Math Duel Problems

1. The n-th square number $S_n := n^2$ while the n-th triangular number is $T_n := n(n+1)/2$. Descrine a method that would find all the square-triangular numbers (i.e. numbers that are both square and triangular)

2. A rational number $p/q < 1$ can be written uniquely in base b as:

$$\frac{p}{q} = 0.\underbrace{x_1 x_2 \cdots x_j}_{\text{Initial part}}\underbrace{\overline{y_1 y_2 \cdots y_k}}_{\text{Periodic part}}$$
$$= \sum_{m=1}^{j} x_j b^m + \sum_{\ell=1}^{\infty} \sum_{n=1}^{k} y_{\ell n} b^{\ell n + j}$$

   Given natural numbers p q and b, how can we find the initial part $x_1 \cdots x_j$, its length j, the periodic part $y_1 \cdots y_k$, and the period k for the fraction p/q in base b?

3. Consider an infinitely long house with rooms numbered as shown in the blueprint in figure 1. The

house have doors between every two adjacent rooms (but not between diagonal rooms). Starting at room 0, in how many ways can you get to a room n if you are required to never go from a room with a larger number to a room from a lower number?

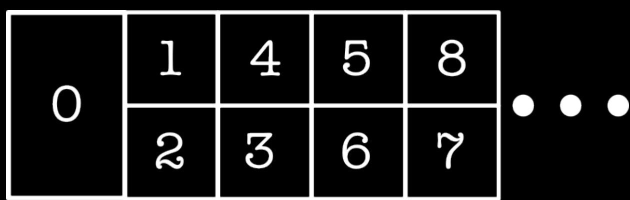

Figure 1: Infinitely long house. The rooms are numbered in an infinite pattern. Each adjacent rooms are connected, for example rooms 1 and 4 are connected, while rooms 3 and 5 are disconnected. You are only allowed to go from a room with a smaller number to a room with a largest number. For example, you can go from room 3 to 6 but not from room 2 to 1.

4. Nuggets are sold in boxes containing either 6, 9, and 20 nuggets. Assuming that you can only buy or sell nuggets in those quantities, what number of nuggets can you get? What if you assume that you can only buy nuggets on those quantities?

5. Cardano's formula for solving the depressed cubic $y^3 = py + q$ tells us:

$$y = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}}$$

Use the formula to solve $y^3 = 2$ and $y^3 = 6y + 6$