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\wp -function

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Let L be a lattice on \mathbb{C} . The Weierstrass \wp function associated to L is given by

$$\wp(z) = \frac{1}{z^2} + \sum_{w \in L \setminus \{0\}} \left(\frac{1}{(z-w)^2} - \frac{1}{w^2} \right).$$

The \wp function is meromorphic and analytic on $\mathbb{C} \setminus L$, whereas at each $w \in L$, it has an order 2 pole. It is also an even function, because $\wp(z) = \wp(-z)$.

Its derivative

$$\wp'(z) = -2 \sum_{w \in L} \frac{1}{(z-w)^3}$$

is also an odd, meromorphic, and elliptic function, analytic at $\mathbb{C} \setminus L$ and having order 3 poles at each $w \in L$.

The functions \wp and \wp' form together a generator set for the field of elliptic functions associated to the lattice L .