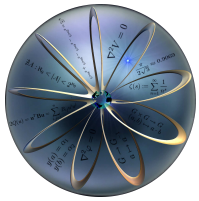

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shift operators in ℓ^p

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Let $\mathbb{F}$ be $\mathbb{R}$ or $\mathbb{C}$, and let $1 \leq p \leq \infty$, let $\ell^p(\mathbb{F}), \|\cdot\|_p$ be as in the parent entry.

The right and left *shift operators* $S_r, S_l \colon \ell^p(\mathbb{F}) \to \ell^p(\mathbb{F})$ as defined as follows. For $a = (a_1, a_2, \ldots) \in \ell^p(\mathbb{F})$,

$$S_r(a) = (0, a_1, a_2, \ldots)$$

and

$$S_l(a) = (a_2, a_3, \ldots).$$

Properties

1. $S_l \circ S_r$ is the identity, but $S_r \circ S_l$ is not.
2. S_r is an isometry; $\|S_r(a)\| = \|a\|$, and $\|S_l(a)\|_p \leq \|a\|$. Both shift operators are therefore bounded (and continuous).