flow

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A *flow* on a set X is a group action of $(\mathbb{R}, +)$ on X.

More explicitly, a flow is a function $\varphi : X \times \mathbb{R} \to X$ satisfying the following properties:

1. $\varphi(x, 0) = x$
2. $\varphi(\varphi(x, t), s) = \varphi(x, s + t)$

for all s, t in $\mathbb{R}$ and $x \in X$.

The set $\mathcal{O}(x, \varphi) = \{\varphi(x, t) : t \in \mathbb{R}\}$ is called the orbit of x by φ.

Flows are usually required to be continuous or differentiable, when the space X has some additional structure (e.g. when X is a topological space or when $X = \mathbb{R}^n$.)

The most common examples of flows arise from describing the solutions of the autonomous ordinary differential equation

$$y' = f(y), \quad y(0) = x \tag{1}$$

as a function of the initial condition x, when the equation has existence and uniqueness of solutions. That is, if (**??**) has a unique solution $\psi_x : \mathbb{R} \to X$ for each $x \in X$, then $\varphi(x, t) = \psi_x(t)$ defines a flow.