Kato-Rellich theorem

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Defines	A-bound

Let $\mathcal{H}$ be a Hilbert space, $A\colon D(A) \subset \mathcal{H} \to \mathcal{H}$ a self-adjoint operator and $B\colon D(B) \subset \mathcal{H} \to \mathcal{H}$ a symmetric operator with $D(A) \subset D(B)$.

We say that B is A-bounded if there are positive constants α, β such that

$$\|Bx\| \leq \alpha\|Ax\| + \beta\|x\|$$

for all $x \in D(A)$, and we say that α is an A-*bound* for B.

Theorem 1. *(Kato-Rellich) If B is A-*bounded *with A-bound smaller than 1, then $A+B$ is self-adjoint on $D(A)$, and essentially self-adjoint on any core of A. Moreover, if A is bounded below, then so is $A+B$.*