amenable group

Canonical name	AmenableGroup
Date of creation	2013-03-22 13:09:26
Last modified on	2013-03-22 13:09:26
Owner	mhale (572)
Last modified by	mhale (572)
Numerical id	9
Author	mhale (572)
Entry type	Definition
Classification	msc 43A07
Related topic	LpSpace
Defines	amenable
Defines	mean

Let G be a locally compact group and $L^\infty(G)$ be the Banach space of all essentially bounded functions $G \to \mathbb{R}$ with respect to the Haar measure.

Definition 1. A linear functional on $L^\infty(G)$ is called a **mean** if it maps the constant function $f(g) = 1$ to 1 and non-negative functions to non-negative numbers.

Definition 2. Let L_g be the left action of $g \in G$ on $f \in L^\infty(G)$, i.e. $(L_g f)(h) = f(g^{-1}h)$. Then, a mean μ is said to be **left invariant** if $\mu(L_g f) = \mu(f)$ for all $g \in G$ and $f \in L^\infty(G)$. Similarly, **right invariant** if $\mu(R_g f) = \mu(f)$, where R_g is the right action $(R_g f)(h) = f(hg)$.

Definition 3. A locally compact group G is **amenable** if there is a left (or right) invariant mean on $L^\infty(G)$.

Example 1 (Amenable groups)
All finite groups and all abelian groups are amenable. Compact groups are amenable as the Haar measure is an (unique) invariant mean.

Example 2 (Non-amenable groups)
If a group contains a free (non-abelian) subgroup on two generators then it is not amenable.