expansive

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Entry type	Definition
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Defines	expansivity
Defines	positively expansive
Defines	forward expansive

If (X, d) is a metric space, a homeomorphism $f\colon X \to X$ is said to be **expansive** if there is a constant $\varepsilon_0 > 0$, called the **expansivity constant**, such that for any two points of X, their n-th iterates are at least ε_0 apart for some integer n; i.e. if for any pair of points $x \neq y$ in X there is $n \in \mathbb{Z}$ such that $d(f^n(x), f^n(y)) \geq \varepsilon_0$.

The space X is often assumed to be compact, since under that assumption expansivity is a topological property; i.e. any map which is topologically conjugate to f is expansive if f is expansive (possibly with a different expansivity constant).

If $f\colon X \to X$ is a continuous map, we say that X is **positively expansive** (or forward expansive) if there is ε_0 such that, for any $x \neq y$ in X, there is $n \in \mathbb{N}$ such that $d(f^n(x), f^n(y)) \geq \varepsilon_0$.

Remarks. The latter condition is much stronger than expansivity. In fact, one can prove that if X is compact and f is a positively expansive homeomorphism, then X is finite (`http://planetmath.org/OnlyCompactMetricSpacesThatAdmitAPos`