global characterization of hypergeometric function

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Owner	rspuzio (6075)
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Author	rspuzio (6075)
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Riemann noted that the hypergeometric function can be characterized by its global properties, without reference to power series, differential equations, or any other sort of explicit expression. His characterization is conveniently restated in terms of sheaves:

Suppose that we have a sheaf of holomorphic functions over $\mathbb{C} \setminus \{0, 1\}$ which satisfy the following properties:

- It is closed under analytic continuation.
- It is closed under taking linear combinations.
- The space of function elements over any open set is two dimensional.
- There exists a neighborhood D_0 such that $0 \in D)$, holomorphic functions ϕ_0, ψ_0 defined on D_0, and complex numbers α_0, β_0 such that, for an open set of d_0 not containing 0, it happens that $z \mapsto z^{\alpha_0}\phi(z)$ and $z \mapsto z^{\beta_0}\psi(z)$ belong to our sheaf.

Then the sheaf consists of solutions to a hypergeometric equation, hence the function elements are hypergeometric functions.