structural stability

Canonical name	StructuralStability
Date of creation	2013-03-22 13:48:45
Last modified on	2013-03-22 13:48:45
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	8
Author	Koro (127)
Entry type	Definition
Classification	msc 37C20
Classification	msc 34D30
Synonym	structurally stable

Given a metric space (X, d) and an homeomorphism $f \colon X \to X$, we say that f is structurally stable if there is a neighborhood $\mathscr{V}$ of f in $\mathrm{Homeo}(X)$ (the space of all homeomorphisms mapping X to itself endowed with the compact-open topology) such that every element of $\mathscr{V}$ is topologically conjugate to f.

If M is a compact smooth manifold, a $\mathcal{C}^k$ diffeomorphism f is said to be $\mathcal{C}^k$ structurally stable if there is a neighborhood of f in $\mathrm{Diff}^k(M)$ (the space of all $\mathcal{C}^k$ diffeomorphisms from M to itself endowed with the strong $\mathcal{C}^k$ topology) in which every element is topologically conjugate to f.

If X is a vector field in the smooth manifold M, we say that X is $\mathcal{C}^k$-structurally stable if there is a neighborhood of X in $\mathscr{X}^k(M)$ (the space of all $\mathcal{C}^k$ vector fields on M endowed with the strong $\mathcal{C}^k$ topology) in which every element is topologically equivalent to X, i.e. such that every other field Y in that neighborhood generates a flow on M that is topologically equivalent to the flow generated by X.

Remark. The concept of structural stability may be generalized to other spaces of functions with other topologies; the general idea is that a function or flow is structurally stable if any other function or flow close enough to it has similar dynamics (from the topological viewpoint), which essentially means that the dynamics will not change under small perturbations.