L = $\int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x$
Coordenadas Polares

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r^2 = x^2 + y^2$$

$$L^2 = \int_R e^{-\frac{x^2}{2}}\ \mathrm{d}x \cdot \int_R e^{-\frac{y^2}{2}}\ \mathrm{d}y = \iint_{R^2} e^{-\frac{1}{2}(x^2+y^2)}\ \mathrm{d}x$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

$$L^2 = \iint_{R^2} e^{-\frac{r^2}{2}} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-\frac{r^2}{2}} r\ \mathrm{d}r \cdot \int_0^{2\pi}\ \mathrm{d}\theta$$

$$u = -\frac{r^2}{2} \qquad -du = r\ \mathrm{d}r$$

$$u(\infty) = -\infty \qquad u(0) = 0$$

$$-\int_0^{-t} e^u\ \mathrm{d}u = \lim_{t\to\infty} [e^u]_0^{-t} = -(0-1) = 1$$

$$-\int_0^\infty e^u\ \mathrm{d}u \cdot 2\pi = 1 \cdot 2\pi$$

Então a integral é $\sqrt{2\pi}$
L = $\int_R e^{-x^2}\ \mathrm{d}x$

$$L^2 = \int_R e^{-x^2}\ \mathrm{d}x \cdot \int_R e^{-y^2}\ \mathrm{d}y = \iint_{R^2} e^{-(x^2+y^2)}\ \mathrm{d}x$$

$$\int_0^\infty e^{-r^2} r\ \mathrm{d}r = -\frac{1}{2}\lim_{t\to\infty}\left[e^{-r^2}\right]_0^t = -\frac{1}{2}(0-1) = \frac{1}{2}$$

$$L^2 = \iint_{R^2} e^{-r^2} r\ \mathrm{d}r\,\mathrm{d}\theta = \int_0^\infty e^{-r^2} r\ \mathrm{d}r \cdot \int_0^{2\pi}\ \mathrm{d}\theta = \frac{1}{2}\cdot 2\pi = \pi$$

Então a integral é $\sqrt{\pi}$
Comprimento de uma arco:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ \mathrm{d}x$$

$$L = \int\ \mathrm{d}s$$

$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ se $y = f(x)$, $a \le x \le b$
$\mathrm{d}s = \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\ \mathrm{d}y$ se $x = h(y)$, $c \le x \le d$

$$L = \int\ \mathrm{d}s$$

1.

Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

$$f'(\ln(\sec x)) = \tan(x)$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)} = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)}\ \mathrm{d}x = \int_0^{\frac{\pi}{4}} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec x + \tan x}{\sec x + \tan x} \sec(x)\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\ \mathrm{d}x$$

$$u = \sec x + \tan x \qquad du = \sec^2 x + \sec x \tan x dx$$

$$u(0) = 1 \qquad u(\frac{\pi}{4}) = \sqrt{2} + 1$$

$$= \int_1^{\sqrt{2}+1} \frac{1}{u}\ \mathrm{d}u$$

$$= [\ln |u|]_1^{\sqrt{2}+1} = \ln(\sqrt{2} + 1)$$

2.

Determine the length of x=$\frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

$$f'(\frac{2}{3}(y-1)^{\frac{3}{2}}) = \sqrt{(y-1)}$$

$$L = \int_1^4 \sqrt{1 + (\sqrt{(y-1)})^2}\ \mathrm{d}y = \int_1^4 \sqrt{y}\ \mathrm{d}y$$

$$= \left[\frac{2}{3} y^{\frac{3}{2}}\right]_1^4 = \frac{2}{3} 4^{\frac{3}{2}} - \frac{2}{3} = \frac{14}{3}$$

3.

Redo the previous example using the function in the form $y = f(x)$ instead.

4.

Determine the length of $x = \frac{1}{2}y^2$ for $0 \leq x \leq \frac{1}{2}$ .Assume that y is positive.

Comprimento de uma curva:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ \mathrm{d}t = \int_a^b ||\vec{r}'(t)||\ \mathrm{d}t$$

1.

Determine the length of the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ on the interval $0 \leq t \leq 2\pi$.

2.

Determine the arc length function for $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$.

3.

Where on the curve $\vec{r}(t) = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ are we after traveling for a distance of $\frac{\pi\sqrt{10}}{3}$?

Área de superfície:

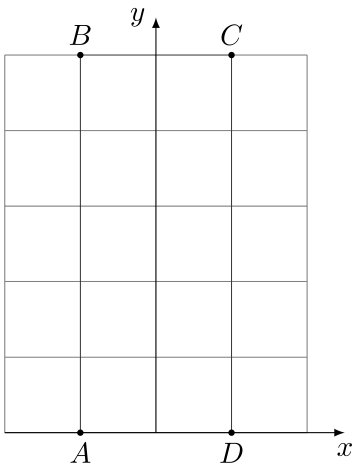
$$A(S) = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\ \mathrm{d}x\,\mathrm{d}y$$

Valores Médios:

$$A = \iint_D \mathrm{d}x\,\mathrm{d}y$$

$$\bar{f} = \frac{\iint_D (x, y)\ \mathrm{d}x\,\mathrm{d}y}{A}$$

$$\bar{f} = \frac{1}{A(R)} \iint_R f(x, y)\ \mathrm{d}A$$



$$A(R) = 2 \cdot 5 = 10$$

$$\bar{f} = \frac{1}{10} \int_{-1}^{1} \int_{0}^{5} x^2 y \; \mathrm{d}y \, \mathrm{d}x$$

$$= \frac{1}{10} \int_{-1}^{1} \left[x^2 \frac{y^2}{2} \right]_0^5 \mathrm{d}x = \frac{1}{10} \int_{-1}^{1} \frac{25}{2} x^2 \; \mathrm{d}x = \frac{25}{20} \left[\frac{x^3}{3} \right]_{-1}^{1} = \frac{5}{6}$$

Centro de Massa:

$$M = \iint_R \rho(x,y) \; \mathrm{d}A \qquad M_x = \iint_R y\rho(x,y) \; \mathrm{d}A \qquad M_y = \iint_R x\rho(x,y) \; \mathrm{d}A$$

$$R = (\bar{x}.\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M} \right)$$

$$\bar{x} = \frac{1}{A} \iint_D x f(x,y) \; \mathrm{d}x \, \mathrm{d}y$$

$$\bar{y} = \frac{1}{A} \iint_D y f(x,y) \; \mathrm{d}x \, \mathrm{d}y$$

Find the volume of the solid that is bounded about by $f(x,y) = y \sin(xy)$ and below $R = [1,2] \times [0,\pi]$

$$V = \int_0^{\pi} \int_1^2 y \sin(xy) \; \mathrm{d}x \, \mathrm{d}y$$

$$= \int_0^{\pi} \left[-\frac{y}{y} \cos(xy) \right]_1^2 \mathrm{d}y = \int_0^{\pi} -\cos(2y) + \cos(y) \; \mathrm{d}y$$

$$= \left[-\frac{1}{2} \sin(2y) + \sin(y) \right]_0^{\pi} = 0$$

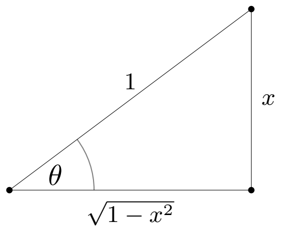
If $R = \{(x,y) | -1 \leq x \leq 1, -2 \leq y \leq 2\}$, evaluate $\iint_R \sqrt{1-x^2}\ \mathrm{d}A$
Método 1:
Como $z = \sqrt{1-x^2}$ é metade de um cilindro então:

$$V_{cilindro} = \pi r^2 h$$

$$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi 1 \cdot 4 = 2\pi$$

Método 2:

$$\int_{-2}^{2}\int_{-1}^{1} \sqrt{1-x^2}\ \mathrm{d}x\,\mathrm{d}y$$



Substituição trigonométrica:

$$x = \sin\theta \qquad \mathrm{d}x = \cos\theta\ \mathrm{d}\theta$$

$$\cos\theta = \sqrt{1-x^2}$$

Se $x = -1$ então $\theta = -\frac{\pi}{2}$
Se $x = 1$ então $\theta = \frac{\pi}{2}$

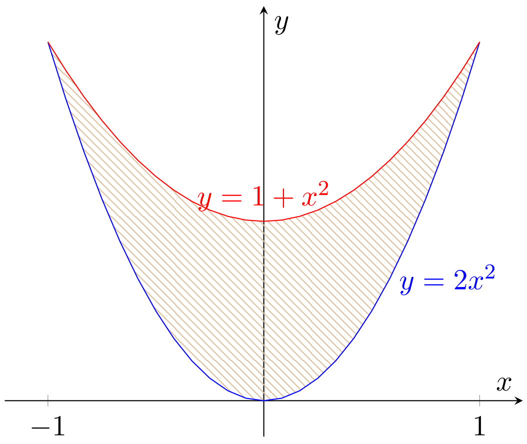
$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta\,\mathrm{d}y$$

$$\cos 2\theta = \sin^2\theta - \cos^2\theta \Leftrightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_{-2}^{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\left(1+\cos 2\theta\right)\ \mathrm{d}\theta\,\mathrm{d}y = \int_{-2}^{2}\frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\ \mathrm{d}y$$

$$= \left[\frac{\pi}{2}y\right]_{-2}^{2} = 2\pi$$

Evaluate $\iint_D (x+2y)\ \mathrm{d}A$ where D is the region bounded by $y = 2x^2$ and $y = 1 + x^2$



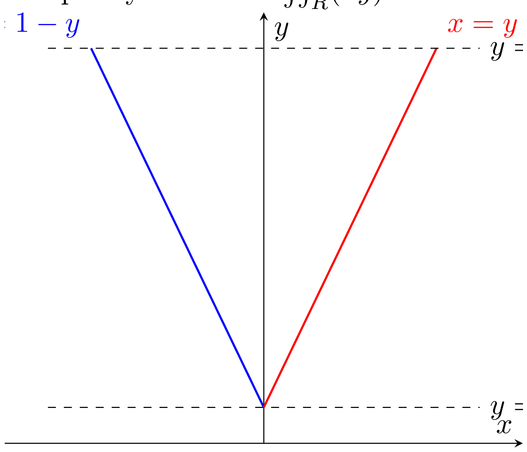
$$\int_{-1}^{1}\int_{2x^2}^{1+x^2}(x+2y)\;\mathrm{d}y\,\mathrm{d}x = \int_{-1}^{1}\left[xy+y^2\right]_{2x^2}^{1+x^2}\;\mathrm{d}x$$

$$= \int_{-1}^{1}\left[(x(1+x^2)+(1+x^2)^2)-(x(2x^2)+(2x^2)^2)\right]\;\mathrm{d}x$$

$$= \int_{-1}^{1}(-x^3-3x^4+x+2x^2+1)\;\mathrm{d}x = \left[-\frac{1}{4}x^4-\frac{3}{5}x^5+\frac{1}{2}x^2+\frac{2}{3}x^3+x\right]_{-1}^{1}$$

$$= \left[(-\frac{1}{4}-\frac{3}{5}+\frac{1}{2}+\frac{2}{3}+1)-(-\frac{1}{4}+\frac{3}{5}+\frac{1}{2}-\frac{2}{3}-1)\right] = \frac{32}{15}$$

Setup only! Evaluate $\iint_R (xy)\;\mathrm{d}A$ where R is the region bounded by $y=-x+1, y=x+1$ and $y=3$



Horizontal fixamos o x

$$\int_{1}^{3}\int_{1-y}^{y-1}(xy)\;\mathrm{d}x\,\mathrm{d}y$$

Find the volume of the solid that lies under $z = xy$ and and about D where D is the region bounded by

$$\int_{-2}^{4}\int_{\frac{1}{2}y^3-3}^{y+1}(xy)\ \mathrm{d}x\,\mathrm{d}y$$

Calculate $\iiint_R = (x+y+2z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ $R: x^2+z^2=4, y=2, y=3.$
Coordenadas cilíndricas:

$$x = r\cos(\theta) \qquad z = r\sin\theta$$

$$\int_{2}^{3}\int_{0}^{2\pi}\int_{0}^{2}(r\cos(\theta)+y+2r\sin(\theta))\,r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\int_{0}^{2\pi}\int_{0}^{2}(r^2\cos(\theta)+yr+2r^2\sin(\theta))\ \mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\int_{0}^{2\pi}\left[(\frac{1}{3}r^3\cos(\theta)+\frac{1}{2}yr^2+\frac{2}{3}r^3\sin(\theta))\right]_0^2\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\int_{0}^{2\pi}(\frac{8}{3}\cos(\theta)+2y+\frac{16}{3}\sin(\theta))\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\left[(\frac{8}{3}\sin(\theta)+2y\theta-\frac{16}{3}\cos(\theta))\right]_0^{2\pi}\ \mathrm{d}y$$

$$= \left[2\pi y^2\right]_2^3 = 18\pi - 8\pi = 10\pi$$

Calculate the volume of the region $\iiint_R = (x+y+2z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, $R: x^2+z^2=4, y=2, y=3.$

$$\int_{2}^{3}\int_{0}^{2\pi}\int_{0}^{2} r\ \mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\int_{0}^{2\pi}\left[\frac{1}{2}r^2\right]_0^2\ \mathrm{d}\theta\,\mathrm{d}y$$

$$= \int_{2}^{3}\left[2\theta\right]_0^{2\pi}\ \mathrm{d}y$$

$$= \left[4\pi y\right]_2^3 = 12\pi - 8\pi = 4\pi$$

Calculate $\iiint_R = y\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ $R: x^2+y^2=3, z=-1, z=2.$
Coordenadas cilíndricas:

$$y = r\sin\theta$$

$$\int_{-1}^{2}\int_{0}^{2\pi}\int_{0}^{\sqrt{3}} r\sin(\theta)\,r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} r^2 \sin(\theta) \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \left[\frac{1}{3} r^3 \sin(\theta) \right]_0^{\sqrt{3}} \mathrm{d}\theta \, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \sqrt{3} \sin(\theta) \, \mathrm{d}\theta \, \mathrm{d}z$$

$$= [0]_{-1}^{2} = 0$$

Calculate the volume of the region $\iiint_R = y \ \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$, $R : x^2 + y^2 = 3, z = -1, z = 2$.

$$\int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} 1 \, r \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{3}} r \ \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z$$

$$= \int_{-1}^{2} \int_{0}^{2\pi} \left[\frac{1}{2} r^2 \right]_0^{\sqrt{3}} \mathrm{d}\theta \, \mathrm{d}z$$

$$= \int_{-1}^{2} \left[\frac{3}{2} \theta \right]_0^{2\pi} \mathrm{d}z$$

$$= [3\pi z]_{-1}^{2} = 9\pi$$