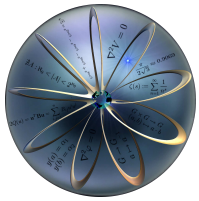

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Grammian determinant

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The Grammian determinant provides a necessary and sufficient method of determining whether a set of continuous functions $f_1, f_2, \ldots, f_n$ is linearly independent on an interval $I = [a, b]$ with respect to the inner product

$$\langle f_i | f_j \rangle = \int_I f_i f_j$$

It is defined as:

$$G(f_1, f_2, \ldots, f_n) = \begin{vmatrix} \int_I (f_1)^2 & \int_I f_1 f_2 & \cdots & \int_I f_1 f_n \\ \int_I f_2 f_1 & \int_I (f_2)^2 & \cdots & \int_I f_2 f_n \\ \vdots & \vdots & \ddots & \vdots \\ \int_I f_n f_1 & \int_I f_n f_2 & \cdots & \int_I (f_n)^2 \end{vmatrix}$$

If the functions are continuous on I, then $G = 0$ if and only if the set of functions is linearly dependent. Note that the Grammian determinant is a special case of the more general Gram determinant.