

(1)

$$f_x(x) = \int_x^1 f(x, y) dy = 2 - 2x, 0 < x < 1$$
$$f_y(y) = \int_0^y f(x, y) dx = 2y, 0 < y < 1$$

若 $f(x, y) = f_x(x)f_y(y)$ 則 X, Y independent

但 $2 \neq 4(1-x)(y)$ 所以不獨立

(2)

$$f(x|y) = f(x, y)/f_y(y) = \frac{1}{y}$$
$$P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{3}{4}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} f_x(x|Y = \frac{3}{4}) dx$$
$$= \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{4}{3} dx = \frac{1}{3}$$