differential equation of circles

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All circles of the plane form a three-parametric family

$$(x - a)^2 + (y - b)^2 = r^2.$$

The parametres a, b, r may be eliminated by using successive differentiations, when one gets

$$x - a + (y - b)y' = 0,$$
$$1 + y'^2 + (y - b)y'' = 0,$$
$$3y'y'' + (y - b)y''' = 0.$$

The two last equations allow to eliminate also b, yielding the differential equation of all circles of the plane:

$$(1 + y'^2)y''' - 3y'y''^2 = 0$$

It is of three, corresponding the number of parametres.