



planetmath.org

Math for the people, by the people.

basic properties of seminorms

Canonical name	BasicPropertiesOfSeminorms
Date of creation	2013-03-22 14:38:57
Last modified on	2013-03-22 14:38:57
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	5
Author	matte (1858)
Entry type	Theorem
Classification	msc 46B20

Proposition 1. *Suppose $p: V \rightarrow \mathbb{R}$ is a seminorm on a real (or complex) vector space V . Then*

1. $p(0) = 0$,
2. $p(v) \geq 0$ for all $v \in V$.

Proof. Property 1 follows using homogeneity;

$$p(0) = p(0 \cdot 0) = |0|p(0) = 0.$$

Property 2 follows using sublinearity and Property 1;

$$0 = p(0) = p(v - v) \leq p(v) + p(-v) = 2p(v).$$

□