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Sobolev inequality

Canonical name	SobolevInequality
Date of creation	2013-03-22 15:05:14
Last modified on	2013-03-22 15:05:14
Owner	paolini (1187)
Last modified by	paolini (1187)
Numerical id	10
Author	paolini (1187)
Entry type	Theorem
Classification	msc 46E35
Synonym	Sobolev embedding
Synonym	sobolev immersion
Synonym	Gagliardo Nirenberg inequality
Related topic	LpSpace
Defines	Sobolev conjugate
Defines	Sobolev exponent

For $1 \le p < n$, define the *Sobolev conjugate* of p as

$$p^* := \frac{np}{n-p}.$$

Note that $-n/p^* = 1 - n/p$.

In the following statement ∇ represent the weak derivative and $W^{1,p}(\Omega)$ is the Sobolev space of functions $u \in L^p(\Omega)$ whose weak derivative ∇u is itself in $L^p(\Omega)$.

Theorem 1 *Assume that $p \in [1, n)$ and let Ω be a bounded, open subset of $\mathbb{R}^n$ with Lipschitz boundary. Then there is a constant $C > 0$ such that, for all $u \in W^{1,p}(\Omega)$ one has*

$$\|u\|_{L^{p^*}(\Omega)} \le C\|\nabla u\|_{L^p(\Omega)}.$$

We can restate the previous Theorem by saying that the Sobolev space $W^{1,p}(\Omega)$ is a subspace of the Lebesgue space $L^{p^*}(\Omega)$ and that the inclusion map $i\colon W^{1,p}(\Omega) \to L^{q^*}(\Omega)$ is continuous.