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topological complement

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Defines	topologically complemented

0.0.1 Definition

Let X be a topological vector space and $M \subseteq X$ a http://planetmath.org/ClosedSetclosed subspace.

If there exists a closed subspace $N \subseteq X$ such that

$$M \oplus N = X$$

we say that M is **topologically complemented**.

In this case N is said to be a **topological complement** of M, and also M and N are said to be **topologically complementary** subspaces.

0.0.2 Remarks

- It is known that every subspace $M \subseteq X$ has an algebraic complement, i.e. there exists a subspace $N \subseteq X$ such that $M \oplus N = X$. The existence of topological complements, however, is not always assured.

- If X is an Hilbert space, then each closed subspace $M \subseteq X$ is topologically complemented by its orthogonal complement $M^{\perp}$, i.e.

$$M \oplus M^{\perp} = X.$$

- Moreover, for Banach spaces the converse of the last paragraph also holds, i.e. if each closed subspace is topologically complemented then X is isomorphic a Hilbert space. This is the http://planetmath.org/CharacterizationOfAHil Tzafriri theorem.