



planetmath.org

Math for the people, by the people.

heat equation

Canonical name	HeatEquation
Date of creation	2013-03-22 12:45:36
Last modified on	2013-03-22 12:45:36
Owner	drini (3)
Last modified by	drini (3)
Numerical id	5
Author	drini (3)
Entry type	Definition
Classification	msc 35Q99
Related topic	DifferentialEquation
Related topic	Laplacian

The heat equation in 1-dimension (for example, along a metal wire) is a partial differential equation of the following form:

$$\frac{\partial u}{\partial t} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$$

also written as

$$u_t = c^2 \cdot u_{xx}$$

Where $u : \mathbb{R}^2 \rightarrow \mathbb{R}$ is the function giving the temperature at time t and position x and c is a real valued constant. This can be easily extended to 2 or 3 dimensions as

$$u_t = c^2 \cdot (u_{xx} + u_{yy})$$

and

$$u_t = c^2 \cdot (u_{xx} + u_{yy} + u_{zz})$$

Note that in the steady state, that is when $u_t = 0$, we are left with the Laplacian of u :

$$\Delta u = 0$$