mixing action

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Let X be a topological space and let G be a semigroup. An action $\phi = \{\phi_g\}_{g \in G}$ of G on X is (topologically) mixing if, given any two *open* subsets U, V of X, the intersection $U \cap \phi_g(V)$ is nonempty for all $g \in G$ except at most finitely many.

Example 1. Let $F : X \to X$ be a continuous function. Then F is topologically mixing if and only if the action of the monoid $\mathbb{N}$ on X defined by $\phi_n(x) = F^n(x)$ is mixing according to the definition given above.

Example 2. Suppose X is a discrete nonempty set and G is a group; endow X^G with the product topology. The action of G on X^G defined by

$$\phi_g(c)(z) = c(g \cdot z) \ \ \forall c : G \to X$$

is mixing.

To prove this fact, we may suppose without loss of generality that U and V are two *cylindric sets* of the form:

$$\begin{array}{rcl} U & = & \{c \in X^G \mid c|_E = u|_E\} \\ V & = & \{c \in X^G \mid c|_F = v|_F\} \end{array}$$

for suitable finite subsets $E, F \subseteq G$ and functions $u, v : G \to X$. Then the only chance for $U \cap \phi_g(V)$ to be empty, is that $e = gf$ for some $e \in E$, $f \in F$ such that $u(e) \neq v(f)$: but then, $g \in EF^{-1}$, which is finite.