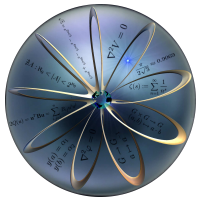

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Minkowski functional

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Let X be a normed space and let K an absorbing convex subset of X such that 0 is in the interior of K. Then the *Minkowski functional* $\rho\colon X \to \mathbb{R}$ is defined as

$$\rho(x) = \inf\{\lambda > 0 \colon x \in \lambda K\}.$$

We put $\rho(x) = 0$ whenever $x = 0$. Clearly $\rho(x) \geq 0$ for all x.

It is important to note that in general $\rho(x) \neq \rho(-x)$.

Properties

ρ is positively 1- homogeneous. This means that

$$\rho(s \cdot x) = s \cdot \rho(x)$$

for $s > 0$.