bounded function

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Defines	supremum norm
Defines	sup norm
Defines	sup-norm
Defines	uniform norm
Defines	bounded function
Defines	unbounded function

Definition Suppose X is a nonempty set. Then a function $f : X \to \mathbb{C}$ is a if there exist a $C < \infty$ such that $|f(x)| < C$ for all $x \in X$. The set of all bounded functions on X is usually denoted by $B(X)$ ([**?**], pp. 61).

Under standard point-wise addition and point-wise multiplication by a scalar, $B(X)$ is a complex vector space.

If $f \in B(X)$, then the *sup-norm*, or *uniform norm*, of f is defined as

$$||f||_\infty = \sup_{x \in X} |f(x)|.$$

It is straightforward to check that $|| \cdot ||_\infty$ makes $B(X)$ into a normed vector space, i.e., to check that $|| \cdot ||_\infty$ satisfies the assumptions for a norm.

0.0.1 Example

Suppose X is a compact topological space. Further, let $C(X)$ be the set of continuous complex-valued functions on X (with the same vector space structure as $B(X)$). Then $C(X)$ is a vector subspace of $B(X)$.

References

[1] C.D. Aliprantis, O. Burkinshaw, *Principles of Real Analysis*, 2nd ed., Academic Press, 1990.