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fundamental theorem of demography

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Let  $A_t$  be a sequence of  $n \times n$  nonnegative primitive matrices. Suppose that  $A_t \rightarrow A_\infty$ , with  $A_\infty$  also a nonnegative primitive matrix. Define the sequence  $x_{t+1} = A_t x_t$ , with  $x_t \in \mathbb{R}^n$ . If  $x_0 \geq 0$ , then

$$\lim_{t \rightarrow \infty} \frac{x_t}{\|x_t\|} = p$$

where  $p$  is the normalized ( $\|p\| = 1$ ) eigenvector associated to the eigenvalue of  $A_\infty$  (also called the Perron-Frobenius eigenvector of  $A_\infty$ ).