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## necessary and sufficient conditions for a normed vector space to be a Banach space

 $Canonical\ name \qquad Necessary And Sufficient Conditions For ANormed Vector Space To Be ABanach Space To Be$ 

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Author asteroid (17536)

Entry type Theorem Classification msc 46B99 **Theorem 1 -** Let  $(X, \|\cdot\|)$  be a normed vector space. X is a Banach space if and only if every absolutely convergent series in X is convergent, i.e., whenever  $\sum_n \|x_n\| < \infty$ ,  $\sum_n x_n$  converges in X. **Theorem 2 -** Let X, Y be normed vector spaces,  $X \neq 0$ . Let B(X, Y)

**Theorem 2** - Let X, Y be normed vector spaces,  $X \neq 0$ . Let B(X, Y) be the space of bounded operators  $X \longrightarrow Y$ . Then Y is a Banach space if and only if B(X, Y) is a Banach space.