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**linear convergence**

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Defines	superlinear convergence

A sequence  $\{x_i\}$  is said to converge linearly to  $x^*$  if there is a constant  $1 > c > 0$  such that  $\|x_{i+1} - x^*\| \leq c\|x_i - x^*\|$  for all  $i > N$  for some natural number  $N > 0$ .

An alternative definition is that  $\|x_{i+1} - x_i\| \leq c\|x_i - x_{i-1}\|$  for all  $i$ .

Notice that if  $N = 1$ , then by iterating the first inequality we have

$$\|x_{i+1} - x^*\| \leq c^i \|x_1 - x^*\|.$$

That is, the error decreases exponentially with the index  $i$ .

If the inequality holds for all  $c > 0$  then we say that the sequence  $\{x_i\}$  has *superlinear convergence*.