generalized Hölder inequality

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Author	Mathprof (13753)
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Theorem Let $1 \leq r < \infty$ and $1 \leq p_j < \infty$, where $\sum_{j=1}^{n} \frac{1}{p_j} = \frac{1}{r}$. If $f_j \in L^{p_j}$ for $1 \leq j \leq n$, then

$\prod_{j=1}^{n} f_j \in L^r$ and

$$||\prod_{j=1}^{n} f_j||_r \leq \prod_{j=1}^{n} ||f_j||_{p_j}.$$

The usual Hölder inequality has $n = 2$ and $r = 1$.

Let X be a finite set, say $X = \{x_1, \ldots, x_m\}$ and μ is the counting measure on X, so that $\mu(\{x_i\}) = 1$ for all i. Let $f_j(x_i) = a_{ij} \geq 0$ for $j = 1, \ldots, n$ and take $r = 1$. Then the inequality becomes:

$$\sum_{i=1}^{m} \prod_{j=1}^{n} a_{ij} \leq \prod_{j=1}^{n} (\sum_{i=1}^{m} a_{ij}{}^{p_j})^{\frac{1}{p_j}} \quad .$$

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Now let $\alpha_j = \frac{1}{p_j}$, and $b_{ij} = a_{ij}{}^{p_j}$. Then the inequality becomes:

$$\sum_{i=1}^{m} \prod_{j=1}^{n} b_{ij}{}^{\alpha_j} \leq \prod_{j=1}^{n} (\sum_{i=1}^{m} b_{ij})^{\alpha_j}.$$