes integrais

1)

$$\int_{-1}^{1} e^{\arcsin(x)}dx$$

$$\sin(x) : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$$

$$x = \sin(t) \qquad dx = \cos(t)dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\arcsin(\sin(t))} \cos(t)dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt$$

$$f' = \cos(t) \qquad f = \sin(t)$$

$$g = e^t \qquad g' = e^t$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt = (\sin(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \sin(t)dt$$

$$f' = \sin(t) \qquad f = -\cos(t)$$

$$g = e^t \qquad g' = e^t$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}} - [(-\cos(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -e^t \cos(t)dt]$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}} + (\cos(t)e^t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt$$

$$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^t \cos(t)dt = e^{\frac{\pi}{2}} - e^{\frac{-\pi}{2}}$$

2)

$$\int_0^1 \frac{x^2}{\sqrt{x+1}}dx$$

$$u = x+1$$

$$u-1 = x$$

$$dx = du$$

$$u(0) = 1 \qquad u(1) = 2$$

$$\int_1^2 \frac{(u-1)^2}{\sqrt{u}}du$$

$$\int_1^2 (u-1)^2 u^{-\frac{1}{2}} du$$

$$\int_1^2 (u^2-2u+1)u^{-\frac{1}{2}} du$$

$$\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\Big|_1^2$$

$$u^{\frac{1}{2}}(\frac{2}{5}u^2 - \frac{4}{3}u + 2)\Big|_1^2$$

$$2^{\frac{1}{2}}(\frac{2}{5}2^2 - \frac{4}{3}2 + 2) - (1^{\frac{1}{2}}(\frac{2}{5}1^2 - \frac{4}{3}1 + 2)) = 0.253265991548222$$

3)

$$\int_0^{\frac{3}{2}} 2^{\sqrt{2x+1}} dx$$

$$u = \sqrt{2x+1}$$

$$\frac{1}{2}(u^2-1) = x$$

$$dx = udu$$

$$\int_0^2 2^u udu$$

$$f' = 2^u \qquad f = \frac{2^u}{\ln{(2)}}$$

$$g = u \qquad g' = 1$$

$$\frac{2^u}{\ln{(2)}} \cdot u\Big|_1^2 - \int_1^2 \frac{2^u}{\ln{(2)}}du$$

$$\frac{2^u}{\ln{(2)}} \cdot u\Big|_1^2 - \frac{1}{\ln{(2)}}\int_1^2 2^u du$$

$$\frac{2^u}{\ln{(2)}} \cdot u\Big|_1^2 - \frac{1}{\ln^2{(2)}}2^u\Big|_1^2$$

4)

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$x = \sin(t)$$

$$dx = \cos(t)dt$$

$$\arcsin(\frac{\sqrt{2}}{2}) = \arcsin(\sin(t))$$

$$t = \frac{\pi}{4}$$

$$\arcsin(0) = \arcsin(\sin(t))$$

$$t = 0$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin^2(t)}{\cancel{\cos(t)}}\cancel{\cos(t)}dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1-\cos(2t)}{2}dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} - \frac{\cos(2t)}{2}dt$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2}dt - \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos(2t)dt$$

$$\frac{1}{2}t\Big|_0^{\frac{\pi}{4}} - \frac{1}{4}\sin(2t)\Big|_0^{\frac{\pi}{4}}$$

$$\frac{1}{2}\cdot\frac{\pi}{4} - \frac{\pi}{4}\cdot\sin(\cancel{2}\cdot\frac{\pi}{\cancel{4}})$$

$$\frac{\pi-2}{8}$$

4)

Represente graficamente o conjunto A dado e calcule a sua área.

a)

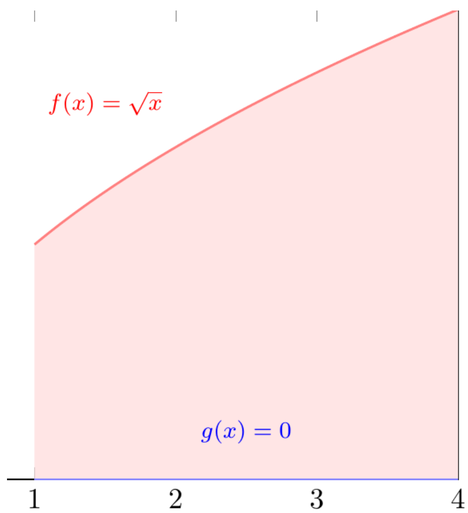
A é o conjunto do plano limitado pelas rectas $x = 1$, $x = 4$, $y = 0$ e pela curva de $f(x) = \sqrt{x}$.

$$\int_1^4 \sqrt{x} - 0dx$$

$$\int_1^4 x^{\frac{1}{2}}dx$$

$$\frac{2}{3}x^{\frac{3}{2}}\Big|_1^4$$

$$\frac{2}{3}4^{\frac{3}{2}} - \frac{2}{3} = 4.666666667$$



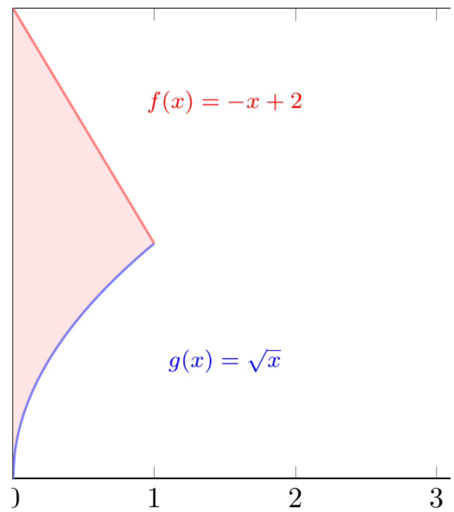
b)

A $= \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 1$ **e** $\sqrt{x} \leq y \leq -x + 2\}$.

$$\int_0^1 (-x + 2) - \sqrt{x}dx$$

$$-\frac{x^2}{2} + 2x - \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1$$

$$-\frac{1}{2} + 2 - \frac{2}{3} = 0.833333333$$



c)

A é o conjunto do plano limitado superiormente pela parábola de equação $y = -x^2 + \frac{7}{2}$ e inferiormente pela parábola de equação $y = x^2 - 1$.

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} -x^2 + \frac{7}{2} - (x^2 - 1)dx$$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} -2x^2 + \frac{9}{2}dx$$

$$-2\frac{x^3}{3} + \frac{9}{2}x\Big|_{-\frac{3}{2}}^{\frac{3}{2}}$$

$$-\frac{4x^3 - 27x}{6}\Big|_{-\frac{3}{2}}^{\frac{3}{2}}$$

$$-\frac{4 \cdot (\frac{3}{2})^3 - 27 \cdot (\frac{3}{2})}{6} - \frac{4 \cdot (-\frac{3}{2})^3 - 27 \cdot (-\frac{3}{2})}{6} = 9$$