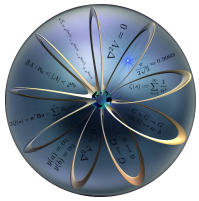

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## Dulac's criteria

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Let

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

be a planar system where $\mathbf{f} = (\mathbf{X}, \mathbf{Y})^t$ and $\mathbf{x} = (x, y)^t$. Furthermore $\mathbf{f} \in C^1(E)$ where $E$ is a simply connected region of the plane. If there exists a function $p(x, y) \in C^1(E)$ such that $\frac{\partial(p(x,y)\mathbf{X})}{\partial x} + \frac{\partial(p(x,y)\mathbf{Y})}{\partial y}$ *(the divergence of the vector field* $p(x, y)\boldsymbol{f}$*,* $\nabla \cdot p(x, y)\boldsymbol{f}$*)* is always of the same sign but not identically zero then there are no periodic solution in the region $E$ of the planar system. In addition, if $A$ is an annular region contained in $E$ on which the above condition is satisfied then there exists at most one periodic solution in $A$.