topological divisor of zero

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Let A be a normed ring. An element $a \in A$ is said to be a *left topological divisor of zero* if there is a sequence a_n with $\|a_n\| = 1$ for all n such that

$$\lim_{n \to \infty} \|aa_n\| = 0.$$

Analogously, a is a if

$$\lim_{n \to \infty} \|b_n a\| = 0,$$

for some sequence b_n with $\|b_n\| = 1$. The element a is a *topological divisor of zero* if it is both a left and a topological divisor of zero.

Remarks.

- Any zero divisor is a topological divisor of zero.

- If a is a (left) topological divisor of zero, then ba is a (left) topological divisor of zero. As a result, a is never a unit, for if b is its inverse, then $1 = ba$ would be a topological divisor of zero, which is impossible.

- In a commutative Banach algebra A, an element is a topological divisor of zero if it lies on the boundary of $U(A)$, the group of units of A.