midpoint rule

Canonical name	MidpointRule
Date of creation	2013-03-22 15:57:44
Last modified on	2013-03-22 15:57:44
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	16
Author	Wkbj79 (1863)
Entry type	Theorem
Classification	msc 41-01
Classification	msc 28-00
Classification	msc 26A42
Related topic	LeftHandRule
Related topic	RightHandRule
Related topic	RiemannSum
Related topic	ExampleOfEstimatingARiemannIntegral

The *midpoint rule* for computing the Riemann integral $\int_a^b f(x)\,dx$ is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{j=1}^{n} f\left(a+\left(j-\frac{1}{2}\right)\left(\frac{b-a}{n}\right)\right)\left(\frac{b-a}{n}\right).$$

If the Riemann integral is considered as a measure of area under a curve, then the expressions $f\left(a+\left(j-\frac{1}{2}\right)\left(\frac{b-a}{n}\right)\right)$ the of the rectangles, and $\frac{b-a}{n}$ is the common of the rectangles.

The Riemann integral can be approximated by using a definite value for n rather than taking a limit. In this case, the partition is $\left\{\left[a, a+\frac{b-a}{n}\right), \ldots, \left[a+\frac{(b-a)(n-1)}{n}, b\right]\right.$ and the function is evaluated at the midpoints of each of these intervals. Note that this is a special case of a Riemann sum in which the x_j's are evenly spaced and the c_j's chosen are the midpoints.

If f is Riemann integrable on $[a, b]$ such that $|f''(x)| \leq M$ for every $x \in [a, b]$, then

$$\left|\int_a^b f(x)\,dx - \sum_{j=1}^{n} f\left(a+\left(j-\frac{1}{2}\right)\left(\frac{b-a}{n}\right)\right)\left(\frac{b-a}{n}\right)\right| \leq \frac{M(b-a)^3}{24n^2}.$$