proof of Poincaré recurrence theorem 1

Canonical name	ProofOfPoincareRecurrenceTheorem1
Date of creation	2013-03-22 14:29:56
Last modified on	2013-03-22 14:29:56
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	5
Author	Koro (127)
Entry type	Proof
Classification	msc 37A05
Classification	msc 37B20

Let $A_n = \cup_{k=n}^{\infty} f^{-k}E$. Clearly, $E \subset A_0$ and $A_i \subset A_j$ when $j \leq i$. Also, $A_i = f^{j-i}A_j$, so that $\mu(A_i) = \mu(A_j)$ for all $i, j \geq 0$, by the f-invariance of μ. Now for any $n > 0$ we have $E - A_n \subset A_0 - A_n$, so that

$$\mu(E - A_n) \leq \mu(A_0 - A_n) = \mu(A_0) - \mu(A_n) = 0.$$

Hence $\mu(E - A_n) = 0$ for all $n > 0$, so that $\mu(E - \cap_{n=1}^{\infty} A_n) = \mu(\cup_{n=1}^{\infty} E - A_n) = 0$. But $E - \cap_{n=1}^{\infty} A_n$ is precisely the set of those $x \in E$ such that for some n and for all $k > n$ we have $f^k(x) \notin E$. $\square$