superconvergence

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A sequence $x_0, x_1, \ldots$ *superconverges to 0* if, when the x_i are written in base 2, then each number x_i starts with $2^i - 1 \approx 2^i$ zeroes. For example, the following sequence is superconverging to 0.

$$
\begin{array}{rccl}
x_{n+1} & = & x_n^2 & (x_n)_{10} \quad (x_n)_2 \\
x_0 & = & & \tfrac{1}{2} \qquad .1 \\
x_1 & = & & \tfrac{1}{4} \qquad .01 \\
x_2 & = & & \tfrac{1}{16} \qquad .0001 \\
x_3 & = & & \tfrac{1}{256} \qquad .00000001 \\
x_4 & = & & \tfrac{1}{65536} \qquad .0000000000000001 \\
\end{array}
$$

In this case it is easy to see that the number of binary 0's doubles each x_n.

A sequence $\{x_i\}$ *superconverges to x* if $\{x_i - x\}$ superconverges to 0, and a sequence $\{y_i\}$ is said to be *superconvergent* if there exists a y to which the sequence superconverges.