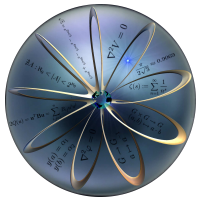

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Banach-Steinhaus theorem

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Synonym	Uniform Boundedness Principle

Let X be a Banach space and Y a normed space. If a family $\mathcal{F} \subset \mathscr{B}(X, Y)$ of bounded operators from X to Y satisfies

$$\sup\{\|T(x)\| : T \in \mathcal{F}\} < \infty$$

for each $x \in X$, then

$$\sup\{\|T\| : T \in \mathcal{F}\} < \infty,$$

i.e. $\mathcal{F}$ is a bounded subset of $\mathscr{B}(X, Y)$ with the usual operator norm. In other words, there exists a constant c such that for all $x \in X$ and $T \in \mathcal{F}$,

$$\|Tx\| \leq c\|x\|.$$