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midpoint

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The concept of <http://planetmath.org/Midpoint> midpoint of line segment is a special case of the midpoint of a curve or arbitrary figure in \mathbb{R}^2 or \mathbb{R}^3 .

A point T is a *midpoint* of the figure f , if for each point A of f there is a point B of f such that T is the midpoint of the line segment AB . One says also that f is symmetric about the point T .

Given the equation of a curve in \mathbb{R}^2 or of a surface f in \mathbb{R}^3 , one can, if , take a new point T for the origin by using the linear substitutions of the form

$$x := x' + a, \quad y := y' + b \quad \text{etc.}$$

Thus one may test whether the origin is the midpoint of f by checking whether f always contains along with any point (x, y, z) also the point $(-x, -y, -z)$.

It is easily verified the

Theorem. If the origin is the midpoint of a quadratic curve or a quadratic surface, then its equation has no <http://planetmath.org/BasicPolynomial> terms of degree 1.

Similarly one can verify the generalisation, that if the origin is the midpoint of an algebraic curve or surface of degree n , the equation has no terms of degree $n-1$, $n-3$ and so on.

Note. Some curves and surfaces have infinitely many midpoints (see <http://planetmath.org/QuadraticSurfaces> quadratic surfaces).

References

- [1] FELIX IVERSEN: *Analyttisen geometrian oppikirja*. Tiedekirjasto Nr. 19. Second edition. Kustannusosakeyhtiö Otava, Helsinki (1963).