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Laplace transform of derivative

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Theorem. If the real function $t \mapsto f(t)$ and its derivative are Laplace-transformable and f is continuous for $t > 0$, then

$$\mathcal{L}\{f'(t)\} \;=\; s\,F(s) - \lim_{t\to 0+} f(t). \tag{1}$$

Proof. By the definition of Laplace transform and using integration by parts, the left hand side of (1) may be written

$$\int_0^\infty e^{-st} f'(t)\,dt \;=\; \Big/_{\!\!t=0}^{\,\infty} e^{-st} f(t) + s\int_0^\infty e^{-st} f(t)\,dt \;=\; \lim_{t\to\infty} e^{-st} f(t) - \lim_{t\to 0} e^{-st} f(t) + s\,F(s).$$

The Laplace-transformability of f implies that $e^{-st}f(t)$ tends to zero as t increases boundlessly. Thus the last expression leads to the right hand side of (1).