resolvent matrix

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Defines	resolvent

The *resolvent matrix* of a matrix A is defined as

$$R_A(s) = (sI - A)^{-1}.$$

Note: I is the identity matrix and s is a complex variable. Also note that $R_A(s)$ is undefined on $Sp(A)$ (the spectrum of A).

More generally, let A be a unital algebra over the field of complex numbers $\mathbb{C}$. The *resolvent R_x* of an element $x \in A$ is a function from $\mathbb{C} - Sp(x)$ to A given by

$$R_x(s) = (s \cdot 1 - x)^{-1}$$

where $Sp(x)$ is the spectrum of x: $Sp(x) = \{t \in \mathbb{C} \mid t \cdot 1 - x \text{ is not invertible in } A\}$.

If A is commutative and $s \notin Sp(x) \cup Sp(y)$, then $R_x(s) - R_y(s) = R_x(s)R_y(s)(x - y)$.