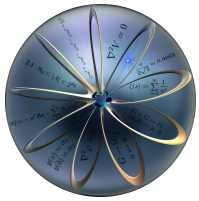

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## residues of tangent and cotangent

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|---|---|
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We will determine the residues of the tangent and the cotangent at their poles, which by the `http://planetmath.org/node/9074`parent entry are `http://planetmath.org/SimplePole`simple.

By the rule in the entry coefficients of Laurent series, in a simple pole $z = a$ of $f$ one has

$$\operatorname{Res}(f;\, a) \;=\; \lim_{z\to a}(z-a)f(z).$$

- We get first

$$\operatorname{Res}(\cot;\, 0) \;=\; \lim_{z\to 0} z\cot z \;=\; \lim_{z\to 0}\frac{\cos z}{\frac{\sin z}{z}} \;=\; \frac{1}{1} \;=\; 1. \qquad (1)$$

- All the poles of cotangent are $n\pi$ with $n \in \mathbb{Z}$. Since $\pi$ is the period of cotangent, we could infer that the residues in all poles are the same as (1). We may also calculate (with the change of variable $z - n\pi = w$) directly

$$\operatorname{Res}(\cot;\, n\pi) \;=\; \lim_{z\to n\pi}(z-n\pi)\cot z \;=\; \lim_{w\to 0} w\cot(w+n\pi) \;=\; \lim_{w\to 0} w\cot w \;=\; 1.$$

- In the `http://planetmath.org/ComplexTangentAndCotangent`parent entry, the complement formula for the tangent function is derived. Using it, we can find the residues of tangent at its poles $\frac{\pi}{2} + n\pi$, which are . For example,

$$\operatorname{Res}(\tan;\, \frac{\pi}{2}) \;=\; \lim_{z\to\frac{\pi}{2}}\left(z-\frac{\pi}{2}\right)\cot\left(\frac{\pi}{2}-z\right) \;=\; \lim_{w\to 0} w\cot(-w) \;=\; -\operatorname{Res}(\cot;\, 0) \;=\; -1.$$

  Similarly as above, the residues in other poles are $-1$.

Consequently, the residues of cotangent are equal to 1 and the residues of tangent equal to $-1$.