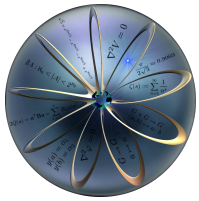

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Bohr-Mollerup theorem

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Let $f : \mathbb{R}^+ \to \mathbb{R}^+$ be a function with the following properties:

1. $\log f(x)$ is a convex function (i.e. f is logarithmically convex);
2. $f(x+1) = xf(x)$ for all $x > 0$;
3. $f(1) = 1$.

Then $f(x) = \Gamma(x)$ for all $x > 0$.
That is, the only function satisfying those properties is the gamma function (restricted to the positive reals.)