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Ekeland’s variational principle

Canonical name	EkelandsVariationalPrinciple
Date of creation	2013-03-22 15:19:16
Last modified on	2013-03-22 15:19:16
Owner	ncrom (8997)
Last modified by	ncrom (8997)
Numerical id	8
Author	ncrom (8997)
Entry type	Theorem
Classification	msc 49J40

Let (M, d) be a complete metric space and let $\psi : M \rightarrow (-\infty, +\infty]$, $\psi \not\equiv +\infty$, be a lower semicontinuous function which is bounded from below. Then the following hold: For every $\varepsilon > 0$ and for any $z_0 \in M$ there exists $z \in M$ such that

- (i) $\psi(z) \leq \psi(z_0) - \varepsilon d(z, z_0)$;
- (ii) $\psi(x) \geq \psi(z) - \varepsilon d(x, z)$, for any $x \in M$.