topological vector space

Canonical name	TopologicalVectorSpace
Date of creation	2013-03-22 12:16:55
Last modified on	2013-03-22 12:16:55
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	22
Author	yark (2760)
Entry type	Definition
Classification	msc 46A99
Synonym	TVS
Synonym	linear topological space
Synonym	topological linear space
Related topic	TopologicalRing
Related topic	FrechetSpace

Definition

A *topological vector space* is a pair $(V, \mathcal{T})$, where V is a vector space over a topological field K, and $\mathcal{T}$ is a topology on V such that under $\mathcal{T}$ the scalar multiplication $(\lambda, v) \mapsto \lambda v$ is a continuous function $K \times V \to V$ and the vector addition $(v, w) \mapsto v + w$ is a continuous function $V \times V \to V$, where $K \times V$ and $V \times V$ are given the respective product topologies.

We will also require that $\{0\}$ is closed (which is equivalent to requiring the topology to be Hausdorff), though some authors do not make this requirement. Many authors require that K be either $\mathbb{R}$ or $\mathbb{C}$ (with their usual topologies).

Topological vector spaces as topological groups

A topological vector space is necessarily a topological group: the definition ensures that the group operation (vector addition) is continuous, and the inverse operation is the same as multiplication by -1, and so is also continuous.

Finite-dimensional topological vector spaces

A finite-dimensional vector space inherits a natural topology. For if V is a finite-dimensional vector space, then V is isomorphic to K^n for some n; then let $f\colon V \to K^n$ be such an isomorphism, and suppose that K^n has the product topology. Give V the topology where a subset A of V is open in V if and only if $f(A)$ is open in K^n. This topology is independent of the choice of isomorphism f, and is the `http://planetmath.org/Coarser`finest topology on V that makes it into a topological vector space.