uniqueness of Fourier expansion

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If a real function f, Riemann integrable on the interval $[-\pi,\,\pi]$, may be expressed as sum of a trigonometric series

$$f(x) = \frac{a_0}{2}+\sum_{m=1}^{\infty}(a_m\cos mx+b_m\sin mx) \tag{1}$$

where the series $a_1+b_1+a_2+b_2+a_3+b_3+\ldots$ of the coefficients converges absolutely, then the series (1) converges uniformly on the interval and can be `http://planetmath.org/SumFunctionOfSeries`integrated termwise. The same concerns apparently the series which are gotten by multiplying the equation (1) by $\cos nx$ and by $\sin nx$; the results of the integrations determine for the coefficients a_n and b_n the unique values

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx,$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

for any n. So the Fourier series of f is unique.

As a consequence, we can infer that the well-known goniometric formula

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

presents the Fourier series of the even function $\sin^2 x$.