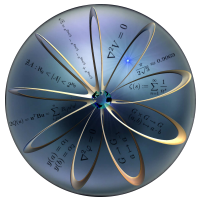

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hyperbolic isomorphism

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Entry type	Definition
Classification	msc 37D05
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Synonym	linear hyperbolic isomorphism

Let X be a Banach space and $T : X \to X$ a continuous linear isomorphism. We say that T is an *hyperbolic isomorphism* if its spectrum is disjoint with the unit circle, i.e. $\sigma(T) \cap \{z \in \mathbb{C} : |z| = 1\} = \emptyset$.

If this is the case, by the spectral theorem there is a splitting of X into two invariant subspaces, $X = E^s \oplus E^u$ (and therefore, a corresponding splitting of T into two operators $T^s : E^s \to E^s$ and $T_u : E^u \to E^u$, i.e. $T = T_s \oplus T_u$), such that $\sigma(T_s) = \sigma(T) \cap \{z : |z| < 1\}$ and $\sigma(T_u) = \sigma(T) \cap \{z : |z| > 1\}$. Also, for any λ greater than the spectral radius of both T_s and T_u^{-1} there exists an equivalent (box-type) norm $\|\cdot\|_1$ such that

$$\|T_s\|_1 < \lambda \text{ and } \|T_u^{-1}\|_1 < \lambda$$

and

$$\|x\|_1 = \max\{\|x_u\|_1, \|x_s\|_1\}.$$

In particular, λ can be chosen smaller than 1, so that T_s and T_u^{-1} are contractions.