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proof of Hilbert space is uniformly convex space

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We prove that in fact an inner product space is uniformly convex. Let $\epsilon > 0$, $u, v \in H$ such that $\|u\| \leq 1$, $\|v\| \leq 1$, $\|u - v\| \geq \epsilon$. Put $\delta = 1 - \frac{1}{2}\sqrt{4 - \epsilon^2}$. Then $\delta > 0$ and by the parallelogram law

$$\begin{aligned}
\|u + v\|^2 &= \|u + v\|^2 + \|u - v\|^2 - \|u - v\|^2 \\
&= 2\|u\|^2 + 2\|v\|^2 - \|u - v\|^2 \\
&\leq 4 - \epsilon^2 \\
&= 4(1 - \delta)^2.
\end{aligned}$$

Hence, $\|\frac{u+v}{2}\| \leq 1 - \delta$.

Since a Hilbert space is an inner product space, a Hilbert space the conditions of a uniformly convex space.