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gradient system

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A *gradient system* in \mathbb{R}^n is an autonomous ordinary differential equation

$$\dot{x} = -\text{grad } V(x) \tag{1}$$

defined by the gradient of V where $V : \mathbb{R}^n \rightarrow \mathbb{R}$ and $V \in C^\infty$. The following results can be deduced from the definition of a gradient system.

Properties:

- The eigenvalues of the linearization of (??) evaluated at equilibrium point are real.
- If x_0 is an isolated minimum of V then x_0 is an asymptotically stable solution of (??)
- If $x(t)$ is a solution of (??) that is not an equilibrium point then $V(x(t))$ is a strictly decreasing function and is perpendicular to the level curves of V .
- There does not exist periodic solutions of (??).

References

- [HSD] Hirsch, W. Morris, Smale, Stephen, Devaney, L. Robert: Differential Equations, Dynamical Systems & An Introduction to Chaos. Elsevier Academic Press, New York, 2004.