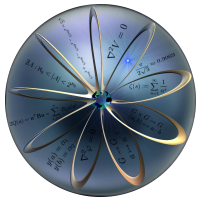

planetmath.org

Math for the people, by the people.

example of Fourier series

Canonical name	ExampleOfFourierSeries
Date of creation	2013-03-22 13:57:13
Last modified on	2013-03-22 13:57:13
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	10
Author	alozano (2414)
Entry type	Example
Classification	msc 42A16
Synonym	example of Fourier coefficients
Related topic	ValueOfTheRiemannZetaFunctionAtS2
Related topic	FourierSineAndCosineSeries

Here we present an example of Fourier series:

Example:

Let $f\colon (-\pi,\pi) \to \mathbb{R}$ be the "identity" function, defined by

$$f(x) = x, \text{ for all } x \in (-\pi,\pi).$$

We will compute the Fourier coefficients for this function. Notice that $\cos(nx)$ is an even function, while f and $\sin(nx)$ are odd functions.

$$\begin{aligned}
a_0^f &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x dx = 0 \\
a_n^f &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos(nx)dx = 0 \\
b_n^f &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)dx = \\
&= \frac{2}{\pi}\int_{0}^{\pi} x\sin(nx)dx = \frac{2}{\pi}\left(\left[-\frac{x\cos(nx)}{n}\right]_0^{\pi} + \left[\frac{\sin(nx)}{n^2}\right]_0^{\pi} =\right) = (-1)^{n+1}\frac{2}{n}
\end{aligned}$$

Notice that a_0^f, a_n^f are 0 because x and $x\cos(nx)$ are odd functions. Hence the Fourier series for $f(x) = x$ is:

$$\begin{aligned}
f(x) = x &= a_0^f + \sum_{n=1}^{\infty}(a_n^f\cos(nx) + b_n^f\sin(nx)) = \\
&= \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n}\sin(nx), \quad \forall x \in (-\pi,\pi)
\end{aligned}$$

For an application of this Fourier series, see value of the Riemann zeta function at $s = 2$.