Poincaré recurrence theorem

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Let $(X, \mathscr{S}, \mu)$ be a probability space and let $f\colon X \to X$ be a measure preserving transformation.

Theorem 1. *For any $E \in \mathscr{S}$, the set of those points x of E such that $f^n(x) \notin E$ for all $n > 0$ has zero measure. That is, almost every point of E returns to E. In fact, almost every point returns infinitely often; i.e.*

$$\mu\left(\{x \in E : \text{ there exists } N \text{ such that } f^n(x) \notin E \text{ for all } n > N\}\right) = 0.$$

The following is a topological version of this theorem:

Theorem 2. *If X is a second countable Hausdorff space and $\mathscr{S}$ contains the Borel sigma-algebra, then the set of recurrent points of f has full measure. That is, almost every point is recurrent.*