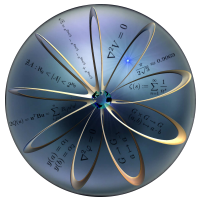

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converse to Taylor's theorem

Canonical name	ConverseToTaylorsTheorem
Date of creation	2013-03-22 15:05:42
Last modified on	2013-03-22 15:05:42
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	6
Author	jirka (4157)
Entry type	Theorem
Classification	msc 41A58
Synonym	Taylor's theorem converse
Related topic	TaylorSeries
Related topic	BorelLemma

Let $U \subset \mathbb{R}^n$ be an open set.

Theorem. *Let $f\colon U \to \mathbb{R}$ be a function such that there exists a constant $C > 0$ and an integer $k \geq 0$ such that for each $x \in U$ there is a polynomial $p_x(y)$ of k where*

$$|f(x+y) - p_x(y)| \leq C|y|^{k+1}$$

for y near 0. Then $f \in C^k(U)$ (f is k continuously differentiable) and the `http://planetmath.org/TaylorSeries`*Taylor expansion of k of f about any $x \in U$ is given by p_x.*

Note that when $k = 0$ the hypothesis of the theorem is just that f is Lipschitz in U which certainly makes it continuous in U.

References

[1] Steven G. Krantz, Harold R. Parks. . Birkhäuser, Boston, 2002.