proof of Jacobi's identity for ϑ functions

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We start with the Fourier transform of $f(x) = e^{i\pi\tau x^2 + 2ixz}$:

$$\int_{-\infty}^{+\infty} e^{i\pi\tau x^2 + 2ixz} e^{2\pi ixy}\, dx = (-i\tau)^{-1/2} e^{-i\frac{(z+\pi y)^2}{\pi\tau}}$$

Applying the Poisson summation formula, we obtain the following:

$$\sum_{n=-\infty}^{+\infty} e^{i\pi\tau n^2 + 2inz} = (-i\tau)^{-1/2} \sum_{n=-\infty}^{+\infty} e^{-i\frac{(z+\pi n)^2}{\pi\tau}}$$

The left hand equals $\vartheta_3(z \mid \tau)$. The right hand can be rewritten as follows:

$$\sum_{n=-\infty}^{+\infty} e^{-i\frac{(z+\pi n)^2}{\pi\tau}} = e^{-i\frac{z^2}{\pi\tau}} \sum_{n=-\infty}^{+\infty} e^{-i\frac{\pi n^2}{\tau} - \frac{2inz}{\tau}} = e^{-i\frac{z^2}{\pi\tau}} \vartheta_3(z/\tau \mid -1/\tau)$$

Combining the two expressions yields

$$\vartheta_3(z \mid \tau) = e^{-i\frac{z^2}{\pi\tau}} \vartheta_3(z/\tau \mid -1/\tau)$$