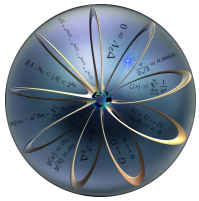

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C^*-algebras have approximate identities

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In this entry $\leq$ has three different meanings:

1. - The http://planetmath.org/OrderingOfSelfAdjointsordering of self-adjoint elements of a given http://planetmath.org/CAlgebraC^*-algebra.

2. - The usual http://planetmath.org/PartialOrderorder in $\mathbb{R}$.

3. - The of a directed set taken as the domain of a given net.

It will be clear from the context which one is being used.

Theorem - Every C^*-algebra has an approximate identity $(e_\lambda)_{\lambda\in\Lambda}$. Moreover, the approximate identity $(e_\lambda)_{\lambda\in\Lambda}$ can be chosen to the following :

- $0 \leq e_\lambda \quad \forall_{\lambda\in\Lambda}$
- $\|e_\lambda\| \leq 1 \quad \forall_{\lambda\in\Lambda}$
- $\lambda \leq \mu \ \Rightarrow \ e_\lambda \leq e_\mu$, i.e. $(e_\lambda)_{\lambda\in\Lambda}$ is increasing.

For http://planetmath.org/Separableseparable C^*-algebras the approximate identity can be chosen as an increasing sequence $0 \leq e_1 \leq e_2 \leq \ldots$ of norm-one elements.