Sharkovskii's theorem

Canonical name	SharkovskiisTheorem
Date of creation	2013-03-22 13:16:11
Last modified on	2013-03-22 13:16:11
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	7
Author	Koro (127)
Entry type	Definition
Classification	msc 37E05
Synonym	Sharkovsky's theorem
Defines	Sharkovskii's ordering
Defines	Sharkovsky's theorem

Every natural number can be written as $2^r p$, where p is odd, and r is the maximum exponent such that 2^r `http://planetmath.org/Divisibility`divides the given number. We define the *Sharkovskii ordering* of the natural numbers in this way: given two odd numbers p and q, and two nonnegative integers r and s, then $2^r p \succ 2^s q$ if

1. $r < s$ and $p > 1$;
2. $r = s$ and $p < q$;
3. $r > s$ and $p = q = 1$.

This defines a linear ordering of $\mathbb{N}$, in which we first have $3, 5, 7, \dots$, followed by $2 \cdot 3$, $2 \cdot 5, \dots$, followed by $2^2 \cdot 3$, $2^2 \cdot 5, \dots$, and so on, and finally $2^{n+1}, 2^n, \dots, 2, 1$. So it looks like this:

$$3 \succ 5 \succ \cdots \succ 3 \cdot 2 \succ 5 \cdot 2 \succ \cdots \succ 3 \cdot 2^n \succ 5 \cdot 2^n \succ \cdots \succ 2^2 \succ 2 \succ 1.$$

Sharkovskii's theorem. Let $I \subset \mathbb{R}$ be an interval, and let $f : I \to \mathbb{R}$ be a continuous function. If f has a periodic point of least period n, then f has a periodic point of least period k, for each k such that $n \succ k$.