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Feynman-Kac formula

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Let X_t be the n-dimensional Itō process satisfying the stochastic differential equation

$$dX_t = \mu(X_t)\,dt + \sigma(X_t)\,dW_t$$

and let A be its infinitesimal generator.

Further suppose that q is a lower-bounded continuous function on $\mathbb{R}^n$, and f is a twice-differentiable function on $\mathbb{R}^n$ with compact support. Then

$$u(t,x) = \mathbb{E}\Big[e^{-\int_0^t q(X_s)\,ds}\, f(X_t) \mid X_0 = x\Big]\,, \quad t \geq 0\,, x \in \mathbb{R}^n$$

is a solution to the partial differential equation

$$\frac{\partial u}{\partial t} = Au(x) - uq(x)$$

with initial condition $u(0,x) = f(x)$.

(The expectation for u is to be taken with respect to the probability measure under which W_t is a Brownian motion.)

References

[1] Bernt Øksendal. , *An Introduction with Applications*. 5th ed., Springer 1998.

[2] Hui-Hsiung Kuo. *Introduction to Stochastic Integration*. Springer 2006.