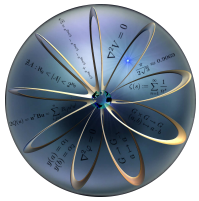

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Mazur-Ulam theorem

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Theorem. *Every* `http://planetmath.org/Isometry`*isometry between normed vector spaces over* $\mathbb{R}$ *is an affine transformation.*

Note that we consider isometries to be surjective by definition. The result is not in general true for non-surjective isometric mappings.

The result does not extend to normed vector spaces over $\mathbb{C}$, as can be seen from the fact that complex conjugation is an isometry $\mathbb{C} \to \mathbb{C}$ but is not affine over $\mathbb{C}$. (But complex conjugation is clearly affine over $\mathbb{R}$, and in general any normed vector space over $\mathbb{C}$ can be considered as a normed vector space over $\mathbb{R}$, to which the theorem can be applied.)

This theorem was first proved by Mazur and Ulam.[**?**] A simpler proof has been given by Jussi Väisälä.[**?**]

References

[1] S. Mazur and S. Ulam, *Sur les transformations isométriques d'espaces vectoriels normés*, C. R. Acad. Sci., Paris 194 (1932), 946–948.

[2] Jussi Väisälä, *A proof of the Mazur–Ulam theorem*, Amer. Math. Mon. 110, #7 (2003), 633–635. (A preprint is `http://www.helsinki.fi/%7Ejvaisala/mazurulam.pdf`available on Väisälä's website.)