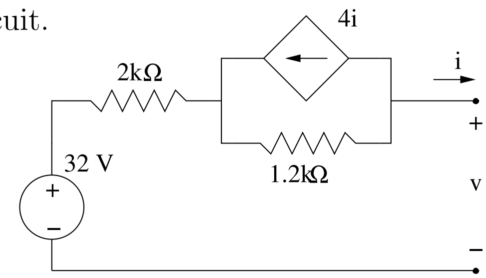
Thevenin Equivalent of Subcircuits with Controlled Sources

Two-terminal subcircuits containing controlled sources reduce to Thevenin form. However, care should be taken in doing so. We discussed three methods to find equivalent of a subcircuit. Our first method, source transformation and circuit reduction, does not work with controlled sources. The second method, directly find i-v characteristics of the subcircuit works but is cumbersome (we may have to use for some subcircuits with controlled sources). The third method was to find two of three parameters: R_T (by killing independent sources), $v_t = v_{oc}$ and $i_N = i_{sc}$. Most of the times, the best choice for subcircuits containing controlled sources is to find $v_t = v_{oc}$ and $i_N = i_{sc}$ as described in the example below.

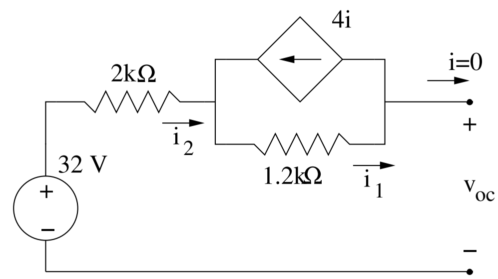
Example: Find the Thevenin equivalent of this subcircuit.



Since the circuit has a controlled source, it is preferred to calculate v_{oc} and i_{sc}.

<u>Finding v_{oc}</u>

Since the circuit is simple, we proceed to solve it with KVL and KCL (noting $i = 0$):



KCL: $\quad -i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 0$

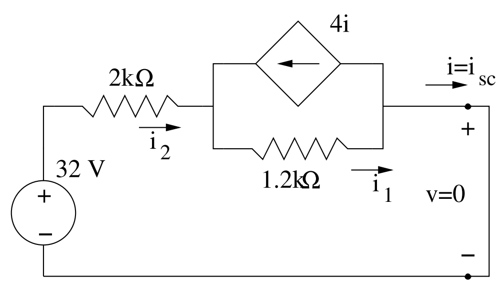
KCL: $\quad -i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = 0$

KVL: $\quad -32 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 + v_{oc} = 0$

$$v_T = v_{oc} = 32 \text{ V}$$

<u>Finding i_{sc}</u>

Again, using KVL and KCL:



KCL: $\quad -i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 5i_{sc}$

KCL: $\quad -i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = i_{sc}$

KVL: $\quad -32 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 = 0$

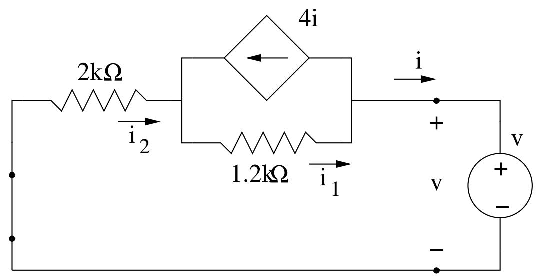
$$-32 + 2 \times 10^3 i_{sc} + 6 \times 10^3 i_{sc} = 0 \quad \rightarrow \quad i_N = i_{sc} = 4 \times 10^{-3} \text{ A} = 4 \text{ mA}$$

Therefore, $v_T = 32$ V, $i_N = 4$ mA, and $R_T = v_T/i_N = 8$ kΩ.

While finding v_{oc} and i_{sc} is preferred method for most circuits, in some cases, the Thevenin equivalent of the subcircuit is only a resistor (you will find $v_{oc} = 0$ and $i_{sc} = 0$), or only a voltage source (you will find $v_{oc} \neq 0$ but finding i_{sc} leads to inconsistent or illegal circuits), or only a current source (you will find $i_{sc} \neq 0$ but finding v_{oc} leads to inconsistent or illegal circuits). For these cases, one has to either find R_T directly and/or directly find i-v characteristics of the subcircuits as is shown below for the circuit of previous example.

Finding R_T

To find R_T, we kill all independent sources in the circuit. The resulting circuit cannot be reduced to a simple resistor by series/parallel formulas. This is why finding v_{oc} and i_{sc} is the preferred choices for subcircuits containing controlled sources. We can find R_T by attaching an ideal voltage source with a known voltage of v and calculate i. Since the subcircuit should be reduced to a resistor (R_T), we should get $i = -v/(constant)$ where the *constant* is R_T. (Negative sign comes from active sign convention used for Thevenin subcircuit).



Since the circuit is simple, we proceed to solve it with KVL and KCL:

$$\text{KCL:} \quad -i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 5i$$

$$\text{KCL:} \quad -i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = i$$

$$\text{KVL:} \quad 0 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 + v = 0$$

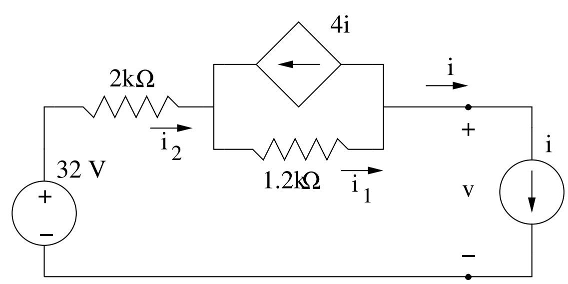
$$2 \times 10^3 i + 6 \times 10^3 i + v = 0 \quad \rightarrow \quad i = -\frac{v}{8 \times 10^3}$$

Therefore, $R_T = 8 \times 10^3\ \Omega = 8\ \text{k}\Omega$.

Note that we could have attached an ideal "current" source with strength of i to the problem, proceeded to calculate v, and woould have got $v = -8 \times 10^3 i$.

Finding i-v Characteristics Equation:

As mentioned above, in some cases, we have to directly find the i-v characteristics equation in order to find the Thevenin equivalent of a subcircuit. The procedure is similar to finding R_T. Attach an ideal voltage source to the circuit. Assume v is known and proceed to calculate i in terms of v. Alternatively, one can attach an ideal current source, assume i is known and find v in terms of i. The final expression should look like $v = v_T - iR_T$ and v_T and R_T can be read directly:



Since the circuit is simple, we proceed to solve it with KVL and KCL:

$$\text{KCL:} \quad -i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 5i$$

$$\text{KCL:} \quad -i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = i$$

$$\text{KVL:} \quad -32 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 + v_{oc} = 0$$

$$-32 + 2 \times 10^3 i + 6 \times 10^3 i + v = 0 \quad \rightarrow \quad v = 32 - 8 \times 10^3 i$$

which is the characteristics equation for the subcircuit and leads to $v_T = 32$ V, $R_T = 8 \times 10^3\ \Omega = 8\ \text{k}\Omega$, and $i_N = v_T/R_T = 4$ mA.