harmonic division

Canonical name	HarmonicDivision
Date of creation	2013-03-22 17:34:29
Last modified on	2013-03-22 17:34:29
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Definition
Classification	msc 51N20
Classification	msc 51M04
Related topic	BisectorsTheorem
Related topic	ApolloniusCircle
Defines	harmonically
Defines	divide harmonically

- If the point X is on the line segment AB and $XA : XB = p : q$, then X divides AB *internally* in the ratio $p : q$.

- If the point Y is on the extension of line segment AB and $YA : YB = p : q$, then Y divides AB *externally* in the ratio $p : q$.

- If $p : q$ is the same in both cases, then the points X and Y divide AB *harmonically* in the ratio $p : q$.

Theorem 1. The bisectors of an angle of a triangle and its linear pair divide the opposite side of the triangle harmonically in the ratio of the adjacent sides.

Theorem 2. If the points X and Y divide the line segment AB harmonically in the ratio $p : q$, then the circle with diameter the segment XY (the so-called Apollonius' circle) is the locus of such points whose distances from A and B have the ratio $p : q$.

The latter theorem may be proved by using analytic geometry.