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direct sum of Hilbert spaces

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Let $\{H_i\}_{i \in I}$ be a family of Hilbert spaces indexed by a set I . The direct sum of this family of Hilbert spaces, denoted as

$$\bigoplus_{i \in I} H_i$$

consists of all elements v of the <http://planetmath.org/GeneralizedCartesianProductCartesianProduct> of $\{H_i\}_{i \in I}$ such that $\sum \|v_i\|^2 < \infty$. Of course, for the previous sum to be finite only at most a countable number of v_i can be non-zero.

Vector addition and scalar multiplication are defined termwise: If $u, v \in \bigoplus_{i \in I} H_i$, then $(u + v)_i = u_i + v_i$ and $(sv)_i = sv_i$.

The inner product of two vectors is defined as

$$\langle u, v \rangle = \sum_{i \in I} \langle u_i, v_i \rangle$$

Linked PDF file:

<http://images.planetmath.org/cache/objects/6363/pdf/DirectSumOfHilbertSpaces.pdf>