properties of the adjoint operator

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Let A and B be linear operators in a Hilbert space, and let $\lambda \in \mathbb{C}$. Assuming all the operators involved are densely defined, the following properties hold:

1. If A^{-1} exists and is densely defined, then $(A^{-1})^* = (A^*)^{-1}$;
2. $(\lambda A)^* = \overline{\lambda} A^*$;
3. $A \subset B$ implies $B^* \subset A^*$;
4. $A^* + B^* \subset (A + B)^*$;
5. $B^* A^* \subset (AB)^*$;
6. $(A + \lambda I)^* = A^* + \overline{\lambda} I$;
7. A^* is a closed operator.

Remark. The notation $A \subset B$ for operators means that B is an of A, i.e. A is the `http://planetmath.org/RestrictionOfAFunction`restriction of B to a smaller domain.

Also, we have the following

Proposition 1 *If A admits a `http://planetmath.org/ClosedOperator`closure $\overline{A}$, then A^* is densely defined and $(A^*)^* = \overline{A}$.*