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subadditive

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Suppose  $V$  is a vector space (over a field), and  $f$  is a function  $f: V \rightarrow \mathbb{R}$ . Then  $f$  is *subadditive* if

$$f(x + y) \leq f(x) + f(y), \quad x, y \in V.$$

### Examples

1. Any linear function  $V \rightarrow \mathbb{R}$  is subadditive.
2. If  $\|\cdot\|$  is a norm on  $V$ ,  $a \geq 0$ , then

$$f(x) = a + \|x\|$$

is subadditive.

### Properties

Suppose  $f$  is subadditive.

1. If  $f$  is positively 1-homogeneous, then  $f$  is convex.
2. The sum of two subadditive functions is subadditive.