stable manifold theorem

Canonical name	StableManifoldTheorem
Date of creation	2013-03-22 12:57:17
Last modified on	2013-03-22 12:57:17
Owner	jarino (552)
Last modified by	jarino (552)
Numerical id	4
Author	jarino (552)
Entry type	Theorem
Classification	msc 34C99

Let E be an open subset of $\mathbb{R}^n$ containing the origin, let $f \in C^1(E)$, and let ϕ_t be the flow of the nonlinear system $x' = f(x)$.

Suppose that $f(x_0) = 0$ and that $Df(x_0)$ has k eigenvalues with negative real part and $n-k$ eigenvalues with positive real part. Then there exists a k-dimensional differentiable manifold S tangent to the stable subspace E^S of the linear system $x' = Df(x)x$ at x_0 such that for all $t \geq 0$, $\phi_t(S) \subset S$ and for all $y \in S$,

$$\lim_{t \to \infty} \phi_t(y) = x_0$$

and there exists an $n-k$ dimensional differentiable manifold U tangent to the unstable subspace E^U of $x' = Df(x)x$ at x_0 such that for all $t \leq 0$, $\phi_t(U) \subset U$ and for all $y \in U$,

$$\lim_{t \to -\infty} \phi_t(y) = x_0.$$