polarization identity

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Theorem [polarization identity] **-** Let X be an inner product space over $\mathbb{R}$. The following identity holds for every $x, y \in X$:

$$\langle x, y \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)$$

If X is an inner product space over $\mathbb{C}$ instead, the identity becomes

$$\langle x, y \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2) + \frac{1}{4}i(\|x + iy\|^2 - \|x - iy\|^2)$$

Remark - This result shows that the inner product of X is determined by the norm. Moreover, it can be shown that if a normed space V the parallelogram law, the above formulas define an inner product compatible with the norm of V.