state

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Defines	pure state
Defines	tracial state

A **state** Ψ on a C^*-algebra A is a positive linear functional $\Psi\colon A \to \mathbb{C}$, $\Psi(a^*a) \geq 0$ for all $a \in A$, with unit norm. The norm of a positive linear functional is defined by

$$\|\Psi\| = \sup_{a\in A}\{|\Psi(a)| : \|a\| \leq 1\}. \tag{1}$$

For a unital C^*-algebra, $\|\Psi\| = \Psi(\mathbb{1})$.

The space of states is a convex set. Let Ψ_1 and Ψ_2 be states, then the convex combination

$$\lambda\Psi_1 + (1-\lambda)\Psi_2, \quad \lambda \in [0,1], \tag{2}$$

is also a state.

A state is **pure** if it is not a convex combination of two other states. Pure states are the extreme points of the convex set of states. A pure state on a commutative C^*-algebra is equivalent to a character.

A state is called a **tracial state** if it is also a trace.

When a C^*-algebra is represented on a Hilbert space $\mathcal{H}$, every unit vector $\psi \in \mathcal{H}$ determines a (not necessarily pure) state in the form of an **expectation value**,

$$\Psi(a) = \langle \psi, a\psi \rangle. \tag{3}$$

In physics, it is common to refer to such states by their vector ψ rather than the linear functional Ψ. The converse is not always true; not every state need be given by an expectation value. For example, delta functions (which are distributions not functions) give pure states on $C_0(X)$, but they do not correspond to any vector in a Hilbert space (such a vector would not be square-integrable).

References

[1] G. Murphy, *C^*-Algebras and Operator Theory.* Academic Press, 1990.