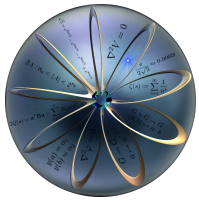

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rules for Laplace transform

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If $\mathcal{L}\{f(t)\} = F(s)$, then

- $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ for $s > a$,
- $\mathcal{L}\{f(\frac{t}{a})\} = a\,F(as)$ for $a > 0$.

For deriving these rules, we start from the definition of Laplace transform. In the first case, we shall use the notation $s-a = r$:

$$\mathcal{L}\{e^{at}f(t)\} = \int_0^\infty e^{-st}e^{at}f(t)\,dt = \int_0^\infty e^{-(s-a)t}f(t)\,dt = \int_0^\infty e^{-rt}f(t)\,dt = F(r) = F(s–a).$$

In the second case, we make the change of variable $\frac{t}{a} = u$ and later use the notation $sa = r$:

$$\mathcal{L}\{f(\frac{t}{a})\} = \int_0^\infty e^{-st}f(\frac{t}{a})\,dt = a\int_0^\infty e^{-sau}f(u)\,du = a\int_0^\infty e^{-ru}f(u)\,du = aF(r) = a\,F(as).$$