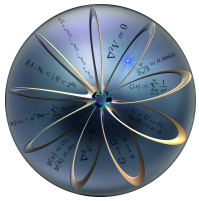

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proof of generalization of the parallelogram law

Canonical name	ProofOfGeneralizationOfTheParallelogramLaw
Date of creation	2013-03-22 16:08:58
Last modified on	2013-03-22 16:08:58
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	7
Author	Mathprof (13753)
Entry type	Proof
Classification	msc 46C05

Let $g(x,y) = \|x+y\|^2 - \|x\|^2$ and $m(x,y) = \langle x, y\rangle + \langle y, x\rangle$. Then

$$g(x,y) = \|y\|^2 + m(x,y).$$

Hence, taking $x_1 = x_4 = x, x_2 = y, x_3 = z$ we have:

$$\begin{aligned}
\sum_{i=1}^{3} \|x_i + x_{i+1}\|^2 - \sum_{i=1}^{3} \|x_i\|^2 &= \sum_{i=1}^{3} g(x_i, x_{i+1}) \\
&= \sum_{i=1}^{3} \|x_i\|^2 + \sum_{i=1}^{3} m(x_i, x_{i+1}) \\
&= \Big\| \sum_{i=1}^{3} x_i \Big\|^2.
\end{aligned}$$