# Weierstrass sigma function

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**Definition 1.** *Let* $\Lambda \subset \mathbb{C}$ *be a lattice. Let* $\Lambda^*$ *denote* $\Lambda - \{0\}$.

1. *The* Weierstrass sigma function *is defined as the product*

$$\sigma(z;\Lambda) = z \prod_{w \in \Lambda^*} \left(1 - \frac{z}{w}\right) e^{z/w + \frac{1}{2}(z/w)^2}$$

2. *The* Weierstrass zeta function *is defined by the sum*

$$\zeta(z;\Lambda) = \frac{\sigma'(z;\Lambda)}{\sigma(z;\Lambda)} = \frac{1}{z} + \sum_{w \in \Lambda^*} \left(\frac{1}{z-w} + \frac{1}{w} + \frac{z}{w^2}\right)$$

*Note that the Weierstrass zeta function is basically the derivative of the logarithm of the sigma function. The zeta function can be rewritten as:*

$$\zeta(z;\Lambda) = \frac{1}{z} - \sum_{k=1}^{\infty} \mathcal{G}_{2k+2}(\Lambda) z^{2k+1}$$

*where* $\mathcal{G}_{2k+2}$ *is the Eisenstein series of weight* $2k+2$.

3. *The* Weierstrass eta function *is defined to be*

$$\eta(w;\Lambda) = \zeta(z+w;\Lambda) - \zeta(z;\Lambda), \textit{for any } z \in \mathbb{C}$$

*(It can be proved that this is well defined, i.e.* $\zeta(z+w;\Lambda) - \zeta(z;\Lambda)$ *only depends on* $w$*). The Weierstrass eta function must not be confused with the Dedekind eta function.*