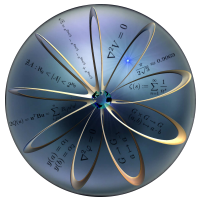

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differential-difference equations for hypergeometric function

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The hypergeometric function satisfies several equations which relate derivatives with respect to the argument z to shifting the parameters a, b, c, d by unity (Here, the prime denotes derivative with respect to z.):

$$
\begin{aligned}
zF'(a,b;c;z) + aF(a,b;c;z) &= F(a+1,b;c;z) \\
zF'(a,b;c;z) + bF(a,b;c;z) &= F(a,b+1;c;z) \\
zF'(a,b;c;z) + (c-1)F(a,b;c;z) &= F(a,b;c-1;z) \\
(1-z)zF'(a,b;c;z) &= (c-a)F(a-1,b;c;z) + (a-c+bz)F(a,b;c;z) \\
(1-z)zF'(a,b;c;z) &= (c-b)F(a,b-1;c;z) + (b-c+az)F(a,b;c;z) \\
(1-z)zF'(a,b;c;z) &= z(c-a)(c-b)F(a,b;c+1;z) + zc(a+b-c)F(a,b;c;z)
\end{aligned}
$$

These equations may readily be verified by differentiating the series which defines the hypergeometric equation. By eliminating the derivatives between these equations, one obtains the contiguity relations for the hypergeometric function. By differentiating them once more and taking suitable linear combinations, one may obtain the differential equation of the hypergeometric function.