derivation of Euler-Lagrange differential equation (advanced)

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Suppose that $x_0 \in D$. Choose r such that the closed ball of radius r about x_0 is contained in D. Let q be any function whose support lies in this closed ball.

By the definition of F,

$$\frac{\partial}{\partial \lambda} F(q_0 + \lambda q) = \frac{\partial}{\partial \lambda} \int_D L(x, q_0 + \lambda q, dq_0 + \lambda dq)\, d^m x$$

$$= \frac{\partial}{\partial \lambda} \left(\int_{|x-x_0| \leq r} L(x, q_0 + \lambda q, dq_0 + \lambda dq)\, d^m x + \int_{\substack{x \in D \\ |x-x_0| > r}} L(x, q_0 + \lambda q, dq_0 + \lambda dq)\, d^m x \right)$$

By the condition imposed on q, the derivative of the second integral is zero. Since the integrand of the first integral and its first derivatives are continuous and the closed ball is compact, the integrand and its first derivatives are uniformly continuous, so it is permissible to interchange differentiation and integration. Hence,

$$\frac{\partial}{\partial \lambda} F(q_0 + \lambda q) = \int_{|x-x_0| \leq r} \frac{\partial L(x, q_0 + \lambda q, dq_0 + \lambda dq)}{\partial \lambda}\, d^m x$$