norm and spectral radius in C^*-algebras

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Let $\mathcal{A}$ be a `http://planetmath.org/CAlgebra`C^*-algebra. Let $R_\sigma(a)$ denote the spectral radius of the element $a \in \mathcal{A}$.

Theorem - For every $a \in \mathcal{A}$ we have that $\|a\| = \sqrt{R_\sigma(a^*a)}$.

This result shows that the norm in a C^*-algebra has a purely   nature. Moreover, the norm in a C^*-algebra is unique (in the sense that there is no other norm for which the algebra is a C^*-algebra).