# evaluation of beta function using Laplace transform

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| Canonical name | EvaluationOfBetaFunctionUsingLaplaceTransform |
| Date of creation | 2013-03-22 14:37:36 |
| Last modified on | 2013-03-22 14:37:36 |
| Owner | rspuzio (6075) |
| Last modified by | rspuzio (6075) |
| Numerical id | 10 |
| Author | rspuzio (6075) |
| Entry type | Derivation |
| Classification | msc 33B15 |

The beta integral can be evaluated elegantly using the http://planetmath.org/LaplaceTransf
theorem for Laplace transforms.

Start with the following Laplace transform:

$$s^{-\alpha} = \mathcal{L}\left[\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right] = \int_0^\infty e^{-st}\frac{t^{\alpha-1}}{\Gamma(\alpha)}dt$$

Since $s^{-q}s^{-p} = s^{-q-p}$, the convolution theorem imples that

$$\frac{t^{q-1}}{\Gamma(q)} * \frac{t^{p-1}}{\Gamma(p)} = \frac{t^{q+p-1}}{\Gamma(q+p)}$$

Writing out the definition of convolution, this becomes

$$\int_0^t \frac{(t-s)^{q-1}}{\Gamma(q)}\frac{s^{p-1}}{\Gamma(p)}ds = \frac{t^{q+p-1}}{\Gamma(p+q)}$$

Setting $t = 1$ and simplifying, we conclude that

$$\int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

QED