# approximation property

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| Classification | msc 46B99 |
| Synonym | approximation by finite rank operators |
| Defines | Schauder basis and approximation by finite rank operators |

Let $Y$ be a Banach space and $B(Y)$ the algebra of bounded operators in $Y$. We say that $Y$ has the **approximation property** if there is a sequence $(P_n)$ of `http://planetmath.org/RankLinearMapping`finite rank operators in $B(Y)$ such that

$$P_n y \longrightarrow y \quad \forall_{y \in Y}$$

i.e. $(P_n)$ converges in the strong operator topology to the identity operator.

The fundamental fact about spaces with the approximation property is that every compact operator is the norm limit of finite rank operators.

**Theorem -** Let $X$ be a normed vector space and $Y$ a Banach space with the approximation property. Then every compact operator $T : X \longrightarrow Y$ is the norm limit of operators of finite rank.

**Examples :**

- Separable Hilbert spaces have the approximation property. Note however that compact operators on Hilbert spaces (not just separable ones) are always norm limit of finite rank operators.
- The `http://planetmath.org/Lp`$\ell^p$-spaces have the approximation property.

Moreover,

**Theorem -** If $Y$ is a Banach space with a Schauder basis then it has the approximation property.