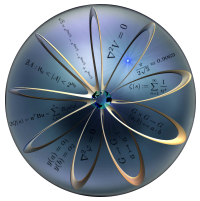

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modular discriminant

Canonical name	ModularDiscriminant
Date of creation	2013-03-22 13:54:09
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Entry type	Definition
Classification	msc 33E05
Synonym	delta function
Related topic	EllipticFunction
Related topic	JInvariant
Related topic	WeierstrassSigmaFunction
Related topic	Discriminant
Related topic	DiscriminantOfANumberField
Related topic	RamanujanTauFunction
Defines	modular discriminant
Defines	Dedekind eta function

Definition 1. *Let $\Lambda \subset \mathbb{C}$ be a lattice.*

1. *Let $q_\tau = e^{2\pi i \tau}$. The* Dedekind eta function *is defined to be*

$$\eta(\tau) = q_\tau^{1/24} \prod_{n=1}^{\infty} (1 - q_\tau^n)$$

The Dedekind eta function should not be confused with the Weierstrass eta function, $\eta(w; \Lambda)$.

2. *The j-invariant, as a function of lattices, is defined to be:*

$$j(\Lambda) = \frac{g_2^3}{g_2^3 - 27 g_3^2}$$

where g_2 and g_3 are certain multiples of the Eisenstein series of weight 4 and 6 (see `http://planetmath.org/encyclopedia/ExamplesOfEllipticFunctions.html` *entry).*

3. *The Δ* function *(*delta function *or* modular discriminant*) is defined to be*

$$\Delta(\Lambda) = g_2^3 - 27 g_3^2$$

Let Λ_τ be the lattice generated by $1, \tau$. The Δ function for Λ_τ has a product expansion

$$\Delta(\tau) = \Delta(\Lambda_\tau) = (2\pi i)^{12} q_\tau \prod_{n=1}^{\infty} (1 - q_\tau^n)^{24} = (2\pi i)^{12} \eta(\tau)^{24}$$