weak-* topology of the space of Radon measures

Canonical name	WeakTopologyOfTheSpaceOfRadonMeasures
Date of creation	2013-03-22 15:22:58
Last modified on	2013-03-22 15:22:58
Owner	stevecheng (10074)
Last modified by	stevecheng (10074)
Numerical id	4
Author	stevecheng (10074)
Entry type	Example
Classification	msc 46A03
Classification	msc 28A33
Related topic	LocallyCompactHausdorffSpace

Let X be a locally compact Hausdorff space. Let $M(X)$ denote the space of complex Radon measures on X, and $C_0(X)^*$ denote the dual of the $C_0(X)$, the complex-valued continuous functions on X vanishing at infinity, equipped with the uniform norm. By the Riesz Representation Theorem, $M(X)$ is isometric to $C_0(X)^*$, The isometry maps a measure μ into the linear functional $I_\mu(f) = \int_X f \, d\mu$.

The weak-* topology (also called the vague topology) on $C_0(X)^*$, is simply the topology of pointwise convergence of I_μ: $I_{\mu_\alpha} \to I_\mu$ if and only if $I_{\mu_\alpha}(f) \to I_\mu(f)$ for each $f \in C_0(X)$.

The corresponding topology on $M(X)$ induced by the isometry from $C_0(X)^*$ is also called the weak-* or vague topology on $M(X)$. Thus one may talk about "weak convergence" of measures $\mu_n \to \mu$. One of the most important applications of this notion is in probability theory: for example, the central limit theorem is essentially the statement that if μ_n are the distributions for certain sums of independent random variables. then μ_n converge weakly to a normal distribution, i.e. the distribution μ_n is "approximately normal" for large n.

References

[1] G.B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed, John Wiley & Sons, Inc., 1999.