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nuclear space

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If E is a Frchet space and (p_j) an increasing sequence of semi-norms on E defining the topology of E, we have

$$E = \lim_{\longleftarrow} \widehat{E}_{p_j},$$

where $\widehat{E}_{p_j}$ is the Hausdorff completion of (E, p_j) and $\widehat{E}_{p_{j+1}} \to \widehat{E}_{p_j}$ the canonical morphism. Here $\widehat{E}_{p_j}$ is a Banach space for the induced norm $\widehat{p}_j$.

A Frchet space E is said to be *nuclear* if the topology of E can be defined by an increasing sequence of semi-norms p_j such that each canonical morphism $\widehat{E}_{p_{j+1}} \to \widehat{E}_{p_j}$ of Banach spaces is nuclear.

Recall that a morphism $f\colon E \to F$ of complete locally convex spaces is said to be nuclear if f can be written as

$$f(x) = \sum \lambda_j \xi_j(x) y_j$$

where (λ_j) is a sequence of scalars with $\sum |\lambda_j| < +\infty$,$\xi_j \in E'$ an equicontinuous sequence of linear forms and $y_j \in F$ a bounded sequence.