Beltrami differential equation

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Suppose that $\mu : G \subset \mathbb{C} \to \mathbb{C}$ is a measurable function, then the partial differential equation

$$f_{\bar{z}}(z) = \mu(z) f_z(z)$$

is called the *Beltrami differential equation*.

If furthermore $|\mu(z)| < 1$ and in fact $|\mu(z)|$ has a uniform bound less then 1 over the domain of definition, then the solution is a quasiconformal mapping with `http://planetmath.org/QuasiconformalMapping`complex dilation $\mu(z)$ and `http://planetmath.org/QuasiconformalMapping`maximal small dilatation $d_f = \sup_z |\mu(z)|$.

A conformal mapping has $f_{\bar{z}} \equiv 0$ and so the solution can be conformal if and only if $\mu \equiv 0$.

The partial derivatives f_z and $f_{\bar{z}}$ (where $\bar{z}$ is the complex conjugate of z) can here be given in terms of the real and imaginary parts of $f = u + iv$ as

$$\begin{aligned} f_z &= \frac{1}{2}(u_x + v_y) + \frac{i}{2}(v_x - u_y), \\ f_{\bar{z}} &= \frac{1}{2}(u_x - v_y) + \frac{i}{2}(v_x + u_y). \end{aligned}$$

References

[1] L. V. Ahlfors. . Van Nostrand-Reinhold, Princeton, New Jersey, 1966