Laplace transform of logarithm

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Theorem. The Laplace transform of the natural logarithm function is

$$\mathcal{L}\{\ln t\} \;=\; \frac{\Gamma'(1)-\ln s}{s}$$

where Γ is Euler's gamma function.

Proof. We use the http://planetmath.org/LaplaceTransformOfPowerFunctionLaplace transform of the power function

$$\int_0^\infty e^{-st}t^a\,dt \;=\; \frac{\Gamma(a+1)}{s^{a+1}}$$

by differentiating it with respect to the parametre a:

$$\int_0^\infty e^{-st}t^a\ln t\,dt \;=\; \frac{\Gamma'(a+1)s^{a+1}-\Gamma(a+1)s^{a+1}\ln s}{(s^{a+1})^2} \;=\; \frac{\Gamma'(a+1)-\Gamma(a+1)\ln s}{s^{a+1}}$$

Setting here $a=0$, we obtain

$$\mathcal{L}\{\ln t\} \;=\; \int_0^\infty e^{-st}\ln t\,dt \;=\; \frac{\Gamma'(1)-1\cdot\ln s}{s},$$

Q.E.D.

Note. The number $\Gamma'(1)$ is equal the of the http://planetmath.org/EulersConstantEuler–Mascheroni constant, as is seen in the entry digamma and polygamma functions.