symmetry of a solution of an ordinary differential equation

Canonical name	SymmetryOfASolutionOfAnOrdinaryDifferentialEquation
Date of creation	2013-03-22 13:42:26
Last modified on	2013-03-22 13:42:26
Owner	Daume (40)
Last modified by	Daume (40)
Numerical id	11
Author	Daume (40)
Entry type	Definition
Classification	msc 34-00
Synonym	symmetry of a periodic solution solution of an ordinary differential equation

Let γ be a http://planetmath.org/SymmetryOfAnOrdinaryDifferentialEquationsymmetry of the ordinary differential equation and x_0 be a steady state solution of $\dot{x} = f(x)$. If

$$\gamma x_0 = x_0$$

then γ is called a *symmetry of the solution of* x_0.

Let γ be a symmetry of the ordinary differential equation and $x_0(t)$ be a periodic solution of $\dot{x} = f(x)$. If

$$\gamma x_0(t - t_0) = x_0(t)$$

for a certain t_0 then (γ, t_0) is called a *symmetry of the periodic solution of* $x_0(t)$.

lemma: If γ is a symmetry of the ordinary differential equation and let $x_0(t)$ be a solution*(either steady state or periodic)* of $\dot{x} = f(x)$. Then $\gamma x_0(t)$ is a solution of $\dot{x} = f(x)$.
proof: If $x_0(t)$ is a solution of $\frac{dx}{dt} = f(x)$ implies $\frac{dx_0(t)}{dt} = f(x_0(t))$. Let's now verify that $\gamma x_0(t)$ is a solution, with a substitution into $\frac{dx}{dt} = f(x)$. The left hand side of the equation becomes $\frac{d\gamma x_0(t)}{dt} = \gamma \frac{dx_0(t)}{dt}$ and the right hand side of the equation becomes $f(\gamma x_0(t)) = \gamma f(x_0(t))$ since γ is a symmetry of the differential equation. Therefore we have that the left hand side equals the right hand side since $\frac{dx_0(t)}{dt} = f(x_0(t))$. qed

References

[GSS] Golubitsky, Martin. Stewart, Ian. Schaeffer, G. David: Singularities and Groups in Bifurcation Theory *(Volume II)*. Springer-Verlag, New York, 1988.