differential equations of Jacobi ϑ functions

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The theta functions the following partial differential equation:

$$\frac{\pi i}{4} \frac{\partial^2 \vartheta_i}{\partial z^2} + \frac{\partial \vartheta_i}{\partial \tau} = 0$$

It is easy to check that each in the series which define the theta functions this differential equation. Furthermore, by the Weierstrass M-test, the series obtained by differentiating the series which define the theta functions term-by-term converge absolutely, and hence one may compute derivatives of the theta functions by taking derivatives of the series term-by-term.

Students of mathematical physics will recognize this equation as a one-dimensional diffusion equation. Furthermore, as may be seen by examining the series defining the theta functions, the theta functions approach periodic delta distributions in the limit $\tau \to 0$. Hence, the theta functions are the Green's functions of the one-dimensional diffusion equation with periodic boundary conditions.