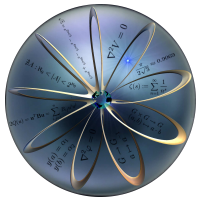

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proof of parallelogram law

Canonical name	ProofOfParallelogramLaw1
Date of creation	2013-03-22 16:08:15
Last modified on	2013-03-22 16:08:15
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	6
Author	Wkbj79 (1863)
Entry type	Proof
Classification	msc 46C05
Related topic	ProofOfParallelogramLaw
Related topic	AlternateProofOfParallelogramLaw

The proof supplied here for the parallelogram law uses the properties of norms and inner products. See the entries about these for more details regarding the following calculations.

Proof.

$$\begin{aligned}
\|x+y\|^2 + \|x-y\|^2 &= \langle x+y, x+y\rangle + \langle x-y, x-y\rangle \\
&= \overline{\langle x, x+y\rangle} + \overline{\langle y, x+y\rangle} + \overline{\langle x, x-y\rangle} - \overline{\langle y, x-y\rangle} \\
&= \overline{\langle x+y, x\rangle} + \overline{\langle x+y, y\rangle} + \overline{\langle x-y, x\rangle} - \overline{\langle x-y, y\rangle} \\
&= \overline{\langle x, x\rangle + \langle y, x\rangle} + \overline{\langle x, y\rangle + \langle y, y\rangle} + \overline{\langle x, x\rangle - \langle y, x\rangle} - \left(\overline{\langle x, y\rangle - \langle y, y\rangle}\right) \\
&= \overline{\langle x, x\rangle} + \overline{\langle y, x\rangle} + \overline{\langle x, y\rangle} + \overline{\langle y, y\rangle} + \overline{\langle x, x\rangle} - \overline{\langle y, x\rangle} - \overline{\langle x, y\rangle} + \overline{\langle y, y\rangle} \\
&= \langle x, x\rangle + \langle y, y\rangle + \langle x, x\rangle + \langle y, y\rangle \\
&= 2\langle x, x\rangle + 2\langle y, y\rangle \\
&= 2\|x\|^2 + 2\|y\|^2.
\end{aligned}$$

□