# sine integral at infinity

| | |
|---|---|
| Canonical name | SineIntegralAtInfinity |
| Date of creation | 2013-03-22 15:17:22 |
| Last modified on | 2013-03-22 15:17:22 |
| Owner | pahio (2872) |
| Last modified by | pahio (2872) |
| Numerical id | 18 |
| Author | pahio (2872) |
| Entry type | Derivation |
| Classification | msc 44A10 |
| Classification | msc 30A99 |
| Synonym | limit of sine integral |
| Related topic | SineIntegral |
| Related topic | SincFunction |
| Related topic | SubstitutionNotation |
| Related topic | IncompleteGammaFunction |
| Related topic | ExampleOfSummationByParts |
| Related topic | SignumFunction |

The value of the improper integral (one of the *Dirichlet integrals*)

$$\int_0^\infty \frac{\sin x}{x}\,dx = \lim_{x\to\infty} \operatorname{Si} x,$$

where Si means the http://planetmath.org/SineIntegralsine integral function, is most simply determined by using Laplace transform which may be aimed to the integrand (see integration of Laplace transform with respect to parameter). Therefore the integrand must be equipped with an additional parametre $t$:

$$\mathcal{L}\{\int_0^\infty \frac{1}{x}\sin tx\,dx\} = \int_0^\infty \frac{1}{x}\cdot\frac{x}{s^2+x^2}\,dx = \int_0^\infty \frac{dx}{s^2+x^2} = \frac{1}{s}\Big/_{x=0}^{\infty} \arctan\frac{x}{s} = \frac{\pi}{2}\cdot\frac{1}{s}$$

The obtained transform $\frac{\pi}{2}\cdot\frac{1}{s}$ corresponds (see the inverse Laplace transformation) to the function $t \mapsto \frac{\pi}{2}$ because $\mathcal{L}\{1\} = \frac{1}{s}$. Thus we have the result

$$\int_0^\infty \frac{\sin x}{x}\,dx \;=\; \frac{\pi}{2}. \tag{1}$$

**Note 1.** Since $x \mapsto \frac{\sin x}{x}$ or $x \mapsto \operatorname{sinc} x$ is an even function, the result (1) may be written also

$$\int_{-\infty}^\infty \operatorname{sinc} x\,dx = \pi;$$

see the http://planetmath.org/SincFunctionsinc-function.

**Note 2.** The result (1) may be easily generalised to

$$\int_0^\infty \frac{\sin ax}{x}\,dx \;=\; \frac{\pi}{2} \qquad (a > 0) \tag{2}$$

and to

$$\int_0^\infty \frac{\sin ax}{x}\,dx \;=\; (\operatorname{sgn} a)\frac{\pi}{2} \qquad (a \in \mathbb{R}). \tag{3}$$