no-cycles condition

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Let X be a metric space and let $f\colon X \to X$ be a homeomorphism. Suppose $\mathcal{F} = \{\Lambda_1, \ldots, \Lambda_k\}$ is a family of compact invariant sets for f. Define a relation $\to$ on $\mathcal{F}$ by $\Lambda_i \to \Lambda_j$ if

$$W^u(\Lambda_i) \cap W^s(\Lambda_j) - \bigcup_{l=1}^{k} \Lambda_l \neq \emptyset,$$

that is, if the unstable set of Λ_i intersects the stable set of Λ_j outside the union of the Λ_l's.

A cycle for $\mathcal{F}$ is a sequence $\{n_i : i = 1, \ldots, j\}$ such that

$$\Lambda_{n_i} \to \Lambda_{n_{i+1}}$$

for $1 \leq i < j$ and

$$\Lambda_{n_j} \to \Lambda_{n_1}.$$

With some abuse of notation, we can write this as

$$\Lambda_{n_1} \to \Lambda_{n_2} \to \cdots \to \Lambda_{n_j} \to \Lambda_{n_1}.$$

If $\mathcal{F}$ has no cycles, then we say that it satisfies the *no-cycles condition.*