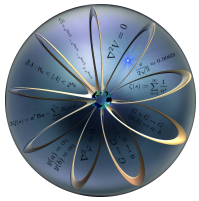

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all norms are not equivalent

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Let V be the vector space of continuous functions $[-1,1] \to \mathbb{R}$ that are differentiable at 0. Then we can define norms

$$\|f\| = \max_{x \in [-1,1]} |f|,$$

and

$$\|f\|' = \|f\| + |f'(0)|.$$

It is not difficult to find a sequence of functions $f_1, f_2, \ldots$ in V such that

1. $f_k'(0) = k$ for $k = 1, 2, \ldots$,
2. $\|f_k\| = 1$.

Then $\|f_k\| = 1$, and $\|f_k\|' = 1 + k$, so there is no $C > 1$ such that

$$\|f\|' \leq C\|f\| \quad f \in V,$$

and $\|\cdot\|$ and $\|\cdot\|'$ cannot be .