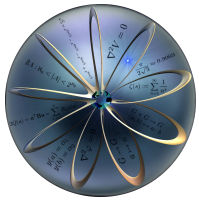

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prismatoid

Canonical name	Prismatoid
Date of creation	2013-03-22 17:12:03
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Owner	Mathprof (13753)
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Numerical id	10
Author	Mathprof (13753)
Entry type	Definition
Classification	msc 51-00
Related topic	SimpsonsRule
Related topic	Volume2
Related topic	TruncatedCone
Defines	altitude
Defines	bases
Defines	prismoidal formula

A *prismatoid* is a polyhedron, possibly not convex, whose vertices all lie in one or the other of two parallel planes. The perpendicular distance between the two planes is called the *altitude* of the prismatoid. The faces that lie in the parallel planes are called the *bases* of the prismatoid. The *midsection* is the polygon formed by cutting the prismatoid by a plane parallel to the bases halfway between them.

The volume of a prismatoid is given by the *prismoidal formula*:

$$V = \frac{1}{6}h(B_1 + B_2 + 4M)$$

where h is the altitude, B_1 and B_2 are the areas of the bases and M is the area of the midsection.

An alternate formula is :

$$V = \frac{1}{4}h(B_1 + 3S)$$

where S is the area of the polygon that is formed by cutting the prismatoid by a plane parallel to the bases but 2/3 of the distance from B_1 to B_2.

A proof of the prismoidal formula for the case where the prismatoid is convex is in [**?**]. It is also proved in [**?**] for any prismatoid. The alternate formula is proved in [**?**].

Some authors impose the condition that the lateral faces must be triangles or trapezoids. However, this condition is unnecessary since it is easily shown to hold.

References

[1] A. Day Bradley, Prismatoid, Prismoid, Generalized Prismoid, *The American Math. Monthly,* **86**, (1979), 486-490.

[2] G.B. Halsted, *Rational Geometry: A textbook for the Science of Space. Based on Hilbert's Foundations*, second edition, John Wiley and Sons, New York, 1907