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# bounds for e

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If $n$ and $m$ are positive integers and $n > m$, we have the following inequality:
$$\left(1 + \frac{1}{n}\right)^n < \frac{n}{n+1}\left(1 + \frac{1}{m}\right)^{m+1}$$

Taking the limit as $n \to \infty$, we obtain an upper bound for $e$. Combining this with the fact that the $(1 + 1/n)^n$ is an increasing sequence, we have the following bounds for $e$:

$$\left(1 + \frac{1}{m}\right)^m < e < \left(1 + \frac{1}{m}\right)^{m+1}$$

This can be used to show that $e$ is not an integer – if we take $m = 5$, we obtain $2.48832 < e < 2.985984$, for instance.