Hilbert's 16th problem for quadratic vector fields

Canonical name	Hilberts16thProblemForQuadraticVectorFields
Date of creation	2013-03-22 14:03:35
Last modified on	2013-03-22 14:03:35
Owner	Daume (40)
Last modified by	Daume (40)
Numerical id	11
Author	Daume (40)
Entry type	Conjecture
Classification	msc 34C07

Find a maximum natural number $H(2)$ and relative position of limit cycles of a vector field

$$\begin{aligned}\dot{x} = p(x,y) &= \sum_{i+j=0}^{2} a_{ij}x^i y^j \\ \dot{y} = q(x,y) &= \sum_{i+j=0}^{2} b_{ij}x^i y^j\end{aligned}$$

[?].
As of now neither part of the problem *(i.e. the bound and the positions of the limit cycles)* are solved. Although R. Bamòn in 1986 showed [?] that a quadratic vector field has finite number of limit cycles. In 1980 Shi Songling [?] and also independently Chen Lan-Sun and Wang Ming-Shu [?] showed an example of a quadratic vector field which has four limit cycles *(i.e.* $H(2) \geq 4$*)*.

Example by Shi Songling:
The following system

$$\begin{aligned}\dot{x} &= \lambda x - y - 10x^2 + (5+\delta)xy + y^2 \\ \dot{y} &= x + x^2 + (-25 + 8\epsilon - 9\delta)xy\end{aligned}$$

has four limit cycles when $0 < -\lambda \ll -\epsilon \ll -\delta \ll 1$. [?]

Example by Chen Lan-sun and Wang Ming-Shu:
The following system

$$\begin{aligned}\dot{x} &= -y - \delta_2 x - 3x^2 + (1-\delta_1)xy + y^2 \\ \dot{y} &= x(1 + \tfrac{2}{3}x - 3y)\end{aligned}$$

has four limit cycles when $0 < \delta_2 \ll \delta_1 \ll 1$. [?]

References

[DRR] Dumortier, F., Roussarie, R., Rousseau, C.: Hilbert's 16th Problem for Quadratic Vector Fields. Journal of Differential Equations 110, 86-133, 1994.

[BR] R. Bamòn: Quadratic vector fields in the plane have a finite number of limit cycles, Publ. I.H.E.S. 64 (1986), 111-142.

[SS] Shi Songling, A concrete example of the existence of four limit cycles for plane quadratic systems, Scientia Sinica 23 (1980), 154-158.

[ZTWZ] Zhang Zhi-fen, Ding Tong-ren, Huang Wen-zoa, Dong Zhen-xi. Qualitative Theory of Differential Equations. American Mathematical Society, Providence, 1992.