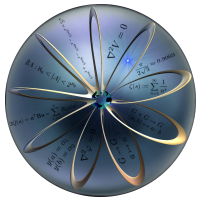

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yet another proof of parallelogram law

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Define $g(\epsilon) = \langle x + \epsilon y, x + \epsilon y \rangle$, where ϵ is real. Then $g(\epsilon) = \langle x, x \rangle + \epsilon(\langle y, x \rangle + \langle x, y \rangle) + \epsilon^2 \langle y, y \rangle$. Hence,

$$\|x + y\|^2 + \|x - y\|^2 = g(1) + g(-1) = 2\langle x, x \rangle + 2\langle y, y \rangle = 2\|x\|^2 + 2\|y\|^2.$$