Simpson's 3/8 rule

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Simpson's $\frac{3}{8}$ rule is a method for approximating a definite integral by evaluating the integrand at finitely many points. The formal rule is given by

$$\int_{x_0}^{x_3} f(x)\,dx \;\approx\; \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

where $x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h$.

Simpson's $\frac{3}{8}$ rule is the third Newton-Cotes quadrature formula. It has degree of precision 3. This means it is exact for polynomials of degree less than or equal to three. Simpson's $\frac{3}{8}$ rule is an improvement to the traditional Simpson's rule. The extra function evaluation gives a slightly more accurate approximation . We can see this with an example.

Using the fundamental theorem of the calculus, one shows

$$\int_0^{\pi} \sin(x)\,dx = 2.$$

In this case Simpson's rule gives,

$$\int_0^{\pi} \sin(x)\,dx \;\approx\; \frac{\pi}{6}\left[\sin(0) + 4\sin\left(\frac{\pi}{2}\right) + \sin(\pi)\right] \;=\; 2.094$$

However, Simpson's $\frac{3}{8}$ rule does slightly better.

$$\int_0^{\pi} \sin(x)\,dx \;\approx\; \left(\frac{3}{8}\right)\frac{\pi}{3}\left[\sin(0) + 3\sin\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{2\pi}{3}\right) + \sin(\pi)\right] \;=\; 2.040$$