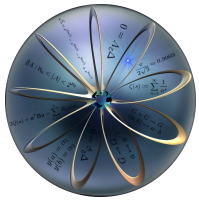

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cycle

Canonical name	Cycle12
Date of creation	2013-03-22 15:00:51
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Numerical id	6
Author	Daume (40)
Entry type	Definition
Classification	msc 34A12
Classification	msc 34C07
Synonym	periodic solution
Synonym	stable periodic solution
Synonym	unstable periodic solution
Synonym	asymptotically stable periodic solution
Defines	period
Defines	stable cycle
Defines	unstable cycle
Defines	asymptotically stable cycle

Let

$$\dot{x} = f(x)$$

be an autonomous ordinary differential equation defined by the vector field $f\colon V \to V$ then $x(t) \in V$ a solution of the system is a *cycle(or* periodic solution*)* if it is a closed solution which is not an equilibrium point. The *period* of a cycle is the smallest positive T such that $x(t) = x(t+T)$.
Let $\phi_t(x)$ be the flow defined by the above ODE and d be the metric of V then:
A cycle, Γ, is a *stable cycle* if for all $\epsilon > 0$ there exists a neighborhood U of Γ such that for all $x \in U$, $d(\phi_t(x), \Gamma) < \epsilon$.
A cycle, Γ, is *unstable cycle* if it is not a stable cycle.
A cycle, Γ, is *asymptotically stable cycle* if for all $x \in U$ where U is a neighborhood of Γ, $\lim_{t\to\infty} d(\phi_t(x), \Gamma) = 0$.[**?**]

example:
Let

$$\begin{aligned} \dot{x} &= -y \\ \dot{y} &= x \end{aligned}$$

then the above autonomous ordinary differential equations with initial value condition $(x(0), y(0)) = (1, 0)$ has a solution which is a stable cycle. Namely the solution defined by

$$\begin{aligned} x(t) &= \cos t \\ y(t) &= \sin t \end{aligned}$$

which has a period of 2π.

References

[PL] Perko, Lawrence: Differential Equations and Dynamical Systems *(Third Edition)*. Springer, New York, 2001.