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set of sampling

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Defines	sampling operator

Definition Let F be a Hilbert space of functions defined on a domain D. Let $T = \{t_i\}_{i \in I}$ be a finite or infinite sequence of points in D. T is said to be a *set of sampling* for F if the sampling operator $S : F \to l^2_{|T|}$ defined by

$$S : f \mapsto \begin{pmatrix} f(t_1) \\ f(t_2) \\ \vdots \end{pmatrix}$$

is bounded (i.e. continuous) and bounded below; i.e. if

$$\exists A, B > 0 \text{ such that } \forall f \in F, A\|f\|^2 \leq \sum_{i=1}^{|T|} |f(t_i)|^2 \leq B\|f\|^2.$$

Relation to Frames Using the Riesz Representation Theorem, it is easy to show that every set of sampling determines a unique frame in such a way that the analysis operator of that frame is the sampling operator associated with the set of sampling. In fact, let $t = \{t_i\}$ be a set of sampling with sampling operator S_t. Use the Riesz representation theorem to rewrite S_t in terms of vectors $\{g_i\}$ in F:

$$S : f \mapsto \begin{pmatrix} f(t_1) \\ f(t_2) \\ \vdots \end{pmatrix} = \begin{pmatrix} \langle f, g_1 \rangle \\ \langle f, g_2 \rangle \\ \vdots \end{pmatrix}$$

then note that

$$\forall f \in F, A\|f\|^2 \leq \sum_i |\langle f, g_i \rangle|^2 \leq B\|f\|^2,$$

so the $\{g_i\}$ form a frame with bounds A, B, and $S_t = \theta_g$.

Reconstruction Particularly nice sets of sampling are those that correspond to tight frames, because then $\theta_g^* \theta_g = \theta_g^* S_t = AI$, and it is possible to reconstruct the function f, given its values over the set of sampling:

$$f = \frac{1}{A} \sum_i f(t_i) g_i.$$

Sets of sampling which correspond to tight frames are referred to as tight sets of sampling.