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balanced set

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Entry type	Definition
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Related topic	AbsorbingSet
Defines	balanced subset
Defines	balanced hull
Defines	balanced envelope
Defines	circled
Defines	équilibré

Definition [?, ?, ?, ?] Let V be a vector space over \mathbb{R} (or \mathbb{C}), and let S be a subset of V . If $\lambda S \subset S$ for all scalars λ such that $|\lambda| \leq 1$, then S is a **balanced set** in V . The **balanced hull** of S , denoted by $\text{eq}(S)$, is the smallest balanced set containing S .

In the above, $\lambda S = \{\lambda s \mid s \in S\}$, and $|\cdot|$ is the absolute value (in \mathbb{R}), or the modulus of a complex number (in \mathbb{C}).

0.0.1 Examples and properties

1. Let V be a normed space with norm $\|\cdot\|$. Then the unit ball $\{v \in V \mid \|v\| \leq 1\}$ is a balanced set.
2. Any vector subspace is a balanced set. Thus, in \mathbb{R}^3 , lines and planes passing through the origin are balanced sets.

0.0.2 Notes

A balanced set is also sometimes called **circled** [?]. The term **balanced envelope** is also used for the balanced hull [?]. Bourbaki uses the term **équilibré** [?], c.f. $\text{eq}(A)$ above. In [?], a balanced set is defined as above, but with the condition $|\lambda| = 1$ instead of $|\lambda| \leq 1$.

References

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