property of uniformly convex Banach Space

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Theorem 1. *Let X be a uniformly convex Banach space. Let (x_n) be a sequence in X such that $\lim x_n = x$ in the weak-topology $(w(X, X^*))$ and $\limsup \|x_n\| \leq \|x\|$. Then x_n converges to x.*

Proof. For $x = 0$ the claim is obvious, so suppose that $x \neq 0$.The sequence $(x_n)_{n\geq 1}$ converges to x for $w - topology \Rightarrow \|x\| \leq \liminf \|x_n\|$.So let $\lambda_n = \max\{\|x\|, \|x_n\|\}$ and we have that $\lim \lambda_n = \|x\|$. Define $y_n = \frac{x_n}{\lambda_n}$ and $y = \frac{x}{\|x\|}$.Then y_n converges to y in $w-topology$. We conclude that $\|y\| \leq \liminf \|\frac{y_n+y}{2}\|$. Also, $\|y\| = 1, \|y_n\| \leq 1$ so we have that $\lim \|\frac{y_n+y}{2}\| = 1$. As the Banach space is uniformly convex one can easily see that $\lim \|y_n - y\| = 0$. Therefore x_n converges to x. The proof is complete. $\square$