best approximation

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One of the problems in approximation theory is to determine points that minimize distances (to a given point or subset). More precisely,

Problem - Let X be a metric space and $S \subseteq X$ a subset. Given $x_0 \in X$ we want to know if there exists a point in S that minimizes the distance to x_0, i.e. if there exists $y_0 \in S$ such that

$$d(x_0, y_0) = \inf_{y \in S} d(x_0, y)$$

Definition - A point y_0 that the above conditions is called a **best approximation** of x_0 in S.

In general, best approximations do not exist. Thus, the problem is usually to identify classes of spaces X and S where the existence of best approximations can be assured.

Example : When S is compact, best approximations of a given point $x_0 \in X$ in S always exist.

After one assures the existence of a best approximation, one can question about its uniqueness and how to calculate it explicitly.

Remark - There is no reason to restrict to metric spaces. The definition of best approximation can be given for pseudo-metric spaces, semimetric spaces or any other space where a suitable notion of "distance" can be given.