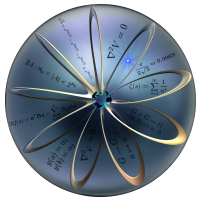

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table of Laplace transforms

Canonical name	TableOfLaplaceTransforms
Date of creation	2014-03-10 19:51:28
Last modified on	2014-03-10 19:51:28
Owner	CWoo (3771)
Last modified by	pahio (2872)
Numerical id	57
Author	CWoo (2872)
Entry type	Feature
Classification	msc 44A10
Related topic	LaplaceTransformOfIntegralSine
Related topic	TelegraphEquation
Related topic	InverseLaplaceTransformOfDerivatives
Related topic	IntegralTables
Related topic	UsingLaplaceTransformToInitialValueProblems
Related topic	RulesForLaplaceTransform
Related topic	ExponentialIntegral
Related topic	IntegrationOfLaplaceTransformWithRespectToParameter
Related topic	Lapla

Below are tables of http://planetmath.org/LaplaceTransformLaplace transforms; one lists some of the common properties, and the other lists some common examples.

Properties

Original	Transformed	comment	
$af(t) + bg(t)$	$a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$	linearity	
$f(t) * g(t)$	$\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$	convolution property	
$\int_a^b f(t,x)\,dx$	$\int_a^b \mathcal{L}\{f(t,x)\}\,dx$	integration with respect to a parametre	http://pla
$\frac{\partial}{\partial x} f(t,x)$	$\frac{\partial}{\partial x}\mathcal{L}\{f(t,x)\}$	diffentiation with respect to a parameter	http://planet
$f(\frac{t}{a})$	$aF(as)$	$\mathcal{L}\{f(t)\} = F(s)$	
$e^{at}f(t)$	$F(s-a)$	$\mathcal{L}\{f(t)\} = F(s)$	
$f(t-a)$	$e^{-as}F(s)$	$\mathcal{L}\{f(t)\} = F(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\mathcal{L}\{f(t)\} = F(s)$	
$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$	$\mathcal{L}\{f(t)\} = F(s)$	
$\int_0^t f(u)\,du$	$\frac{F(s)}{s}$	$\mathcal{L}\{f(t)\} = F(s)$	
$f'(t)$	$sF(s) - \lim_{x\to 0+} f(x)$	$\mathcal{L}\{f(t)\} = F(s)$	
$f''(t)$	$s^2F(s) - s\lim_{x\to 0+} f(x) - \lim_{x\to 0+} f'(x)$	$\mathcal{L}\{f(t)\} = F(s)$	

Examples

$f(t)$	$\mathcal{L}\{f(t)\}$	conditions	explanation	
e^{at}	$\frac{1}{s-a}$	$s > a$		
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$		http://planetmath.
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$		http://planetmath.
$\cosh at$	$\frac{s}{s^2-a^2}$	$s > \lvert a\rvert$		http://planetmath.
$\sinh at$	$\frac{a}{s^2-a^2}$	$s > \lvert a\rvert$		http://planetmath.
$\frac{\sin t}{t}$	$\arctan\frac{1}{s}$	$s > 0$	See sinc function	http://planetmath.
t^r	$\frac{\Gamma(r+1)}{s^{r+1}}$	$r > -1,\ \ s > 0$	gamma function Γ	http://planetmath.
$e^{a^2t}\operatorname{erf} a\sqrt{t}$	$\frac{a}{(s-a^2)\sqrt{s}}$	$s > a^2$	See error function	http://planetmath.org/
$e^{a^2t}\operatorname{erfc} a\sqrt{t}$	$\frac{1}{(a+\sqrt{s})\sqrt{s}}$	$s > 0$	See error function	http://planetmath.org/
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$	$s > 0$		http://planetmath.
$J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}$	$s > 0$	Bessel function J_0	http://planetmath.org
e^{-t^2}	$\frac{\sqrt{\pi}}{2}e^{\frac{s^2}{4}}\operatorname{erfc}\left(\frac{s}{2}\right)$	$s > 0$	See error function	http://planetmath.or
$\ln t$	$-\frac{\gamma+\ln s}{s}$	$s > 0$	Euler'sconstant γ	http://planetmat
$\delta(t)$	1		Dirac delta function	http://planetmath

Rational Functions

$f(t)$	$\mathcal{L}\{f(t)\}$	conditions	explanation	c
1	$\frac{1}{s}$			
t	$\frac{1}{s^2}$			http://planetmath.org/
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$			http://planetmath.org/
$\frac{1}{t+a}$	$e^{as}\mathrm{E}_1(as)$	$a>0$	exponential integral E_1	http://planetmath.c
$\frac{1}{(t+a)^2}$	$\frac{1}{a}-se^{as}\mathrm{E}_1(as)$	$a>0$		http://planetmath.c
$\frac{1}{(t+a)^n}$	$a^{1-n}e^{as}E_n(as)$	$a>0,\ \ n\in\mathbb{N}$	?	
$L_n(t)$	$\frac{1}{s}\left(\frac{s-1}{s}\right)^n$	$s>0$	Laguerre polynomial L_n	