Maxwell's equations

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Maxwell's equations are a set of four partial differential equations first combined by James Clerk Maxwell. They may also be written as integral equations. Two other important equations, the electromagnetic wave equation and the equation of conservation of charge, may be derived from them.

0.1 Notation

As this article considers merely the mathematical aspects of the equations, natural units have been used throughout. For their use in physics see any classical electromagnetism textbook.

$$\mathbf{E} = \text{Electric field strength}$$

$$\mathbf{B} = \text{Magnetic flux density}$$

0.2 Gauss' Law of Electrostatics

$$\nabla \cdot \mathbf{E} = 0$$
$$\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{S} = 0$$

0.3 Gauss' Law of Magnetostatics

$$\nabla \cdot \mathbf{B} = 0$$
$$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$$

0.4 Faraday's Law

Differential form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Integral form

$$\oint_C \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}\right)$$

0.5 Ampère's Law

Differential form

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

Integral form

$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \int_S \frac{\partial \mathbf{E}}{\partial t} \cdot \mathrm{d}\mathbf{S}$$

0.6 Properties of Maxwell's Equations

These four equations together have several interesting properties:

- Lorentz invariance
- The fields **E** and **B** may be Helmholtz decomposed into irrotational and solenoidal potentials. A gauge transformation in these variables does not affect the values of the fields.