compact operator

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Let X and Y be two Banach spaces. A **compact operator** (completely continuous operator) is a linear operator $T\colon X \to Y$ that maps the unit ball in X to a set in Y with compact closure. It can be shown that a compact operator is necessarily a bounded operator.

The set of all compact operators on X, commonly denoted by $\mathbb{K}(X)$, is a closed two-sided ideal of the set of all bounded operators on X, $\mathbb{B}(X)$.

Any bounded operator which is the norm limit of a sequence of finite rank operators is compact. In the case of Hilbert spaces, the converse is also true. That is, any compact operator on a Hilbert space is a norm limit of finite rank operators.

Example 1 (Integral operators)
Let $k(x,y)$, with $x, y \in [0,1]$, be a continuous function. The operator defined by

$$(T\psi)(x) = \int_0^1 k(x,y)\psi(y)\,\mathrm{d}y, \qquad \psi \in C([0,1])$$

is compact.