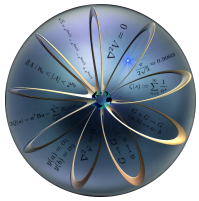

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smooth linear partial differential equation without solution

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Cauchy-Kowalewski theorem says that real analytic partial differential equations with real analytic initial data always have solutions. On the other hand Hans Lewy showed in 1957 that this is not true if the equation is only smooth. The example is obvious once we have the following theorem.

Theorem (Lewy)**.** *Let x, y, z be independent real variables. Let f be a C^1 real function. Suppose that there exists a C^1 solution u to the following equation*

$$\left[-\frac{\partial}{\partial x} - i\frac{\partial}{\partial y} + 2i(x+iy)\frac{\partial}{\partial z}\right] u = f'(z),$$

in some neighbourhood of a point $(0, 0, z_0)$. Then f is real analytic at z_0.

Hence we need only pick f which is smooth and not real analytic at z_0 and we have an example. For example, let $z_0 = 0$ and $f(x) = \int_0^x e^{-1/t}\, dt$.

References

[1] Lewy, Hans. Ann. of Math. (2) **66** (1957), 155–158.