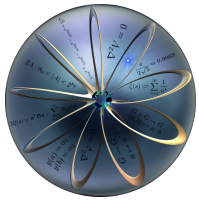

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convex hull of S is open if S is open

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Theorem If S is an open set in a topological vector space, then the convex hull $\mathrm{co}(S)$ is open.

As the next example shows, the corresponding result does not hold for a closed set.

Example (Valentine, p. 14) If

$$S = \{(x, 1/|x|) \in \mathbb{R}^2 \mid x \in \mathbb{R} \setminus \{0\}\},$$

then S is closed, but $\mathrm{co}(S)$ is the open half-space $\{(x, y) \mid x \in \mathbb{R}, y \in (0, \infty)\}$, which is not closed (points on the x-axis are accumulation points not in the set, or also can be seen by checking the complement is not open). $\square$

Reference

F.A. Valentine, *Convex sets*, McGraw-Hill book company, 1964.