wave equation

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Defines	d'Alembert's solution to the wave equation

The *wave equation* is a partial differential equation which describes certain kinds of waves. It arises in various physical situations, such as vibrating , waves, and electromagnetic waves.

The wave equation in one is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

The general solution of the one-dimensional wave equation can be obtained by a change of coordinates: $(x, t) \longrightarrow (\xi, \eta)$, where $\xi = x - ct$ and $\eta = x + ct$. This gives $\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$, which we can integrate to get *d'Alembert's solution*:

$$u(x, t) = F(x - ct) + G(x + ct)$$

where F and G are twice differentiable functions. F and G represent waves traveling in the positive and negative x directions, respectively, with velocity c. These functions can be obtained if appropriate initial conditions and boundary conditions are given. For example, if $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = g(x)$ are given, the solution is

$$u(x, t) = \frac{1}{2}[f(x - ct) + f(x + ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) \mathrm{d}s.$$

In general, the wave equation in n is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u.$$

where u is a function of the location variables $x_1, x_2, \ldots, x_n$, and time t. Here, ∇^2 is the Laplacian with respect to the location variables, which in Cartesian coordinates is given by $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_n^2}$.