Lagrange multiplier method

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The Lagrange multiplier method is used when one needs to find the extreme or stationary points of a function on a set which is a subset of the domain.

Method

Suppose that $f(\mathbf{x})$ and $g_i(\mathbf{x}), i = 1, ..., m$ $(\mathbf{x} \in \mathbb{R}^n)$ are differentiable functions that map $\mathbb{R}^n \mapsto \mathbb{R}$, and we want to solve

$$\min f(\mathbf{x}), \max f(\mathbf{x}) \quad \text{such that} \quad g_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m$$

By a calculus theorem, if the constaints are independent, the gradient of f, ∇f, must satisfy the following equation at the stationary points:

$$\nabla f = \sum_{i=1}^{m} \lambda_i \nabla g_i$$

The constraints are said to be independent iff all the gradients of each constraint are linearly independent, that is:

$\{\nabla g_1(\mathbf{x}), \ldots, \nabla g_m(\mathbf{x})\}$ is a set of linearly independent vectors on all points where the constraints are verified.

Note that this is equivalent to finding the stationary points of:

$$f(\mathbf{x}) - \sum_{i=1}^{m} \lambda_i (g_i(\mathbf{x}))$$

for $\mathbf{x}$ in the domain and the *Lagrange multipliers* λ_i without restrictions.

After finding those points, one applies the g_i constraints to get the actual stationary points subject to the constraints.