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The C Clustering Library for cDNA microarray data.

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1 Introduction

Clustering is widely used in gene expression data analysis. By grouping genes together based on the similarity between their gene expression profiles, functionally related genes may be found. Such a grouping suggests the function of presently unknown genes.

The C Clustering Library is a collection of numerical routines that implement the clustering algorithms that are most commonly used. The routines can be applied both to genes and to arrays. The clustering algorithms are:

- Hierarchical clustering (pairwise centroid-, single-, complete-, and average-linkage);
- *k*-means clustering;
- Self-Organizing Maps;
- Principal Component Analysis.

To measure the similarity or distance between gene expression data, eight distance measures are available:

- Pearson correlation;
- Absolute value of the Pearson correlation;
- Uncentered Pearson correlation (equivalent to the cosine of the angle between two data vectors);
- Absolute uncentered Pearson correlation (equivalent to the cosine of the smallest angle between two data vectors);
- Spearman's rank correlation;
- Kendall's τ ;
- Euclidean distance;
- City-block distance.

This library was written in ANSI C and can therefore be easily linked to other C/C++ programs. Cluster 3.0 (http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster) is an example of such a program. This library may be particularly useful when called from a scripting language such as Python (http://www.python.org), Perl (http://www.perl.org), or Ruby (http://www.ruby.org). The C Clustering Library contains wrappers for Python and Perl; interfaces to other scripting languages may be generated using SWIG (http://www.swig.org).

This manual contains a description of clustering techniques, their implementation in the C Clustering Library, the Python and Perl modules that give access to the C Clustering Library, and information on how to use the routines in the library from other C or C++ programs.

The C Clustering Library was released under the Python License.

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2 Distance functions

In order to cluster gene expression data into groups with similar genes or microarrays, we should first define what exactly we mean by *similar*. In the C Clustering Library, eight distance functions are available to measure similarity, or conversely, distance:

'c' Pearson correlation coefficient;

'a' Absolute value of the Pearson correlation coefficient;

'u' Uncentered Pearson correlation (equivalent to the cosine of the angle between two data vectors);

'x' Absolute uncentered Pearson correlation;

's' Spearman's rank correlation;

'k' Kendall's τ ;

'e' Euclidean distance;

'b' City-block distance.

The first six of these distance measures are related to the correlation coefficient, while the remaining three are related to the Euclidean distance. The characters in front of the distance measures are used as mnemonics to be passed to various routines in the C Clustering Library.

One of the properties one would like to see in a distance function is that it satisfies the triangle inequality:

$$d(\underline{u},\underline{v}) \le d(\underline{u},\underline{w}) + d(\underline{w},\underline{v}) \text{ for all } \underline{u},\underline{v},\underline{w}.$$

In everyday language, this equation means that the shortest distance between two points is a straight line.

Correlation-based distance functions usually define the distance d in terms of the correlation r as

$$d = 1 - r.$$

All correlation-based similarity measures are converted to a distance using this definition. Note that this distance function does not satisfy the triangle inequality. As an example, try

$$\underline{u} = (1, 0, -1);$$

 $\underline{v} = (1, 1, 0).$
 $\underline{w} = (0, 1, 1);$

Using the Pearson correlation, we find $d(\underline{u},\underline{w}) = 1.8660$, while $d(\underline{u},\underline{v}) + d(\underline{v},\underline{w}) = 1.6340$. None of the distance functions based on the correlation coefficient satisfy the triangle inequality; this is a general characteristic of the correlation coefficient. The Euclidean distance and the city-block distance, which are *metrics*, do satisfy the triangle inequality. The correlation-based distance functions are sometimes called *semi-metric*.

2.1 Data handling

The input to the distance functions contains two arrays and two row or column indices, instead of two data vectors. This makes it easier to calculate the distance between two columns in the gene expression data matrix. If the distance functions would require two vectors, we would have to extract two columns from the matrix and save them in two vectors to be passed to the distance function. In order to specify if the distance between rows or between columns is to be calculated, each distance function has a flag transpose. If transpose==0, then the distance between two rows is calculated. Otherwise, the distance between two columns is calculated.

2.2 Weighting

For most of the distance functions available in the C Clustering Library, a weight vector can be applied. The weight vector contains weights for the elements in the data vector. If the weight for element i is w_i , then that element is treated as if it occurred w_i times in the data. The weight do not have to be integers. For the Spearman rank correlation and Kendall's τ , discussed below, the weights do not have a well-defined meaning and are therefore not implemented.

2.3 Missing Values

Often in microarray experiments, some of the data values are missing. In the distance functions, we therefore use an additional matrix mask which shows which data values are missing. If mask[i][j]==0, then data[i][j] is missing, and is not included in the distance calculation.

2.4 The Pearson correlation coefficient

The Pearson correlation coefficient is defined as

$$r = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma_x} \right) \left(\frac{y_i - \bar{y}}{\sigma_y} \right)$$

in which \bar{x}, \bar{y} are the sample mean of x and y respectively, and σ_x, σ_y are the sample standard deviation of x and y. The Pearson correlation coefficient is a measure for how well a straight line can be fitted to a scatterplot of x and y. If all the points in the scatterplot lie on a straight line, the Pearson correlation coefficient is either +1 or -1, depending on whether the slope of line is positive or negative. If the Pearson correlation coefficient is equal to zero, there is no correlation between x and y.

The Pearson distance is then defined as

$$d_{\rm P} \equiv 1 - r$$
.

As the Pearson correlation coefficient lies between -1 and 1, the Pearson distance lies between 0 and 2.

Note that the Pearson correlation automatically centers the data by subtracting the mean, and normalizes them by dividing by the standard deviation. While such normalization may be useful in some situations (e.g., when clustering gene expression levels directly instead of gene expression ratios), information is being lost in this step. In particular, the magnitude of changes in gene expression is being ignored. This is in fact the reason that the Pearson distance does not satisfy the triangle inequality.

2.5 Absolute Pearson correlation

By taking the absolute value of the Pearson correlation, we find a number between zero and one. If the absolute value is one, all the points in the scatter plot lie on a straight line with either a positive or a negative slope. If the absolute value is equal to zero, there is no correlation between x and y.

The distance is defined as usual as

$$d_{\rm A} \equiv 1 - |r|$$
,

where r is the Pearson correlation coefficient. As the absolute value of the Pearson correlation coefficient lies between 0 and 1, the corresponding distance lies between 0 and 1 as well.

In the context of gene expression experiments, note that the absolute correlation is equal to one if the gene expression data of two genes/microarrays have a shape that is either exactly the same or exactly opposite. The absolute correlation coefficient should therefore be used with care.

2.6 Uncentered correlation (cosine of the angle)

In some cases, it may be preferable to use the uncentered correlation instead of the regular Pearson correlation coefficient. The uncentered correlation is defined as

$$r_{\rm U} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i}{\sigma_x^{(0)}} \right) \left(\frac{y_i}{\sigma_y^{(0)}} \right),$$

where

$$\sigma_x^{(0)} = \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2};$$

$$\sigma_y^{(0)} = \sqrt{\frac{1}{n} \sum_{i=1}^n y_i^2}.$$

This is the same expression as for the regular Pearson correlation coefficient, except that the sample means \bar{x}, \bar{y} are set equal to zero. The uncentered correlation may be appropriate if there is a zero reference state. For instance, in the case of gene expression data given in terms of log-ratios, a log-ratio equal to zero corresponds to the green and red signal being equal, which means that the experimental manipulation did not affect the gene expression.

The distance corresponding to the uncentered correlation coefficient is defined as

$$d_{\rm U} \equiv 1 - r_{\rm U},$$

where $r_{\rm U}$ is the uncentered correlation. As the uncentered correlation coefficient lies between -1 and 1, the corresponding distance lies between 0 and 2.

The uncentered correlation is equal to the cosine of the angle of the two data vectors in n-dimensional space, and is often referred to as such. (From this viewpoint, it would make more sense to define the distance as the arc cosine of the uncentered correlation coefficient).

2.7 Absolute uncentered correlation

As for the regular Pearson correlation, we can define a distance measure using the absolute value of the uncentered correlation:

$$d_{\rm AU} \equiv 1 - |r_{\rm U}|$$
,

where $r_{\rm U}$ is the uncentered correlation coefficient. As the absolute value of the uncentered correlation coefficient lies between 0 and 1, the corresponding distance lies between 0 and 1 as well.

Geometrically, the absolute value of the uncentered correlation is equal to the cosine between the supporting lines of the two data vectors (i.e., the angle without taking the direction of the vectors into consideration).

2.8 Spearman rank correlation

The Spearman rank correlation is an example of a non-parametric similarity measure. It is useful because it is more robust against outliers than the Pearson correlation.

To calculate the Spearman rank correlation, we replace each data value by their rank if we would order the data in each vector by their value. We then calculate the Pearson correlation between the two rank vectors instead of the data vectors.

Weights cannot be suitably applied to the data if the Spearman rank correlation is used, especially since the weights are not necessarily integers. The calculation of the Spearman rank correlation in the C Clustering Library therefore does not take any weights into consideration.

As in the case of the Pearson correlation, we can define a distance measure corresponding to the Spearman rank correlation as

$$d_{\rm S} \equiv 1 - r_{\rm S}$$

where $r_{\rm S}$ is the Spearman rank correlation.

2.9 Kendall's τ

Kendall's τ is another example of a non-parametric similarity measure. It is similar to the Spearman rank correlation, but instead of the ranks themselves only the relative ranks are used to calculate τ (see Snedecor & Cochran). As in the case of the Spearman rank correlation, the weights are ignored in the calculation.

We can define a distance measure corresponding to Kendall's τ as

$$d_{\rm K} \equiv 1 - \tau$$
.

As Kendall's τ is defined such that it will lie between -1 and 1, the corresponding distance will be between 0 and 2.

2.10 Euclidean distance

The Euclidean distance is a true metric, as it satisfies the triangle inequality. In this software package, we define the Euclidean distance as

$$d = \frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2.$$

Only those terms are included in the summation for which both x_i and y_i are present. The denominator n is chosen accordingly.

In this formula, the expression data x_i and y_i are subtracted directly from each other. We should therefore make sure that the expression data are properly normalized when using the Euclidean distance, for example by converting the measured gene expression levels to log-ratios.

Unlike the correlation-based distance functions, the Euclidean distance takes the magnitude of the expression data into account. It therefore preserves more information about the data and may be preferable. De Hoon, Imoto, Miyano (2002) give an example of the use of the Euclidean distance for k-means clustering.

2.11 City-block distance

The city-block distance, alternatively known as the Manhattan distance, is related to the Euclidean distance. Whereas the Euclidean distance corresponds to the length of the shortest path between two points, the city-block distance is the sum of distances along each dimension. As gene expression data tend to have missing values, in the C Clustering Library we define the city-block distance as the sum of distances divided by the number of dimensions:

$$d = \frac{1}{n} \sum_{i=1}^{n} |x_i - y_i|.$$

This is equal to the distance you would have to walk between two points in a city, where you have to walk along city blocks. The city-block distance is a metric, as it satisfies the triangle inequality. As for the Euclidean distance, the expression data are subtracted directly from each other, and we should therefore make sure that they are properly normalized.

2.12 Calculating the distance between clusters

In the hierarchical clustering methods, the distance matrix between all genes/microarrays is first calculated, and at successive steps of the algorithm the new distance matrix is calculated from the previous distance matrix. In some cases, however, we would like to calculate the distance between clusters directly, given their members. For this purpose, the function clusterdistance can be used. This function can also be used to calculate the distance between two genes/microarrays by defining two clusters consisting of one gene/microarray each. While this function is not used internally in the C Clustering Library, it may be used by other C applications, and can also be called from Python (See (undefined) [Using the C Clustering Library with Python: Pycluster and Bio.Cluster], page (undefined) and Perl (See (undefined) [Using the C Clustering Library with Perl: Algorithm::Cluster], page (undefined)).

The distance between two clusters can be defined in several ways. The distance between the arithmetic means of the two clusters is used in pairwise centroid-linkage clustering and in k-means clustering. For the latter, the distance between the medians of the two clusters can be used alternatively. The shortest pairwise distance between elements of the two clusters is used in pairwise single-linkage clustering, while the longest pairwise distance is used in pairwise maximum-linkage clustering. In pairwise average-linkage clustering, the distance between two clusters is defined as the average over the pairwise distances.

Prototype

double clusterdistance (int nrows, int ncolumns, double** data, int** mask, double weight[], int n1, int n2, int index1[], int index2[], char dist, char method, int transpose);

Arguments

• int nrows;

The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.

• int ncolumns;

The number of columns in the *data* matrix, equal to the number of microarrays in the gene expression experiment.

• double** data;

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].

• int** mask;

This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].

double weight[];

The weights that are used to calculate the distance. Dimension: [ncolumns] if transpose==0; [nrows] if transpose==1.

• int *n1*;

The number of elements in the first cluster.

• int n2:

The number of elements in the second cluster.

• int index1[];

Contains the indices of the elements belonging to the first cluster. Dimension: [n1].

• int index2[];

Contains the indices of the elements belonging to the second cluster. Dimension: [n2].

• char dist

Specifies which distance measure is used. See (undefined) [Distance functions], page (undefined).

char method;

Specifies how the distance between clusters is defined:

'a' Distance between the two cluster centroids (arithmetic mean);

'm' Distance between the two cluster centroids (median);

- 's' Shortest pairwise distance between elements in the two clusters;
- 'x' Longest pairwise distance between elements in the two clusters;
- 'v' Average over the pairwise distances between elements in the two clusters.

• int transpose;

If transpose==0, the distances between rows in the data matrix are calculated. Otherwise, the distances between columns are calculated.

Return value

The distance between two clusters (double).

2.13 The distance matrix

The first step in clustering problems is usually to calculate the distance matrix. This matrix contains all the distances between the items that are being clustered. As the distance functions are symmetric, the distance matrix is also symmetric. Furthermore, the elements on the diagonal are zero, as the distance of an item to itself is zero. The distance matrix can therefore be stored as a ragged array, with the number of columns in each row equal to the (zero-offset) row number. The distance between items i and j is stored in location [i][j] if j < i, in [j][i] if j > i, while it is zero if j = i. Note that the first row of the distance matrix is empty. It is included for computational convenience, as including an empty row requires minimal storage.

Prototype

double** distancematrix (int nrows, int ncolumns, double** data, int** mask,
double weight[], char dist, int transpose);

Arguments

- int nrows;
 - The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.
- int ncolumns;
 - The number of columns in the data matrix, equal to the number of microarrays in the gene expression experiment.
- double** data;
 - The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].
- int** mask:
 - This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].
- double weight[];
 - The weights that are used to calculate the distance. Dimension: [ncolumns] if transpose==0; [nrows] if transpose==1.
- char dist;
 - Specifies which distance measure is used. See \(\)undefined \(\) [Distance functions], page \(\)undefined \(\).

• int transpose;

If transpose==0, the distances between the rows in the data matrix are calculated. Otherwise, the distances between the columns are calculated.

Return value

A pointer to the distance matrix stored as a newly allocated ragged array. If insufficient memory is available to store the distance matrix, a NULL pointer is returned, and all memory allocated thus far is freed.

3 Partitioning algorithms

Partitioning algorithms divide items into k clusters such that the sum of distances over the items to their cluster centers is minimal. The number of clusters k is specified by the user. In the C Clustering Library, three partitioning algorithms are available:

- k-means clustering
- k-medians clustering
- k-medoids clustering

These algorithms differ in how the cluster center is defined. In k-means clustering, the cluster center is defined as the mean data vector averaged over all items in the cluster. Instead of the mean, in k-medians clustering the median is calculated for each dimension in the data vector. Finally, in k-medoids clustering the cluster center is defined as the item which has the smallest sum of distances to the other items in the cluster. This clustering algorithm is suitable for cases in which the distance matrix is known but the original data matrix is not available, for example when clustering proteins based on their structural similarity.

The expectation-maximization (EM) algorithm is commonly used to find the partitioning into k groups. The first step in the EM algorithm is to create k clusters and randomly assign items (genes or microarrays) to them. We then iterate:

- Calculate the centroid of each cluster;
- For each item, determine which cluster centroid is closest;
- Reassign the item to that cluster.

The iteration is stopped if no further item reassignments take place.

As the initial assignment of items to clusters is done randomly, usually a different clustering solution is found each time the EM algorithm is executed. To find the optimal clustering solution, the *k*-means algorithm is repeated many times, each time starting from a different initial random clustering. The sum of distances of the items to their cluster center is saved for each run, and the solution with the smallest value of this sum will be returned as the overall clustering solution.

How often the EM algorithm should be run depends on the number of items being clustered. As a rule of thumb, we can consider how often the optimal solution was found. This number is returned by the partitioning algorithms as implemented in this library. If the optimal solution was found many times, it is unlikely that better solutions exist than the one that was found. However, if the optimal solution was found only once, there may well be other solutions with a smaller within-cluster sum of distances.

3.1 Initialization

The k-means algorithm is initialized by randomly assigning items (genes or microarrays) to clusters. To ensure that no empty clusters are produced, we use the binomial distribution to randomly choose the number of items in each cluster to be one or more. We then randomly permute the cluster assignments to items such that each item has an equal probability to be in any cluster. Each cluster is thus guaranteed to contain at least one item.

3.2 Finding the cluster centroid

The centroid of a cluster can be defined in different ways. For k-means clustering, the centroid of a cluster is defined as the mean over all items in a cluster for each dimension separately. For robustness against outliers, in k-medians clustering the median is used instead of the mean. In k-medoids clustering, the cluster centroid is the item with the smallest sum of distances to the other items in the cluster. The C Clustering Library provides routines to calculate the cluster mean, the cluster median, and the cluster medoid.

3.2.1 Finding the cluster mean or median

The routine getclustercentroids calculates the centroids of the clusters by calculating the mean or median for each dimension separately over all items in a cluster. Missing data values are not included in the calculation of the mean or median. Missing values in the cluster centroids are indicated in the array cmask. If for cluster i the data values for dimension j are missing for all items, then cmask[i][j] (or cmask[j][i] if transpose==1) is set equal to zero. Otherwise, it is set equal to one. The argument method determines if the means or medians are calculated. If a memory allocation error occurred, getclustercentroids returns 0; this can happen only if the cluster medians are being calculated. If no errors occur, getclustercentroids returns 1;

Prototype

int getclustercentroids(int nclusters, int nrows, int ncolumns, double** data,
int** mask, int clusterid[], double** cdata, int** cmask, int transpose, char
method);

Arguments

- int nclusters;
 The number of clusters.
- int nrows;

The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.

- int ncolumns;
 - The number of columns in the data matrix, equal to the number of microarrays in the gene expression experiment.
- double** data;

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].

- int** mask:
 - This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].
- int clusterid[];

The cluster number to which each item belongs. Each element in this array should be between 0 and nclusters-1 inclusive. Dimension: [nrows] if transpose==0, or [ncolumns] if transpose==1.

• double** cdata;

This matrix stores the centroid information. Space for this matrix should be allocated

before calling getclustercentroids. Dimension: [nclusters] [ncolumns] if transpose==0 (row-wise clustering), or [nrows] [nclusters] if transpose==1 (column-wise clustering).

• int** cmask;

This matrix stores which values in *cdata* are missing. If *cmask*[i][j]==0, then *cdata*[i][j] is missing. Space for *cmask* should be allocated before calling getclustercentroids. Dimension: [nclusters][ncolumns] if transpose==0 (row-wise clustering), or [nrows][nclusters] if transpose==1 (column-wise clustering).

• int transpose;

This flag indicates whether row-wise (gene) or column-wise (microarray) clustering is being performed. If transpose==0, rows (genes) are being clustered. Otherwise, columns (microarrays) are being clustered.

• char method;

If method=='a', the cluster centroids are calculated as the arithmetic mean over each dimension. For method=='m', the cluster centroids are calculated as the median over each dimension.

Return value

This routine returns 1 if successful, and 0 in case of a memory allocation error.

3.2.2 Finding the cluster medoid

The cluster medoid is defined as the item which has the smallest sum of distances to the other items in the cluster. The getclustermedoids routine calculates the cluster medoids, given to which cluster each item belongs.

Prototype

void getclustermedoids(int nclusters, int nelements, double** distance, int
clusterid[], int centroids[], double errors[]);

Arguments

- int nclusters;
 - The number of clusters.
- int nelements:

The total number of elements that are being clustered.

• double** distmatrix;

The distance matrix. The distance matrix is symmetric and has zeros on the diagonal. To save space, the distance matrix is stored as a ragged array. Dimension: [nelements][] as a ragged array. The number of columns in each row is equal to the row number (starting from zero). Accordingly, the first row always has zero columns.

• int clusterid[];

The cluster number to which each element belongs. Dimension: [nelements].

• int centroid[];

For each cluster, the element of the item that was determined to be its centroid. Dimension: [nclusters].

double errors[];

For each cluster, the sum of distances between the items belonging to the cluster and the cluster centroid. Dimension: [nclusters].

3.3 The EM algorithm

The EM algorithm as implemented in the C Clustering Library first randomly assigns items to clusters, followed by iterating to find a clustering solution with a smaller within-cluster sum of distances. During the iteration, first we find the centroids of all clusters, where the centroids are defined in terms of the mean, the median, or the medoid. The distances of each item to the cluster centers are calculated, and we determine for each item which cluster is closest. We then reassign the items to their closest clusters, and recalculate the cluster centers.

All items are first reassigned before recalculating the cluster centroids. If unchecked, clusters may become empty if all their items are reassigned. For k-means and k-medians clustering, the EM routine therefore keeps track of the number of items in each cluster at all times, and prohibits the last remaining item in a cluster from being reassigned to a different cluster. For k-medoids clustering, such a check is not needed, as the item that functions as the cluster centroid has a zero distance to itself, and will therefore not be reassigned to a different cluster.

The EM algorithm terminates when no further reassignments take place. We noticed, however, that for some sets of initial cluster assignments, the EM algorithm fails to converge due to the same clustering solution reappearing periodically after a small number of iteration steps. In the EM algorithm as implemented in the C Clustering Library, the occurrence of such periodic solutions is checked for. After a given number of iteration steps, the current clustering result is saved as a reference. By comparing the clustering result after each subsequent iteration step to the reference state, we can determine if a previously encountered clustering result is found. In such a case, the iteration is halted. If after a given number of iterations the reference state has not yet been encountered, the current clustering solution is saved to be used as the new reference state. Initially, ten iteration steps are executed before resaving the reference state. This number of iteration steps is doubled each time, to ensure that periodic behavior with longer periods can also be detected.

3.4 Finding the optimal solution

3.4.1 k-means and k-medians

The optimal solution is found by executing the EM algorithm repeatedly and saving the best clustering solution that was returned by it. This can be done automatically by calling the routine kcluster. The routine to calculate the cluster centroid and the distance function are selected based on the arguments passed to kcluster.

The EM algorithm is then executed repeatedly, saving the best clustering solution that was returned by these routines. In addition, kcluster counts how often the EM algorithm found this solution. If it was found many times, we can assume that there are no other solutions possible with a smaller within-cluster sum of distances. If, however, the solution was found only once, it may well be that better clustering solutions exist.

Prototype

void kcluster (int nclusters, int nrows, int ncolumns, double** data, int**
mask, double weight[], int transpose, int npass, char method, char dist, int
clusterid[], double* error, int* ifound);

Arguments

• int nclusters;

The number of clusters k.

• int nrows:

The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.

• int ncolumns;

The number of columns in the data matrix, equal to the number of microarrays in the gene expression experiment.

• double** data;

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].

int** mask;

This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].

• double weight[];

The weights that are used to calculate the distance. Dimension: [ncolumns] if transpose==0; [nrows] if transpose==1.

• int transpose;

This flag indicates whether row-wise (gene) or column-wise (microarray) clustering is being performed. If **transpose==0**, rows (genes) are being clustered. Otherwise, columns (microarrays) are being clustered.

• int npass;

The number of times the EM algorithm should be run. If npass > 0, each run of the EM algorithm uses a different (random) initial clustering. If npass == 0, then the EM algorithm is run with an initial clustering specified by *clusterid*. Reassignment to a different cluster is prevented for the last remaining item in the cluster in order to prevent empty clusters. For npass == 0, the EM algorithm is run only once, using the initial clustering as specified by *clusterid*.

• char method;

Specifies whether the arithmetic mean (method=='a') or the median (method=='m') should be used to calculate the cluster center.

• char dist;

Specifies which distance function should be used. The character should correspond to one of the distance functions that are available in the C Clustering Library. See \(\text{undefined} \) [Distance functions], page \(\text{undefined} \).

• int clusterid[];

This array will be used to store the cluster number to which each item was assigned by the clustering algorithm. Space for clusterid should be allocated before calling kcluster. If npass==0, then the contents of clusterid on input is used as the

initial assignment of items to clusters; on output, *clusterid* contains the optimal clustering solution found by the EM algorithm. Dimension: [nrows] if transpose==0, or [ncolumns] if transpose==1.

• double* error;

The sum of distances of the items to their cluster center after k-means clustering, which can be used as a criterion to compare clustering solutions produced in different calls to kcluster.

• int* ifound;

Returns how often the optimal clustering solution was found. In case of an inconsistency in the input arguments (specifically, if *nclusters* is larger than the number of elements to be clustered), *ifound is set to 0. If a memory allocation error occurred, *ifound is set to -1.

3.4.2 k-medoids

The kmedoids routine performs k-medoids clustering on a given set of elements, using the distance matrix and the number of clusters passed by the user. Multiple passes are being made to find the optimal clustering solution, each time starting from a different initial clustering.

Prototype

void kmedoids (int nclusters, int nelements, double** distance, int npass, int
clusterid[], double* error, int* ifound);

Arguments

• int nclusters;

The number of clusters to be found.

• int nelements;

The number of elements to be clustered.

• double** distmatrix;

The distance matrix. The distance matrix is symmetric and has zeros on the diagonal. To save space, the distance matrix is stored as a ragged array. Dimension: [nelements][] as a ragged array. The number of columns in each row is equal to the row number (starting from zero). Accordingly, the first row always has zero columns.

• int npass;

The number of times the EM algorithm should be run. If npass > 0, each run of the EM algorithm uses a different (random) initial clustering. If npass == 0, then the EM algorithm is run with an initial clustering specified by clusterid.

• int clusterid[];

This array will be used to store the cluster number to which each item was assigned by the clustering algorithm. Space for clusterid should be allocated before calling kcluster. On input, if npass==0, then clusterid contains the initial clustering assignment from which the clustering algorithm starts; all numbers in clusterid should be between 0 and nelements-1 inclusive. If npass!=0, clusterid is ignored on input. On output, clusterid contains the number of the cluster to which each item was assigned in the optimal clustering solution. The number of a cluster is defined as the item number of the centroid of the cluster. Dimension: [nelements].

• double* error;

The sum of distances of the items to their cluster center after k-means clustering, which can be used as a criterion to compare clustering solutions produced in different calls to kmedoids.

• int* ifound;

Returns how often the optimal clustering solution was found. In case of an inconsistency in the input arguments (specifically, if *nclusters* is larger than *nelements*), *ifound* is set to 0. If a memory allocation error occurred, *ifound* is set to -1.

3.5 Choosing the distance measure

Whereas all eight distance measures are accepted for k-means, k-medians, and k-medoids clustering, using a distance measure other than the Euclidean distance or city-block distance with k-means or k-medians is in a sense inconsistent. When using the distance measures based on the Pearson correlation, the data are effectively normalized when calculating the distance. However, no normalization is applied when calculating the centroid in the k-means or k-medians algorithm. From a theoretical viewpoint, it is best to use the Euclidean distance for the k-means algorithm, and the city-block distance for k-medians.

4 Hierarchical clustering

Hierarchical clustering methods are inherently different from the k-means clustering method. In hierarchical clustering, the similarity in the expression profile between genes or experimental conditions are represented in the form of a tree structure. This tree structure can be shown graphically by programs such as TreeView and Java TreeView, which has contributed to the popularity of hierarchical clustering in the analysis of gene expression data.

The first step in hierarchical clustering is to calculate the distance matrix, specifying all the distances between the items to be clustered. Next, we create a node by joining the two closest items. Subsequent nodes are created by pairwise joining of items or nodes based on the distance between them, until all items belong to the same node. A tree structure can then be created by retracing which items and nodes were merged. Unlike the EM algorithm, which is used in k-means clustering, the complete process of hierarchical clustering is deterministic.

Several flavors of hierarchical clustering exist, which differ in how the distance between subnodes is defined in terms of their members. In the C Clustering Library, pairwise single, maximum, average, and centroid linkage are available.

- In pairwise single-linkage clustering, the distance between two nodes is defined as the shortest distance among the pairwise distances between the members of the two nodes.
- In pairwise maximum-linkage clustering, alternatively known as pairwise completelinkage clustering, the distance between two nodes is defined as the longest distance among the pairwise distances between the members of the two nodes.
- In pairwise average-linkage clustering, the distance between two nodes is defined as the average over all pairwise distances between the elements of the two nodes.
- In pairwise centroid-linkage clustering, the distance between two nodes is defined as the distance between their centroids. The centroids are calculated by taking the mean over all the elements in a cluster. As the distance from each newly formed node to existing nodes and items need to be calculated at each step, the computing time of pairwise centroid-linkage clustering may be significantly longer than for the other hierarchical clustering methods. Another peculiarity is that (for a distance measure based on the Pearson correlation), the distances do not necessarily increase when going up in the clustering tree, and may even decrease. This is caused by an inconsistency between the centroid calculation and the distance calculation when using the Pearson correlation: Whereas the Pearson correlation effectively normalizes the data for the distance calculation, no such normalization occurs for the centroid calculation.

For pairwise single-, complete-, and average-linkage clustering, the distance between two nodes can be found directly from the distances between the individual items. Therefore, the clustering algorithm does not need access to the original gene expression data, once the distance matrix is known. For pairwise centroid-linkage clustering, however, the centroids of newly formed subnodes can only be calculated from the original data and not from the distance matrix.

The implementation of pairwise single-linkage hierarchical clustering is based on the SLINK algorithm (R. Sibson, 1973), which is much faster and more memory-efficient than a straightforward implementation of pairwise single-linkage clustering. The clustering result produced by this algorithm is identical to the clustering solution found by the conventional

single-linkage algorithm. The single-linkage hierarchical clustering algorithm implemented in this library can be used to cluster large gene expression data sets, for which conventional hierarchical clustering algorithms fail due to excessive memory requirements and running time.

4.1 Representing a hierarchical clustering solution

The result of hierarchical clustering consists of a tree of nodes, in which each node joins two items or subnodes. Usually, we are not only interested in which items or subnodes are joined at each node, but also in their similarity (or distance) as they are joined. To store one node in the hierarchical clustering tree, we make use of a struct Node, which has the following members:

- int left; First item or subnode.
- int right; Second item or subnode.
- double distance;

 The distance between the two items or subnodes that were joined.

Each item and subnode is represented by an integer. For hierarchical clustering of n items, we number the original items $\{0, \ldots, n-1\}$, nodes are numbered $\{-1, \ldots, -(n-1)\}$. Note that the number of nodes is one less than the number of items.

A hierarchical clustering tree can now be written as an array of Node structs. The treecluster routine allocates this array, and returns a Node* pointer to the first element. The calling function is responsible for deallocating the Node* array.

4.2 Performing hierarchical clustering: treecluster

The treecluster routine implements pairwise single-, complete, average-, and centroid-linkage clustering. A pointer distmatrix to the distance matrix can be passed as one of the arguments to treecluster; if this pointer is NULL, the treecluster routine will calculate the distance matrix from the gene expression data using the arguments data, mask, weight, and dist. For pairwise single-, complete-, and average-linkage clustering, the treecluster routine ignores these four arguments if distmatrix is given, as the distance matrix by itself is sufficient for the clustering calculation. For pairwise centroid-linkage clustering, on the other hand, the arguments data, mask, weight, and dist are always needed, even if distmatrix is available.

The treecluster routine will complete faster if it can make use of a previously calculated distance matrix passed as the *distmatrix* argument. Note, however, that newly calculated distances are stored in the distance matrix, and its elements may be rearranged during the clustering calculation. Therefore, in order to save the original distance matrix, it should be copied before treecluster is called. The memory that was allocated by the calling routine for the distance matrix will not be deallocated by treecluster, and should be deallocated by the calling routine after treecluster returns. If *distmatrix* is NULL, however, treecluster takes care both of the allocation and the deallocation of memory for the distance matrix. In that case, treecluster may fail if not enough memory can be allocated for the distance matrix, in which case treecluster returns NULL.

Prototype

tree* treecluster (int nrows, int ncolumns, double** data, int** mask, double
weight[], int transpose, char dist, char method, double** distmatrix);

Arguments

• int nrows;

The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.

• int ncolumns;

The number of columns in the *data* matrix, equal to the number of microarrays in the gene expression experiment.

double** data;

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].

int** mask;

This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].

• double weight[];

The weights that are used to calculate the distance. Dimension: [ncolumns] if transpose==0; [nrows] if transpose==1.

• int transpose;

This flag indicates whether row-wise (gene) or column-wise (microarray) clustering is being performed. If transpose==0, rows (genes) are being clustered. Otherwise, columns (microarrays) are being clustered.

• char dist;

Specifies which distance measure is used. See (undefined) [Distance functions], page (undefined).

• char method;

Specifies which type of hierarchical clustering is used:

- 's': pairwise single-linkage clustering
- 'm': pairwise maximum- (or complete-) linkage clustering
- 'a': pairwise average-linkage clustering
- 'c': pairwise centroid-linkage clustering

double** distmatrix;

The distance matrix, stored as a ragged array. This argument is optional; if the distance matrix is not available, it can be passed as NULL. In that case, treecluster will allocate memory space for the distance matrix, calculate it from the gene expression data, and deallocate the memory space before returning. If the distance matrix happens to be available, the hierarchical clustering calculation can be completed faster by passing it as the distance matrix argument. Note that the contents of the distance matrix will be modified by the clustering algorithm in treecluster. The memory that was allocated for the distance matrix should be deallocated by the calling routine after treecluster returns. Dimension: Ragged array, [nrows][] if transpose==0, or [ncolumns][] if

transpose == 1. In both cases, the number of columns in each row is equal to the row number (starting from zero). Accordingly, the first row always has zero columns.

Return value

A pointer to a newly allocated tree structure that describes the calculated hierarchical clustering solution. If treecluster fails due to a memory allocation error, it returns NULL.

4.3 Cutting a hierarchical clustering tree: cuttree

The tree structure generated by the hierarchical clustering routine treecluster can be further analyzed by dividing the genes or microarrays into n clusters, where n is some positive integer less than or equal to the number of items that were clustered. This can be achieved by ignoring the top n-1 linking events in the tree structure, resulting in n separated subnodes. The items in each subnode are then assigned to the same cluster. The routine cuttree determines to which cluster each item is assigned, based on the hierarchical clustering result stored in the tree structure.

Prototype

void cuttree (int nelements, Node* tree, int nclusters, int clusterid[]);

Arguments

- int nelements;
 - The number of elements whose clustering results are stored in the hierarchical clustering result tree.
- Node* tree;

The hierarchical clustering solution. Each node tree[i] in the array describes one linking event, with tree[i].left and tree[i].right containing the numbers of the nodes that were joined. The original elements are numbered $\{0, \ldots, nelements-1\}$, nodes are numbered $\{-1, \ldots, -(nelements-1)\}$. Note that the number of nodes is one less than the number of elements. The cuttree function performs no error checking of the tree structure. Dimension: [nelements-1].

- int nclusters;
 - The desired number of clusters. The number of clusters should be positive, and less than or equal to *nelements*.
- int clusterid[];
 - The cluster number to which each element is assigned. Memory space for *clusterid* should be allocated before cuttree is called. Dimension: [nelements].

5 Self-Organizing Maps

Self-Organizing Maps (SOMs) were invented by Kohonen to describe neural networks (see for instance Kohonen, 1997). Tamayo (1999) first applied Self-Organizing Maps to gene expression data.

SOMs organize items into clusters that are situated in some topology. Usually a rectangular topology is chosen. The clusters generated by SOMs are such that neighboring clusters in the topology are more similar to each other than clusters far from each other in the topology.

The first step to calculate a SOM is to randomly assign a data vector to each cluster in the topology. If genes are being clustered, then the number of elements in each data vector is equal to the number of microarrays in the experiment.

An SOM is then generated by taking genes one at a time, and finding which cluster in the topology has the closest data vector. The data vector of that cluster, as well as those of the neighboring clusters, are adjusted using the data vector of the gene under consideration. The adjustment is given by

$$\Delta \underline{x}_{\text{cell}} = \tau \cdot (\underline{x}_{\text{gene}} - \underline{x}_{\text{cell}})$$
.

The parameter τ is a parameter that decreases at each iteration step. We have used a simple linear function of the iteration step:

$$\tau = \tau_{\text{init}} \cdot \left(1 - \frac{i}{n}\right),\,$$

in which τ_{init} is the initial value of τ as specified by the user, i is the number of the current iteration step, and n is the total number of iteration steps to be performed. While changes are made rapidly in the beginning of the iteration, at the end of iteration only small changes are made.

All clusters within a radius R are adjusted to the gene under consideration. This radius decreases as the calculation progresses as

$$R = R_{\max} \cdot \left(1 - \frac{i}{n}\right),\,$$

in which the maximum radius is defined as

$$R_{\text{max}} = \sqrt{N_x^2 + N_y^2},$$

where (N_x, N_y) are the dimensions of the rectangle defining the topology.

The routine somcluster carries out the complete SOM algorithm. First it initializes the random number generator. The distance function to be used is specified by dist. The node data are then initialized using the random number generator. The order in which genes or microarrays are used to modify the SOM is also randomized. The total number of iterations is specified by niter, given by the user.

Prototype

void somcluster (int nrows, int ncolumns, double** data, int** mask, double
weight[], int transpose, int nxgrid, int nygrid, double inittau, int niter,
char dist, double*** celldata, int clusterid[][2]);

Arguments

• int nrows;

The number of rows in the data matrix, equal to the number of genes in the gene expression experiment.

• int ncolumns;

The number of columns in the data matrix, equal to the number of microarrays in the gene expression experiment.

• double** data;

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise. Dimension: [nrows] [ncolumns].

int** mask;

This array shows which elements in the data array, if any, are missing. If mask[i][j]==0, then data[i][j] is missing. Dimension: [nrows][ncolumns].

• double weight[];

The weights that are used to calculate the distance. Dimension: [ncolumns] if transpose==0; [nrows] if transpose==1.

• int transpose;

This flag indicates whether row-wise (gene) or column-wise (microarray) clustering is being performed. If *transpose*==0, rows (genes) are being clustered. Otherwise, columns (microarrays) are being clustered.

• int nxgrid;

The number of cells horizontally in the rectangular topology containing the clusters.

• int nygrid;

The number of cells vertically in the rectangular topology containing the clusters.

double inittau;

The initial value for the parameter τ that is used in the SOM algorithm. A typical value for *inittau* is 0.02, which was used in Michael Eisen's Cluster/TreeView program.

• int niter;

The total number of iterations.

• char dist;

Specifies which distance measure is used. See $\langle undefined \rangle$ [Distance functions], page $\langle undefined \rangle$.

• double*** celldata;

The data vectors of the clusters in the rectangular topology that were found by the SOM algorithm. These correspond to the cluster centroids. The first dimension is the horizontal position of the cluster in the rectangle, the second dimension is the vertical position of the cluster in the rectangle, while the third dimension is the dimension along the data vector. The somcluster routine does not allocate storage space for the celldata array. Space should be allocated before calling somcluster. Alternatively, if celldata is equal to NULL, the somcluster routine allocates space for celldata and frees it before returning. In that case, somcluster does not return the data vectors of the clusters that were found. Dimension: [nxgrid][nygrid][ncolumns] if transpose==0, or [nxgrid][nygrid][nrows] if transpose==1.

• int clusterid[][2];

Specifies the cluster to which a gene or microarray was assigned, using two integers to identify the horizontal and vertical position of a cell in the grid for each gene or microarray. Gene or microarrays are assigned to clusters in the rectangular grid by determining which cluster in the rectangular topology has the closest data vector. Space for the *clusterid* argument should be allocated before calling somcluster. If *clusterid* is NULL, the somcluster routine ignores this argument and does not return the cluster assignments. Dimension: [nrows][2] if transpose==0; [ncolumns][2] if transpose==1.

6 Principal Component Analysis

Principal Component Analysis (PCA) is a widely used technique for analyzing multivariate data. In PCA, the data vectors are written as a linear sum over principal components. The number of principal components is equal to the number of dimensions of the data vectors.

The principal components are chosen such that they maximally explain the variance in the data vectors. For example, in case of 3D data vectors, the data can be represented as an ellipsoidal cloud of points in three dimensional space. The first principal component would be the longest axis of the ellipsoid, the second principal component would be the shortest axis. In other words, the principal components are ordered by the amount of variance they explain.

Each data point can be reconstructed by a suitable linear combination of the principal components. However, in order to reduce the dimensionality of the data, usually only the most important principal components are used. The remaining variance present in the data is then regarded as unexplained variance.

The principal components can be found by calculating the eigenvectors of the covariance matrix of the data. The corresponding eigenvalues determine how much of the variance present in the data is explained by each principal component.

The eigenvectors are found by calculating the singular value decomposition of the data matrix. For this purpose, we have included a routine to calculate the singular value decomposition of a matrix. This subroutine is a translation in C of the Algol procedure svd (Golub and Reinsch, 1970). Python users can instead use the svd routine in the linalg package of Numerical Python.

A practical example of applying Principal Component Analysis to gene expression data is presented by Yeung and Ruzzo (2001).

Prototype

void svd (int nrows, int ncolumns, double** U, double S[], double** V, int* ierror);

calculates the singular value decomposition $U_{\text{input}} = U_{\text{output}} \cdot S \cdot V^{\text{T}}$. Householder bidiagonalization and a variant of the QR algorithm are used.

Arguments

• int nrows

The number of rows in U. The number of rows should be greater than or equal to the number of columns.

• int ncolumns

The number of columns in U and the order of V.

• double** U;

On input: U is the rectangular matrix to be decomposed. On output: The contents of U is replaced such that $U_{\text{input}} = U_{\text{output}} \cdot S \cdot V^{\text{T}}$. Dimension: [nrows] [ncolumns].

double* S;

The *ncolumns* non-negative singular values of the matrix U, unordered. If an error exit is made, the singular values should be correct for indices { *ierror, *ierror+1, ..., ncolumns-1}. Dimension: [ncolumns].

• double** V;

The orthogonal matrix V of the decomposition. If an error exit is made, the columns of V corresponding to indices of correct singular values should be correct. Dimension: [ncolumns][ncolumns].

• int* ierror;

Error exit:

- *ierror==0 for normal return;
- *ierror==k if the k^{th} singular value has not been determined after 30 iterations;
- \bullet *ierror==-1 if svd fails to allocate sufficient memory to complete.

7 The random number generator

The random number generator in the C Clustering Library is used to initialize the *k*-means/medians/medoids clustering algorithm and Self-Organizing Maps (SOMs), as well as to randomly select a gene or microarray in the calculation of a SOM.

We need both a generator of uniform random deviates, and a generator whose deviates follow the binomial distribution. The latter can be built, given a uniform random number generator, using the BTPE algorithm by Kachitvichyanukul and Schmeiser (1988).

The uniform random number generator in the C Clustering Library is described by L'Ecuyer (1988). The random number generator is initialized automatically during its first call. As the random number generator by L'Ecuyer uses a combination of two multiplicative linear congruential generators, two (integer) seeds are needed for initialization, for which we use the system-supplied random number generator rand (in the C standard library). We initialize this generator by calling srand with the epoch time in seconds, and use the first two random numbers generated by rand as seeds for the uniform random number generator in the C Clustering Library.

8 Using the C Clustering Library with Python: Pycluster and Bio.Cluster

The C Clustering Library is particularly useful when used as a module to a scripting language. This chapter describes the routines in the C Clustering Library as seen from Python.

To make the routines available to Python, install Pycluster (see Section 10.1 [Installing the C Clustering Library for Python], page 71). From Python, then type

>>> from Pycluster import *

Pycluster is also available as part of Biopython (http://www.biopython.org), in which case you should use

>>> from Bio.Cluster import *

This will give you access to the clustering routines in the C Clustering Library directly from Python.

8.1 Partitioning algorithms

8.1.1 k-means and k-medians clustering: kcluster

clusterid, error, nfound = kcluster (data, nclusters=2, mask=None, weight=None, transpose=0, npass=1, method='a', dist='e', initialid=None) implements the k-means and k-medians clustering algorithms.

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• nclusters

The number of clusters k.

• mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

• weight

contains the weights to be used when calculating distances. If weight==None, then equal weights are assumed.

transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

npass

The number of times the k-means clustering algorithm is performed, each time with a different (random) initial condition. If initialid is given, the value of npass is ignored and the clustering algorithm is run only once, as it behaves deterministically in that case.

• method

describes how the center of a cluster is found:

- method=='a': arithmetic mean;
- method=='m': median.

For other values of method, the arithmetic mean is used.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation:
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance:
- dist=='b': City-block distance.

• initialid

Specifies the initial clustering to be used for the EM algorithm. If initialid==None, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is not None, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns a tuple (clusterid, error, nfound).

• clusterid

An array containing the number of the cluster to which each gene/microarray was assigned.

error

The within-cluster sum of distances for the optimal clustering solution.

nfound

The number of times the optimal solution was found.

8.1.2 k-medoids clustering: kmedoids

```
clusterid, error, nfound = kmedoids (distance, nclusters=2, npass=1,
initialid=None)
```

implements the k-medoids clustering algorithm.

Arguments

• distance

The matrix containing the distances between the elements. You can specify the distance matrix in three ways:

• as a 2D Numerical Python array (in which only the left-lower part of the array will be accessed):

```
distance = array([[0.0, 1.1, 2.3], [1.1, 0.0, 4.5], [2.3, 4.5, 0.0]])
```

• as a 1D Numerical Python array containing consecutively the distances in the left-lower part of the distance matrix:

```
distance = array([1.1, 2.3, 4.5])
```

• as a list containing the rows of the left-lower part of the distance matrix:

These three expressions correspond to the same distance matrix.

• nclusters

The number of clusters k.

• npass

The number of times the k-medoids clustering algorithm is performed, each time with a different (random) initial condition. If initialid is given, the value of npass is ignored, as the clustering algorithm behaves deterministically in that case.

• initialid

Specifies the initial clustering to be used for the EM algorithm. If initialid==None, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is not None, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns a tuple (clusterid, error, nfound).

• clusterid

An array containing the number of the cluster to which each item was assigned, where the cluster number is defined as the item number of the item representing the cluster centroid.

• error

The within-cluster sum of distances for the optimal k-medoids clustering solution.

nfound

The number of times the optimal solution was found.

8.2 Hierarchical clustering

The pairwise single-, maximum-, average-, and centroid-linkage clustering methods are accessible through the function treecluster. The hierarchical clustering routines can be applied either on the original gene expression data, or (except for centroid-linkage clustering) on the distance matrix directly. The tree structure generated by treecluster can be cut in order to separate the elements into a given number of clusters.

8.2.1 Representing a hierarchical clustering solution

A hierarchical clustering solution is represented using two Python classes: Node and Tree.

The class Node

The Python class Node corresponds to the C struct Node described above. A Node object has three attributes:

- left.
- right
- distance

Here, left and right are integers referring to the two items or subnodes that are joined at this node, and distance is the distance between them. The items being clustered are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items-1).

To create a new Node object, we need to specify left and right; distance is optional.

```
>>> from Pycluster import *
>>> Node(2,3)
(2, 3): 0
>>> Node(2,3,0.91)
(2, 3): 0.91
```

The attributes left, right, and distance of an existing Node object can be modified directly:

```
>>> node = Node(4,5)
>>> node.left = 6
>>> node.right = 2
>>> node.distance = 0.73
>>> node
(6, 2): 0.73
```

An error is raised if left and right are not integers, or if distance cannot be converted to a floating-point value.

The class Tree

The Python class Tree represents a full hierarchical clustering solution. A Tree object can be created from a list of Node objects:

```
>>> nodes = [Node(1,2,0.2), Node(0,3,0.5), Node(-2,4,0.6), Node(-1,-3,0.9)]
>>> tree = Tree(nodes)
>>> print tree
(1, 2): 0.2
(0, 3): 0.5
(-2, 4): 0.6
(-1, -3): 0.9
    The Tree initializer checks if the list of nodes is a valid hierarchical clustering result:
>>> nodes = [Node(1,2,0.2), Node(0,2,0.5)]
>>> Tree(nodes)
Traceback (most recent call last):
File "<stdin>", line 1, in ?
cluster.error: Inconsistent tree
```

Individual nodes in a Tree object can be accessed using square brackets:

```
>>> nodes = [Node(1,2,0.2), Node(0,-1,0.5)]
>>> tree = Tree(nodes)
>>> tree[0]
(1, 2): 0.2
>>> tree[1]
(0, -1): 0.5
>>> tree[-1]
(0, -1): 0.5
```

As a Tree object is read-only, we cannot change individual nodes in a Tree object. However, we can convert the tree to a list of nodes, modify this list, and create a new tree from this list:

```
>>> tree = Tree([Node(1,2,0.1), Node(0,-1,0.5), Node(-2,3,0.9)])
>>> print tree
(1, 2): 0.1
(0, -1): 0.5
(-2, 3): 0.9
>>> nodes = tree[:]
>>> nodes[0] = Node(0,1,0.2)
>>> nodes[1].left = 2
>>> tree = Tree(nodes)
>>> print tree
(0, 1): 0.2
(2, -1): 0.5
(-2, 3): 0.9
```

This guarantees that any Tree object is always well-formed.

A Tree object has two methods (scale and cut, described below). The treecluster function returns Tree objects.

8.2.2 Performing hierarchical clustering: treecluster

```
tree = treecluster(data=None, mask=None, weight=None, transpose=0,
method='m', dist='e', distancematrix=None)
implements the hierachical clustering methods.
```

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise. Either data or distancematrix should be None.

mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

• weight

contains the weights to be used when calculating distances. If weight==None, then equal weights are assumed.

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• method

defines the linkage method to be used:

- method=='s': pairwise single-linkage clustering
- method=='m': pairwise maximum- (or complete-) linkage clustering
- method=='c': pairwise centroid-linkage clustering
- method=='a': pairwise average-linkage clustering

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

• distancematrix

The distance matrix. Either data or distancematrix should be None. If data is None and distancematrix is given, the arguments mask, weights, transpose, and dist are ignored. Note that pairwise single-, maximum-, and average-linkage clustering can be calculated from the distance matrix, but pairwise centroid-linkage cannot.

If not None, distancematrix should correspond to a distance matrix, which can be specified in three ways:

• as a 2D Numerical Python array (in which only the left-lower part of the array will be accessed):

```
distance = array([[0.0, 1.1, 2.3], [1.1, 0.0, 4.5], [2.3, 4.5, 0.0]])
```

• as a 1D Numerical Python array containing consecutively the distances in the left-lower part of the distance matrix:

```
distance = array([1.1, 2.3, 4.5])
```

• as a list containing the rows of the left-lower part of the distance matrix:

These three expressions correspond to the same distance matrix. As treecluster may shuffle the values in the distance matrix as part of the clustering algorithm, be sure to save this array in a different variable before calling treecluster if you need it later.

Return values

This function returns a Tree object. This object contains number of items - 1 nodes, where the number of items is the number of genes if genes were clustered, or the number of microarrays if microarrays were clustered. Each node describes a pairwise linking event, where the node attributes left and right each contain the number of one gene/microarray or subnode, and distance the distance between them. Genes/microarrays are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items - 1).

8.2.3 Scaling a hierarchical clustering tree: tree.scale

To display a hierarchical clustering solution with Java TreeView, it is better to scale all node distances such that they are between zero and one. This can be accomplished by calling the scale method on an existing Tree object.

tree.scale()

scales the distances in the hierarchical clustering tree such that they are all between zero and one. This function takes no arguments, and returns None.

8.2.4 Cutting a hierarchical clustering tree: tree.cut

clusterid = tree.cut(nclusters=1)

groups the items into nclusters clusters based on the tree structure generated by the hierarchical clustering routine treecluster.

Arguments

• tree

The Tree object tree contains the hierarchical clustering result generated by treecluster. Each node in this tree describes a pairwise linking event, where the node attributes left and right contain the number of the two items or subnodes. Items are numbered from 0 to (number of elements - 1), while clusters are numbered -1 to -(number of elements-1).

• nclusters

The desired number of clusters; nclusters should be positive, and less than or equal to the number of elements.

Return values

This function returns the array clusterid.

• clusterid

An array containing the number of the cluster to which each gene/microarray is assigned.

8.3 Self-Organizing Maps: somcluster

clusterid, celldata = somcluster(data, mask=None, weight=None, transpose=0,
nxgrid=2, nygrid=1, inittau=0.02, niter=1, dist='e')
implements a Self-Organizing Map on a rectangular grid.

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

• weight

contains the weights to be used when calculating distances. If weight==None, then equal weights are assumed.

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• nxgrid, nygrid

The number of cells horizontally and vertically in the rectangular grid, on which the Self-Organizing Map is calculated.

• inittau

The initial value for the parameter τ that is used in the SOM algorithm. The default value for inittau is 0.02, which was used in Michael Eisen's Cluster/TreeView program.

niter

The number of iterations to be performed.

• dist

defines the distance function to be used:

- dist=='c': correlation:
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This function returns the tuple clusterid, celldata.

clusterid

An array with two columns, where the number of rows is equal to the number of genes or the number of microarrays depending on whether genes or microarrays are being clustered. Each row contains the x and y coordinates of the cell in the rectangular SOM grid to which the gene or microarray was assigned.

• celldata

An array with dimensions (nxgrid, nygrid, number of microarrays) if genes are being clustered, or (nxgrid, nygrid, number of genes) if microarrays are being clustered.

Each element [ix][iy] of this array is a 1D vector containing the gene expression data for the centroid of the cluster in the grid cell with coordinates (ix,iy).

8.4 Finding the cluster centroids: clustercentroids

cdata, cmask = clustercentroids(data, mask=None, clusterid=None, method='a',
transpose=0)

calculates the cluster centroids.

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

• clusterid

Vector of integers showing to which cluster each element belongs. If clusterid is not given, then all elements are assumed to belong to the same cluster.

• method

Specifies whether the arithmetic mean (method=='a') or the median (method=='m') is used to calculate the cluster center.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the tuple cdata, cmask.

• cdata

A 2D array containing the centroid data. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered. Each row (if genes were clustered) or column (if microarrays were clustered) contains the averaged gene expression data for corresponding to the centroid of one cluster that was found.

• cmask

This matrix stores which values in cdata are missing. If cmask[i][j]==0, then cdata[i][j] is missing. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered.

8.5 The distance between two clusters: clusterdistance

distance = clusterdistance(data, mask=None, weight=None, index1=[0],
index2=[0], method='a', dist='e', transpose=0)
calculates the distance between two clusters.

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

• weight

contains the weights to be used when calculating distances. If weight==None, then equal weights are assumed.

• index1

is a list containing the indices of the elements belonging to the first cluster. A cluster containing only one element i can be represented as either a list [i], or as an integer i.

• index2

is a list containing the indices of the elements belonging to the second cluster. A cluster containing only one element i can be represented as either a list [i], or as an integer i.

method

Specifies how the distance between clusters is defined:

- method=='a': Distance between the two cluster centroids (arithmetic mean);
- method=='m': Distance between the two cluster centroids (median);
- method=='s': Shortest pairwise distance between elements in the two clusters;
- method=='x': Longest pairwise distance between elements in the two clusters;
- method=='v': Average over the pairwise distances between elements in the two clusters.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the distance between the two clusters.

8.6 Calculating the distance matrix: distancematrix

matrix = distancematrix(data, mask=None, weight=None, transpose=0, dist='e')
returns the distance matrix between gene expression data.

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

mask

Array of integers showing which data are missing. If mask[i][j]==0, then data[i][j] is missing. If mask==None, then there are no missing data.

weight

contains the weights to be used when calculating distances. If weight==None, then equal weights are assumed.

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

• matrix is a list of 1D arrays containing the distance matrix between the gene expression data. The number of columns in each row is equal to the row number. Hence, the first row has zero elements. An example of the return value is

This corresponds to the distance matrix

$$\begin{pmatrix} 0 & 1 & 7 & 4 \\ 1 & 0 & 3 & 2 \\ 7 & 3 & 0 & 6 \\ 4 & 2 & 6 & 0 \end{pmatrix}$$

8.7 Handling Cluster/TreeView-type files

Cluster/TreeView are GUI-based codes for clustering gene expression data. They were originally written by Michael Eisen (http://rana.lbl.gov) while at Stanford University. Pycluster contains routines for reading and writing data files that correspond to the format specified for Cluster/TreeView. In particular, by saving a clustering result in that format, TreeView can be used to visualize the clustering results. We recommend using Alok Saldanha's Java TreeView program (http://jtreeview.sourceforge.net/), which can display hierarchical as well as k-means clustering results.

8.7.1 The Record class

An object of the class Record contains all information stored in a Cluster/TreeView-type data file. To store the information contained in the data file in a Record object, we first open the file and then read it:

```
>>> import Pycluster
>>> handle = open("mydatafile.txt")
>>> record = Pycluster.read(handle)
This two-step process gives you some flexibility in the source of the data. For example, you can use
>>> import gzip # Python standard library
>>> handle = gzip.open("mydatafile.txt.gz")
to open a gzipped file, or
>>> import urllib # Python standard library
>>> handle = urllib.urlopen("http://somewhere.org/mydatafile.txt")
to open a file stored on the Internet before calling read.
    If you're using the C Clustering Library from Biopython, you should use:
>>> from Bio import Cluster
>>> record = Cluster.read(handle)
```

The read command reads the tab-delimited text file mydatafile.txt containing gene expression data in $_{
m the}$ format specified for Michael Cluster/TreeView program. For a description of this file format, manual to Cluster/TreeView. It is available at Michael Eisen's lab website (http://rana.lbl.gov/manuals/ClusterTreeView.pdf) and (http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster/cluster3.pdf).

A Record object stores the following information:

• data

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise.

mask

This array shows which elements in the data array, if any, are missing. If mask[i,j]==0, then data[i,j] is missing. If no data were found to be missing, mask is set to None.

• geneid

This is a list containing a unique description for each gene (i.e., ORF numbers).

• genename

This is a list containing a description for each gene (i.e., gene name). If not present in the data file, genename is set to None.

• gweight

The weights that are to be used to calculate the distance in expression profile between genes. If not present in the data file, gweight is set to None.

• gorder

The preferred order in which genes should be stored in an output file. If not present in the data file, gorder is set to None.

• expid

This is a list containing a description of each microarray, e.g. experimental condition.

• eweight

The weights that are to be used to calculate the distance in expression profile between microarrays. If not present in the data file, eweight is set to None.

• eorder

The preferred order in which microarrays should be stored in an output file. If not present in the data file, eorder is set to None.

• uniqid

The string that was used instead of UNIQID in the data file.

After loading a Record object, each of these attributes can be accessed and modified directly. For example, the data can be log-transformed by taking the logarithm of record.data.

8.7.2 Performing hierarchical clustering

tree = record.treecluster(transpose=0, method='m', dist='e')
applies hierarchical clustering to the data contained in the Record object.

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• method

defines the linkage method to be used:

- method=='s': pairwise single-linkage clustering
- method=='m': pairwise maximum- (or complete-) linkage clustering
- method=='c': pairwise centroid-linkage clustering
- method=='a': pairwise average-linkage clustering

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;

- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This method returns a Tree object containing number of items - 1 nodes, where the number of items is the number of genes if genes were clustered, or the number of microarrays if microarrays were clustered. Each node describes a pairwise linking event, where the node attributes left and right each contain the number of one gene/microarray or subnode, and distance the distance between them. Genes/microarrays are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items-1).

8.7.3 Performing k-means or k-medians clustering

clusterid, error, nfound = record.kcluster(nclusters=2, transpose=0, npass=1, method='a', dist='e', initialid=None) applies k-means or k-medians clustering to the data contained in the Record object.

Arguments

• nclusters

The number of clusters k.

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

npass

The number of times the k-means clustering algorithm is performed, each time with a different (random) initial condition. If **initialid** is given, the value of **npass** is ignored and the clustering algorithm is run only once, as it behaves deterministically in that case.

method

describes how the center of a cluster is found:

- method=='a': arithmetic mean;
- method=='m': median.

For other values of method, the arithmetic mean is used.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;

• dist=='b': City-block distance.

• initialid

Specifies the initial clustering to be used for the EM algorithm. If initialid==None, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is not None, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns a tuple (clusterid, error, nfound).

clusterid

An array containing the number of the cluster to which each gene/microarray was assigned.

• error

The within-cluster sum of distances for the optimal clustering solution.

• nfound

The number of times the optimal solution was found.

8.7.4 Calculating a Self-Organizing Map

clusterid, celldata = record.somcluster(transpose=0, nxgrid=2, nygrid=1,
inittau=0.02, niter=1, dist='e')

calculates a Self-Organizing Map on a rectangular grid, using the gene expression data in the Record object record.

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• nxgrid, nygrid

The number of cells horizontally and vertically in the rectangular grid, on which the Self-Organizing Map is calculated.

• inittau

The initial value for the parameter τ that is used in the SOM algorithm. The default value for inittau is 0.02, which was used in Michael Eisen's Cluster/TreeView program.

niter

The number of iterations to be performed.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;

- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This function returns the tuple clusterid, celldata.

• clusterid

An array with two columns, where the number of rows is equal to the number of genes or the number of microarrays depending on whether genes or microarrays are being clustered. Each row contains the x and y coordinates of the cell in the rectangular SOM grid to which the gene or microarray was assigned.

• celldata

An array with dimensions (nxgrid, nygrid, number of microarrays) if genes are being clustered, or (nxgrid, nygrid, number of genes) if microarrays are being clustered. Each element [ix] [iy] of this array is a 1D vector containing the gene expression data for the centroid of the cluster in the grid cell with coordinates (ix,iy).

8.7.5 Finding the cluster centroid

cdata, cmask = record.clustercentroids(clusterid=None, method='a',
transpose=0)
calculates the cluster centroids.

Arguments

• clusterid

Vector of integers showing to which cluster each element belongs. If clusterid is not given, then all elements are assumed to belong to the same cluster.

• method

Specifies whether the arithmetic mean (method=='a') or the median (method=='m') is used to calculate the cluster center.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the tuple cdata, cmask.

• cdata

A 2D array containing the centroid data. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered. Each row (if genes were clustered) or column (if microarrays were clustered) contains the averaged gene expression data for corresponding to the centroid of one cluster that was found.

• cmask

This matrix stores which values in cdata are missing. If cmask[i][j]==0, then

cdata[i][j] is missing. The dimensions of this array are (number of clusters,
number of microarrays) if genes were clustered, or (number of genes, number of
clusters) if microarrays were clustered.

8.7.6 Calculating the distance between two clusters

distance = record.clusterdistance(index1=[0], index2=[0], method='a',
dist='e', transpose=0)
calculates the distance between two clusters.

Arguments

• index1

is a list containing the indices of the elements belonging to the first cluster. A cluster containing only one element i can be represented as either a list [i], or as an integer i.

index2

is a list containing the indices of the elements belonging to the second cluster. A cluster containing only one element i can be represented as either a list [i], or as an integer i.

• method

Specifies how the distance between clusters is defined:

- method=='a': Distance between the two cluster centroids (arithmetic mean);
- method=='m': Distance between the two cluster centroids (median);
- method=='s': Shortest pairwise distance between elements in the two clusters;
- method=='x': Longest pairwise distance between elements in the two clusters;
- method=='v': Average over the pairwise distances between elements in the two clusters.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the distance between the two clusters.

8.7.7 Calculating the distance matrix

matrix = record.distancematrix(transpose=0, dist='e')
returns the distance matrix between gene expression data.

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance:
- dist=='b': City-block distance.

Return values

• matrix is a list of 1D arrays containing the distance matrix between the gene expression data. The number of columns in each row is equal to the row number. Hence, the first row has zero elements. An example of the return value is

This corresponds to the distance matrix

$$\begin{pmatrix}
0 & 1 & 7 & 4 \\
1 & 0 & 3 & 2 \\
7 & 3 & 0 & 6 \\
4 & 2 & 6 & 0
\end{pmatrix}$$

8.7.8 Saving the clustering result

record.save(jobname, geneclusters, expclusters)

writes the text file jobname.cdt, jobname.gtr, jobname.atr, jobname*.kgg, and/or jobname*.kag for subsequent reading by the Java TreeView program. If geneclusters and expclusters are both None, this method only writes the text file jobname.cdt; this file can subsequently be read into a new Record object.

Arguments

• jobname

The string jobname is used as the base name for names of the files that are to be saved.

• geneclusters

This argument describes the gene clustering result. In case of k-means clustering, this is a 1D array containing the number of the cluster each gene belongs to. It can be calculated using kcluster. In case of hierarchical clustering, geneclusters is a Tree object.

• expclusters

This argument describes the clustering result for the experimental conditions. In case of k-means clustering, this is a 1D array containing the number of the cluster each experimental condition belongs to. It can be calculated using kcluster. In case of hierarchical clustering, expclusters is a Tree object.

8.7.9 Example calculation

This is an example of a hierarchical clustering calculation, using single linkage clustering for genes and maximum linkage clustering for experimental conditions. As the Euclidean distance is being used for gene clustering, it is necessary to scale the node distances <code>genetree</code> such that they are all between zero and one. This is needed for the Java TreeView code to display the tree diagram correctly. To cluster the experimental conditions, the uncentered correlation is being used. No scaling is needed in this case, as the distances in <code>exptree</code> are already between zero and two. The example data <code>cyano.txt</code> can be found in the <code>data</code> subdirectory.

```
>>> from Pycluster import *
>>> handle = open("cyano.txt")
>>> record = read(handle)
>>> genetree = record.treecluster(method='s')
>>> genetree.scale()
>>> exptree = record.treecluster(dist='u', transpose=1)
>>> record.save("cyano_result", genetree, exptree)
This will create the files cyano_result.cdt, cyano_result.gtr, and cyano_result.atr.
  Similarly, we can save a k-means clustering solution:
>>> from Pycluster import *
>>> handle = open("cyano.txt")
>>> record = read(handle)
>>> (geneclusters, error, ifound) = record.kcluster(nclusters=5, npass=1000)
>>> (expclusters, error, ifound) = record.kcluster(nclusters=2, npass=100,
transpose=1)
>>> record.save("cyano_result", geneclusters, expclusters)
This will create the files cyano_result_K_G2_A2.cdt, cyano_result_K_G2.kgg, and
cyano_result_K_A2.kag.
```

8.8 Auxiliary routines

```
median(data) returns the median of the 1D array data.

mean(data) returns the mean of the 1D array data.
```

version()

returns the version number of the C Clustering Library as a string.

9 Using the C Clustering Library with Perl: Algorithm::Cluster

Algorithm::Cluster is a Perl wrapper extension of the C Clustering Library written by John Nolan of the University of California, Santa Cruz.

Algorithm::Cluster requires Perl 5.6.0 at a minimum. It will not compile properly with 5.005_03 or previous versions or Perl. It has been tested on Win32, Mac OS X, Linux, OpenBSD and Solaris.

To install Algorithm::Cluster on UNIX, Linux, Mac OS X, or Cygwin, you can download the source code from http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster. You will need an ANSI C compiler; GNU's free gcc compiler will do. Unpack the distribution and type the following familiar commands to compile, test and install the module: perl Makefile.PL

make

make test

make install

You can also use the CPAN shell from within Perl to download the module and install it.

If you use ActiveState Perl on Windows, use the Perl Package Manager ppm to install Algorithm::Cluster by executing the following command from the DOS command prompt.

ppm install http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster/AlgorithmCluster.ppd

Algorithm::Cluster offers the following functions:

- kcluster
- kmedoids
- treecluster
- somcluster
- clustercentroids
- clusterdistance
- distancematrix
- mean
- median

You can also use Algorithm::Cluster to read expression data files and to write the clustering solution to a file; see below for details.

9.1 Using named parameters

Most of the interface functions in Algorithm::Cluster expect named parameters. This is implemented by passing a hash as a parameter to the function. For example, if a Perl function sign_up() accepts three named parameters, name, program and class, then you can invoke the function like this:

```
'program' => 'Biology',
'class' => 'Intro to Molecular Biology',
);
```

When the function parses its parameters, it will create a hash, on the fly, with three keys. The function can access the values by referring to the hash.

This is convenient for several reasons. First, it means that you can pass the parameters in any order. Both invocations below are valid:

```
$return_value = sign _up(
    'class' => 'Intro to Molecular Biology',
    'name' => 'Ms. Jones',
    'program' => 'Biology',
);
$return_value = sign _up(
    'name' => 'Miss Chen',
    'program' => 'Biology',
    'class' => 'Intro to Molecular Biology',
);
```

If the function defines default values for parameters, you can also leave some parameters out, and the function will still know which parameter is which:

You can define the hash on your own, and pass this to the function. This is useful if your function accepts a long list of parameters, and you intend to call it several times, (mostly) reusing the same values. You can implement your own defaults:

9.2 References to arrays, and two-dimensional matrices

Perl implements two-dimensional matrices using references. For example, a reference to a one-dimensional array (a row) can be defined like this:

```
$row = [1, 2];
```

In this example, \$row itself is not an array, but a reference to the array (1, 2). (The square brackets indicate that we want to create a reference.) \$row->[0] equals 1 and \$row->[1] equals 2. A 3×2 matrix of integers can be defined like this:

```
$row0 = [ 1, 2 ];
$row1 = [ 3, 4 ];
$row2 = [ 5, 6 ];
$data = [ $row0, $row1, $row2 ];

Or, more succinctly:

$data = [
       [ 1, 2 ],
       [ 3, 4 ],
       [ 5, 6 ],
];
```

In this example, \$\data->[0]->[1] equals 2, while \$\data->[2]->[0] equals 5.

Many of the functions available in Algorithm::Cluster expect data to be in the form of a two dimensional array, like the example above. Some functions also return references to data structures like this.

9.3 Partitioning algorithms

9.3.1 The k-means clustering algorithm: kcluster

The function kcluster() implements the k-means clustering algorithm. In the example invocation below, we have created \$param{data} as an empty matrix, because that is the default value, but you must populate \$param{data} with real data, in order to invoke kcluster() properly.

Arguments

• data

A reference to a two-dimensional matrix containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• nclusters

The number of clusters k.

mask

A reference to a two-dimensional matrix of integers showing which data are missing. If \$param{mask}->[i]->[j]==0, then \$param{data}->[i]->[j] is missing. If mask is '' (i.e., the null string, and not a reference at all), then there are no missing data.

• weight

A reference to an array containing the weights to be used when calculating distances. If weight equals '' (i.e., the null string, and not a reference at all), then equal weights are assumed. If transpose==0, the length of this array must equal the number of columns in the data matrix. If transpose==1, the length of the weight array should equal the number of rows in the data matrix. If weight has a different length, the entire array will be ignored.

• transpose

Determines if genes or microarrays are being clustered. If \$param{transpose}==0, genes (rows) are being clustered. If \$param{transpose}==1, microarrays (columns) are clustered.

• npass

The number of times the k-means clustering algorithm is performed, each time with a different (random) initial condition. If the argument initialid is given, the value of npass is ignored and the clustering algorithm is run only once, as it behaves deterministically in that case.

method

A one-character flag, indicating how the center of a cluster is found:

- 'a': arithmetic mean
- 'm': median

For any other values of method, the arithmetic mean is used.

• dist

A one-character flag, defining the distance function to be used:

- 'c': correlation
- 'a': absolute value of the correlation
- 'u': uncentered correlation
- 'x': absolute uncentered correlation
- 's': Spearman's rank correlation
- 'k': Kendall's τ ;
- 'e': Euclidean distance
- 'b': City-block distance

For other values of dist, the default (Euclidean distance) is used.

• initialid

An optional parameter defining the initial clustering to be used for the EM algorithm. If initialid is not specified, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is specified, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns a list of three items: \$clusterid, \$error, \$nfound.

• \$clusterid

A reference to an array whose length is equal to the number of rows in the data array. Each element in the *clusterid* array contains the number of the cluster to which each gene/microarray was assigned.

• \$error

The within-cluster sum of distances of the optimal clustering solution that was found.

\$nfound

The number of times the optimal solution was found.

9.3.2 The k-medoids algorithm: kmedoids

The function kmedoids() implements the k-means clustering algorithm. In the example invocation below, we have created the distance matrix \$param{distances} as an empty matrix, because that is the default value, but you must populate \$param{distances} with real data, in order to invoke kmedoids() properly.

Arguments

• nclusters

The number of clusters k.

distances

A list containing the distance matrix between the elements. An example of a distance matrix is:

```
$distances = [[], [1.1], [1.0, 4.5], [2.3, 1.8, 6.1]];
```

• npass

The number of times the k-medoids clustering algorithm is performed, each time with a different (random) initial condition. If initialid is given, the value of npass is ignored, as the clustering algorithm behaves deterministically in that case.

• initialid

Specifies the initial clustering to be used for the EM algorithm. If initialid is not specified, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is specified, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns a list of three items: \$clusterid, \$error, \$nfound.

• \$clusterid

\$clusterid is a reference to an array whose length is equal to the number of rows of the distance matrix. Each element in the clusterid array contains the number of the cluster to which each gene/microarray was assigned. The cluster number is defined as the number of the gene/microarray that is the centroid of the cluster.

• \$error

\$error is the within-cluster sum of distances of the optimal clustering solution that was found.

\$nfound

\$nfound is the number of times the optimal solution was found.

9.4 Hierarchical clustering: treecluster

The pairwise single-, maximum-, average-, and centroid-linkage clustering methods are accessible through the function treecluster. The hierarchical clustering routines can be applied either on the original gene expression data, or (except for centroid-linkage clustering) on the distance matrix directly. The tree structure generated by treecluster can be cut in order to separate the elements into a given number of clusters.

9.4.1 Representing a hierarchical clustering solution

A hierarchical clustering solution is represented using two Perl classes: Node and Tree.

The class Node

The Perl class Node corresponds to the C struct Node described above. A Node object has three attributes:

- left
- right
- distance

Here, left and right are integers referring to the two items or subnodes that are joined at this node, and distance is the distance between them. The items being clustered are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items-1).

To create a new Node object, we need to specify left, right, and distance:

```
use Algorithm::Cluster;
my $node = Algorithm::Cluster::Node->new(3, 4, 2.7);
```

```
print "Left:", $node->left, "\n";
print "Right:", $node->right, "\n";
print "Distance:", $node->distance, "\n";

Executing this code prints

Left:3
Right:4
Distance:2.7

The attributes left, right, and distance of an existing Node object can be modified by using the set_left, set_right, and set_distance methods:
use Algorithm::Cluster;
my $node = Algorithm::Cluster::Node->new(3, 4, 2.7);
$node->set_left(2);
$node->set_right(1);
$node->set_distance(2.1);
```

The class Tree

The Perl class Tree represents a full hierarchical clustering solution. A Tree object can be created from an array of Node objects:

```
use Algorithm::Cluster;
my $node1 = Algorithm::Cluster::Node->new(1,2,3.1);
my $node2 = Algorithm::Cluster::Node->new(-1,3,5.3);
my $node3 = Algorithm::Cluster::Node->new(4,0,5.9);
my $node4 = Algorithm::Cluster::Node->new(-2,-3,7.8);
my @nodes = [$node1,$node2,$node3,$node4];
my $tree = Algorithm::Cluster::Tree->new(@nodes);

The Tree initializer checks if the list of nodes is a valid hierarchical clustering result:
use Algorithm::Cluster;
my $node1 = Algorithm::Cluster::Node->new(1,2,0.3);
my $node2 = Algorithm::Cluster::Node->new(1,3,0.7);
my @nodes = [$node1,$node2];
my $tree = Algorithm::Cluster::Tree->new(@nodes);
```

results in the following error message:

the array of nodes passed to Algorithm::Cluster::Tree::new do not represent a valid tree

Individual nodes in a Tree object can be accessed using the get method. Continuing the previous example, the following code prints 0.7:

```
my $node3 = $tree->get(1);
print $node3->distance;
```

As a Tree object is read-only, we cannot change individual nodes in a Tree object. However, we can convert the tree to a list of nodes, modify this list, and create a new tree from this list:

```
use Algorithm::Cluster;
my $node1 = Algorithm::Cluster::Node->new(0,1,0.3);
my $node2 = Algorithm::Cluster::Node->new(2,3,0.7);
my $node3 = Algorithm::Cluster::Node->new(-1,-2,0.9);
my $nodes = [$node1, $node2, $node3];
my $tree = Algorithm::Cluster::Tree->new($nodes);
my $i;
my $n = $tree->length;
my $node;
my \quad podes2 = [];
for ($i = 0; $i < $n; $i++) {
        $nodes2->[$i] = $tree->get($i);
}
node = nodes2 -> [1];
$node->set_left(-1);
node = nodes2 -> [2];
$node->set_left(2);
my $tree2 = Algorithm::Cluster::Tree->new($nodes2);
```

This guarantees that any Tree object is always well-formed.

A Tree object has two methods (scale and cut, described below). The treecluster function returns Tree objects.

9.4.2 Performing hierarchical clustering: treecluster

The pairwise single-, maximum-, average-, and centroid-linkage clustering methods are accessible through the function treecluster.

Arguments

• data

A reference to a two-dimensional matrix containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise. It is also possible to pass a user-defined distance matrix as a lower-diagonal matrix, for example:

If the data argument has this form, it will be interpreted as a distance matrix instead of a raw data matrix, and the arguments mask, weight, transpose, and dist will be ignored.

• mask

A referene to a two-dimensional matrix of integers showing which data are missing. If \$param{mask}->[i]->[j]==0, then \$param{data}->[i]->[j] is missing. If mask is '' (i.e., the null string, and not a reference at all), then there are no missing data.

• weight

A reference to an array containing the weights to be used when calculating distances. If weight equals '' (i.e., the null string, and not a reference at all), then equal weights are assumed. If transpose==0, the length of this array must equal the number of columns in the data matrix. If transpose==1, the length of the weight array should equal the number of rows in the data matrix. If weight has a different length, the entire array will be ignored.

• transpose

Determines if genes or microarrays are being clustered. If \$param{transpose}==0, genes (rows) are being clustered. If \$param{transpose}==1, microarrays (columns) are clustered.

• dist

A one-character flag, defining the distance function to be used:

- 'c': correlation
- 'a': absolute value of the correlation
- 'u': uncentered correlation
- 'x': absolute uncentered correlation
- 's': Spearman's rank correlation
- 'k': Kendall's tau
- 'e': Euclidean distance
- 'b': City-block distance

• method

Specifies which type of hierarchical clustering is used:

- 's': pairwise single-linkage clustering
- 'm': pairwise maximum- (or complete-) linkage clustering
- 'a': pairwise average-linkage clustering
- 'c': pairwise centroid-linkage clustering

Pairwise centroid-linkage clustering is not available if a user-defined distance matrix is passed via data.

Return values

This function returns a Tree object. This object contains number of items - 1 nodes, where the number of items is the number of genes if genes were clustered, or the number of microarrays if microarrays were clustered. Each node describes a pairwise linking event, where the node attributes left and right each contain the number of one gene/microarray or subnode, and distance the distance between them. Genes/microarrays are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items - 1).

9.4.3 Scaling a hierarchical clustering tree: \$tree->scale

To display a hierarchical clustering solution with Java TreeView, it is better to scale all node distances such that they are between zero and one. This can be accomplished by calling the scale method on an existing Tree object.

\$tree->scale

scales the distances in the hierarchical clustering tree such that they are all between zero and one. This function takes no arguments.

9.4.4 Cutting a hierarchical clustering tree: \$tree.cut

```
$clusterid = $tree->cut($nclusters)
```

groups the items into *\$nclusters* clusters based on the tree structure generated by the hierarchical clustering routine treecluster.

Arguments

• \$tree

The Tree object *\$tree* contains the hierarchical clustering result generated by treecluster. Each node in this tree describes a pairwise linking event, where the node attributes left and right contain the number of the two items or subnodes. Items are numbered from 0 to (number of elements - 1), while clusters are numbered -1 to -(number of elements-1).

• \$nclusters

The desired number of clusters; *\$nclusters* should be positive, and less than or equal to the number of elements.

Return values

This function returns the anonymous array \$clusterid.

• \$clusterid

An anonymous array containing the number of the cluster to which each gene/microarray is assigned.

Example

```
use Algorithm::Cluster;
my $node1 = Algorithm::Cluster::Node->new(0,1,0.3);
my $node2 = Algorithm::Cluster::Node->new(2,3,0.7);
my $node3 = Algorithm::Cluster::Node->new(-1,-2,0.9);
my $nodes = [$node1, $node2, $node3];
```

9.5 Self-Organizing Maps: somcluster

The somcluster() function implements a Self-Organizing Map on a rectangular grid.

Arguments

• data

A reference to a two-dimensional matrix containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

mask

A reference to a two-dimensional matrix of integers showing which data are missing. If \$param{mask}->[i]->[j]==0, then \$param{data}->[i]->[j] is missing. If mask is '', (i.e., the null string, and not a reference at all), then there are no missing data.

• weight

A reference to an array containing the weights to be used when calculating distances. If weight equals '' (i.e., the null string, and not a reference at all), then equal weights are assumed. If transpose==0, the length of this array must equal the number of columns in the data matrix. If transpose==1, the length of the weight array should equal the number of rows in the data matrix. If weight has a different length, the entire array will be ignored.

• transpose

Determines if genes or microarrays are being clustered. If \$param{transpose}==0,

genes (rows) are being clustered. If \$param{transpose}==1, microarrays (columns) are clustered.

• nxgrid, nygrid

Both parameters are integers, indicating the number of cells horizontally and vertically in the rectangular grid, on which the Self-Organizing Map is calculated.

• inittau

The initial value for the neighborhood function, as given by the parameter τ . The default value for inittau is 0.02, which was used in Michael Eisen's Cluster/TreeView program.

• niter

The number of iterations to be performed.

• dist

A one-character flag, defining the distance function to be used:

- 'c': correlation
- 'a': absolute value of the correlation
- 'u': uncentered correlation
- 'x': absolute uncentered correlation
- 's': Spearman's rank correlation
- 'k': Kendall's tau
- 'e': Euclidean distance
- 'b': City-block distance

Return values

This function returns one value, \$clusterid\$, which is a reference to a two-dimensional matrix. If \$param{transpose}==0\$, then the number of rows in \$clusterid\$ equals the number of rows (genes) in the original data array; if \$param{transpose}==1\$, then then the number of rows in \$clusterid\$ equals the number of columns (microarrays) in the original data array. Each row in the array clusterid contains the x and y coordinates of the cell in the rectangular SOM grid to which the gene or microarray was assigned.

9.6 Finding the cluster centroids: clustercentroids

Arguments

• data

Array containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

mask

Array of integers showing which data are missing. If \$param{mask}->[\$i]->[\$j]==0, then \$param{data}->[\$i]->[\$j] is missing. If mask is '', then there are no missing data.

• clusterid

Vector of integers showing to which cluster each element belongs. If clusterid is not given, then all elements are assumed to belong to the same cluster.

• method

Specifies whether the arithmetic mean (method=>'a') or the median (method=>'m') is used to calculate the cluster center.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the list cdata, cmask.

• \$cdata

A 2D array containing the centroid data. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered. Each row (if genes were clustered) or column (if microarrays were clustered) contains the averaged gene expression data for corresponding to the centroid of one cluster that was found.

• \$cmask

This matrix stores which values in \$cdata are missing. If \$cmask[\$i][\$j]==0, then \$cdata[\$i][\$j] is missing. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered.

9.7 The distance between two clusters: clusterdistance

The clusterdistance routine calculates the distance between two clusters, between a cluster and an item, or between two items.

my (\$distance) = Algorithm::Cluster::clusterdistance(%param);

Arguments

• data

A reference to a two-dimensional matrix containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• mask

A reference to a two-dimensional matrix of integers showing which data are missing. If \$param{mask}->[i]->[j]==0, then \$param{data}->[i]->[j] is missing. If mask is '' (i.e., the null string, and not a reference at all), then there are no missing data.

• weight

A reference to an array containing the weights to be used when calculating distances. If weight equals '' (i.e., the null string, and not a reference at all), then equal weights are assumed. If transpose==0, the length of this array must equal the number of columns in the data matrix. If transpose==1, the length of the weight array should equal the number of rows in the data matrix. If weight has a different length, the entire array will be ignored.

• cluster1

contains the indices of the elements belonging to the first cluster, or alternatively an integer to refer to a single item.

• cluster2

contains the indices of the elements belonging to the second cluster, or alternatively an integer to refer to a single item.

• dist

A one-character flag, defining the distance function to be used:

- 'c': correlation
- 'a': absolute value of the correlation
- 'u': uncentered correlation
- 'x': absolute uncentered correlation
- 's': Spearman's rank correlation
- 'k': Kendall's tau
- 'e': Euclidean distance
- 'b': City-block distance

• method

A one-character flag, indicating how the center of a cluster is found:

- 'a': Distance between the two cluster centroids (arithmetic mean)
- 'm': Distance between the two cluster centroids (median)
- 's': Shortest distance between elements in the two clusters
- 'x': Longest pairwise distance between elements in the two clusters
- 'v': Average over the pairwise distances between elements in the two clusters.

For any other values of method, the arithmetic mean is used.

• transpose

Determines if the distance between genes or between microarrays should be calculated. If \$param{transpose}==0, the function calculates the distance between the genes (rows) specified by \$param{cluster1} and \$param{cluster2}. If \$param{transpose}==1, the function calculates the distance between the microarrays (columns) specified by \$param{cluster1} and \$param{cluster2}.

Return values

The distance between the clusters indicated by cluster1 and cluster2.

9.8 Calculating the distance matrix: distancematrix

The function distancematrix() calculates the distance matrix between the gene expression data and returns it as a ragged array.

Arguments

• data

A reference to a two-dimensional matrix containing the gene expression data, where genes are stored row-wise and microarray experiments column-wise.

• mask

A reference to a two-dimensional matrix of integers showing which data are missing. If \$param{mask}->[i]->[j]==0, then \$param{data}->[i]->[j] is missing. If mask is '' (i.e., the null string, and not a reference at all), then there are no missing data.

• weight

A reference to an array containing the weights to be used when calculating distances. If weight equals '' (i.e., the null string, and not a reference at all), then equal weights are assumed. If transpose==0, the length of this array must equal the number of columns in the data matrix. If transpose==1, the length of the weight array should equal the number of rows in the data matrix. If weight has a different length, the entire array will be ignored.

• transpose

Determines if the distances between genes or microarrays should be calculated. If \$param{transpose}==0, the distances between genes (rows) are calculated. If \$param{transpose}==1, the distances are calculated between microarrays (columns).

• dist

A one-character flag, defining the distance function to be used:

- 'c': correlation
- 'a': absolute value of the correlation

- 'u': uncentered correlation
- 'x': absolute uncentered correlation
- 's': Spearman's rank correlation
- 'k': Kendall's τ ;
- 'e': Euclidean distance
- 'b': City-block distance For other values of dist, the default (Euclidean distance) is used.

Return values

This function returns the \$distancematrix, an array of rows containing the distance matrix between the gene expression data. The number of columns in each row is equal to the row number. Hence, the first row has zero elements.

9.9 Auxiliary routines

```
median($data)
```

Returns the median of the data. \$data is a reference to a (one-dimensional) array of numbers.

mean(\$data)

Returns the mean of the data. \$data is a reference to a (one-dimensional) array of numbers. version()

Returns the version number of the C Clustering Library as a string.

9.10 Handling Cluster/TreeView-type files

Cluster/TreeView are GUI-based codes for clustering gene expression data. They were originlly written by Michael Eisen (http://rana.lbl.gov) while at Stanford University. Algorithm::Cluster contains routines for reading and writing data files in the format specifies for Cluster/TreeView. In particular, by saving a clustering result in that format, we can use TreeView to visualize the clustering results. We recommend using Alok Saldanha's Java TreeView program (http://jtreeview.sourceforget.net), which can display hierarchical as well as k-means clustering clustering results.

9.10.1 The Record class

An object of the class Record contains all information stored in a Cluster/TreeView-type data file. To store the information contained in the data file in a Record object, we create a Record object, open the file, and then read the file contents into the Record object:

```
use Algorithm::Cluster::Record;
my $record = Algorithm::Cluster::Record->new();
open INPUT, "mydatafile.txt";
$record->read(*INPUT);
```

The read command reads the tab-delimited text file mydatafile.txt containing gene expression data in the format specified for Michael Eisen's

Cluster/TreeView program. For a description of this file format, see the manual to Cluster/TreeView. It is available at Michael Eisen's lab website (http://rana.lbl.gov/manuals/ClusterTreeView.pdf) and at our website (http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster/cluster3.pdf).

A Record object stores the following information:

data

The data array containing the gene expression data. Genes are stored row-wise, while microarrays are stored column-wise.

mask

This array shows which elements in the data array, if any, are missing. If \$record{mask}[\$i][\$j]==0, then \$record{data}[\$i][\$j] is missing. If no data were found to be missing, \$record{mask} remains undef.

• geneid

This is a list containing a unique description for each gene (i.e., ORF numbers).

• genename

This is a list containing a description for each gene (i.e., gene name). If not present in the data file, \$record{genename} remains undef.

• gweight

The weights that are to be used to calculate the distance in expression profile between genes. If not present in the data file, \$record{gweight} remains undef.

• gorder

The preferred order in which genes should be stored in an output file. If not present in the data file, <code>\$record{gorder}</code> remains undef.

expid

This is a list containing a description of each microarray, e.g. experimental condition.

• eweight

The weights that are to be used to calculate the distance in expression profile between microarrays. If not present in the data file, \$record{eweight} remains undef.

• eorder

The preferred order in which microarrays should be stored in an output file. If not present in the data file, \$record{eorder} remains undef.

uniqid

The string that was used instead of UNIQID in the data file.

After loading a Record object, each of these attributes can be accessed and modified directly. For example, the data can be log-transformed by taking the logarithm of <code>record->data</code>.

9.10.2 Performing hierarchical clustering

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• method

defines the linkage method to be used:

- method=='s': pairwise single-linkage clustering
- method=='m': pairwise maximum- (or complete-) linkage clustering
- method=='c': pairwise centroid-linkage clustering
- method=='a': pairwise average-linkage clustering
- dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This function returns a Tree object. This object contains number of items - 1 nodes, where the number of items is the number of genes if genes were clustered, or the number of microarrays if microarrays were clustered. Each node describes a pairwise linking event, where the node attributes left and right each contain the number of one gene/microarray or subnode, and distance the distance between them. Genes/microarrays are numbered from 0 to (number of items - 1), while clusters are numbered -1 to -(number of items - 1).

9.10.3 Performing k-means or k-medians clustering

Arguments

• nclusters

The number of clusters k.

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• npass

The number of times the k-means clustering algorithm is performed, each time with a different (random) initial condition. If initialid is given, the value of npass is ignored and the clustering algorithm is run only once, as it behaves deterministically in that case.

method

describes how the center of a cluster is found:

- method=='a': arithmetic mean;
- method=='m': median.

For other values of method, the arithmetic mean is used.

dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

• initialid

Specifies the initial clustering to be used for the EM algorithm. If initialid==None, then a different random initial clustering is used for each of the npass runs of the EM algorithm. If initialid is not None, then it should be equal to a 1D array containing the cluster number (between 0 and nclusters-1) for each item. Each cluster should contain at least one item. With the initial clustering specified, the EM algorithm is deterministic.

Return values

This function returns three variables (\$clusterid, \$error, \$nfound).

• \$clusterid

An array containing the number of the cluster to which each gene/microarray was assigned.

• \$error

The within-cluster sum of distances for the optimal clustering solution.

• \$nfound

The number of times the optimal solution was found.

9.10.4 Calculating a Self-Organizing Map

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• nxgrid, nygrid

The number of cells horizontally and vertically in the rectangular grid, on which the Self-Organizing Map is calculated.

• inittau

The initial value for the parameter τ that is used in the SOM algorithm. The default value for inittau is 0.02, which was used in Michael Eisen's Cluster/TreeView program.

• niter

The number of iterations to be performed.

• dist

defines the distance function to be used:

- dist=='c': correlation;
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This function returns an array with two columns, where the number of rows is equal to the number of genes or the number of microarrays depending on whether genes or microarrays are being clustered. Each row contains the x and y coordinates of the cell in the rectangular SOM grid to which the gene or microarray was assigned.

9.10.5 Finding the cluster centroid

Arguments

• clusterid

Vector of integers showing to which cluster each element belongs. If clusterid is not given, then all elements are assumed to belong to the same cluster.

method

Specifies whether the arithmetic mean (method=='a') or the median (method=='m') is used to calculate the cluster center.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the list \$cdata, \$cmask.

• \$cdata

A 2D array containing the centroid data. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered. Each row (if genes were clustered) or column (if microarrays were clustered) contains the averaged gene expression data for corresponding to the centroid of one cluster that was found.

• \$cmask

This matrix stores which values in cdata are missing. If \$cmask[\$i][\$j]==0, then \$cdata[\$i][\$j] is missing. The dimensions of this array are (number of clusters, number of microarrays) if genes were clustered, or (number of genes, number of clusters) if microarrays were clustered.

9.10.6 Calculating the distance between two clusters

Arguments

• index1

is a list containing the indices of the elements belonging to the first cluster, or alternatively an integer to refer to a single item.

• index2

is a list containing the indices of the elements belonging to the second cluster, or alternatively an integer to refer to a single item.

• method

Specifies how the distance between clusters is defined:

- method=='a': Distance between the two cluster centroids (arithmetic mean);
- method=='m': Distance between the two cluster centroids (median);
- method=='s': Shortest pairwise distance between elements in the two clusters;
- method=='x': Longest pairwise distance between elements in the two clusters;
- method=='v': Average over the pairwise distances between elements in the two clusters.

• dist

defines the distance function to be used:

- dist=='c': correlation:
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation;
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

• transpose

Determines if gene or microarray clusters are being considered. If transpose==0, then we are considering clusters of genes (rows). If transpose==1, then we are considering clusters of microarrays (columns).

Return values

This function returns the distance between the two clusters.

9.10.7 Calculating the distance matrix

Arguments

• transpose

Determines if genes or microarrays are being clustered. If transpose==0, genes (rows) are being clustered. If transpose==1, microarrays (columns) are clustered.

• dist

defines the distance function to be used:

- dist=='c': correlation:
- dist=='a': absolute value of the correlation;
- dist=='u': uncentered correlation:
- dist=='x': absolute uncentered correlation;
- dist=='s': Spearman's rank correlation;
- dist=='k': Kendall's τ ;
- dist=='e': Euclidean distance;
- dist=='b': City-block distance.

Return values

This function returns the \$distancematrix, an array of rows containing the distance matrix between the gene expression data. The number of columns in each row is equal to the row number. Hence, the first row has zero elements.

9.10.8 Saving the clustering result

writes the text file jobname.cdt, jobname.gtr, jobname.atr, jobname*.kgg, and/or jobname*.kag for subsequent reading by the Java TreeView program. If geneclusters and expclusters are both None, this method only writes the text file jobname.cdt; this file can subsequently be read into a new Record object.

Arguments

• jobname

The string jobname is used as the base name for names of the files that are to be saved.

• geneclusters

This argument describes the gene clustering result. In case of k-means clustering, this is a 1D array containing the number of the cluster each gene belongs to. It can be calculated using kcluster. In case of hierarchical clustering, geneclusters is a Tree object.

• expclusters

This argument describes the clustering result for the experimental conditions. In case of k-means clustering, this is a 1D array containing the number of the cluster each

experimental condition belongs to. It can be calculated using kcluster. In case of hierarchical clustering, expclusters is a Tree object.

9.10.9 Example calculation

This is an example of a hierarchical clustering calculation, using single linkage clustering for genes and maximum linkage clustering for experimental conditions. As the Euclidean distance is being used for gene clustering, it is necessary to scale the node distances genetree such that they are all between zero and one. This is needed for the Java TreeView code to display the tree diagram correctly. To cluster the experimental conditions, the uncentered correlation is being used. No scaling is needed in this case, as the distances in exptree are already between zero and two. The example data cyano.txt can be found in the data subdirectory.

```
use Algorithm::Cluster::Record;
my $record = Algorithm::Cluster::Record->new();
open INPUT, "cyano.txt";
$record->read(*INPUT);
my $genetree = $record->treecluster(method=>'s');
my $exptree = $record->treecluster(dist=>'u', transpose=>1);
$record->save(jobname=>"cyano_result", geneclusters=>$genetree,
expclusters=>$exptree);
This will create the files cyano_result.cdt, cyano_result.gtr, and cyano_result.atr.
  Similarly, we can save a k-means clustering solution:
use Algorithm::Cluster::Record;
my $record = Algorithm::Cluster::Record->new();
open INPUT, "cyano.txt";
$record->read(*INPUT);
my ($geneclusters, $error, $ifound) = $record->kcluster(nclusters=>5,
npass=>1000);
my ($expclusters, $error, $ifound) = $record->kcluster(nclusters=>2,
npass=>100, transpose=>1);
$record->save(jobname=>"cyano_result", geneclusters=>$geneclusters,
expclusters=>$expclusters);
This will create the files cyano_result_K_G2_A2.cdt, cyano_result_K_G2.kgg, and
cyano_result_K_A2.kag.
```

10 Compiling and linking

In the instructions below, <version> refers to the version number. The C Clustering Library complies with the ANSI-C standard since version 1.04. As of version 1.06, the C Clustering Library makes use of autoconf/automake to make the installation process easier. To install the library for use with Python, use the setup.py script instead, as described below. To install the library for use with Perl, use the Makefile.PL script.

10.1 Installing the C Clustering Library for Python

Pycluster is available as part of the Biopython distribution and as a separate package. As of version 1.41, Pycluster uses the "new" Numerical Python (version 1.1.1 or later), which you should install before installing Pycluster.

To install Pycluster as a separate package, download Pycluster-<version>.tar.gz from http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster. Unpack this file:

```
gunzip Pycluster-<version>.tar.gz
tar -xvf Pycluster-<version>.tar
and change to the directory Pycluster-<version>. Type
python setup.py install
```

from this directory. This will compile the library and install it for use with Python. To test your installation, you can run python setup.py test

If the installation was successful, you can remove the directory Pycluster-<version>. For Python on Windows (run from a DOS command window, or with a graphical user interface such as IDLE, PyCrust, PyShell, or PythonWin), a binary installer is available from http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster.

Installation instructions for Biopython are available from the Biopython website (http://www.biopython.org).

10.2 Installing the C Clustering Library for Perl

gunzip Algorithm-Cluster-<version>.tar.gz
tar -xvf Algorithm-Cluster-<version>.tar
and change to the directory Algorithm-Cluster-<version>. Type
perl Makefile.PL

which will create a Makefile. To compile and install, type make

make install

from this directory. You can execute

make test

to run some scripts that test the Algorithm::Cluster module. Some example Perl scripts can be found in the perl/examples subdirectory. If the installation was successful, you can remove the directory Algorithm-Cluster-<version>.

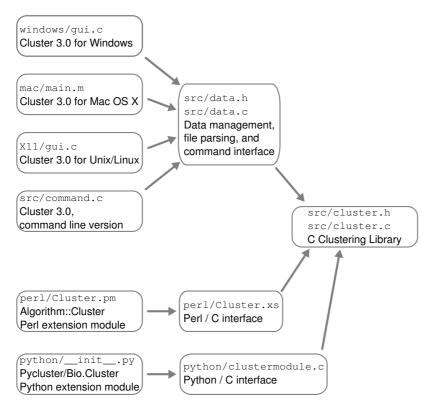
If you use ActiveState Perl on Windows, you can use the Perl Package Manager by executing

ppm install http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster/AlgorithmCluster.ppd

from the DOS command prompt. This assumes that you are using Perl 5.8 or 5.10 from ActiveState.

10.3 Accessing the C Clustering Library from C/C++

To call the routines in the C Clustering Library from your own C or C++ program, simply collect the relevant source files and compile them together with your program. The figure below shows the dependency structure for the source files in the C Clustering Library.



To use the routines in the C Clustering Library, put #include <cluster.h>
in your source code. If your program is written in C++, use extern "C" {
#include <cluster.h>
}

instead. To compile a C or C++ program with the C Clustering Library, add the relevant source files to the compile command. For example, a C program myprogram.c can be compiled and linked by

gcc -o myprogram myprogram.c cluster.c

An example C program that makes use of the C Clustering Library can be found in the example subdirectory.

10.4 Installing Cluster 3.0 for Windows

The easiest way to install Cluster 3.0 for Windows is to use the Windows installer (http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster). The executable cluster.com can be used both as a GUI and as a command line program. To start Cluster 3.0 as a GUI program, simply double-click on cluster.com. If you want to use Cluster 3.0 from the command prompt as a command line program, you may need to give the full path to the executable (e.g., C:\Program Files\Stanford University\Cluster 3.0\cluster.com; the exact path may be different on your computer). Type cluster.com --help for an overview of all command line options.

If you want to compile Cluster 3.0 from the source, change to the windows directory, and type make. This will compile the C Clustering Library, the Cluster 3.0 GUI, the Windows help files and the documentation. To compile the GUI, you need an ANSI C compiler such as GNU gcc. To compile the resources needed for the GUI, you will need the GNU program windres. To generate the help files, you need the HTML Help SDK, which can be downloaded from Microsoft. You will also need GNU makeinfo.

To generate the Windows installer, type make clustersetup.exe

For this, you will need the Inno Setup Compiler, which can be downloaded from http://www.jrsoftware.org.

10.5 Installing Cluster 3.0 for Mac OS X

Cluster 3.0 for Mac OS X can be installed most easily by using the prebuilt package that is available at http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster. After installing, you can start Cluster 3.0 as a GUI program by double-clicking on its icon. To run Cluster 3.0 as a command line program (e.g., from the Terminal), most likely you will need to give the full path to the executable (e.g., /Applications/Cluster.app/Contents/MacOS/Cluster). If you want to recompile Cluster 3.0, it is easiest to use the Project Builder and Interface Builder that are part of Mac OS X. The directory mac contains the project file that was used.

10.6 Installing Cluster 3.0 for Linux/Unix

Cluster 3.0 was ported to Linux/Unix using the Motif libraries. Motif is installed on most Linux/Unix computers. You will need a version compliant with Motif 2.1, such as Open Motif (http://www.opengroup.org), which is available at http://www.motifzone.net. Currently, LessTif (http://www.lesstif.org) does not work correctly with Cluster 3.0.

To install Cluster 3.0 on Linux/Unix, type

./configure

make

make install

This will create the executable cluster and install it in /usr/local/bin. Some auxiliary files will be installed in /usr/local/cluster. The executable can be used both as a GUI program and as a command line program. Type

cluster --help

for more information about running Cluster 3.0 as a command line program.

10.7 Installing Cluster 3.0 as a command line program

Cluster 3.0 can also be installed without GUI support. In this case, Cluster 3.0 can only be run as a command line program, in which the action taken by the program depends on the command line parameters. To install Cluster 3.0 as a command line program, the Motif libraries are not needed. Simply download the source code for the C Clustering Library from our website http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster, unpack and untar the file, and change to the directory cluster-<version>. Then type ./configure --without-x

make

make install

For the last step, you may need superuser privileges. For more information about the command line options, check the Cluster 3.0 manual.

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