# isogonal trajectory

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| Defines | isogonal trajectory |

Let a one-parametric family of plane curves $\gamma$ have the differential equation

$$F(x,\, y,\, \frac{dy}{dx}) \;=\; 0. \tag{1}$$

We want to determine the *isogonal trajectories* of this family, i.e. the curves $\iota$ intersecting all members of the family under a given angle, which is denoted by $\omega$. For this purpose, we denote the slope angle of any curve $\gamma$ at such an intersection point by $\alpha$ and the slope angle of $\iota$ at the same point by $\beta$. Then

$$\beta - \alpha \;=\; \omega \quad \text{(or alternatively } \;-\omega),$$

and accordingly

$$\frac{dy}{dx} \;=\; \tan\alpha \;=\; \frac{\tan\beta - \tan\omega}{1 + \tan\beta\tan\omega} \;=\; \frac{y' - \tan\omega}{1 + y'\tan\omega},$$

where $y'$ means the slope of $\iota$. Thus the equation

$$F(x,\, y,\, \frac{y' - \tan\omega}{1 + y'\tan\omega}) \;=\; 0 \tag{2}$$

is satisfied by the derivative $y'$ of the ordinate of $\iota$. In other , (2) is the differential equation of all isogonal trajectories of the given family of curves.

**Note.** In the special case $\omega = \frac{\pi}{2}$, it's a question of orthogonal trajectories.