# pointwise limit of bounded operators is bounded

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The following result is a corollary of the principle of uniform boundedness.

**Theorem -** Let $X$ be a Banach space and $Y$ a normed vector space. Let $(T_n) \in B(X,Y)$ be a sequence of bounded operators from $X$ to $Y$. If $(T_n x)$ converges for every $x \in X$, then the operator

$$T : X \longrightarrow Y$$

$$Tx = \lim_{n\to\infty} T_n x$$

is linear and . Moreover, the sequence $(\|T_n\|)$ is http://planetmath.org/Boundedbounded.

**Proof :** It is clear that the operator $T$ is linear.

For each $x \in X$ we have $\sup_n \|T_n x\| < \infty$ since $(T_n x)$ is . By the http://planetmath.org/BanachSteinhausTheoremprinciple of uniform boundedness we must also have $M := \sup_n \|T_n\| < \infty$.

Then for each $x \in X$ we have

$$\|Tx\| = \lim_{n\to\infty} \|T_n x\| \leq M\|x\|$$

which means that $T$ is . $\square$