Poisson's equation

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Poisson's equation is a second-order partial differential equation which arises in physical problems such as finding the electric potential of a given charge distribution. Its general form in n is

$$\nabla^2\phi(\mathbf{r}) = \rho(\mathbf{r})$$

where ∇^2 is the Laplacian and $\rho : D \to \mathbb{R}$, often called a *source* , is a given function on some subset D of $\mathbb{R}^n$. If ρ is identically zero, the Poisson equation reduces to the Laplace equation.

The Poisson equation is linear, and therefore obeys the *superposition principle*: if $\nabla^2\phi_1 = \rho_1$ and $\nabla^2\phi_2 = \rho_2$, then $\nabla^2(\phi_1 + \phi_2) = \rho_1 + \rho_2$. This fact can be used to construct solutions to Poisson's equation from *fundamental solutions*, or *Green's functions*, where the source distribution is a delta function.

A very important case is the one in which $n = 3$, D is all of $\mathbb{R}^3$, and $\phi(\mathbf{r}) \to 0$ as $|\mathbf{r}| \to \infty$. The general solution is then given by

$$\phi(\mathbf{r}) = -\frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \mathrm{d}^3\mathbf{r}'.$$