$$E = \{f : [0,1] \mapsto \mathbb{R} \text{ contínua}\}$$

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt$$

$$f(t) = t^2 - t + 1$$

$$u(t) = 1 \qquad w(t) = t$$

Calcula a projeção ortogonal de f(t) sobre o plano gerado por u(t) e w(t).

$$Usando\,Gram - Schmidt:$$

$$v_1 := u$$

$$v_2 := w - proj_{v_1}w \implies \forall t \in [0,1] : v_2(t) = w(t) - (proj_{v_1}w)(t) = t - \frac{\int_0^1 s\,\mathrm{d}s}{\int_0^1 1\,\mathrm{d}s}u(t) = t - 1/2$$

$$\|v_2\| = \sqrt{\langle v_2(t), v_2(t) \rangle} = \sqrt{\int_0^1 (t^2 + \frac{1}{4})\,\mathrm{d}t} = \sqrt{\frac{7}{12}}$$

$$\frac{v_2(t)}{\|v_2\|} = \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$b_1 := 1 \qquad b_2 := \frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}$$

$$proj_{b_1}f + proj_{b_1}f = \langle f, b_1 \rangle b_1 + \langle f, b_2 \rangle b_2$$

$$= (\int_0^1 (t^2 - t + 1)\,\mathrm{d}t \cdot 1 + (\int_0^1 (t^2 - t + 1\frac{t - \frac{1}{2}}{\sqrt{\frac{7}{12}}}\,\mathrm{d}t) \cdot (t^2 - t + 1)$$