derivation of Coulomb's Law from Gauss' Law

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As an example of the statement that Maxwell's equations completely define electromagnetic phenomena, it will be shown that Coulomb's Law may be derived from Gauss' law for electrostatics. Consider a point charge. We can obtain an expression for the electric field surrounding the charge. We surround the charge with a "virtual" sphere of radius R, then use Gauss' law in integral form:

$$\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q}{\epsilon_0}$$

We rewrite this as a volume integral in spherical polar coordinates over the "virtual" sphere mentioned above, which has the point charge at its centre. Since the electric field is spherically symmetric (by assumption) the electric field is constant over this volume.

$$\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \int_0^R \int_0^{2\pi} \int_0^{\pi} Er \sin\theta \, dr \, d\theta \, d\phi$$

Hence

$$4\pi R^2 E = \frac{q}{\epsilon_0}$$

Or

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

The usual form can then be recovered from the Lorentz force law, $\mathbf{F} = \mathbf{E}q + \mathbf{v} \times \mathbf{B}$ noting the absence of magnetic field.