divided difference

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Let f be a real (or complex) function. Given distinct real (or complex) numbers $x_0, x_1, x_2, \ldots$, the *divided differences* of f are defined recursively as follows:

$$\Delta^1 f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$
$$\Delta^{n+1} f[x_0, x_1, \ldots, x_{n+1}] = \frac{\Delta^n f[x_1, x_2, \ldots, x_{n+1}] - \Delta^n f[x_0, x_2, \ldots, x_{n+1}]}{x_1 - x_0}$$

It is also convenient to define the zeroth divided difference of f to be f itself:

$$\Delta^0 f[x_0] = f[x_0]$$

Some important properties of divided differences are:

1. Divided differences are invariant under permutations of $x_0, x_1, x_2, \ldots$

2. If f is a polynomial of order m and $m < n$, then the n-th divided differences of f vanish identically

3. If f is a polynomial of order $m + n$, then $\Delta^n(x, x_1, \ldots, x_n)$ is a polynomial in x of order m.

Divided differences are useful for interpolating functions when the values are given for unequally spaced values of the argument.

Becuse of the first property listed above, it does not matter with respect to which two arguments we compute the divided difference when we compute the $n + 1$-st divided difference from the n-th divided difference. For instance, when computing the divided difference table for tabulated values of a function, a convenient choice is the following:

$$\Delta^{n+1} f[x_0, x_1, \ldots, x_{n+1}] = \frac{\Delta^n f[x_1, x_2, \ldots, x_{n+1}] - \Delta^n f[x_0, x_1, \ldots, x_n]}{x_{n+1} - x_0}$$