locally compact groupoids

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1 Locally compact groupoids

This is a specific topic entry defining the basics of locally compact groupoids and related concepts.

Let us first recall the related concepts of groupoid and *topological groupoid*, together with the appropriate notations needed to define a *locally compact groupoid*.

1.0.1 Groupoids and topological groupoids: categorical definitions

Recall that a groupoid G is a small category with inverses over its set of objects $X = Ob(\mathsf{G})$. One writes G_x^y for the set of morphisms in G from x to y .

A topological groupoid consists of a space G, a distinguished subspace $\mathsf{G}^{(0)} = \mathrm{Ob}(\mathsf{G}) \subset \mathsf{G}$, called *the space of objects* of G, together with maps

$$r, s \ : \ \mathsf{G} \underset{s}{\overset{r}{\rightrightarrows}} \mathsf{G}^{(0)} \tag{1.1}$$

called the *range* and *source maps* respectively, together with a law of composition

$$\circ \ : \ \mathsf{G}^{(2)} := \mathsf{G} \times_{\mathsf{G}^{(0)}} \mathsf{G} = \{ \ (\gamma_1, \gamma_2) \in \mathsf{G} \times \mathsf{G} \ : \ s(\gamma_1) = r(\gamma_2) \ \} \longrightarrow \mathsf{G} \ , \tag{1.2}$$

such that the following hold :

(1) $s(\gamma_1 \circ \gamma_2) = r(\gamma_2)$, $r(\gamma_1 \circ \gamma_2) = r(\gamma_1)$, for all $(\gamma_1, \gamma_2) \in \mathsf{G}^{(2)}$.

(2) $s(x) = r(x) = x$, for all $x \in \mathsf{G}^{(0)}$.

(3) $\gamma \circ s(\gamma) = \gamma$, $r(\gamma) \circ \gamma = \gamma$, for all $\gamma \in \mathsf{G}$.

(4) $(\gamma_1 \circ \gamma_2) \circ \gamma_3 = \gamma_1 \circ (\gamma_2 \circ \gamma_3)$.

(5) Each γ has a two–sided inverse γ^{-1} with $\gamma\gamma^{-1} = r(\gamma)$, $\gamma^{-1}\gamma = s(\gamma)$.

Furthermore, only for topological groupoids the inverse map needs be continuous. It is usual to call $\mathsf{G}^{(0)} = Ob(\mathsf{G})$ *the set of objects* of G . For $u \in Ob(\mathsf{G})$, the set of arrows $u \longrightarrow u$ forms a group G_u, called the *isotropy group of* G *at* u.

Thus, as is well kown, a topological groupoid is just a groupoid internal to the `http://planetmath.org/Cat` of topological spaces and continuous maps. The notion of internal groupoid has proved significant in a number of fields, since groupoids generalize bundles of groups, group actions, and equivalence relations. For a further study of groupoids we refer the reader to ref. [**?**].

1.1 Locally compact and analytic groupoids

Definition 1.1. A *locally compact groupoid* G_{lc} is defined as a groupoid that has also the topological structure of a second countable, `http://planetmath.org/LocallyCompactHausdorffSpace`locally compact Hausdorff space, and if the product and also inversion maps are continuous. Moreover, each G_{lc}^u as well as the unit space G_{lc}^0 is closed in G_{lc}.

Remark 1.1. The locally compact Hausdorff second countable spaces are *analytic*.

One can therefore say also that G_{lc} is analytic.

When the groupoid G_{lc} has only one object in its object space, that is, when it becomes a group, the above definition is restricted to that of a *locally compact topological group*; it is then a special case of a one-object category with all of its morphisms being invertible, that is also endowed with a locally compact, topological structure.

References

[1] R. Brown. (2006). *Topology and Groupoids*. BookSurgeLLC