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matrix inversion lemma

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These frequently used formulae allow to quickly calculate the inverse of a slight modification of an operator (matrix) x, given that x^{-1} is already known.

The matrix inversion lemma states that

$$(x + s\sigma z^*)^{-1} = x^{-1} - x^{-1}s\left(\sigma^{-1} + z^* x^{-1} s\right)^{-1} z^* x^{-1},$$

where x, s, z^* and σ are operators (matrices) of appropriate size. The formula especially is convenient if the rank of the regular σ is 1, or small in comparison to x's rank.

This identity, involving the inverse of Schur's complement $d - z^* x^{-1} s$ (hopefully this may be easily computed) holds as well:

$$\begin{bmatrix} x & s \\ z^* & d \end{bmatrix}^{-1} = \begin{bmatrix} x^{-1} + x^{-1}s(d - z^* x^{-1} s)^{-1} z^* x^{-1} & -x^{-1}s(d - z^* x^{-1} s)^{-1} \\ -(d - z^* x^{-1} s)^{-1} z^* x^{-1} & (d - z^* x^{-1} s)^{-1} \end{bmatrix}.$$