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## limit cycle

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Entry type	Definition
Classification	msc 34A12
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Synonym	$\omega$ -limit cycle
Synonym	$\alpha$ -limit cycle
Related topic	OmegaLimitSet
Defines	stable limit cycle
Defines	unstable limit cycle

Let

$$\dot{x} = f(x)$$

be a planar autonomous ordinary differential equation and  $\Gamma$  be a periodic solution of the system. If the [`http://planetmath.org/OmegaLimitSet`](http://planetmath.org/OmegaLimitSet) $\alpha$ -limit set or the [`http://planetmath.org/OmegaLimitSet`](http://planetmath.org/OmegaLimitSet) $\omega$ -limit set of a solution with initial value not on  $\Gamma$  and the respective limit set is  $\Gamma$  then  $\Gamma$  is a *limit cycle*. In simpler terms a limit cycle is an isolated periodic solution of the system.

A limit cycle,  $\Gamma$ , is a *stable limit cycle* (or  $\omega$ -limit cycle) if  $\Gamma$  is the  $\omega$ -limit set of all solutions in some neighborhood of  $\Gamma$ .

A limit cycle,  $\Gamma$ , is a *unstable limit cycle* (or  $\alpha$ -limit cycle) if  $\Gamma$  is the  $\alpha$ -limit set of all solutions in some neighborhood of  $\Gamma$ .<sup>[?]</sup>

## References

- [PL] Perko, Lawrence: Differential Equations and Dynamical Systems (*Third Edition*). Springer, New York, 2001.