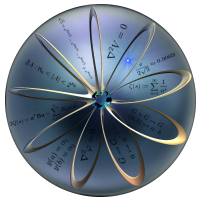

planetmath.org

Math for the people, by the people.

hyperbolic identities

Canonical name	HyperbolicIdentities
Date of creation	2013-03-22 17:50:42
Last modified on	2013-03-22 17:50:42
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	11
Author	Wkbj79 (1863)
Entry type	Topic
Classification	msc 33B10
Classification	msc 26A09
Synonym	hyperbolic formulas
Synonym	hyperbolic formulae
Related topic	GoniometricFormulae
Related topic	AdditionAndSubtractionFormulasForHyperbolicFunctions
Related topic	ExamplesOfPeriodicFunctions
Related topic	TaylorSeriesOfHyperbolicFunctions

There are many formulas involving hyperbolic functions, many of which are to formulas for trigonometric functions. Below is a list of some of these formulas (usually for real arguments).

1. Hyperbolic version of Pythagorean identities
   - $\cosh^2 x - \sinh^2 x = 1$
   - $1 - \tanh^2 x = \operatorname{sech}^2 x$
   - $\coth^2 x - 1 = \operatorname{csch}^2 x$
2. Fractional identities
   - $\tanh x = \dfrac{\sinh x}{\cosh x}$
   - $\coth x = \dfrac{\cosh x}{\sinh x}$
   - $\coth x = \dfrac{1}{\tanh x}$
   - $\tanh x = \dfrac{1}{\coth x}$
   - $\operatorname{csch} x = \dfrac{1}{\sinh x}$
   - $\operatorname{sech} x = \dfrac{1}{\cosh x}$
3. Hyperbolic functions of a purely imaginary number
   - $\sinh(ix) = i \sin x$
   - $\cosh(ix) = \cos x$
   - $\tanh(ix) = i \tan x$
   - $\cosh(ix) = i \cot x$
   - $\operatorname{csch}(ix) = i \csc x$
   - $\operatorname{sech}(ix) = \sec x$
4. `http://planetmath.org/AdditionAndSubtractionFormulasForHyperbolicFunctionsAd` formulas and subtraction formulas

- $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
- $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
- $\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

5. Formulas for hyperbolic functions of a complex number

   - $\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y$
   - $\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y$
   - $\tanh(x + iy) = \dfrac{\tanh x + i \tan y}{1 + i \tanh x \tan y}$

6. Opposite formulas

   - $\sinh(-x) = -\sinh x$
   - $\cosh(-x) = \cosh x$
   - $\tanh(-x) = -\tanh x$

7. Double argument formulas

   - $\sinh(2x) = 2 \sinh x \cosh x$
   - $\cosh(2x) = \cosh^2 x + \sinh^2 x = 2 \cosh^2 x - 1 = 1 + 2 \sinh^2 x$
   - $\tanh(2x) = \dfrac{2 \tanh x}{1 + \tanh^2 x}$

8. `http://planetmath.org/Periodic`Periodicity formulas

   - $\sinh(z + 2\pi i) = \sinh z$
   - $\cosh(z + 2\pi i) = \cosh z$
   - $\tanh(z + \pi i) = \tanh z$

   `http://planetmath.org/Cf`Cf. the periodicity of exponential function.

9. `http://planetmath.org/ExponentialFunction`Exponential formulas

   - $\cosh x = \dfrac{e^x + e^{-x}}{2}$

- $\sinh x = \dfrac{e^x - e^{-x}}{2}$
- $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$
- $e^x = \cosh x + \sinh x$
- $e^{-x} = \cosh x - \sinh x$

Note that the first three formulas given in this are definitions.