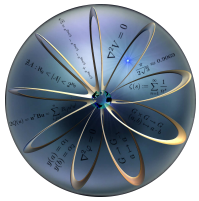

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Banach algebra

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Definition 1. A **Banach algebra** $\mathcal{A}$ is a Banach space (over $\mathbb{C}$) with an multiplication law compatible with the norm which turns $\mathcal{A}$ into an algebra. Compatibility with the norm means that, for all $a, b \in \mathcal{A}$, it is the case that the following product inequality holds:

$$\|ab\| \leq \|a\| \, \|b\|$$

Definition 2. A **Banach *-algebra** is a Banach algebra $\mathcal{A}$ with a map $^*\colon \mathcal{A} \to \mathcal{A}$ which satisfies the following properties:

$$\begin{aligned} a^{**} &= a, && (1)\\ (ab)^* &= b^* a^*, && (2)\\ (a+b)^* &= a^* + b^*, && (3)\\ (\lambda a)^* &= \bar{\lambda} a^* \quad \forall \lambda \in \mathbb{C}, && (4)\\ \|a^*\| &= \|a\|, && (5) \end{aligned}$$

where $\bar{\lambda}$ is the complex conjugation of λ. In other words, the operator * is an involution.

Example 1
The algebra of bounded operators on a Banach space is a Banach algebra for the operator norm.