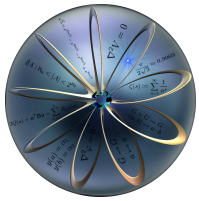

planetmath.org

Math for the people, by the people.

von Neumann algebras contain the range projections of its elements

Canonical name	VonNeumannAlgebrasContainTheRangeProjectionsOfItsElements
Date of creation	2013-03-22 17:28:57
Last modified on	2013-03-22 17:28:57
Owner	asteroid (17536)
Last modified by	asteroid (17536)
Numerical id	5
Author	asteroid (17536)
Entry type	Result
Classification	msc 46L10
Classification	msc 47A05

- Let T be an operator in a von Neumann algebra $\mathcal{M}$ acting on an Hilbert space H. Then the orthogonal projection onto the range of T and the orthogonal projection onto the kernel of T both belong to $\mathcal{M}$.

Proof : Let $T = VR$ be the polar decomposition of T with $KerV = KerR$.

By the result on the `http://planetmath.org/PolarDecompositionInVonNeumannAlgebras`pa entry we see that $V \in \mathcal{M}$.

As V is a partial isometry, VV^* is the () projection onto the range of T, and $I - V^*V$ is the () projection onto the kernel of T, where I is the identity operator in $\mathcal{M}$.

Therefore the () projections onto the range and kernel of T both belong to $\mathcal{M}$. $\square$