nth root by Newton's method

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The Newton's method is very suitable for computing approximate values of higher n^{th} http://planetmath.org/NthRootroots of positive numbers (and odd roots of negative numbers!).

The general recurrence formula

$$x_{k+1} \;=\; x_k - \frac{f(x_k)}{f'(x_k)}$$

of the method for determining the zero of a function f, applied to

$$f(x) \;:=\; x^n - \alpha$$

whose zero is $\sqrt[n]{\alpha}$, reads

$$x_{k+1} \;=\; \frac{1}{n}\left[(n-1)x_k + \frac{\alpha}{x_k^{n-1}}\right]. \tag{1}$$

For a radicand α, beginning from some initial value x_0 and using (1) repeatedly with successive values of k, one obtains after a few steps a sufficiently accurate value of $\sqrt[n]{\alpha}$ if x_0 was not very far from the searched root.

Especially for cube root $\sqrt[3]{\alpha}$, the formula (1) is

$$x_{k+1} \;=\; \frac{1}{3}\left[2x_k + \frac{\alpha}{x_k^2}\right]. \tag{2}$$

For example, if one wants to compute $\sqrt[3]{2}$ and uses $x_0 = 1$, already the fifth step gives

$$x_5 \;=\; 1.259921049894873$$

which decimals.