Kingman's subadditive ergodic theorem

Canonical name	KingmansSubadditiveErgodicTheorem
Date of creation	2014-03-18 14:34:03
Last modified on	2014-03-18 14:34:03
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Last modified by	Filipe (28191)
Numerical id	5
Author	Filipe (28191)
Entry type	Theorem
Related topic	birkhoff ergodic theorem

Let $(M, \mathcal{A}, \mu)$ be a probability space, and $f : M \to M$ be a measure preserving dynamical system. let $\phi_n : M \to \mathbf{R}$, $n \geq 1$ be a subadditive sequence of measurable functions, such that ϕ_1^+ is integrable, where $\phi_1^+ = \max\{\phi, 0\}$. Then, the sequence $(\frac{\phi_n}{n})_n$ converges μ almost everywhere to a function $\phi : M \to [-\infty, \infty)$ such that:

ϕ^+ is integrable

ϕ is f invariant, that is, $\phi(f(x)) = \phi(x)$ for μ almost all x, and

$$\int \phi d\mu = \lim_n \frac{1}{n} \int \phi_n d\mu = \inf_n \frac{1}{n} \int \phi_n d\mu \in [-\infty, \infty)$$

The fact that the limit equals the infimum is a consequence of the fact that the sequence $\int \phi_n d\mu$ is a subadditive sequence and Fekete's subadditive lemma.

A superadditive version of the theorem also exists. Given a superadditive sequence φ_n, then the symmetric sequence is subadditive and we may apply the original version of the theorem.

Every additive sequence is subadditive. As a consequence, one can prove the Birkhoff ergodic theorem from Kingman's subadditive ergodic theorem.