integration of Laplace transform with respect to parameter

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We use the curved from the Laplace-transformed functions to the corresponding initial functions.

If

$$f(t,\, x) \;\backsim\; F(s,\, x),$$

then one can integrate both functions with respect to the parametre x between the same which may be also infinite provided that the integrals converge:

$$\int_a^b f(t,\, x)\, dx \;\backsim\; \int_a^b F(s,\, x)\, dx \tag{1}$$

(1) may be written as

$$\mathcal{L}\{\int_a^b f(t,\, x)\, dx\} \;=\; \int_a^b \mathcal{L}\{f(t,\, x)\}\, dx. \tag{2}$$

Proof. Using the definition of the Laplace transform, we can write

$$\int_a^b f(t,\, x)\, dx \;\backsim\; \int_0^\infty \left(e^{-st} \int_a^b f(s,\, x)\, dx\right) dt \;=\; \int_0^\infty \left(\int_a^b e^{-st} f(s,\, x)\, dx\right) dt.$$

We change the of integration in the last double integral and use again the definition, obtaining

$$\int_a^b f(t,\, x)\, dx \;\backsim\; \int_a^b \left(\int_0^\infty e^{-st} f(s,\, x)\, dt\right) dx \;=\; \int_a^b F(s,\, t)\, dx,$$

Q.E.D.