delta distribution

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Let U be an open subset of $\mathbb{R}^n$ such that $0 \in U$. Then the *delta distribution* is the mapping

$$\begin{array}{rcl} \delta : \mathcal{D}(U) & \to & \mathbb{C} \\ u & \mapsto & u(0). \end{array}$$

Claim The delta distribution is a distribution of zeroth order, i.e., $\delta \in \mathcal{D}'^0(U)$.

Proof. With obvious notation, we have

$$\begin{array}{rcl} \delta(u+v) & = & (u+v)(0) = u(0) + v(0) = \delta(u) + \delta(v), \\ \delta(\alpha u) & = & (\alpha u)(0) = \alpha u(0) = \alpha \delta(u), \end{array}$$

so δ is linear. To see that δ is continuous, we use condition (3) on this `http://planetmath.org/Distribution4`this page. Indeed, if K is a compact set in U, and $u \in \mathcal{D}_K$, then

$$|\delta(u)| = |u(0)| \leq ||u||_\infty,$$

where $||\cdot||_\infty$ is the supremum norm. $\square$