second order ordinary differential equation

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A second order ordinary differential equation $F(x,\,y,\,\frac{dy}{dx},\,\frac{d^2y}{dx^2}) = 0$ can often be written in the form

$$\frac{d^2y}{dx^2} \;=\; f\left(x,\,y,\,\frac{dy}{dx}\right). \tag{1}$$

By its general solution one means a function $x \mapsto y = y(x)$ which is at least on an interval twice differentiable and satisfies

$$y''(x) \;\equiv\; f(x,\,y(x),\,y'(x)).$$

By setting $\frac{dy}{dx} := z$, one has $\frac{d^2y}{dx^2} = \frac{dz}{dx}$, and the equation (1) reads $\frac{dz}{dx} = f(x,\,y,\,z)$. It is easy to see that solving (1) is http://planetmath.org/Equivalent3equivalent with solving the system of simultaneous http://planetmath.org/ODEfirst order differential equations

$$\begin{cases} \frac{dy}{dx} = z, \\ \frac{dz}{dx} = f(x,\,y,\,z), \end{cases} \tag{2}$$

the so-called *normal system* of (1).

The system (2) is a special case of the general *normal system of second order*, which has the form

$$\begin{cases} \frac{dy}{dx} = \varphi(x,\,y,\,z), \\ \frac{dz}{dx} = \psi(x,\,y,\,z), \end{cases} \tag{3}$$

where y and z are unknown functions of the variable x. The existence theorem of the solution

$$\begin{cases} y = y(x), \\ z = z(x) \end{cases} \tag{4}$$

is as follows; cf. the http://planetmath.org/PicardsTheorem2Picard–Lindelöf theorem.

Theorem. If the functions φ and ψ are continuous and have continuous partial derivatives with respect to y and z in a neighbourhood of a point $(x_0,\,y_0,\,z_0)$, then the normal system (3) has one and (as long as $|x-x_0|$ does not exceed a certain) only one solution (4) which satisfies the initial conditions $y(x_0) = y_0$, $z(x_0) = z_0$. The functions (4) are continuously differentiable in a neighbourhood of x_0.

References

[1] E. Lindelöf: *Differentiali- ja integralilasku III 1.* Mercatorin Kirjapaino Osakeyhtiö, Helsinki (1935).