



equilibrium point

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Entry type	Definition
Classification	msc 34C99
Synonym	steady state solution
Synonym	fixed point
Synonym	singular point
Defines	hyperbolic equilibrium
Defines	nonhyperbolic equilibrium
Defines	stable
Defines	unstable
Defines	asymptotically stable

Consider an autonomous differential equation

$$\dot{x} = f(x). \tag{1}$$

An *equilibrium point* x_0 of (??) is such that $f(x_0) = 0$. Conversely a *regular point* of (??) is such that $f(x_0) \neq 0$.

If the linearization $Df(x_0)$ has no eigenvalue with zero real part, x_0 is said to be a hyperbolic equilibrium, whereas if there exists an eigenvalue with zero real part, the equilibrium point is nonhyperbolic.

An equilibrium point x_0 is said to be *stable* if for every neighborhood x_0, U there exists a neighborhood of x_0 , $U' \subset U$ such that every solution of (??) with initial condition in U' (*i.e.* $x(0) \in U'$), satisfies

$$x(t) \in U$$

for all $t \geq 0$.

Consequently an equilibrium point x_0 is said to be *unstable* if it is not stable.

Moreover an equilibrium point x_0 is said to be *asymptotically stable* if it is stable and there exists U'' such that every solution of (??) with initial condition in U'' (*i.e.* $x(0) \in U''$) satisfies

$$\lim_{t \rightarrow \infty} x(t) = x_0.$$