example of derivative as parameter

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Owner	pahio (2872)
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Author	pahio (2872)
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For solving the (nonlinear) differential equation

$$x \;=\; \frac{y}{3p}-2py^2 \tag{1}$$

with $p = \frac{dy}{dx}$, according to **III** in the `http://planetmath.org/DerivativeAsParameterForSolvin` entry, we differentiate both sides in regard to y, getting first

$$\frac{1}{p} \;=\; \frac{1}{3p}-\left(\frac{y}{3p^2}+2y^2\right)\frac{dp}{dy}-4py.$$

Removing the denominators, we obtain

$$2p+(y+6p^2y^2)\frac{dp}{dy}+12p^3y = 0.$$

The left hand side can be factored:

$$(y\frac{dp}{dy}+2p)(1+6p^2y) = 0 \tag{2}$$

Now we may use the zero rule of product; the first factor of the product in (2) yields $y\frac{dp}{dy} = -2p$, i.e.

$$2\int\frac{dy}{y} = -\int\frac{dp}{p}+\ln C,$$

whence $y^2 = \frac{C}{p}$, i.e. $p = \frac{C}{y^2}$. Substituting this into the original equation (1) we get $x = \dfrac{y^3}{3C}-2C$. Hence the general solution of (1) may be written

$$y^3 = 3Cx+6C^2.$$

The second factor in (2) yields $6p^2y = -1$, which is substituted into (1) multiplied by $3p$:

$$3px = y-(-y)$$

Thus we see that $p = \frac{2y}{3x}$, which is again set into (1), giving

$$x = \frac{y\cdot 3x}{3\cdot 2y}-\frac{4y^3}{3x}.$$

Finally, we can write it

$$3x^2 = -8y^3,$$

which (a variant of the so-called semicubical parabola) is the singular solution of (1).