approximate identity

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Entry type	Definition
Classification	msc 46H05
Synonym	approximate unit
Defines	left approximate identity
Defines	right approximate identity

Let $\mathcal{A}$ be a Banach algebra.

A **left approximate identity** for $\mathcal{A}$ is a net $(e_\lambda)_{\lambda\in\Lambda}$ in $\mathcal{A}$ which :

1. $\|e_\lambda\| < C \quad \forall_{\lambda\in\Lambda}$, for some constant C.
2. $e_\lambda a \longrightarrow a$, for every $a \in \mathcal{A}$.

Similarly, a **right approximate identity** for $\mathcal{A}$ is a net $(e_\lambda)_{\lambda\in\Lambda}$ in $\mathcal{A}$ which :

1. $\|e_\lambda\| < C \quad \forall_{\lambda\in\Lambda}$, for some constant C.
2. $ae_\lambda \longrightarrow a$, for every $a \in \mathcal{A}$.

An **approximate identity** for a $\mathcal{A}$ is a net $(e_\lambda)_{\lambda\in\Lambda}$ in $\mathcal{A}$ which is both a left and right approximate identity.

0.0.1 Remarks:

- There are examples of Banach algebras that do not have approximate .
- If $\mathcal{A}$ has an identity element e, then clearly e itself is an approximate identity for $\mathcal{A}$.