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values of $(1 + 1/n)^n$ for $0 < n < 26$

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The following table gives the numerator and denominator of $(1 + \frac{1}{n})^n$ as well as the decimal expansion to 20 places.

n	Numerator of $(1 + \frac{1}{n})^n$	Denominator of $(1 + \frac{1}{n})^n$	De
1	2	1	2.0
2	9	4	2.2
3	64	27	2.3
4	625	256	2.4
5	7776	3125	2.4
6	117649	46656	2.5
7	2097152	823543	2.5
8	43046721	16777216	2.5
9	1000000000	387420489	2.5
10	25937424601	10000000000	2.5
11	743008370688	285311670611	2.6
12	23298085122481	8916100448256	2.6
13	793714773254144	302875106592253	2.6
14	29192926025390625	11112006825558016	2.6
15	1152921504606846976	437893890380859375	2.6
16	48661191875666868481	18446744073709551616	2.6
17	2185911559738696531968	827240261886336764177	2.6
18	104127350297911241532841	39346408075296537575424	2.6
19	5242880000000000000000000	1978419655660313589123979	2.6
20	278218429446951548637196401	10485760000000000000000000	2.6
21	15519448971100888972574851072	5842587018385982521381124421	2.6
22	907846434775996175406740561329	341427877364219557396646723584	2.6
23	55572324035428505185378394701824	20880467999847912034355032910567	2.6
24	3552713678800500929355621337890625	1333735776850284124449081472843776	2.6
25	236773830007967588876795164938469376	88817841970012523233890533447265625	2.6

With a large enough value of n , this formula approximates the natural log base e . For example, with n set to ten million, we get 2.7182816925449662712, which is 0.000000135914078964161737245574 short of 2.7182818284590452354 (this calculation took almost four minutes with Mathematica 4.2). It is less computationally intensive to use

$$\sum_{i=0}^n \frac{1}{i!},$$

which with n set to 100 gives in less than a second a result to 20 places that is indistinguishable from e .