diagonalizable operator

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The expression "*diagonalizable operator*" has several meanings in operator theory. The purpose of this entry is to present some commonly used concepts where this terminology appears.

0.1 Definition 1

Let H be a finite dimensional Hilbert space. A linear operator $T : H \longrightarrow H$ is said to be **diagonalizable** if the corresponding matrix (in a given basis) is a http://planetmath.org/Diagonalizable2diagonalizable matrix.

The above definition is equivalent to: There exists a basis of H consisting of eigenvectors of T.

Remark - This is a common definition in linear algebra.

0.2 Definition 2

Let H be a finite dimensional Hilbert space. A linear operator $T : H \longrightarrow H$ is said to be **diagonalizable** if there is an orthonormal basis of H in which T is represented by a diagonal matrix.

The above definition is equivalent to: There exists an orthonormal basis of H consisting of eigenvectors of T.

Another equivalent definition is: There exists an orthonormal basis $\{e_1, \dots, e_n\}$ of H and values $\lambda_1, \dots, \lambda_2 \in \mathbb{C}$ such that

$$T(x) = \sum_{i=1}^{n} \lambda_i \langle x, e_i \rangle e_i$$

Remarks -

- In http://planetmath.org/LinearAlgebralinear algebra such operators are also called **unitarily diagonalizable**.
- Diagonalizable operators (in this sense) are always normal operators. The http://planetmath.org/SpectralTheoremForHermitianMatricesSpectral theorem for normal operators assures that the converse is also true.

0.3 Definition 3

Let H be a Hilbert space. A bounded linear operator $T : H \longrightarrow H$ is said to be **diagonalizable** if there exists an orthonormal basis consisting of eigenvectors of T.

An equivalent definition is: There exists an orthonormal basis $\{e_i\}_{i\in J}$ of H and values $\{\lambda_i\}_{i\in J}$ such that

$$T(x) = \sum_{i\in J} \lambda_i \langle x, e_i\rangle e_i$$

Remarks -

- If H is finite dimensional this is the same as definition 2.
- Diagonalizable operators (in this sense) are always normal operators. For compact operators the converse is assured by an appropriate version of the spectral theorem for compact normal operators.

0.4 Definition 4

Let H be a Hilbert space. A linear operator $T : H \longrightarrow H$ is said to be **diagonalizable** if it is to a http://planetmath.org/MultiplicationOperatorOnMathbbL22multiplicat operator in some http://planetmath.org/L2SpacesAreHilbertSpacesL^2-space, i.e. if there exists

- a measure space $(X, \mathcal{B}, \mu)$,
- a unitary operator $U : L^2(X) \longrightarrow H$ and
- a function $f \in$ http://planetmath.org/LpSpace$L^\infty(X)$ such that

$$T = UM_fU^*$$

where $M_f : L^2(X) \longrightarrow L^2(X)$ is the http://planetmath.org/MultiplicationOperatoroperat of multiplication by f

$$M_f(\psi) = f.\psi \quad .$$

Remarks -

- If $H = \mathbb{C}^n$ the above definition is equivalent to say that T is unitarily diagonalizable (Definition 2). Indeed, we can think of $\mathbb{C}^n$ as $L^2(\{1, \ldots, n\})$ with the counting measure. In this case, multiplication operators correspond to diagonal matrices.
- Diagonalizable operators (in this sense) are necessarily normal operators (since multiplication operators are so). The discussion about the converse result is the content of general versions of the spectral theorem.