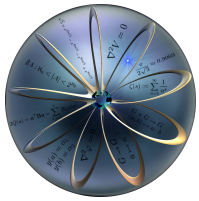

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commutant

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Definition

Let H be an Hilbert Space, $B(H)$ the algebra of bounded operators in H and $\mathcal{F} \subset B(H)$.

The **commutant** of $\mathcal{F}$, usually denoted $\mathcal{F}'$, is the subset of $B(H)$ consisting of all elements that commute with every element of $\mathcal{F}$, that is

$$\mathcal{F}' = \{T \in B(H) : \ TS = ST\,, \quad \forall S \in \mathcal{F}\}$$

The **double commutant** of $\mathcal{F}$ is just $(\mathcal{F}')'$ and is usually denoted $\mathcal{F}''$.

Properties:

- If $\mathcal{F}_1 \subseteq \mathcal{F}_2$, then $\mathcal{F}_2' \subseteq \mathcal{F}_1'$.
- $\mathcal{F} \subseteq \mathcal{F}''$.
- If $\mathcal{A}$ is a subalgebra of $B(H)$, then $\mathcal{A} \cap \mathcal{A}'$ is the `http://planetmath.org/CenterOfARing`cente of $\mathcal{A}$.
- If $\mathcal{F}$ is self-adjoint then $\mathcal{F}'$ is self-adjoint.
- $\mathcal{F}'$ is always a subalgebra of $B(H)$ that contains the identity operator and is closed in the weak operator topology.
- If $\mathcal{F}$ is self-adjoint then $\mathcal{F}'$ is a von Neumann algebra.

Remark: The commutant is a particular case of the more general definition of centralizer.