Dirac delta function

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The Dirac delta "function" $\delta(x)$, or *distribution* is not a true function because it is not uniquely defined for all values of the argument x. Similar to the Kronecker delta symbol, the notation $\delta(x)$ stands for

$$\delta(x) = 0 \text{ for } x \neq 0, \text{ and } \int_{-\infty}^{\infty} \delta(x) dx = 1$$

For any continuous function F:

$$\int_{-\infty}^{\infty} \delta(x) F(x) dx = F(0)$$

or in n dimensions:

$$\int_{\mathbb{R}^n} \delta(x-s) f(s) \, d^n s = f(x)$$

$\delta(x)$ can also be defined as a normalized Gaussian function (normal distribution) in the limit of zero width.

Notes: However, the limit of the normalized Gaussian function is still meaningless as a function, but some people still write such a limit as being equal to the Dirac distribution considered above in the first paragraph.
An example of how the Dirac distribution arises in a physical, classical context is available `http://www.rose-hulman.edu/ rickert/Classes/ma222/Wint0102/dirac.pdfo` line.

Remarks: Distributions play important roles in Dirac's formulation of quantum mechanics.

References

[1] W. Rudin, *Functional Analysis*, McGraw-Hill Book Company, 1973.

[2] L. Hörmander, *The Analysis of Linear Partial Differential Operators I, (Distribution theory and Fourier Analysis)*, 2nd ed, Springer-Verlag, 1990.

[3] Originally from The Data Analysis Briefbook (`http://rkb.home.cern.ch/rkb/titleA.html`http://rkb.home.cern.ch/rkb/titleA.html)