inner product space

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Entry type	Definition
Classification	msc 46C99
Synonym	pre-Hilbert space
Related topic	InnerProduct
Related topic	OrthonormalBasis
Related topic	HilbertSpace
Related topic	EuclideanVectorSpace2
Related topic	AngleBetweenTwoLines
Related topic	FluxOfVectorField
Related topic	CauchySchwarzInequality
Defines	angle between two vectors
Defines	perpendicularity

An *inner product space* (or *pre-Hilbert space*) is a vector space (over $\mathbb{R}$ or $\mathbb{C}$) with an inner product $\langle\cdot,\cdot\rangle$.

For example, $\mathbb{R}^n$ with the familiar dot product forms an inner product space.

Every inner product space is also a normed vector space, with the norm defined by $\|x\| := \sqrt{\langle x,\, x\rangle}$. This norm satisfies the parallelogram law.

If the metric $\|x-y\|$ induced by the norm is `http://planetmath.org/Complete`complete, then the inner product space is called a Hilbert space.

The Cauchy–Schwarz inequality

$$|\langle x,\, y\rangle| \leq \|x\|\cdot\|y\| \tag{1}$$

holds in any inner product space.

According to (1), one can define the *angle between two non-zero vectors* x and y:

$$\cos(x,\, y) := \frac{\langle x,\, y\rangle}{\|x\|\cdot\|y\|}. \tag{2}$$

This provides that the scalars are the real numbers. In any case, the *perpendiculatity* of the vectors may be defined with the condition

$$\langle x,\, y\rangle = 0.$$