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Lagrange multipliers on Banach spaces

Canonical name	LagrangeMultipliersOnBanachSpaces
Date of creation	2013-03-22 15:28:30
Last modified on	2013-03-22 15:28:30
Owner	stevecheng (10074)
Last modified by	stevecheng (10074)
Numerical id	5
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Entry type	Theorem
Classification	msc 49-00
Classification	msc 49K35

Let U be open in a real Banach space X, and Y be another real Banach space. Let $f\colon U \to \mathbb{R}$ and $g\colon U \to Y$ be continuously differentiable functions.

Suppose that a is a minimum or maximum point of f on $M = \{x \in U : g(x) = 0\}$, and the Fréchet derivative $\mathrm{D}\, g(a)\colon X \to Y$ is surjective. Then there exists a Lagrange multiplier vector $\lambda \in Y^*$ such that

$$\mathrm{D}\, f(a) = \mathrm{D}\, g(a)^* \lambda = \lambda \circ \mathrm{D}\, g(a)\,.$$

(The function $\mathrm{D}\, g(a)^*\colon Y^* \to X^*$ denotes the pullback or adjoint by $\mathrm{D}\, g(a)$ on the continuous duals, defined by the second equality.)

If X and Y are finite-dimensional, writing out the above equation in matrix form shows that λ really is the usual Lagrange multiplier vector. The condition that $\mathrm{D}\, g(a)$ is surjective means that $\mathrm{D}\, g(a)$ must have full rank as a matrix.

References

[1] Eberhard Zeidler. *Applied functional analysis: main principles and their applications*. Springer-Verlag, 1995.