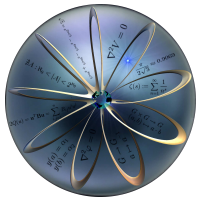

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Hilbert parallelotope

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The *Hilbert parallelotope* I^ω is a closed subset of the Hilbert space $\mathbb{R}\ell^2$ (The symbol '$\mathbb{R}$' has been prefixed to indicate that the field of scalars is $\mathbb{R}$.) defined as

$$I^\omega = \{(a_0, a_1, a_2, \ldots) \mid 0 \leq a_i \leq 1/(i+1)\}$$

As a topological space, I^ω is homeomorphic to the product of a countably infinite number of copies of the closed interval $[0, 1]$. By Tychonoff's theorem, this product is compact, so the Hilbert parallelotope is a compact subset of Hilbert space. This fact also explains the notation I^ω.

The Hilbert parallelotope enjoys a remarkable universality property — every second countable metric space is homeomorphic to a subset of the Hilbert parallelotope. Since second countability is hereditary, the converse is also true — every subset of the Hilbert parallelotope is a second countable metric space.