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method of undetermined coefficients

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| Canonical name | MethodOfUndeterminedCoefficients |
| Date of creation | 2013-03-22 12:52:55 |
| Last modified on | 2013-03-22 12:52:55 |
| Owner | CWoo (3771) |
| Last modified by | CWoo (3771) |
| Numerical id | 10 |
| Author | CWoo (3771) |
| Entry type | Definition |
| Classification | msc 34-00 |
| Related topic | ODE |
| Related topic | DifferentialEquation |

Given a (usually non-homogeneous) ordinary differential equation

$$F(x, f(x), f'(x), \dots, f^{(n)}(x)) = 0,$$

the *method of undetermined coefficients* is a way of finding an exact solution when a guess can be made as to the general form of the solution.

In this method, the form of the solution is guessed with unknown coefficients left as variables. A typical guess might be of the form Ae^{2x} or $Ax^2 + Bx + C$. This can then be substituted into the differential equation and solved for the coefficients. Obviously the method requires knowing the approximate form of the solution, but for many problems this is a feasible requirement.

This method is most commonly used when the formula is some combination of exponentials, polynomials, sin and cos.

Example

Suppose we have the following second order non-homogeneous equation

$$f''(x) - 2f'(x) + f(x) = 2e^{2x}.$$

If we guess that the solution is of the form $f(x) = Ae^{2x}$, then, by substitution, we get

$$4Ae^{2x} - 4Ae^{2x} + Ae^{2x} - 2e^{2x} = 0$$

and therefore $Ae^{2x} = 2e^{2x}$, so $A = 2$, giving $f(x) = 2e^{2x}$ as a solution.