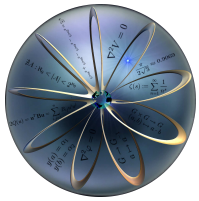

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Gelfand-Naimark theorem

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Let **Haus** be the category of locally compact Hausdorff spaces with continuous proper maps as morphisms. And, let $\mathbf{C^*Alg}$ be the category of commutative C^*-algebras with proper *-homomorphisms (send approximate units into approximate units) as morphisms. There is a contravariant functor $C\colon \mathbf{Haus}^{\mathrm{op}} \to \mathbf{C^*Alg}$ which sends each locally compact Hausdorff space X to the commutative C^*-algebra $C_0(X)$ ($C(X)$ if X is compact). Conversely, there is a contravariant functor $M\colon \mathbf{C^*Alg}^{\mathrm{op}} \to \mathbf{Haus}$ which sends each commutative C^*-algebra A to the space of characters on A (with the Gelfand topology).

The functors C and M are an equivalence of categories.