Pythagorean theorem in inner product spaces

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- Let X be an inner product space (over $\mathbb{R}$ or $\mathbb{C}$) and $x, y \in X$ two orthogonal vectors. Then

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2.$$

Proof : As $x \perp y$ one has $\langle x, y\rangle = 0$. Then

$$\begin{aligned} \|x+y\|^2 &= \langle x+y, x+y\rangle \\ &= \langle x, x\rangle + \langle x, y\rangle + \langle y, x\rangle + \langle y, y\rangle \\ &= \|x\|^2 + \langle x, y\rangle + \overline{\langle x, y\rangle} + \|y\|^2 \\ &= \|x\|^2 + \|y\|^2 \qquad \square \end{aligned}$$

Remark$-$ This theorem is valid (with the same proof) for spaces with a semi-definite inner product.