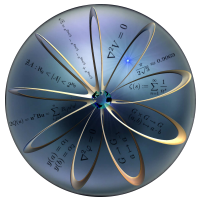

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inverse Gudermannian function

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Since the real Gudermannian function gd is strictly increasing and forms a bijection from $\mathbb{R}$ onto the open interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, it has an inverse function

$$\mathrm{gd}^{-1}\colon\ (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}.$$

The function gd^{-1} is denoted also **arcgd**.

If $x = \mathrm{gd}\, y$, which may be explicitly written e.g.

$$x \ = \ \arcsin(\tanh y),$$

one can solve this for y, getting first $\tanh y = \sin x$ and then

$$y \ = \ \mathrm{artanh}(\sin x)$$

(see the area functions). Hence the *inverse Gudermannian* is expressed as

$$\mathrm{gd}^{-1}(x) \ = \ \mathrm{arcgd}\, x \ = \ \mathrm{artanh}(\sin x) \tag{1}$$

It has other `http://planetmath.org/Equivalent3`equivalent expressions, such as

$$\mathrm{gd}^{-1}(x) \ = \ \mathrm{arsinh}(\tan x) \ = \ \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} \ = \ \int_0^x \frac{dt}{\cos t}. \tag{2}$$

Thus its derivative is

$$\frac{d}{dx}\mathrm{gd}^{-1}(x) \ = \ \frac{1}{\cos x}. \tag{3}$$

Cf. the formulae (1)–(3) with the corresponding ones of gd.