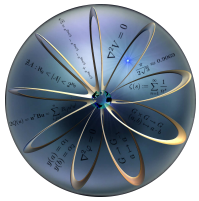

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Ω-stability theorem

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Let M be a differentiable manifold and let $f\colon M \to M$ be a $\mathcal{C}^k$ diffeomorphism. We say that f is $\mathcal{C}^k$-Ω-stable, if there is a neighborhood $\mathcal{U}$ of f in the $\mathcal{C}^k$ topology of $\mathrm{Diff}^k(M)$ such that for any $g \in \mathcal{U}$, $f|_{\Omega(f)}$ is topologically conjugate to $g|_{\Omega(g)}$.

Ω-stability theorem. If f is Axiom A and satisfies the no-cycles condition, then f is $\mathcal{C}^1$-Ω-stable.

Remark. The reciprocal of this theorem is also true (the difficult part is showing that Ω-stability implies Axiom A), but it is unknown whether $\mathcal{C}^k$-Ω-stability implies Axiom A when $k > 1$. This is known as the $\mathcal{C}^k$ *Ω-stability conjecture*.