properties of Minkowski's functional

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Let X be a normed space, K convex subset of X and 0 belongs to the interior of K.Then

1. $\rho_K(x) \geq 0$ for all $x \in X$
2. $\rho_K(0) = 0$
3. $\rho_K(\lambda x) = \lambda \rho_K(x)$, for all $\lambda \geq 0$ and $x \in X$
4. $\rho_K(x + y) \leq \rho_K(x) + \rho_K(y)$ for all $x, y \in K$
5. $\{x \in X \colon \rho_K(x) < 1\} \subset K \subset \{x \in X \colon \rho_K(x) \leq 1\}$
6. $K^0 = \{x \in X \colon \rho_K(x) < 1\}$ where K^0 denotes the interior of K
7. $\bar{K} = \{x \in X \colon \rho_K(x) \leq 1\}$ where $\bar{K}$ denotes the closure of K
8. $Bd(K) = \{x \in X \colon \rho_K(x) = 1\}$ where the $Bd(K)$ denotes the boundary of K.

Minkowski's functional is a useful tool to prove propositions and solve exercises. Let us see an example
Example Let K be a convex subset of X. Show that $Ex(K) \subset Bd(K)$, where $Ex(K)$ denotes the set of extreme points of K.
If $x \in Ex(K)$ then from this follows that $x \in 1K$ and $\rho_K(x) = 1$. Now we hypothesize that $\rho_K(x) < 1$ then there is a real number s such that $\rho_K(x) < s < 1$ and so $\rho_K(\frac{x}{s}) < 1$. Therefore we have that $x = s\frac{x}{s} + (1 - s)0 \in K$, that contradicts to the fact that $x \in Ex(K)$.