Oseledets multiplicative ergodic theorem

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Oseledets multiplicative ergodic theorem, or Oseledets decomposition, considerably extends the results of Furstenberg-Kesten theorem, under the same conditions.

Consider μ a probability measure, and $f : M \to M$ a measure preserving dynamical system. Consider $A : M \to GL(d, \mathbf{R})$, a measurable transformation, where GL(d,$\mathbf{R}$) is the space of invertible square matrices of size d. Consider the multiplicative cocycle $(\phi^n(x))_n$ defined by the transformation A, and assume $\log^+ ||A||$ and $\log^+ ||A^{-1}||$ are integrable.

Then, μ almost everywhere $x \in M$, one can find a natural number $k = k(x)$ and real numbers $\lambda_1(x) > \cdots > \lambda_k(x)$ and a filtration

$$\mathbf{R}^d = V_x^1 > \cdots > V_x^k > V_x^{k+1} = \{0\}$$

such that, for μ almost everywhere and for all $i \in \{1, \ldots, k\}$

1. $k(f(x)) = k(x)$ and $\lambda_i(f(x)) = \lambda_i(x)$ and $A(x) \cdot V_x^i = V_{f(x)}^i$;
2. $\lim_n \frac{1}{n} \log ||\phi^n(x)v|| = \lambda_i(x)$ for all $v \in V_x^i \backslash V_x^{i+1}$;
3. $\lim_n \frac{1}{n} \log |\det \phi^n(x)| = \sum_{i=1}^k d_i(x)\lambda_i(x)$ where $d_i(x) = \dim V_x^i - \dim V_x^{i+1}$

Furthermore, the numbers $k_i(x)$ and the subspaces V_x^i depend measurably on the point x.

The numbers $\lambda_i(x)$ are called *Lyapunov exponents* of A relatively to f at the point x. Each number $d_i(x)$ is called the *multiplicity* of the Lyapunov exponent $\lambda_i(x)$. We also have that $\lambda_1 = \lambda_{\max}$ and $\lambda_k = \lambda_{\min}$, where λ_{max} and $\lambda_{\min}$ are as given by Furstenberg-Kesten theorem.