Laplace transform of power function

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In the defining http://planetmath.org/ImproperIntegralintegral

$$\mathcal{L}\left\{t^r\right\} \;=\; \int_0^\infty e^{-st}t^r\,dt$$

of the Laplace transform of the power function $t \mapsto t^r$, we make the http://planetmath.org/Subst
$u := st$:

$$\mathcal{L}\left\{t^r\right\} \;=\; \int_0^\infty e^{-u}\left(\frac{u}{s}\right)^r\frac{du}{s} \;=\; \frac{1}{s^{n+1}}\int_0^\infty e^{-u}u^{r+1-1}\,du$$

Here we have assumed that $r > -1$ and $s > 0$. According to the definition of the gamma function, the last integral is equal to $\Gamma(r+1)$. Thus we obtain

$$\mathcal{L}\left\{t^r\right\} \;=\; \frac{\Gamma(r+1)}{s^{r+1}}. \tag{1}$$

The special case $r = -\frac{1}{2}$ gives the result

$$\mathcal{L}\left\{\frac{1}{\sqrt{t}}\right\} \;=\; \sqrt{\frac{\pi}{s}}. \tag{2}$$