approximating sums of rational functions

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Given a sum of the form $\sum_{m=n}^{\infty} f(m)$ where f is a rational function, it is possible to approximate it by approximating f by another rational function which can be summed in closed form. Furthermore, the approximation so obtained becomes better as n increases.

We begin with a simple illustrative example. Suppose that we want to sum $\sum_{m=n}^{\infty} 1/m^2$. We approximate m^2 by $m^2 - 1/4$, which factors as $(m + 1/2)(m-1/2)$. Then, upon separating the approximate summand into partial fractions, the sum collapses:

$$\begin{aligned}
\sum_{m=n}^{\infty} \frac{1}{(m+1/2)(m-1/2)} &= \sum_{m=n}^{\infty} \left(\frac{1}{m-1/2} - \frac{1}{m+1/2} \right) \\
&= \sum_{m=n}^{\infty} \frac{1}{m-1/2} - \sum_{m=n+1}^{\infty} \frac{1}{m-1/2} \\
&= \frac{1}{n-1/2}
\end{aligned}$$

Using a similar approach, we may estimate the error of our approximation. [general method to come]