Fekete's subadditive lemma

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Let $(a_n)_n$ be a subadditive sequence in $[-\infty, \infty)$. Then, the following limit exists in $[-\infty, \infty)$ and equals the infimum of the same sequence:

$$\lim_n \frac{a_n}{n} = \inf_n \frac{a_n}{n}$$

Although the lemma is usually stated for subadditive sequences, an analogue conclusion is valid for superadditive sequences. In that case, for $(a_n)_n$ a subadditive sequence in $(-\infty, \infty]$, one has:

$$\lim_n \frac{a_n}{n} = \sup_n \frac{a_n}{n}$$

The proof of the superadditive case is obtained by taking the symmetric sequence $(-a_n)_n$ and applying the subadditive version of the theorem.