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### Jacobi's identity for $\vartheta$ functions

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Jacobi's identities describe how theta functions transform under replacing the period with the negative of its reciprocal. Together with the quasiperiodicity relations, they describe the transformations of theta functions under the modular group.

$$\theta_1(z \mid -1/\tau) = -i(-i\tau)^{1/2} e^{\frac{i\tau z^2}{\pi}} \theta_1(\tau z \mid \tau)$$

$$\theta_2(z \mid -1/\tau) = (-i\tau)^{1/2} e^{\frac{i\tau z^2}{\pi}} \theta_4(\tau z \mid \tau)$$

$$\theta_3(z \mid -1/\tau) = (-i\tau)^{1/2} e^{\frac{i\tau z^2}{\pi}} \theta_3(\tau z \mid \tau)$$

$$\theta_4(z \mid -1/\tau) = (-i\tau)^{1/2} e^{\frac{i\tau z^2}{\pi}} \theta_2(\tau z \mid \tau)$$