basic criterion for self-adjointness

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Let $A\colon D(A) \subset \mathscr{H} \to \mathscr{H}$ be a symmetric operator on a Hilbert space. The following are equivalent:

1. $A = A^*$ (i.e A is self-adjoint);
2. $\mathrm{Ker}(A^* \pm i) = \{0\}$ and A is closed;
3. $\mathrm{Ran}(A \pm i) = \mathscr{H}$.

Remark: $A + \lambda$ represents the operator $A + \lambda I\colon D(A) \subset \mathscr{H} \to \mathscr{H}$, and Ker and Ran stand for kernel and range, respectively.

A similar version for essential self-adjointness is an easy corollary of the above. The following are equivalent:

1. $\overline{A} = A^*$ (i.e. A is essentially self-adjoint);
2. $\mathrm{Ker}(A^* \pm i) = \{0\}$;
3. $\mathrm{Ran}(A \pm i)$ is dense in $\mathscr{H}$.