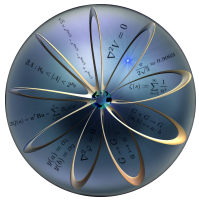

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logarithmic convolution

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Definition The *scale convolution* of two functions $s(t)$ and $r(t)$, also known as their *logarithmic convolution* is defined as the function

$$s *_l r(t) = r *_l s(t) = \int_0^\infty s(\frac{t}{a}) r(a) \frac{da}{a}$$

when this quantity exists.

Results The logarithmic convolution can be related to the ordinary convolution by changing the variable from t to $v = \log t$:

$$\begin{aligned} s *_l r(t) &= \int_0^\infty s(\frac{t}{a}) r(a) \frac{da}{a} = \int_{-\infty}^\infty s(\frac{t}{e^u}) r(e^u) du \\ &= \int_{-\infty}^\infty s(e^{\log t - u}) r(e^u) du \end{aligned}$$

Define $f(v) = s(e^v)$ and $g(v) = r(e^v)$ and let $v = \log t$, then

$$s *_l r(v) = f * g(v) = g * f(v) = r *_l s(v).$$