sesquilinear forms over general fields

Canonical name	SesquilinearFormsOverGeneralFields
Date of creation	2013-03-22 15:58:17
Last modified on	2013-03-22 15:58:17
Owner	Algeboy (12884)
Last modified by	Algeboy (12884)
Numerical id	11
Author	Algeboy (12884)
Entry type	Definition
Classification	msc 47A07
Classification	msc 15A63
Classification	msc 11E39
Classification	msc 51A05
Synonym	Hermitian form
Synonym	Hermitean form
Related topic	ReflexiveNonDegenerateSesquilinear
Related topic	NonDegenerate
Related topic	Polarity2
Related topic	Projectivity
Related topic	ProjectiveGeometry
Related topic	Isometry2
Related topic	ProjectiveGeometry3
Related topic	ClassicalGroups
Defines	sesquilinear form
Defines	Hermitian form
Defines	bilinear form
Defines	Hermitean

Let V be a vector space over a field k. k may be of any characteristic.

1 Sesquilinear Forms

Definition 1. *A function $b : V \times V \to k$ is sesquilinear if it satisfies each of the following:*

1. $b(v, w+u) = b(v, w) + b(v, u)$ *and* $b(v+u, w) = b(v, w) + b(u, w)$ *for all* $u, v, w \in V$*;*
2. *For a given field automorphism θ of k,* $b(v, lw) = l^{\theta} b(v, w)$ *and* $b(lv, w) = lb(v, w)$ *for all* $v, w \in V$ *and* $l \in k$*.*

Remark 2. *It is possible to apply the field automorphism in the first variable but is more common to do so in the second variable. Also, if $\theta = 1$ the form is a bilinear form.*

Sesquilinear forms are commonly ascribed any combination of the following properties:

- non-degenerate,
- reflexive, (commonly required to define perpendicular);
- positive definite (this condition requires the fixed field of θ, k_0, be an ordered field, such as the rationals $\mathbb{Q}$ or reals $\mathbb{R}$).

Non-degenerate sesquilinear and bilinear forms apply to projective geometries as dualities and polarities through the induced $\perp$ operation. (See `http://planetmath.org/Polarity2`polarity.)

2 Hermitian Forms

If $\theta^2 = 1$, it is common to exchange notation at this point and use the same notation of $\bar{l}$ for l^{θ} as is common for complex conjugation – even if k is not $\mathbb{C}$. Then $\bar{\bar{l}} = l$.

In this notation, Hermitian forms may be defined by the property

$$b(v, w) = \overline{b(w, v)}.$$

Remark 3. *It is not uncommon to see hermitian or Hermitean instead of Hermitian. The name is a tribute to Charles Hermite of the Ecole Polytechnique.*