approximation of the log function

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Because

$$\lim_{x \to 0} x \log(x) = \lim_{x \to 0} x^x - 1$$

we can approximate $\log(x)$ for small x:

$$\log(x) \approx \frac{x^x - 1}{x}.$$

A perhaps more interesting and useful result is that for x small we have the approximation

$$\log(1 + x) \approx x.$$

In general, if x is smaller than 0.1 our approximation is practical. This occurs because for small x, the area under the curve (which is what log is a measurement of) is approximately that of a rectangle of height 1 and width x.

Now when we combine this approximation with the formula $\log(ab) = \log(a) + \log(b)$, we can now approximate the logarithm of many positive numbers. In fact, scientific calculators use a (somewhat more precise) version of the same procedure.

For example, suppose we wanted $\log(1.21)$. If we estimate $\log(1.1) + \log(1.1)$ by taking $0.1 + 0.1 = 0.2$, we would be pretty close.