delay theorem

Canonical name	DelayTheorem
Date of creation	2013-03-22 18:02:46
Last modified on	2013-03-22 18:02:46
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	10
Author	pahio (2872)
Entry type	Theorem
Classification	msc 44A10
Synonym	delay theorem of Laplace transform
Related topic	HeavisideStepFunction
Related topic	TelegraphEquation

Theorem. If $f(t) \equiv 0$ for $t < 0$ and $\mathcal{L}\{f(t)\} := F(s)$, one has

$$\mathcal{L}\{f(t-t_0)\} \;=\; e^{-t_0 s}F(s).$$

Proof. Since $f(t-t_0) \equiv 0$ for $t < t_0$, the definition of Laplace transform at first gives

$$\mathcal{L}\{f(t-t_0)\} \;=\; \int_{t_0}^{\infty} e^{-st} f(t-t_0)\, dt.$$

The `http://planetmath.org/SubstitutionForIntegration`substitution $t-t_0 := u$ yields

$$\mathcal{L}\{f(t-t_0)\} \;=\; \int_0^{\infty} e^{-s(u+t_0)} f(u)\, du \;=\; e^{-t_0 s}\int_0^{\infty} e^{-su} f(u)\, du \;=\; e^{-t_0 s}F(s).$$

Corollary. For any $f(t)$ and the Heaviside step function $H(t)$, one has

$$\mathcal{L}\{f(t-a)H(t-a)\} \;=\; e^{-as}F(s).$$