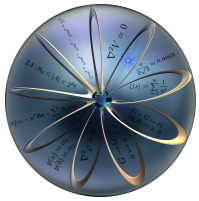

planetmath.org

Math for the people, by the people.

hyperbolic set

Canonical name	HyperbolicSet
Date of creation	2013-03-22 13:40:21
Last modified on	2013-03-22 13:40:21
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	5
Author	Koro (127)
Entry type	Definition
Classification	msc 37D20
Synonym	hyperbolic structure
Synonym	uniformly hyperbolic
Related topic	HyperbolicFixedPoint

Let M be a compact smooth manifold, and let $f : M \to M$ be a diffeomorphism. An f-invariant subset Λ of M is said to be *hyperbolic* (or to have an hyperbolic structure) if there is a splitting of the tangent bundle of M restricted to Λ into a (Whitney) sum of two Df-invariant subbundles, E^s and E^u such that the restriction of $Df|_{E^s}$ is a contraction and $Df|_{E^u}$ is an expansion. This means that there are constants $0 < \lambda < 1$ and $c > 0$ such that

1. $T_\Lambda M = E^s \oplus E^u$;
2. $Df(x)E^s_x = E^s_{f(x)}$ and $Df(x)E^u_x = E^u_{f(x)}$ for each $x \in \Lambda$;
3. $\|Df^n v\| < c\lambda^n \|v\|$ for each $v \in E^s$ and $n > 0$;
4. $\|Df^{-n} v\| < c\lambda^n \|v\|$ for each $v \in E^u$ and $n > 0$.

using some Riemannian metric on M.

If Λ is hyperbolic, then there exists an *adapted* Riemannian metric, i.e. one such that $c = 1$.