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# SNCF metric

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The following two examples of a metric space (one of which is a real tree) obtained their name from the of the French railway system. Especially *malicious* rumour has it that if you want to go by train from $x$ to $y$ in France, the most efficient solution is to reduce the problem to going from $x$ to Paris and then from Paris to $y$.

Since their discovery, the intrinsic laws of the French way of going by train have made it around the world and reached the late-afternoon tutorials of first-term mathematics courses in an effort to lighten the moods in the guise of the following definition:

**Definition 1** (SNCF metric)**.** Let $P$ be a point in a metric space $(F, d)$. Then the *SNCF metric* $d_P$ with respect to $P$ is defined by

$$d_P(x, y) := \begin{cases} 0 & \text{if } x = y \\ d(x, P) + d(P, y) & \text{otherwise.} \end{cases}$$

It is easy to see that $d_P$ is a metric.

Now, what if the train from $x$ to Paris stops over in $y$ during the ride (or the other way round)? Sure, Paris is a beautiful city, but you wouldn't *always* want to go there and back again. To implement this, the geometric notion of "$y$ lies on the straight line defined by $x$ and $P$" is required, so the definition becomes more specialised:

**Definition 2** (SNCF metric, enhanced version)**.** Let $P$ be the origin in the space $\mathbb{R}^n$ with Euclidean norm $\|\cdot\|_2$. Then the *SNCF metric* $d_P$ is defined by

$$d_P(x, y) := \begin{cases} \|x - y\|_2 & \text{if } x \text{ and } y \text{ lie on the same ray from the origin} \\ \|x\|_2 + \|y\|_2 & \text{otherwise} \end{cases}.$$

The metric space $(\mathbb{R}^n, d_P)$ is, in addition, a real tree since if $x$ and $y$ do not lie on the same `http://planetmath.org/node/6962`ray from $P$, the only arc in $(\mathbb{R}^n, d_P)$ joining $x$ and $y$ consists of the two ray `http://planetmath.org/node/5783`segments $xP$ and $yP$. Other injections which are arcs in Euclidean $\mathbb{R}^n$ do not remain continuous in $(\mathbb{R}^n, d_P)$.