mountain pass theorem

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Let X a real Banach space and $F \in C^1(X, \mathbb{R})$. Consider K a compact metric space, and $K^* \subset K$ a closed nonempty subset of K. If $p^* : K^* \to X$ is a continuous mapping, set

$$\mathcal{P} = \{p \in C(K,\, X);\ p = p^* \text{ on } K^*\}.$$

Define

$$c = \inf_{p \in \mathcal{P}} \max_{t \in K} F(p(t)).$$

Assume that

$$c > \max_{t \in K^*} F(p^*(t)). \tag{1}$$

Then there exists a sequence (x_n) in X such that

(i) $\lim\limits_{n \to \infty} F(x_n) = c$;

(ii) $\lim\limits_{n \to \infty} \|F'(x_n)\| = 0$.

The name of this theorem is a consequence of a simplified visualization for the objects from theorem. If we consider the set $K^* = \{A,\, B\}$, where A and B are two villages, $\mathcal{P}$ is the set of all the routes from A to B, and $F(x)$ represents the altitude of point x; then the assumption (**??**) is equivalent to say that the villages A and B are separated with a mountains chain. So, the conclusion of the theorem tell us that exists a route between the villages with a minimal altitude. With other words exists a “mountain pass” .