properties of states

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Let $\mathcal{A}$ be a http://planetmath.org/CAlgebraC^*-algebra and $x \in \mathcal{A}$.

Let $S(\mathcal{A})$ and $P(\mathcal{A})$ denote the http://planetmath.org/Statestate space and the pure state space of $\mathcal{A}$, respectively.

0.1 States

The space is sufficiently large to reveal many of elements of a C^*-algebra.

Theorem 1- We have that

- $S(\mathcal{A})$ separates points, i.e. $x = 0$ if and only if $\phi(x) = 0$ for all $\phi \in S(\mathcal{A})$.
- x is http://planetmath.org/InvolutaryRingself-adjoint if and only if $\phi(x) \in \mathbb{R}$ for all $\phi \in S(\mathcal{A})$.
- x is positive if and only if $\phi(x) \geq 0$ for all $\phi \in S(\mathcal{A})$.
- If x is http://planetmath.org/InvolutaryRingnormal, then $\phi(x) = \|x\|$ for some $\phi \in S(\mathcal{A})$.

0.2 Pure states

The pure state space is also sufficiently large to the of Theorem 1. Hence, we can replace $S(\mathcal{A})$ by $P(\mathcal{A})$, or by any other family of linear functionals F such that $P(\mathcal{A}) \subset F \subset S(\mathcal{A})$, in the previous result.

Theorem 2 - We have that

- $P(\mathcal{A})$ separates points, i.e. $x = 0$ if and only if $\phi(x) = 0$ for all $\phi \in P(\mathcal{A})$.
- x is if and only if $\phi(x) \in \mathbb{R}$ for all $\phi \in P(\mathcal{A})$.
- x is positive if and only if $\phi(x) \geq 0$ for all $\phi \in P(\mathcal{A})$.
- If x is , then $\phi(x) = \|x\|$ for some $\phi \in P(\mathcal{A})$.

- Every multiplicative linear functional on $\mathcal{A}$ is a pure state.