using Laplace transform to solve initial value problems

Canonical name	UsingLaplaceTransformToSolveInitialValueProblems
Date of creation	2015-05-29 15:21:45
Last modified on	2015-05-29 15:21:45
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	10
Author	pahio (2872)
Entry type	Example
Classification	msc 34A12
Classification	msc 44A10
Related topic	TableOfLaplaceTransforms
Related topic	LaplaceTransform

Since the Laplace transforms of the derivatives of $f(t)$ are polynomials in the transform parameter s (see table of Laplace transforms), forming the Laplace transform of a linear differential equation with constant coefficients and initial conditions at $t=0$ yields generally a simple equation (`http://planetmath.org/imageequation`image equation) for solving the transformed function $F(s)$. Since the initial conditions can be taken into consideration instantly, one needs not to determine the general solution of the differential equation.

For example, transforming the equation

$$f''(t)+2f'(t)+f(t)=e^{-t} \qquad (f(0)=0, \;\; f'(0)=1)$$

gives

$$[s^2F(s)-sf(0)-f'(0)]+2[sF(s)-f(0)]+F(s)=\frac{1}{s+1},$$

i.e.

$$(s^2+2s+1)F(s)=1+\frac{1}{s+1},$$

whence

$$F(s)=\frac{1}{(s+1)^2}+\frac{1}{(s+1)^3}.$$

Taking the inverse Laplace transform produces the result

$$f(t) \;=\; te^{-t}+\frac{t^2e^{-t}}{2} \;=\; \frac{e^{-t}}{2}(t^2+2t).$$