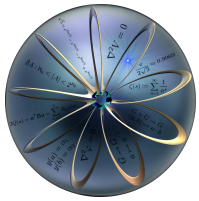

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Lyapunov function

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Synonym	Liapunov function

Suppose we are given an autonomous system of first order differential equations.

$$\frac{dx}{dt} = F(x, y) \quad \frac{dy}{dt} = G(x, y)$$

Let the origin be an isolated critical point of the above system.

A function $V(x, y)$ that is of class C^1 and satisfies $V(0, 0) = 0$ is called a *Lyapunov function* if every open ball $B_\delta(0, 0)$ contains at least one point where $V > 0$. If there happens to exist δ^* such that the function $\dot{V}$, given by

$$\dot{V}(x, y) = V_x(x, y)F(x, y) + V_y(x, y)G(x, y)$$

is positive definite in $B^*_\delta(0, 0)$. Then the origin is an unstable critical point of the system.