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Christoffel-Darboux formula

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Let $\{\phi_i\}_{i=0}^n$ be orthonormal polynomials (the degree of ϕ_k is k) and let k_n be the coefficient of x^n in ϕ_n . Then

$$\sum_{k=0}^n \phi_k(x)\phi_k(y) = \frac{k_n}{k_{n+1}} \left(\frac{\phi_n(y)\phi_{n+1}(x) - \phi_n(x)\phi_{n+1}(y)}{x - y} \right)$$

The reason this formula is interesting is that the left-hand side is the integral kernel for the projection operator to the subspace spanned by the polynomials $\{\phi_i\}_{i=0}^n$.