Taylor series, derivation of

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Let $f(x)$ be given by the following power series:

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n + \cdots = \sum_{k=0}^{\infty} c_k(x-a)^k$$

Now let's compute a few derivatives at $x = a$.

$$f(a) = c_0; \quad f'(a) = c_1; \quad f''(a) = 2c_2; \quad f^{(3)}(a) = 6c_3; \quad f^{(n)}(a) = n!c_n$$

From this, it is clear that $c_n = \dfrac{f^{(n)}(a)}{n!}$, thus the series can be written as:

$$T_n = \sum_{k=0}^{n} c_k(x-a)^k = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

where $f(x) = T_\infty$.