Gabor frame

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Entry type	Definition
Classification	msc 46C99
Defines	Gabor frame
Defines	Gabor super-frame
Defines	Vector-valued Gabor frame

One may be interested in Gabor frames and its related theory if one looks further into the frame framework. First, denote a lattice by $\Lambda = A\mathbb{Z}^{2d}$, where A is an invertible matrix, and let $\pi(\xi, \phi)f = e^{2\pi i\xi x} f(x - \phi)$

Definition. *Let $g \in L^2(\mathbb{R}^d)$ be a nonzero window, and let $\lambda \in \Lambda$, then*

$$G(g, \lambda) = \{\pi(\lambda)g : \lambda \in \Lambda\}$$

is a Gabor system. If $G(g, \lambda)$ is a frame, it's called a Gabor frame for $L^2(\mathbb{R}^d)$

Supose now that one wants to look at a more general framework, and work with functions in $L^2(\mathbb{R}^d, \mathbb{C}^n)$. Then the definition above generalises to

Definition. *Let $\boldsymbol{g} \in L^2(\mathbb{R}^d, \mathbb{C}^n)$ be a nonzero window and let $\lambda \in \Lambda$, then*

$$\boldsymbol{G}(\boldsymbol{g}, \lambda) = \{\pi(\lambda)\boldsymbol{g} : \lambda \in \Lambda\}$$

is a Gabor super-frame if the frame inequalities hold, where

$$\pi(\xi, \phi)\boldsymbol{g} = e^{2\pi \imath x \cdot \xi} \left(g_1(x - \phi), g_2(x - \phi), ..., g_n(x - \phi)\right)$$

and for $\boldsymbol{f}, \boldsymbol{h} \in L^2(\mathbb{R}^d, \mathbb{C}^n)$

$$\langle \boldsymbol{f}, \boldsymbol{h} \rangle_{L^2(\mathbb{R}^d, \mathbb{C}^n)} = \sum_{i=1}^{n} \langle f_i, h_i \rangle_{L^2(\mathbb{R}^d)}$$

References

[1] Karlheinz Grchenig, ”Foundations of Time-Frequency Analysis,” *Birkhhuser* (2000)