# Ascoli-Arzelà theorem

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Let $\Omega$ be a bounded subset of $\mathbb{R}^n$ and $(f_k)$ a sequence of functions $f_k\colon \Omega \to \mathbb{R}^m$. If $\{f_k\}$ is equibounded and uniformly equicontinuous then there exists a uniformly convergent subsequence $(f_{k_j})$.

A more abstract (and more general) version is the following.

Let $X$ and $Y$ be totally bounded metric spaces and let $F \subset \mathcal{C}(X, Y)$ be an uniformly equicontinuous family of continuous mappings from $X$ to $Y$. Then $F$ is totally bounded (with respect to the uniform convergence metric induced by $\mathcal{C}(X, Y)$).

Notice that the first version is a consequence of the second. Recall, in fact, that a subset of a complete metric space is totally bounded if and only if its closure is compact (or sequentially compact). Hence $\Omega$ is totally bounded and all the functions $f_k$ have image in a totally bounded set. Being $F = \{f_k\}$ totally bounded means that $\overline{F}$ is sequentially compact and hence $(f_k)$ has a convergent subsequence.