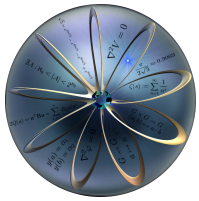

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chaotic dynamical system

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Synonym	chaotic behavior
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Related topic	SystemDefinitions

As Strogatz says in reference [1], "No definition of the term chaos is universally accepted yet, but almost everyone would agree on the three ingredients used in the following working definition".

Chaos is the aperiodic long-term in a deterministic system that exhibits sensitive dependence on initial conditions.

Aperiodic long-term means that there are trajectories which do not settle down to fixed points, periodic `http://planetmath.org/Orbit`orbits, or quasiperiodic as $t \to \infty$. For the purposes of this definition, a trajectory which approaches a limit of ∞ as $t \to \infty$ should be considered to have a fixed point at ∞.

Sensitive dependence on initial conditions means that nearby trajectories separate exponentially fast; `http://planetmath.org/Iei`.e., the system has a positive Liapunov exponent.

Strogatz notes that he favors additional constraints on the aperiodic long-term , but leaves `http://planetmath.org/OpenQuestion`open what form they may take. He suggests two alternatives to fulfill this:

1. Requiring that there exists an open set of initial conditions having aperiodic trajectories, or

2. If one picks a random initial condition $x(0)$ then there must be a nonzero chance of the associated trajectory $x(t)$ being aperiodic.

0.1 Further reading

1. B. Codenotti and Luciano Margara. Chaos in Mathematics, Physics, and Computer Science: Similarities and Dissimilarities. http://pespmc1.vub.ac.be/Einmag_A

0.2 References

1. Steven H. Strogatz, "Nonlinear Dynamics and Chaos". Westview Press, 1994.