weak derivative

Canonical name	WeakDerivative
Date of creation	2013-03-22 14:54:52
Last modified on	2013-03-22 14:54:52
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Last modified by	paolini (1187)
Numerical id	16
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Entry type	Definition
Classification	msc 46E35
Related topic	SobolevSpaces

Let $f\colon \Omega \to \mathbf{R}$ and $g = (g_1, \dots, g_n)\colon \Omega \to \mathbf{R}^n$ be locally integrable functions defined on an open set $\Omega \subset \mathbf{R}^n$. We say that g is the *weak derivative* of f if the equality

$$\int_\Omega f \frac{\partial \phi}{\partial x_i} = -\int_\Omega g_i \phi$$

holds true for all functions $\phi \in \mathcal{C}_c^\infty(\Omega)$ (smooth functions with compact support in Ω) and for all $i = 1, \dots, n$. Notice that the integrals involved are well defined since ϕ is bounded with compact support and because f is assumed to be integrable on compact subsets of Ω.

Comments

1. If f is of class $\mathcal{C}^1$ then the gradient of f is the weak derivative of f in view of Gauss Green Theorem. So the weak derivative is an extension of the classical derivative.

2. The weak derivative is unique (as an element of the Lebesgue space L^1_{loc}) in view of a result about locally integrable functions.

3. The same definition can be given for functions with complex values.