Mellin's inverse formula

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It may be proven, that if a function $F(s)$ has the *inverse Laplace transform* $f(t)$, i.e. a piecewise continuous and exponentially real function f satisfying the condition

$$\mathcal{L}\{f(t)\} = F(s),$$

then $f(t)$ is uniquely determined when not regarded as different such functions which differ from each other only in a point set having Lebesgue measure zero.

The inverse Laplace transform is directly given by *Mellin's inverse formula*

$$f(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} F(s)\, ds,$$

by the Finn R. H. Mellin (1854—1933). Here it must be integrated along a straight line parallel to the imaginary axis and intersecting the real axis in the point γ which must be chosen so that it is greater than the real parts of all singularities of $F(s)$.

In practice, computing the complex integral can be done by using the Cauchy residue theorem.