



Math for the people, by the people.

eigenfunction

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Entry type	Definition
Classification	msc 34B24
Synonym	characteristics function
Defines	solution of system

Consider the Sturm-Liouville system given by

$$\frac{d}{dx} \left[ p(x) \frac{dy}{dx} \right] + q(x)y + \lambda r(x)y = 0 \quad a \leq x \leq b \quad (1)$$

$$a_1 y(a) + a_2 y'(a) = 0, \quad b_1 y(b) + b_2 y'(b) = 0, \quad (2)$$

where  $a_i, b_i \in \mathbb{R}$  with  $i \in \{1, 2\}$  and  $p(x), q(x), r(x)$  are differentiable functions and  $\lambda \in \mathbb{R}$ . A non zero solution of the system defined by (1) and (2) exists in general for a specified  $\lambda$ . The functions corresponding to that specified  $\lambda$  are called eigenfunctions.

More generally, if  $D$  is some linear differential operator, and  $\lambda \in \mathbb{R}$  and  $f$  is a function such that  $Df = \lambda f$  then we say  $f$  is an eigenfunction of  $D$  with eigenvalue  $\lambda$ .