inverse Galois problem

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The inverse Galois problem is extremely to , yet one of the hardest problems for theorists and . It generally asks for descriptions of the of groups that can occur as Galois groups. Of course, a significantly more precise formulation is required, for example, because of a result that that every Galois group is profinite, and every profinite group is a Galois group. Also ambiguous is what field(s) we allow ourselves to include when computing the Galois group. Unfortunately, many of these related questions all go under the heading "the inverse Galois problem," so care must be taken to determine an exact formulation of the question being asked.

As an example of a partial solution to this question, it is known that every finite abelian group occurs as the Galois group of an extension over $\mathbb{Q}$ (by the Kronecker-Weber theorem), though it is *not* known whether or not this is true for every finite (not necessarily abelian) group. This latter question can also be phrased in of the absolute Galois group: "Does every finite group occur as a quotient group of the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$"? Thus, an answer to this question would not only reveal information about the nature of finite Galois groups, but also shed light on one of the most elusive objects in all of algebra and number theory.

It is also known (see Shafarevich' theorem) that every solvable group occurs as a Galois group.