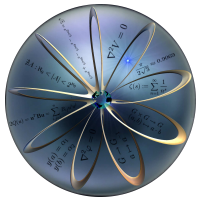

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proof of Poincaré recurrence theorem 2

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Let $\{U_n : n \in \mathbb{N}\}$ be a basis of open sets for X, and for each n define

$$U_n' = \{x \in U_n : \forall n \geq 1,\ f^n(x) \notin U_n\}.$$

From theorem 1 we know that $\mu(U_n') = 0$. Let $N = \bigcup_{n\in\mathbb{N}} U_n'$. Then $\mu(N) = 0$. We assert that if $x \in X - N$ then x is recurrent. In fact, given a neighborhood U of x, there is a basic neighborhood U_n such that $x \subset U_n \subset U$, and since $x \notin N$ we have that $x \in U_n - U_n'$ which by definition of U_n' means that there exists $n \geq 1$ such that $f^n(x) \in U_n \subset U$; thus x is recurrent. $\square$