planetmath.org

Math for the people, by the people.

C^n

Canonical name	Cn
Date of creation	2013-03-22 14:59:43
Last modified on	2013-03-22 14:59:43
Owner	drini (3)
Last modified by	drini (3)
Numerical id	13
Author	drini (3)
Entry type	Definition
Classification	msc 46G05
Classification	msc 26B05
Classification	msc 26A99
Classification	msc 26A24
Classification	msc 26A15
Synonym	C^1
Synonym	C^2
Synonym	C^k
Synonym	C^∞
Related topic	Derivative
Related topic	SmoothFunctionsWithCompactSupport

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function. We say that f is of class C^1 if f' exists and is continuous.

We also say that f is of class C^n if its n-th derivative exists and is continuous (and therefore all other previous derivatives exist and are continuous too).

The class of continuous functions is denoted by C^0. So we get the following relationship among these classes:

$$C^0 \supset C^1 \supset C^2 \supset C^3 \supset \dots$$

Finally, the class of functions that have continuous derivatives of any order is denoted by C^∞ and thus

$$C^\infty = \bigcap_{n=0}^{\infty} C^n.$$

It holds that any function that is differentiable is also continuous (see `http://planetmath.org/Dif` entry). Therefore, $f \in C^\infty$ if and only if every derivative of f exists.

The previous concepts can be extended to functions $f\colon \mathbb{R}^m \to \mathbb{R}$, where f being of class C^n amounts to asking that all the partial derivatives of order n be continuous. For instance, $f\colon \mathbb{R}^m \to \mathbb{R}$ being C^2 means that

$$\frac{\partial^2 f}{\partial x_j \partial x_i}$$

exists and are all continuous for any i, j from 1 to m.

C^n functions on an open set of $\mathbb{R}^m$

Sometimes we need to talk about continuity not globally on $\mathbb{R}$, but on some interval or open set.

If $U \subseteq \mathbb{R}^m$ is an open set, and $f\colon U \to \mathbb{R}$ (or $f\colon U \to \mathbb{C}$) we say that f is of class C^n if $\partial^\alpha f$ exist and are continuous for all multi-indices α with $|\alpha| \le n$. See `http://planetmath.org/MultiIndexNotation`this page for the multi-index notation.