Lenguajes de **P**rogramación

- First-Order Functions

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Adding functions to our language

We want to extend our language with the ability to define and apply functions.

Indeed, most programming languages allow developers to abstract behavior by means of functions (or methods, subroutines, etc) in order to improve modularity and program abstraction.

Although we would like to have functions as values, as in Racket, we will start with a simpler approach: the so-called ***first-order functions***.

We want to run programs such as:

```
{+ 1 {double 2}}
```

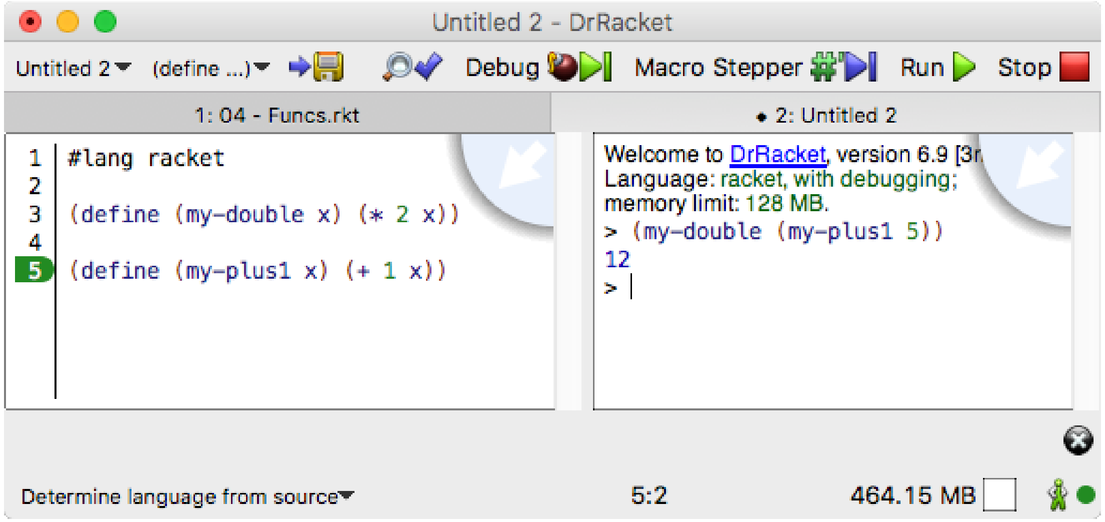
Where `double` is a function defined elsewhere.

3 Modeling approach

Conceptually, the approach is similar to DrRacket:

function(s) definition + expression to evaluate

(window on the left) (window on the right)



The idea is that as function definitions are available to be applied in the REPL, in our language function definitions are available to be applied in users' programs.

Modeling approach

First-order functions

- Functions are <u>not</u> values
- <u>Cannot be anonymous</u>
- Only definable **<u>in a designated portion of the code</u>**, where they must be given names.
- They are defined "before" the program is executed.

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Where `double` is a function defined elsewhere as:

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{fundef 'double 'n (parse '{+ n n})}
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A function is defined by its name, the name of its parameter, and its body. <u>The body is a proper AST</u> (using Expr deftype).

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A function is defined by its name, the name of its parameter, and its body. The body is a proper AST (using Expr deftype).

As a sub-program. We must determine what is the semantics of interpreting function application. We will see this depends on substitution.

How do we extend our language semantics?

EXTEND CONCRETE SYNTAX:

how will developers define functions?
how will developers apply functions?

EXTEND ABSTRACT SYNTAX:

what information will we extract for function definition? and for function application?

EXTEND PARSER:

what elements of the parser we must update?
also update the definition of the substitution function

EXTEND INTERPRETER:

what is the semantics of applying first-order functions?

Extending concrete and abstract syntax

Similar to Racket, we assume any expression that starts with an identifier is a function application.

Extend Concrete Syntax

```
#|
<s-expr> ::= <num>
           | (list '+ <s-expr> <s-expr>)
           | (list '- <s-expr> <s-expr>)
           | (list 'if0 <s-expr> <s-expr> <s-expr>)
           | (list 'with (list <sym> <s-expr>) <s-expr>)
           | <sym>
           | (list <sym> <s-expr>)
|#
```

Extend Abstract Syntax

```
#|
<expr> ::= (num <num>)
         | (add <expr> <expr>)
         | (sub <expr> <expr>)
         | (if0 <expr> <expr> <expr>)
         | (with <sym> <expr> <expr>)
         | (id <sym>)
         | (app <sym> <expr>)
|#
```

We need additional syntax for function definitions:

```
#|
<fundef> ::= (fundef <sym> <sym> <expr>)
|#
```

Note function bodies are full-blown ASTs. They cannot contain other function definitions.

Following the grammars: abstract syntax

```
#|
<expr> ::= (num <num>)
         | (add <expr> <expr>)
         | (sub <expr> <expr>)
         | (if0 <expr> <expr> <expr>)
         | (with <sym> <expr> <expr>)
         | (id <sym>)
         | (app <sym> <expr>)
|#
(deftype Expr
  (num n)
  (add l r)
  (sub l r)
  (if0 c t f)
  (with x e b)
  (id x)
  (app f-name f-arg))
```

Following the grammars: concrete syntax & parser

```
#|
<s-expr> ::= <num>
           | (list '+ <s-expr> <s-expr>)
           | (list '- <s-expr> <s-expr>)
           | (list 'if0 <s-expr> <s-expr> <s-expr>)
           | (list 'with (list <sym> <s-expr>) <s-expr>)
           | <sym>
           | (list <sym> <s-expr>)
|#
(define (parse s-expr)
  (match s-expr
    [(? number? n) (num n)]
    [(? symbol? x) (id x)]
    [(list '+ l r) (add (parse l) (parse r))]
    [(list '- l r) (sub (parse l) (parse r))]
    [(list 'if0 c t f) (if0 (parse c) (parse t) (parse f))]
    [(list 'with (list x e) b) (with x (parse e) (parse b))]
    [(list f-name f-arg) (app f-name (parse f-arg))]))
```

Following the grammars: function definitions

```
#|
<fundef> ::= (fundef <sym> <sym> <expr>)
|#
(deftype FunDef
  (fundef name arg body))
```

```
;; lookup :: symbol listof(FunDef) -> FunDef
;; Searches a function definition within a list of definitions.
(define (lookup f-name defs)
  (match defs
    [(list) (error 'lookup "Function ~a not found" f-name)]
    [(cons head tail) (if (symbol=? f-name (fundef-name head))
                          head
                          (lookup f-name tail))]))
```

Updating the substitution function

Applying substitution to function application nodes simply means to propagate the substitution into the argument expression.

```
;; subst :: Expr Symbol Expr -> Expr
;; (subst in what for) substitutes all free occurrences
;; of identifier 'what' in expression 'in' for expression 'for'.
(define (subst in what for)
  (match in
    [(num n) (num n)]
    [(add l r) (add (subst l what for) (subst r what for))]
    [(sub l r) (sub (subst l what for) (subst r what for))]
    [(if0 c t f) (if0 (subst c what for) (subst t what for) (subst f what for))]
    [(with x e b) (with x (subst e what for) (if (eq? x what) b (subst b what for)))]
    [(id x) (if (eq? x what) for (id x))]
    [(app f-name f-arg) (app f-name (subst f-arg what for))]))
```

Putting it all together

We want to run programs such as:

```
{+ 1 {double 2}}
```

Where **double** is a function defined elsewhere as:

```
{fundef 'double 'n (parse '{+ n n})}
```

```
;; interp :: Expr listof(FunDef) -> number
;; Evaluates arithmetic expression with function calls
(define (interp expr f-list) ...)
```

```
(interp (parse '{+ 1 {double 2}})
        (list (fundef 'double 'n (parse '{+ n n}))))
```

Program results depend on the available function definitions

The interpreter

```
;; interp :: Expr listof(FunDef) -> number
;; Evaluates arithmetic expression with function calls
(define (interp expr f-list)
  (match expr
    [(num n) n]
    [(add l r) (+ (interp l f-list) (interp r f-list))]
    [(sub l r) (- (interp l f-list) (interp r f-list))]
    [(if0 c t f) (if (zero? (interp c f-list)) (interp t f-list) (interp f f-list))]
    [(with x e b) (interp (subst b x (num (interp e f-list))) f-list)]
    [(id x) (error 'interp "Open expression (free occurrence of ~a)" x)]
    [(app f-name f-arg)

                                                                          ]))
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  1  (def (fundef _ the-arg the-body) (lookup f-name f-list))
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1 First, we extract the name of the function argument, as well as the function body.

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                                   ② (num (interp f-arg f-list))         ]))
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2. Interpret the function argument to get its value, which is reinstated as AST node.

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                3                                  2
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2. Interpret the function argument to get its value, which is reinstated as AST node.
3. Substitute free occurrences of the argument name, in the function body, for the computed value.
4. Interpret the function body after the argument substitution.

Example

```
;; run :: s-expr listof(FunDef) -> number
(define (run prog f-list)
  (interp (parse prog) f-list))

(define my-funcs
  (list (fundef 'double 'x (parse '{+ x x}))
        (fundef 'plus1 'n (parse '{+ n 1}))))

(test (run '{+ 1 {double (plus1 4)}} my-funcs) 11)
```

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```

Using our arithmetical language define function my-S

my-S(0) = 0   my-S(n+1) = n+1 + my-S(n)

that sums up the first n natural numbers.

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- A function might be defined multiple times. By our definition of `lookup`, the first (left-most in the list) definition takes precedence.

- Functions are accessible to other functions, and moreover,

- Functions may call one another, regardless of the order in which they are defined.

Lecture material

Bibliography

- Programming Languages: Application and Interpretation (1st Edition)
  Shriram Krishnamurthi [Download]
  Chapters 3, 4

Source code

- Arithmetical language with function calls [Download]

Lenguajes de Programación

- Evaluation by deferred substitution
- Static and dynamic scoping

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- Evaluation by deferred substitution

Recap: language of arithmetical expressions + functions

```
(deftype Expr
  (num n)                          — constants
  (add l r)
  (sub l r)                        — addition, subtraction
  (if0 c t f)                      — conditionals
  (with id named-expr body)
  (id x)                           — local definitions
  (app f-name f-arg))              — function application
```

Recap: language of arithmetical expressions + functions

Our interpreter:

```
(define (interp expr f-list)
  (match expr

    [(num n) ...]

    [(add l r) ...]
    [(sub l r) ...]
    [(if0 c t f) ...]

    [(with id named-expr body) ...]
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    [(id x) ...]

    ))
```

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Our interpreter:

How does it work?

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— error (open expression)

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1. Reduce the named expression **e** recursively.
2. Substitute in body **b** identifier **x** with the value of the named expression **e**.

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```
[(app f-name f-arg)
 (def (fundef _ the-arg the-body) (lookup f-name f-list))
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1. Retrieve argument name `the-arg` and body `the-body` from the function definition.
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ADDS EVALUATION OVERHEAD

```
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Local definitions and function calls are handled using substitution. This substitution adds evaluation overhead.

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the interpreter needs to performs the following substitutions:

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{with {x 3} {with {y 4} {with {z 5} {+ x {+ y z}}}}}
```

the interpreter needs to performs the following substitutions:

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(subst  {with {y 4} {with {z 5} {+ x {+ y z}}}} x 3)
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Evaluation by substitution

Local definitions and function calls are handled using substitution. This substitution adds evaluation overhead.

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which in turn requires “walking through” the three grayed expressions

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- Start up with an empty repository
- When finding a substitution (originated by a local definition or function application), instead of doing it, we add an entry to the repository associating the identifier with its value and continue the evaluation.
- Upon the occurrence of an identifier, we retrieve its value from the repository.

Modeling the repository of deferred substitutions

— interface of the ADT

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;; empty-env  :: Env
;; extend-env :: Symbol Value Env -> Env
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Adapting the interpreter

```
;; interp :: Expr listof(FunDef) Env -> number
;; Evaluates an arithmetic expression with function calls
;; and local definitions deferring the substitutions.
(define (interp expr f-list env)
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               (def new-env (extend-env the-arg (interp e f-list env) empty-env))
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— Evaluate the function's body in the environment only mapping the formal parameter with the value of the actual parameter

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More to come later

```
    [(app f e) (def (fundef _ the-arg the-body) (lookup f f-list))
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Some remarks on the new interpreter

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- The interpreter with deferred substitution, and the original interpreter with explicit substitution return both the same output

- For this to hold true, it is indispensable that when querying the repository for an identifier with multiple bindings, it returns <u>**the most recently added**</u>.

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- The interpreter with deferred substitution, and the original interpreter with explicit substitution return both the same output

- For this to hold true, it is indispensable that when querying the repository for an identifier with multiple bindings, it returns **the most recently added**.

Under this policy: `{with {x 0} {with {x 1} x}}` $\rightsquigarrow$ `1`

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Why do we never remove entries from the repository of deferred substitutions?

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```

What happens if when evaluating a local definition, we use a fresh repository, bounding only the identifier at hand (as in the case of function application)?

Why do we never remove entries from the repository of deferred substitutions?

- Static and dynamic scoping

Static vs dynamic scope

Consider expression

```
{with {n 5} {f 10}}
```

with function definition

```
(list (fundef 'f 'x (id 'n)))
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To which value should it reduce?

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To which value should it reduce?

- **5** — **DYNAMIC SCOPE**
- Error (n is free) — **STATIC (OR LEXICAL) SCOPE**

Our interpreter(s)

Scoping rules determine how identifiers obtain their values

Static scope

In a language with *static scope*, the scope of a binding is a syntactically delimited region.

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{with {n 5} {f 10}}
```

w/ def.

```
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scope of the binding `n => 5`

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The binding `n => 5` has effect only in “text” `{f 10}`.
As there is no occurrence of `n` in `{f 10}`, the binding has indeed no effect.

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As there is no occurrence of `n` in `{f 10}`, the binding has indeed no effect.

- Corresponds to the semantics provided by either of our interpreters
- Used in most modern programming languages: Ada, C, Pascal, Scheme, and Haskell, Python, etc.

Dynamic scope

In a language with *dynamic scope*, the scope of a binding is the entire remaining of the execution during which the binding is in effect.

`{with {n 5} {f 10}}` w/ def. `(list (fundef ‘f ‘x (id ’n)))`

Dynamic scope

In a language with *dynamic scope*, the scope of a binding is the entire remaining of the execution during which the binding is in effect.

`{with {n 5} {f 10}}` w/ def. `(list (fundef ‘f ‘x (id ’n)))`

The binding `n => 5` still has effect during the “execution” of **`f`**.
And during the “execution” of any other function that f invokes.

Dynamic scope

Providing a dynamic scope to our language requires adapting only the evaluation rule of function calls to:

```
(define (interp expr f-list env)
  (match expr
    [(app f e) (def (fundef _ the-arg the-body) (lookup f f-list))
               (def new-env (extend-env the-arg (interp e f-list env) env))
               (interp the-body f-list new-env)]))
```

i.e. for evaluating the function's body, the current environment must be augmented (instead of discarded) with the binding of the function's formal argument

Dynamic scope

∴

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- The same function invocation within two different contexts may reduce to different values.

> **[…] dynamic scope is entirely unreasonable. The problem is that we simply cannot determine what the value of a program will be without knowing everything about its execution history […] We will therefore regard dynamic scope as an error and reject its use.**
>
> **PLAI, Shriram Krishnamurthi**

Static vs dynamic scope

<u>Scoping rules determine how identifiers obtain their values</u>

Static scope

- Based on the **<u>structure of the code</u>** as it is written.
- The scope is determined when the code is written.
- It is predictable. We just have to read the source code.
- The proper default for modern languages.
- Generally has better tooling support and scalability.

Dynamic scope

- Based on the **<u>history of function calls</u>** during program execution.
- The scope is determined at runtime, based on the call stack.
- It's difficult to predict, resulting in weird bugs.
- Useful as an optional and explicit feature.
- Can be used to pass arbitrary parameters without changing function signatures.

Dynamic scope

Is it such an error? No!

Dynamic scope

Is it such an error? No!

- In certain cases, it provides some desired flexibility
  - Configuration: e.g. system-wise STDOUT
  - Adaptation: ability to behave differently according to the context (e.g. desktop vs mobile)

See A Note on Dynamic Scope by Eric Tanter !!!

Dynamic scope

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- In certain cases, it provides some desired flexibility
  - Configuration: e.g. system-wise STDOUT
  - Adaptation: ability to behave differently according to the context (e.g. desktop vs mobile)

See A Note on Dynamic Scope by Eric Tanter !!!

- It is good as an opt-in feature (as in Common Lisp, Scala, Racket.), **but not as the default option.**

Lecture material

Bibliography

- Programming Languages: Application and Interpretation (1st Edition)
  Shriram Krishnamurthi [Download]
  Chapter 5
- A Note on Dynamic Scope by Éric Tanter

Source code

- Arithmetical language with deferred substitution [Download]