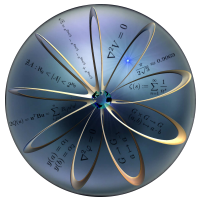

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alternate integral representation of beta function (2)

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Substitute $x := \frac{1}{1+s}$, $dx = \frac{-1}{(1+s)^2}\,ds$:

$$\begin{aligned}\int_0^1 x^{p-1}(1-x)^{q-1}\,dx &= \int_0^\infty \frac{1}{(1+s)^{p+1}}\left(\frac{s}{1+s}\right)^{q-1}\,ds \\ &= \int_0^\infty \frac{s^{q-1}}{(1+s)^{p+q}}\,ds\end{aligned}$$

Since $B(p,q) = B(q,p)$ this gives:

$$\int_0^\infty \frac{s^{p-1}}{(1+s)^{p+q}}\,ds = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$