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$L^1(G)$ has an approximate identity

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Defines	$L^1(G)$ has an identity element iff G is discrete

Let G be a locally compact topological group. In general, the Banach *-algebra $L^1(G)$ (<http://planetmath.org/L1IsABanachAlgebra> parent entry) does not have an identity element. In fact:

- $L^1(G)$ has an identity element if and only if G is discrete.

When G is discrete the identity element of $L^1(G)$ is just the Dirac delta, i.e. the function that takes the value 1 on the identity element of G and vanishes everywhere else.

Nevertheless, $L^1(G)$ has always an approximate identity.

Theorem - $L^1(G)$ has an approximate identity $(e_\lambda)_{\lambda \in \Lambda}$. Moreover the approximate identity $(e_\lambda)_{\lambda \in \Lambda}$ can be chosen to the following :

- e_λ is <http://planetmath.org/InvolutiveRings> self-adjoint,
- $\|e_\lambda\|_1 = 1$,
- $e_\lambda \in C_c(G)$

where $C_c(G)$ stands for the space of continuous functions $G \rightarrow \mathbb{C}$ with compact support.