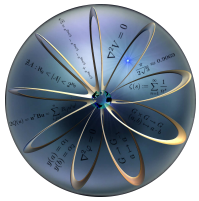

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## Gelfand spectral radius theorem

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For every self-consistent matrix norm, $||\cdot||$, and every square matrix $\mathbf{A}$ we can write

$$\rho(\mathbf{A}) = \lim_{n\to\infty} ||\mathbf{A}^n||^{\frac{1}{n}}.$$

Note: $\rho(\mathbf{A})$ denotes the spectral radius of $\mathbf{A}$.

This theorem also generalizes to infinite dimensions and plays an important role in the theory of operator algebras. If $\mathcal{A}$ is a Banach algebra with norm $||\cdot||$ and $A \in \mathcal{A}$, then we have

$$\rho(\mathbf{A}) = \lim_{n\to\infty} ||\mathbf{A}^n||^{\frac{1}{n}}.$$

It is worth pointing out that the self-consistency condition which was imposed on the matrix norm is part of the definition of a Banach algebra. A common case of the infinite-dimensional generalization occurs when $\mathcal{A}$ is the algebra of bounded operators on a Hilbert space — the operators may be regarded as an infinite-dimensional generalization of the square matrices.