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# Lipschitz inverse mapping theorem

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Let $(E, \|\cdot\|)$ be a Banach space and let $A\colon E \to E$ be a bounded linear isomorphism with bounded inverse (i.e. a topological linear automorphism); let $B(r)$ be the ball with center 0 and radius $r$ (we allow $r = \infty$). Then for any Lipschitz map $\phi\colon B(r) \to E$ such that $\operatorname{Lip}\phi < \|A^{-1}\|^{-1}$ and $\phi(0) = 0$, there are open sets $U \subset E$ and $V \subset B(r)$ and a map $T\colon U \to V$ such that $T(A+\phi) = I|_V$ and $(A+\phi)T = I|_U$. In other words, there is a local inverse of $A + \phi$ near zero. Furthermore, the inverse $T$ is Lipschitz with $\operatorname{Lip} T \leq (\|A\| + \operatorname{Lip}\phi)^{-1}$ and

$$B\left(r(\|A^{-1}\|^{-1} - \operatorname{Lip}\phi)\right) \subset U.$$

*Remark.* The inclusion above implies that $A + \phi\colon E \to E$ is invertible if $r = \infty$.

*Remark.* $\operatorname{Lip}\phi$ denotes the smallest Lipschitz constant of $\phi$.