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Hartman-Grobman theorem

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Consider the differential equation

$$x' = f(x) \tag{1}$$

where f is a C^1 vector field. Assume that x_0 is a hyperbolic equilibrium of f. Denote $\Phi_t(x)$ the flow of (??) through x at time t. Then there exists a homeomorphism $\varphi(x) = x + h(x)$ with h bouded, such that

$$\varphi \circ e^{tDf(x_0)} = \Phi_t \circ \varphi$$

is a sufficiently small neighboorhood of x_0.

This fundamental theorem in the qualitative analysis of nonlinear differential equations states that, in a small neighborhood of x_0, the flow of the nonlinear equation (??) is qualitatively similar to that of the linear system $x' = Df(x_0)x$.