essential boundary

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Let $E \subset \mathbf{R}^n$ be a measurable set. We define the *essential boundary* of E as

$$\partial^* E := \{x \in \mathbf{R}^n \colon 0 < |E \cap B_\rho(x)| < |B_\rho(x)|, \quad \forall \rho > 0\}$$

where $|\cdot|$ is the Lebesgue measure.

Compare the definition of $\partial^* E$ with the definition of the topological boundary ∂E which can be written as

$$\partial E = \{x \in \mathbf{R}^n \colon \emptyset \subsetneq E \cap B_\rho(x) \subsetneq B_\rho(x), \quad \forall \rho > 0\}.$$

Hence one clearly has $\partial^* E \subset \partial E$.

Notice that the essential boundary does not depend on the Lebesgue representative of the set E, in the sense that if $|E \triangle F| = 0$ then $\partial^* E = \partial^* F$. For example if $E = \mathbf{Q}^n \subset \mathbf{R}^n$ is the set of points with rational coordinates, one has $\partial^* E = \emptyset$ while $\partial E = \mathbf{R}^n$.

Nevertheless one can easily prove that $\partial^* E$ is always a closed set (in the usual sense).