weaker version of Stirling's approximation

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One can prove a weaker version of Stirling's approximation without appealing to the gamma function. Consider the graph of $\ln x$ and note that

$$\ln(n-1)! \leq \int_1^n \ln x \, \mathrm{d}x \leq \ln n!$$

But $\int \ln x \, \mathrm{d}x = x \ln x - x$, so

$$\ln(n-1)! \leq n \ln n - n + 1 \leq \ln n!$$

and thus

$$n \ln n - n + 1 + \ln n \geq \ln(n-1)! + \ln n = \ln n! \geq n \ln n - n + 1$$

so

$$\ln n - 1 + \frac{1}{n} + \frac{\ln n}{n} \geq \frac{1}{n} \ln n! \geq \ln n - 1 + \frac{1}{n}$$

As n gets large, the expressions on either end approach $\ln n - 1$, so we have

$$\frac{1}{n} \ln n! \approx \ln n - 1$$

Multiplying through by n and exponentiating, we get

$$n! \approx n^n e^{-n}$$