T_f is a distribution of zeroth order

Canonical name	TfIsADistributionOfZerothOrder
Date of creation	2013-03-22 13:44:28
Last modified on	2013-03-22 13:44:28
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	6
Author	Koro (127)
Entry type	Proof
Classification	msc 46F05
Classification	msc 46-00

To check that T_f is a `http://planetmath.org/Distribution4`distribution of zeroth order, we shall use condition (3) on `http://planetmath.org/Distribution4`this page. First, it is clear that T_f is a linear mapping. To see that T_f is continuous, suppose K is a compact set in U and $u \in \mathcal{D}_K$, i.e., u is a smooth function with support in K. We then have

$$
\begin{aligned}
|T_f(u)| &= |\int_K f(x)u(x)dx| \\
&\leq \int_K |f(x)|\ |u(x)|dx \\
&\leq \int_K |f(x)|dx\ ||u||_\infty.
\end{aligned}
$$

Since f is locally integrable, it follows that $C = \int_K |f(x)|dx$ is finite, so

$$
|T_f(u)| \leq C||u||_\infty.
$$

Thus f is a distribution of zeroth order ([**?**], pp. 381). $\square$

References

[1] S. Lang, *Analysis II*, Addison-Wesley Publishing Company Inc., 1969.