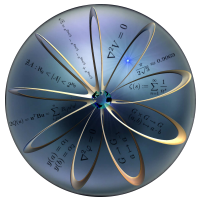

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Alexander trick

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Want to extend a homeomorphism of the circle S^1 to the whole disk D^2?

Let $f\colon S^1 \to S^1$ be a homeomorphism. Then the formula

$$F(x) = ||x|| f(x/||x||)$$

allows you to define a map $F\colon D^2 \to D^2$ which extends f, for if $x \in S^1 \subset D^2$ then $||x|| = 1$ and $F(x) = 1 \cdot f(x/1) = f(x)$. Clearly this map is continuous, save (maybe) the origin, since this formula is undefined there. Nevertheless this is removable.

To check continuity at the origin use: *"A map f is continuous at a point p if and only if for each sequence $x_n \to p$, $f(x_n) \to f(p)$"*.

So take a sequence $u_n \in D^2$ such that $u_n \to 0$ (i.e. which tends to the origin). Then $F(u_n) = ||u_n|| f(u_n/||u_n||)$ and since $f(u_n/||u_n||) \neq 0$, hence $||u_n|| \to 0$ implies $F(u_n) \to 0$, that is F is also continuous at the origin.

The same method works for f^{-1}.

In the same vein one can extend homeomorphisms $S^n \to S^n$ to $D^{n+1} \to D^{n+1}$.