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interval halving converges linearly

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Theorem 1. *The interval halving algorithm converges linearly.*

Proof. To see that interval halving (or bisection) converges linearly we use the alternative definition of linear convergence that says that $|x_{i+1} - x_i| < c|x_i - x_{i-1}|$ for some constant $1 > c > 0$.

In the case of interval halving, $|x_{i+1} - x_i|$ is the length of the interval we should search for the solution in and has x_{i+2} as its midpoint. We have then that this interval has half the length of the previous interval which means, $length_{i+1} = \frac{1}{2}length_i$. Thus $c = 1/2$ and we have exact linear convergence. \square