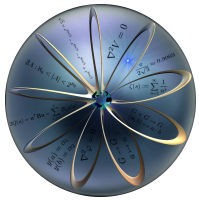

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Fredholm operator

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A **Fredholm operator** is a bounded operator between Banach spaces that has a finite dimensional kernel and cokernel (and closed range). Equivalently, it is invertible modulo compact operators. That is, if $F\colon X \to Y$ is a Fredholm operator between two vector spaces X and Y, then there exists a bounded operator $G\colon Y \to X$ such that

$$GF - \mathbb{1}_X \in \mathbb{K}(X), \quad FG - \mathbb{1}_Y \in \mathbb{K}(Y), \tag{1}$$

where $\mathbb{K}(X)$ denotes the space of compact operators on X. (Another way to say this is that F is invertible in the Calkin algebra). The set of Fredholm operators $\{F\colon X \to X\}$ is an open subset of the Banach algebra of bounded operators $\{T\colon X \to X\}$.

If F is Fredholm then so is its adjoint, F^*. If $T \in \mathbb{K}(X, Y)$ is a compact operator then $F + T$ is also Fredholm.