



Math for the people, by the people.

# integral represetations of Jacobi $\vartheta$ functions

Canonical name	IntegralRepresentationsOfJacobivarthetaFunctions
Date of creation	2013-03-22 14:39:52
Last modified on	2013-03-22 14:39:52
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	12
Author	rspuzio (6075)
Entry type	Theorem
Classification	msc 33E05

The Jacobi theta functions have the following integral representations:

$$\begin{aligned}\vartheta_1(z|\tau) &= -e^{iz+i\pi\tau/4} \int_{i-\infty}^{i+\infty} \frac{e^{i\pi\tau u^2} \cos(2uz + \pi\tau u)}{\sin(\pi u)} du \\ \vartheta_2(z|\tau) &= -ie^{iz+i\pi\tau/4} \int_{i-\infty}^{i+\infty} \frac{e^{i\pi\tau u^2} \cos(2uz + \pi u + \pi\tau u)}{\sin(\pi u)} du \\ \vartheta_3(z|\tau) &= -i \int_{i-\infty}^{i+\infty} \frac{e^{i\pi\tau u^2} \cos(2uz + \pi u)}{\sin(\pi u)} du \\ \vartheta_4(z|\tau) &= -i \int_{i-\infty}^{i+\infty} \frac{e^{i\pi\tau u^2} \cos(2uz)}{\sin(\pi u)} du\end{aligned}$$