wavelet set

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Definition An *(orthonormal dyadic) wavelet set* on $\mathbb{R}$ is a subset $E \subset \mathbb{R}$ such that

1. $\chi_E \in L^2(\mathbb{R})$ (since $\|\chi_E\| = \sqrt{m(E)}$, this implies $m(E) < \infty$).
2. $\frac{\chi_E}{\sqrt{m(E)}}$ is the Fourier transform of an orthonormal dyadic wavelet,

where χ_E is the characteristic function of E, and $m(E)$ is the Lebesgue measure of E.

Characterization $E \subset \mathbb{R}$ is a wavelet set iff

1. $\{E + 2\pi n\}_{n\in\mathbb{Z}}$ is a measurable partition of $\mathbb{R}$; i.e. $\mathbb{R}\setminus \bigcup_{n\in\mathbb{Z}}\{E + 2\pi n\}$ has measure zero, and $\bigcap_{n=i,j}\{E + 2\pi n\}$ has measure zero if $i \neq j$. In short, E is a 2π-translation "tiler" of $\mathbb{R}$
2. $\{2^n E\}_{n\in\mathbb{Z}}$ is a 2-dilation "tiler" of $\mathbb{R}$ (once again modulo sets of measure zero).

Notes There are higher dimensional analogues to wavelet sets in $\mathbb{R}$, corresponding to wavelets in higher dimensions. Wavelet sets can be used to derive wavelets— by creating a set E satisfying the conditions given above, and using the inverse Fourier transform on χ_E, you are guaranteed to recover a wavelet. A particularly interesting open question is: do all wavelets contain wavelet sets in their frequency support?