spherical mean

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Let h be a function (usually real or complex valued) on $\mathbb{R}^n$ ($n \geq 1$). Its *spherical mean* at point x over a sphere of radius r is defined as

$$M_h(x,r) = \frac{1}{A(n-1)} \int_{\|\xi\|=1} h(x + r\xi)\, dS = \frac{1}{A(n-1,r)} \int_{\|\xi\|=|r|} h(x+\xi)\, dS,$$

where the integral is over the surface of the unit $n-1$-sphere. Here $A(n-1)$ is is the area of the unit sphere, while $A(n-1,r) = r^{n-1}A(n-1)$ is the `http://planetmath.org/AreaOfTheNSphere`area of a sphere of radius r. In essense, the spherical mean $M_h(x,r)$ is just the average of h over the surface of a sphere of radius r centered at x, as the name suggests.

The spherical mean is defined for both positive and negative r and is independent of its sign. As $r \to 0$, if h is continuous, $M_h(x,r) \to h(x)$. If h has two continuous derivatives (is in $C^2(\mathbb{R}^n)$) then the following identity holds:

$$\nabla_x^2 M_h(x,r) = \left(\frac{\partial^2}{\partial r^2} + \frac{n-1}{r} \frac{\partial}{\partial r} \right) M_h(x,r),$$

where ∇^2 is the Laplacian.

Spherical means are used to obtain an explicit general solution for the wave equation in n space and one time dimensions.