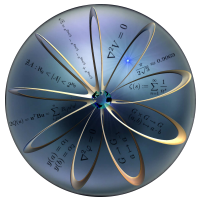

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functional calculus for Hermitian matrices

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Let $I \subset \mathbb{R}$ be a real interval, f a real-valued function on I, and let M be an $n \times n$ real symmetric (and thus Hermitian) matrix whose eigenvalues are contained in I.

By the spectral theorem, we can diagonalize M by an orthogonal matrix O, so we can write $M = ODO^{-1}$ where D is the diagonal matrix consisting of the eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. We then define

$$f(A) = Of(D)O^{-1},$$

where $f(D)$ denotes the diagonal matrix whose diagonal entries are given by $f(\lambda_i)$.

It is easy to verify that $f(A)$ is well-defined, i.e. a permutation of the eigenvalues corresponds to the same definition of $f(A)$.