discrete sine transform

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Related topic	DiscreteCosineTransform
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Defines	DST-I
Defines	DST-II
Defines	DST-III
Defines	DST-IV
Defines	DST-V
Defines	DST-VI
Defines	DST-VII
Defines	DST-VII
Defines	DST-VIII

The are a family of transforms closely related to the discrete cosine transform and the discrete Fourier transform. The set of variants of the DST was first introduced by Wang and Hunt [**?**].

1 Definition

The orthonormal variants of the DST, where x_n is the original vector of N real numbers, C_k is the transformed vector of N real numbers and δ is the Kronecker delta, are defined by the following equations:

1.1 DST-I

$$\begin{aligned} S_k^I &= p \sum_{n=0}^{N-1} x_n \sin \frac{\pi(n+1)(k+1)}{N+1} \qquad k = 0, 1, 2, \ldots, N-1 \\ p &= \sqrt{\frac{2}{N+1}} \end{aligned}$$

The DST-I is its own inverse.

1.2 DST-II

$$\begin{aligned} S_k^{II} &= p_k \sum_{n=0}^{N-1} x_n \sin \frac{\pi\left(n+\frac{1}{2}\right)(k+1)}{N} \qquad k = 0, 1, 2, \ldots, N-1 \\ p_k &= \sqrt{\frac{2-\delta_{k,0}}{N}} \end{aligned}$$

The inverse of DST-II is DST-III.

1.3 DST-III

$$\begin{aligned} S_k^{III} &= p\sum_{n=0}^{N-1} x_n q_n \sin\frac{\pi(n+1)\left(k+\frac{1}{2}\right)}{N} \qquad k=0,1,2,\ldots,N-1 \\ p &= \sqrt{\frac{2}{N}} \\ q_n &= \sqrt{\frac{1}{1+\delta_{n,0}}} \end{aligned}$$

The inverse of DST-III is DST-II.

1.4 DST-IV

$$\begin{aligned} S_k^{IV} &= p\sum_{n=0}^{N-1} x_n \sin\frac{\pi\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)}{N} \qquad k=0,1,2,\ldots,N-1 \\ p &= \sqrt{\frac{2}{N}} \end{aligned}$$

The DST-IV is its own inverse.

1.5 DST-V

$$\begin{aligned} S_k^{V} &= p\sum_{n=0}^{N-1} x_n \sin\frac{\pi(n+1)(k+1)}{N+\frac{1}{2}} \qquad k=0,1,2,\ldots,N-1 \\ p &= \sqrt{\frac{2}{N+\frac{1}{2}}} \end{aligned}$$

The DST-V is its own inverse.

1.6 DST-VI

$$\begin{aligned} S_k^{VI} &= p\sum_{n=0}^{N-1} x_n \sin\frac{\pi\left(n+\frac{1}{2}\right)(k+1)}{N+\frac{1}{2}} \qquad k=0,1,2,\ldots,N-1 \\ p &= \sqrt{\frac{2}{N+\frac{1}{2}}} \end{aligned}$$

The inverse of DST-VI is DST-VII.

1.7 DST-VII

$$
\begin{aligned}
S_k^{VII} &= p \sum_{n=0}^{N-1} x_n \sin \frac{\pi(n+1)\left(k+\frac{1}{2}\right)}{N+\frac{1}{2}} \qquad k = 0, 1, 2, \ldots, N-1 \\
p &= \sqrt{\frac{2}{N+\frac{1}{2}}}
\end{aligned}
$$

The inverse of DST-VII is DST-VI.

1.8 DST-VIII

$$
\begin{aligned}
S_k^{VIII} &= p_k \sum_{n=0}^{N-1} x_n q_n \sin \frac{\pi\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)}{N-\frac{1}{2}} \qquad k = 0, 1, 2, \ldots, N-1 \\
p_k &= \sqrt{\frac{2-\delta_{k,N-1}}{N-\frac{1}{2}}} \\
q_n &= \sqrt{\frac{1}{1+\delta_{n,N-1}}}
\end{aligned}
$$

The DST-VIII is its own inverse.

2 Two-dimensional DST

The DST in two dimensions is simply the one-dimensional transform computed in each row and each column. For example, the DST-II of a $N_1 \times N_2$ matrix is given by

$$
S_{k_1,k_2}^{II} = p_{k_1} p_{k_2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{n_1,n_2} \sin \frac{\pi\left(n_1+\frac{1}{2}\right)(k_1+1)}{N_1} \sin \frac{\pi\left(n_2+\frac{1}{2}\right)(k_2+1)}{N_2}
$$

References

[1] Xuancheng Shao, Steven G. Johnson. Type-II/III DCT/DST algorithms with reduced number of arithmetic operations. 2007.

[2] Markus Päuschel, José M. F. Mouray. The algebraic approach to the discrete cosine and sine transforms and their fast algorithms. 2006.

[3] Z. Wang and B. Hunt, The Discrete W Transform, Applied Mathematics and Computation, 16. 1985.