Fredholm index

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Let P be a Fredholm operator. The **index** of P is defined as

$$\begin{aligned}
\operatorname{index}(P) &= \dim\ker(P) - \dim\operatorname{coker}(P) \\
&= \dim\ker(P) - \dim\ker(P^*).
\end{aligned}$$

Note: this is well defined as $\ker(P)$ and $\ker(P^*)$ are finite-dimensional vector spaces, for P Fredholm.

- $\operatorname{index}(P^*) = -\operatorname{index}(P)$.
- $\operatorname{index}(P+K) = \operatorname{index}(P)$ for any compact operator K.
- If $P_1\colon \mathcal{H}_1 \to \mathcal{H}_2$ and $P_2\colon \mathcal{H}_2 \to \mathcal{H}_3$ are Fredholm operators, then $\operatorname{index}(P_2P_1) = \operatorname{index}(P_1) + \operatorname{index}(P_2)$.
- If $t \to P_t$, $t \in [0,1]$ is a norm continuous path of Fredholm operators, then $\operatorname{index}(P_t) = \operatorname{index}(P_0)$.

Fredholm operators of the form *invertible* + *compact* have index zero.