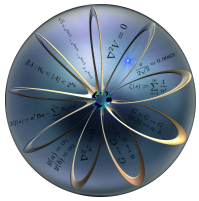

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Maximal ergodic theorem

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Let $(X, \mathcal{B}, \mu)$ be a probability space and $T : X \to X$ a measure preserving transformation. Let f be a $L^1(\mu)$ function. Define the averages

$$f^*(x) = \sup_{N \geq 1} \frac{1}{N} \sum_{i=0}^{N-1} f(T^i(x))$$

Then, for any $\lambda \in \mathbf{R}$, we have:

$$\int_{f^* > \lambda} f d\mu \geq \lambda \mu(\{f^* > \lambda\})$$

This theorem may be used in the proof of the ergodic theorem (also known as Birkhoff ergodic theorem, or pointwise or strong ergodic theorem)