proof that the convex hull of S is open if S is open

Let S be an open set in some topological vector space V. For any sequence of positive real numbers $\Lambda = (\lambda_1, \ldots, \lambda_n)$ with $\sum_{i=1}^{n} \lambda_i = 1$ define

$$S_\Lambda = \left\{ x \in V \text{ such that } x = \sum_{i=1}^{n} \lambda_i s_i \text{ for } s_i \in S \right\}.$$

Then since addition and scalar multiplication are both open maps, each S_Λ is open. Finally, the convex hull is clearly just

$$\bigcup_\Lambda S_\Lambda,$$

which is therefore open.