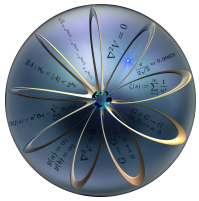

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countable basis

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A *countable basis* β of a vector space V over a field F is a countable subset $\beta \subset V$ with the property that every element $v \in V$ can be written as an infinite series

$$v = \sum_{x \in \beta} a_x x$$

in exactly one way (where $a_x \in F$). We are implicitly assuming, without further comment, that the vector space V has been given a topological structure or normed structure in which the above infinite sum is absolutely convergent (so that it converges to v regardless of the order in which the terms are summed).

The archetypical example of a countable basis is the Fourier series of a function: every continuous real-valued periodic function f on the unit circle $S^1 = \mathbb{R}/2\pi$ can be written as a Fourier series

$$f(x) = \sum_{n=0}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

in exactly one way.

Note: A countable basis is a countable set, but it is not usually a basis.