ergodic theorem

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Let $(X, \mathfrak{B}, \mu)$ be a probability space, $f \in L^1(\mu)$, and $T\colon X \to X$ a measure preserving transformation. Birkhoff's *ergodic theorem* (often called the *pointwise* or *strong* ergodic theorem) states that there exists $f^* \in L^1(\mu)$ such that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} f(T^k x) = f^*(x)$$

for almost all $x \in X$. Moreover, f^* is T-invariant (i.e., $f^* \circ T = f^*$) almost everywhere and

$$\int f^* d\mu = \int f d\mu.$$

In particular, if T is ergodic then the T-invariance of f^* implies that it is constant almost everywhere, and so this constant must be the integral of f^*; that is, if T is ergodic, then

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} f(T^k x) = \int f d\mu$$

for almost every x. This is often interpreted in the following way: for an ergodic transformation, the time average equals the space average almost surely.