ergodicity of a map in terms of its induced operator

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Theorem - Let $(X, \mathfrak{B}, \mu)$ be a probability space and $T : X \longrightarrow X$ a measure-preserving transformation. The following statements are equivalent:

1. - T is ergodic.

2. - If f is a measurable function and $f \circ T = f$ `http://planetmath.org/AlmostSurely`a.e., then f is constant a.e.

3. - If f is a measurable function and $f \circ T \geq f$ a.e., then f is constant a.e.

4. - If $f \in L^2(X)$ and $f \circ T = f$ a.e., then f is constant a.e..

5. - If $f \in L^p(X)$, with $p \geq 1$, and $f \circ T = f$ a.e., then f is constant a.e.

Let U_T denote the `http://planetmath.org/OperatorInducedByAMeasurePreservingMap`oper induced by T, i.e. the operator defined by $U_T f := f \circ T$. The statements above are statements about U_T. The above theorem can be rewritten as follows:

Theorem - Let $(X, \mathfrak{B}, \mu)$ be a probability space and $T : X \longrightarrow X$ a measure-preserving transformation. The following statements are equivalent:

1. - T is ergodic.

2. - The only fixed points of U_T are the functions that are constant a.e.

3. - If f a measurable function and $U_T f \geq f$ a.e., then f is constant a.e.

4. - The eigenspace of U_T (seen as an operator in $L^p(X)$, with $p \geq 1$) associated with the eigenvalue 1, is one-dimensional and consists of functions that are constant a.e.