Stone-Weierstrass theorem for locally compact spaces

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The following results generalize the Stone-Weierstrass theorem (and its http://planetmath.org/StoneWeierstrassTheoremComplexVersioncomplex version) for locally compact spaces. The cost of this generalization is that one no longer deals with all continuous functions, but only those that vanish at infinity.

Real version

Theorem - Let X be a locally compact space and $C_0(X, \mathbb{R})$ the algebra of continuous functions $X \to \mathbb{R}$ that http://planetmath.org/ VanishAtInfinityvanish at infinity, endowed with the sup norm $\| \cdot \|_\infty$. Let $\mathcal{A}$ be a subalgebra of $C_0(X; \mathbb{R})$ for which the following conditions hold:

1. $\forall x, y \in X, x \neq y, \exists f \in \mathcal{A} : f(x) \neq f(y)$, i.e. $\mathcal{A}$ separates points.
2. For each $x \in X$ there exists $f \in \mathcal{A}$ such that $f(x) \neq 0$.

Then $\mathcal{A}$ is dense in $C_0(X; \mathbb{R})$.

Complex version

Theorem - Let X be a locally compact space and $C_0(X)$ the algebra of continuous functions $X \to \mathbb{C}$ that vanish at infinity, endowed with the sup norm $\| \cdot \|_\infty$. Let $\mathcal{A}$ be a subalgebra of $C_0(X)$ for which the following conditions hold:

1. $\forall x, y \in X, x \neq y, \exists f \in \mathcal{A} : f(x) \neq f(y)$, i.e. $\mathcal{A}$ separates points.
2. For each $x \in X$ there exists $f \in \mathcal{A}$ such that $f(x) \neq 0$.
3. If $f \in \mathcal{A}$ then $\overline{f} \in \mathcal{A}$, i.e. $\mathcal{A}$ is a self-adjoint subalgebra of $C(X)$.

Then $\mathcal{A}$ is dense in $C_0(X)$.