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barrel

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Entry type	Definition
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Synonym	barreled space
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Defines	barrelled space
Defines	infrabarrelled space
Defines	ultrabarrelled space

Let V be a topological vector space (TVS). A *barrel* B is a subset of V that is closed, convex, `http://planetmath.org/BalancedSet`balanced, and absorbing. For example, in a Banach space A, any ball $\{v \in A \mid ||v|| \leq r\}$ for some $r > 0$ is a barrel.

A topological vector space is said to be a *barrelled space* if it is `http://planetmath.org/Locally` convex, and every barrel is a neighborhood of 0. Every Banach space is a barrelled space.

A weaker form of a barrelled space is that of an *infrabarrelled space*. A TVS is said to be *infrabarrelled* if it is locally convex, and every barrel that absorbs every bounded set is a neighborhood of 0.

Let V be a vector space and $\mathfrak{T}$ be the set of all those topologies on V making V a TVS. In other words, if $T \in \mathfrak{T}$, then (V, T) is a topological vector space.

Let V and $T \in \mathfrak{T}$ be defined as above. Then (V, T) being barrelled has an equivalent characterization below:

> (*) for any $T_1 \in \mathfrak{T}$ such that there is a `http://planetmath.org/LocalBase`neighborhood base of 0 consisting of T-closed sets, then T_1 is coarser than T.

A variation of a barrelled space is that of an *ultrabarrelled space*. A topological vector space is said to be *ultrabarrelled* if it satisfies (*) above. A locally convex ultrabarrelled space is barrelled.

References

[1] H. H. Schaefer, *Topological Vector Spaces*, Springer-Verlag, New York (1970).

[2] R. E. Edwards, *Functional Analysis, Theory and Applications*, Holt, Reinhart and Winston, New York (1965).