Bessel inequality

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Let $\mathcal{H}$ be a Hilbert space, and suppose $e_1, e_2, \ldots \in \mathcal{H}$ is an orthonormal sequence. Then for any $x \in \mathcal{H}$,

$$\sum_{k=1}^{\infty} \left|\langle x, e_k \rangle\right|^2 \leq \left\|x\right\|^2.$$

Bessel's inequality immediately lets us define the sum

$$x' = \sum_{k=1}^{\infty} \langle x, e_k \rangle \, e_k.$$

The inequality means that the series converges.

For a complete orthonormal series, we have Parseval's theorem, which replaces inequality with equality (and consequently x' with x).