



Math for the people, by the people.

Hermitian form

Canonical name	HermitianForm
Date of creation	2013-03-22 12:25:47
Last modified on	2013-03-22 12:25:47
Owner	djao (24)
Last modified by	djao (24)
Numerical id	8
Author	djao (24)
Entry type	Definition
Classification	msc 47A07
Classification	msc 15A63
Classification	msc 11E39
Synonym	sesquilinear form
Synonym	sesqui-linear form
Related topic	InnerProduct

A *sesquilinear form* over a pair of complex vector spaces (V, W) is a function $B: V \times W \rightarrow \mathbb{C}$ satisfying the following properties:

1. $B(\mathbf{v}_1 + \mathbf{v}_2, \mathbf{w}) = B(\mathbf{v}_1, \mathbf{w}) + B(\mathbf{v}_2, \mathbf{w})$
2. $B(\mathbf{v}, \mathbf{w}_1 + \mathbf{w}_2) = B(\mathbf{v}, \mathbf{w}_1) + B(\mathbf{v}, \mathbf{w}_2)$
3. $B(c\mathbf{v}, d\mathbf{w}) = cB(\mathbf{v}, \mathbf{w})\bar{d}$

for all $\mathbf{v}, \mathbf{v}_1, \mathbf{v}_2 \in V$, $\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2 \in W$, and $c, d \in \mathbb{C}$. The vector spaces V and W are often identical, although the definition does not require them to be the same vector space.

A sesquilinear form $B: V \times V \rightarrow \mathbb{C}$ over a single vector space V is called a *Hermitian form* if it is complex conjugate symmetric: namely, if $B(\mathbf{v}_1, \mathbf{v}_2) = \overline{B(\mathbf{v}_2, \mathbf{v}_1)}$.

An inner product over a complex vector space is a positive definite Hermitian form.