Kac's theorem

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Let $f : M \to M$ be a transformation and μ a finite invariant measure for f. Let E be a subset of M with positive measure. We define the first return map for E:

$$\rho_E(x) = \min\{n \geq 1 : f^n(x) \in E\}$$

If the set on the right is empty, then we define $\rho_E(x) = \infty$. The Poincar recurrence theorem asserts that ρ_E is finite for almost every $x \in R$. We define the following sets:

$$E_0 = \{x \in E : f^n(x) \notin E, n \geq 1\}$$

$$E_0^* = \{x \in M : f^n(x) \notin E, n \geq 0\}$$

By Poincar recurrence theorem, $\mu(E_0) = 0$. Kac's theorem asserts that the function ρ_E is integrable and

$$\int_E \rho_E d\mu = \mu(M) - \mu(E_0^*)$$

When the system is ergodic, then $\mu(E_0^*) = 0$, and Kac's theorem implies:

$$\frac{1}{\mu(E)} \int_E \rho_E d\mu = \frac{\mu(M)}{\mu(E)}$$

This equality can be interpreted as: the mean return time to E s inversely proportional to the measure of E.