every finite dimensional subspace of a normed space is closed

Canonical name	EveryFiniteDimensionalSubspaceOfANormedSpaceIsClosed
Date of creation	2013-03-22 14:58:56
Last modified on	2013-03-22 14:58:56
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	14
Author	Mathprof (13753)
Entry type	Corollary
Classification	msc 46B99
Classification	msc 15A03
Classification	msc 54E52

Theorem 1 *Any finite dimensional subspace of a normed vector space is closed.*

Proof. Let $(V, \|\cdot\|)$ be such a normed vector space, and $S \subset V$ a finite dimensional vector subspace.

Let $x \in V$, and let $(s_n)_n$ be a sequence in S which converges to x. We want to prove that $x \in S$. Because S has finite dimension, we have a basis $\{x_1, ..., x_k\}$ of S. Also, $x \in \text{span}(x_1, ..., x_k, x)$. But, as proved in the case when V is finite dimensional (see this `http://planetmath.org/EverySubspaceOfANormedSpaceOfFinit`
we have that S is closed in $\text{span}(x_1, ..., x_k, x)$ (taken with the norm induced by $(V, \|\cdot\|)$) with $s_n \to x$, and then $x \in S$. QED.

0.0.1 Notes

The definition of a normed vector space requires the ground field to be the real or complex numbers. Indeed, consider the following counterexample if that condition doesn't hold:

$V = \mathbb{R}$ is a $\mathbb{Q}$ - vector space, and $S = \mathbb{Q}$ is a vector subspace of V. It is easy to see that $\dim(S) = 1$ (while $\dim(V)$ is infinite), but S is not closed on V.