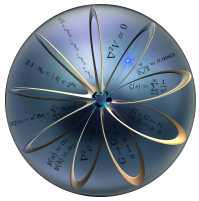

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addition and subtraction formulas for tangent

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The addition formula for tangent will be achieved via brute from the addition formulas for sine and cosine.

$$\begin{aligned}
\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \\
&= \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta} \\
&= \frac{\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\cos\beta}+\frac{\cos\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}}{\frac{\cos\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\cos\beta}-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}} \\
&= \frac{\tan\alpha\cdot 1+1\cdot\tan\beta}{1\cdot 1-\tan\alpha\tan\beta} \\
&= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}
\end{aligned}$$

Note that tan is an odd function, `http://planetmath.org/Iei`.e. $\tan(-x) = -\tan x$. This fact enables us to obtain the subtraction formula for tangent.

$$\tan(\alpha-\beta)=\tan(\alpha+(-\beta))=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$