absorbing set

Canonical name	AbsorbingSet
Date of creation	2013-03-22 15:26:24
Last modified on	2013-03-22 15:26:24
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	10
Author	CWoo (3771)
Entry type	Definition
Classification	msc 46A08
Classification	msc 15A03
Related topic	BalancedSet
Related topic	AbsorbingElement
Defines	absorbing
Defines	absorb
Defines	radial

Let V be a vector space over a field F equipped with a non-discrete valuation $|\cdot| : F \to \mathbb{R}$. Let A, B be two subsets of V. Then A is said to *absorb* B if there is a non-negative real number r such that, for all $\lambda \in F$ with $|\lambda| \geq r$, $B \subseteq \lambda A$. A is said to be an *absorbing set*, or a *radial subset* of V if A absorbs all finite subsets of V.

Equivalently, A is absorbing if for any $x \in V$, there is a non-negative real number r such that $x \in \lambda A$ for all $\lambda \in F$ with $|\lambda| \geq r$. If a finite subset B of V consists of $x_1, \ldots, x_n$, then corresponding to each x_i, there is an $r_i \geq 0$ such that $x_i \in \lambda A$ such that $|\lambda| \geq r_i$, $\forall \lambda \in F$. So $x_i \in \lambda A$ with $|\lambda| \geq r$ if we take $r = \max\{r_1, \ldots, r_n\}$. So A absorbs B.

Example. If $V = \mathbb{R}^n$ and $F = \mathbb{R}$, then any set containing an open ball centered at 0 is absorbing. This implies that an absorbing set does not have to be connected, convex.

A closely related concept is that of a *circled set*, or a *balanced set*. Let V and F be defined as above. A subset C of V is said to be *circled*, or *balanced*, if $\lambda C \subseteq C$ for all $|\lambda| \leq 1$. Clearly, C absorbs itself ($C \subseteq \lambda^{-1} C$, $|\lambda^{-1}| \geq 1$), and $0 \in C$. C is also symmetric ($-C = C$), for $-C \subseteq C$ and $C = -(-C) \subseteq -C$. As an example of a circled set that is neither absorbing nor convex, consider $V = \mathbb{R}^2$ and $F = \mathbb{R}$, and C the union of x and y axes. For an example of an absorbing set that is not circled, take the union of a unit disk and an annulus centered at 0 that is large enough so it is disjoint from the disk.