properties of the transpose operator

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In this entry, whenever V, W are normed vector spaces, $\mathcal{B}(V, W)$ denotes the algebra of bounded linear operators $V \longrightarrow W$.

Let X, Y, Z be normed vector spaces and X', Y', Z' be their continuous dual spaces. Let $T, S \in \mathcal{B}(X, Y)$, $R \in \mathcal{B}(Y, Z)$ and $\lambda \in \mathbb{C}$.

0.0.1 Basic properties

- $T' \in \mathcal{B}(Y', X')$ and $\|T\| = \|T'\|$.

- $(\lambda T)' = \lambda T'$.

- $(S + T)' = S' + T'$.

- $(RT)' = T'R'$.

- If T^{-1} exists and $T^{-1} \in \mathcal{B}(Y, X)$ then $(T')^{-1} \in \mathcal{B}(X', Y')$ and $(T')^{-1} = (T^{-1})'$.

0.0.2 Miscellaneous properties

- If we endow X' and Y' with the weak-* topology then $T' : Y' \longrightarrow X'$ is continuous.

- T is an isometric isomorphism if and only if T' is an isometric isomorphism.

- If T is `http://planetmath.org/CompactOperator`compact then T' is also compact.

- If T' is compact and Y is a Banach space, then T is also compact.

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