angle bisector as locus

Canonical name	AngleBisectorAsLocus
Date of creation	2013-03-22 17:10:41
Last modified on	2013-03-22 17:10:41
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	11
Author	pahio (2872)
Entry type	Definition
Classification	msc 51N20
Related topic	DistanceFromPointToALine
Related topic	ConverseOfIsoscelesTriangleTheorem
Related topic	ConstructionOfTangent
Related topic	LengthsOfAngleBisectors
Related topic	Incenter
Related topic	CenterNormalAndCenterNormalPlaneAsLoci

If $0 < \alpha < 180^o$, then the angle bisector of α is the locus of all such points which are equidistant from both sides of the angle (it is proved by using the AAS and SSA theorems).

The equation of the angle bisectors of all four angles formed by two intersecting lines

$$a_1x+b_1y+c_1 = 0, \qquad a_2x+b_2y+c_2 = 0 \tag{1}$$

is

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}, \tag{2}$$

which may be written in the form

$$x\sin\alpha_1 - y\cos\alpha_1 + h_1 = \pm(x\sin\alpha_2 - y\cos\alpha_2 + h_2) \tag{3}$$

after performing the divisions in (2) termwise; the angles α_1 and α_2 then the slope angles of the lines.

Note. The two lines in (2) are perpendicular, since their slopes $\dfrac{\sin\alpha_1 \pm \sin\alpha_2}{\cos\alpha_1 \pm \cos\alpha_2}$ are opposite inverses of each other.