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goniometric formulas

The *goniometric* (from Greek γωνία "angle" and μετρικός "measuring") concerns the trigonometric functions and their mutual connections. There are a great amount of formulas involving these functions (usually for real arguments).

1. Pythagorean identities

   - $\sin^2 x + \cos^2 x = 1$
   - $\tan^2 x + 1 = \sec^2 x$
   - $1 + \cot^2 x = \csc^2 x$

2. Fractional identities

   - $\tan x = \dfrac{\sin x}{\cos x}$
   - $\cot x = \dfrac{\cos x}{\sin x}$
   - $\cot x = \dfrac{1}{\tan x}$
   - $\tan x = \dfrac{1}{\cot x}$
   - $\csc x = \dfrac{1}{\sin x}$
   - $\sec x = \dfrac{1}{\cos x}$

3. Formulas involving `http://planetmath.org/Radical6`radicals

   - $\sin x = \pm \dfrac{\tan x}{\sqrt{1 + \tan^2 x}}$
   - $\cos x = \pm \dfrac{1}{\sqrt{1 + \tan^2 x}}$

4. Weierstrass substitution formulas and related formula for $\tan x$

   - $\sin x = \dfrac{2 \tan\left(\dfrac{x}{2}\right)}{1 + \tan^2\left(\dfrac{x}{2}\right)}$

1

- $\cos x = \dfrac{1 - \tan^2\left(\dfrac{x}{2}\right)}{1 + \tan^2\left(\dfrac{x}{2}\right)}$

- $\tan x = \dfrac{2\tan\left(\dfrac{x}{2}\right)}{1 - \tan^2\left(\dfrac{x}{2}\right)}$

5. Trigonometric functions of a purely imaginary number

- $\sin(ix) = i\sinh x$
- $\cos(ix) = \cosh x$
- $\tan(ix) = i\tanh x$
- $\cot(ix) = i\coth x$
- $\csc(ix) = i\operatorname{csch} x$
- $\sec(ix) = \operatorname{sech} x$

6. `http://planetmath.org/AdditionFormulasForSineAndCosine`Addition formulas and subtraction formulas

- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

7. Formulas for trigonometric functions of a complex number

- $\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y$
- $\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y$
- $\tan(x + iy) = \dfrac{\tan x + i\tanh y}{1 - i\tan x \tanh y}$

8. Complement formulas

- $\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$
- $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

2

- $\tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

9. Supplement formulas

- $\sin(\pi - x) = \sin x$
- $\cos(\pi - x) = -\cos x$
- $\tan(\pi - x) = -\tan x$

10. Explement formulas

- $\sin(2\pi - x) = -\sin x$
- $\cos(2\pi - x) = \cos x$
- $\tan(2\pi - x) = -\tan x$

11. angle formulas

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\tan(-x) = -\tan x$

12. `http://planetmath.org/Periodic`Periodicity formulas

- $\sin(x + 2\pi) = \sin x$
- $\cos(x + 2\pi) = \cos x$
- $\tan(x + \pi) = \tan x$

13. Double angle formulas

- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \dfrac{2\tan x}{1 - \tan^2 x}$

14. Triple angle formulas

- $\sin(3x) = 3\sin x - 4\sin^3 x = (4\cos^2 x - 1)\sin x$

3

- $\cos(3x) = 4\cos^3 x - 3\cos x = (1 - 4\sin^2 x)\cos x$

- $\tan(3x) = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

15. Half angle formulas

- $\sin\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1 - \cos x}{2}}$

- $\cos\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1 + \cos x}{2}}$

- $\tan\left(\dfrac{x}{2}\right) = \dfrac{\sin x}{1 + \cos x} = \dfrac{1 - \cos x}{\sin x} = \pm\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

16. Prosthaphaeresis formulas

- $\sin x + \sin y = 2\sin\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)$

- $\sin x - \sin y = 2\sin\left(\dfrac{x - y}{2}\right)\cos\left(\dfrac{x + y}{2}\right)$

- $\cos x + \cos y = 2\cos\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)$

- $\cos x - \cos y = -2\sin\left(\dfrac{x + y}{2}\right)\sin\left(\dfrac{x - y}{2}\right)$

17. formulas

- $\sin x \, \sin y = \dfrac{\cos(x - y) - \cos(x + y)}{2}$

- $\cos x \, \sin y = \dfrac{\sin(x + y) - \sin(x - y)}{2}$

- $\cos x \, \cos y = \dfrac{\cos(x - y) + \cos(x + y)}{2}$

18. Other sums and differences

- $\tan x \pm \tan y = \dfrac{\sin(x \pm y)}{\cos x \, \cos y}$

- $\cot x \pm \cot y = \dfrac{\sin(y \pm x)}{\sin x \, \sin y}$

- $\cos x \pm \sin x = \sqrt{2} \sin\left(\dfrac{\pi}{4} \pm x\right) = \sqrt{2} \cos\left(\dfrac{\pi}{4} \mp x\right)$

19. formulas

  - Second power

    - $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$

    - $\cos^2 x = \dfrac{1 + \cos(2x)}{2}$

    - $\tan^2 x = \dfrac{1 - \cos(2x)}{1 + \cos(2x)}$

  - Third power

    - $\sin^3 x = \dfrac{3\sin x - \sin(3x)}{4}$

    - $\cos^3 x = \dfrac{3\cos x + \cos(3x)}{4}$

    - $\tan^3 x = \dfrac{3\sin x - \sin(3x)}{3\cos x + \cos(3x)}$

  - Fourth power

    - $\sin^4 x = \dfrac{\cos(4x) - 4\cos(2x) + 3}{8}$

    - $\cos^4 x = \dfrac{\cos(4x) + 4\cos(2x) + 3}{8}$

    - $\tan^4 x = \dfrac{\cos(4x) - 4\cos(2x) + 3}{\cos(4x) + 4\cos(2x) + 3}$

20. Recursion formulas

  - $\sin[(n+1)x] = 2\cos x \, \sin(nx) - \sin[(n-1)x]$

  - $\cos[(n+1)x] = 2\cos x \, \cos(nx) - \cos[(n-1)x]$

21. `http://planetmath.org/ExponentialFunction`Exponential formulas

  - $e^{ix} = \cos x + i\sin x$

- $e^{-ix} = \cos x - i \sin x$

- $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$

- $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

- $\tan x = \dfrac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$

22. Some special formulas

- $\tan\left(x + \dfrac{\pi}{4}\right) = \dfrac{\cos x + \sin x}{\cos x - \sin x} = \pm\sqrt{\dfrac{1 + \sin 2x}{1 - \sin 2x}}$

- $\tan x + \sec x = \tan\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)$

- $\tan\left(\dfrac{x \pm y}{2}\right) = \dfrac{\sin x \pm \sin y}{\cos x + \cos y} = \dfrac{\cos y - \cos x}{\sin x \mp \sin y}$

- $\tan\left(\dfrac{x + y}{2}\right)\tan\left(\dfrac{x - y}{2}\right) = \dfrac{\cos y - \cos x}{\cos y + \cos x}$

- $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$