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ω-limit set

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Let X be a metric space, and let $f : X \to X$ be a homeomorphism. The *ω-limit set* of $x \in X$, denoted by $\omega(x, f)$, is the set of cluster points of the forward orbit $\{f^n(x)\}_{n\in\mathbb{N}}$. Hence, $y \in \omega(x, f)$ if and only if there is a strictly increasing sequence of natural numbers $\{n_k\}_{k\in\mathbb{N}}$ such that $f^{n_k}(x) \to y$ as $k \to \infty$.

Another way to express this is

$$\omega(x, f) = \bigcap_{n\in\mathbb{N}} \overline{\{f^k(x) : k > n\}}.$$

The *α-limit set* is defined in a similar fashion, but for the backward orbit; i.e. $\alpha(x, f) = \omega(x, f^{-1})$.

Both sets are f-invariant, and if X is compact, they are compact and nonempty.

If $\varphi : \mathbb{R} \times X \to X$ is a continuous flow, the definition is similar: $\omega(x, \varphi)$ consists of those elements y of X for which there exists a strictly increasing sequnece $\{t_n\}$ of real numbers such that $t_n \to \infty$ and $\varphi(x, t_n) \to y$ as $n \to \infty$. Similarly, $\alpha(x, \varphi)$ is the ω-limit set of the reversed flow (i.e. $\psi(x, t) = \phi(x, -t)$). Again, these sets are invariant and if X is compact they are compact and nonempty. Furthermore,

$$\omega(x, f) = \bigcap_{n\in\mathbb{N}} \overline{\{\varphi(x, t) : t > n\}}.$$