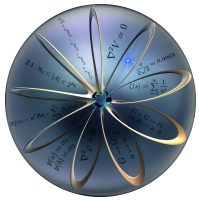

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Banach space

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Defines	dual space

A *Banach space* $(X, \|\cdot\|)$ is a normed vector space such that X is complete under the metric induced by the norm $\|\cdot\|$.

Some authors use the term Banach space only in the case where X is infinite-dimensional, although on Planetmath finite-dimensional spaces are also considered to be Banach spaces.

If Y is a Banach space and X is any normed vector space, then the set of continuous linear maps $f\colon X \to Y$ forms a Banach space, with norm given by the operator norm. In particular, since $\mathbb{R}$ and $\mathbb{C}$ are complete, the continuous linear functionals on a normed vector space $\mathcal{B}$ form a Banach space, known as the *dual space* of $\mathcal{B}$.

Examples:

- `http://planetmath.org/EveryFiniteDimensionalNormedVectorSpaceIsABanachSpaceFi` dimensional normed vector spaces.
- `http://planetmath.org/LpSpace`L^p spaces are by far the most common example of Banach spaces.
- `http://planetmath.org/Lp`ℓ^p spaces are L^p spaces for the counting measure on $\mathbb{N}$.
- Continuous functions on a compact set under the supremum norm.
- `http://planetmath.org/FiniteMeasureSpace`Finite signed measures on a `http://planetmath.org/SigmaAlgebra`σ-algebra.