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vector measure

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Defines	complex measure
Defines	countably additive vector measure

Let S be a set and $\mathcal{F}$ a field of sets of S. Let X be a topological vector space.

A **vector measure** is a function $\mu : \mathcal{F} \longrightarrow X$ that is , i.e. for any two disjoint sets A_1, A_2 in $\mathcal{F}$ we have

$$\mu(A_1 \cup A_2) = \mu(A_1) + \mu(A_2)$$

A vector measure μ is said to be if for any sequence $(A_n)_{n\in\mathbb{N}}$ of disjoint sets in $\mathcal{F}$ such that $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$ one has

$$\mu(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mu(A_n)$$

where the series converges in the topology of X.

In the particular case when $X = \mathbb{C}$, a countably additive vector measure is usually called a **complex measure**.

Thus, vector measures are to measures and signed measures but they take values on a vector space (with a particular topology).

0.0.1 Examples :

- Let $(X, \mathfrak{B}, \lambda)$ be a measure space. Consider the Banach space `http://planetmath.org/LpSpa` with $1 \leq p \leq \infty$. Define the the function $\mu : \mathfrak{B} \longrightarrow L^p(X, \mathfrak{B}, \mu)$ by

$$\mu(A) := \chi_A$$

  where χ_A denotes the characteristic function of the measurable set A. It is easily seen that μ is a vector measure, which is countably additive if $1 \leq p < \infty$ (in case $p = \infty$, countably additiveness fails).

- spectral measures are vector measures in the σ-algebra of Borel sets in $\mathbb{C}$ whose values are projections on some Hilbert space. They are used in general formulations of the spectral theorem.