Dini derivative

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The **upper Dini derivative** of a continuous function, $f : \mathbf{R} \mapsto \mathbf{R}$, denoted by f'_+, is defined as

$$f'_+(t) = \lim_{h \to 0^+} \sup \frac{f(t+h) - f(t)}{h}.$$

The **lower Dini derivative**, f'_-, is defined as

$$f'_-(t) = \lim_{h \to 0^+} \inf \frac{f(t+h) - f(t)}{h}.$$

Remark: Sometimes the notation $D^+ f(t)$ is used instead of $f'_+(t)$, and $D^- f(t)$ is used instead of $f'_-(t)$.

Remark: Like conventional derivatives, Dini derivatives do not always exist.

If f is defined on a vector space, then the upper Dini derivative at t in the direction d is denoted

$$f'_+(t, d) = \lim_{h \to 0^+} \sup \frac{f(t+hd) - f(t)}{h}.$$

If f is locally Lipschitz then $D^+ f$ is finite. If f is differentiable at t then the Dini derivative at t is the derivative at t.