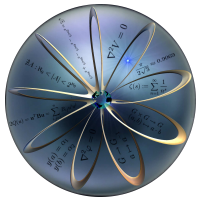

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volume of ellipsoid

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Let us determine the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \;=\; 1.$$

Suppose $-a \leqq x \leqq a$. When we cut the ellipsoid with a plane parallel to the yz-plane, that is, let x be , we get the ellipse

$$\frac{y^2}{b^2}+\frac{z^2}{c^2} \;=\; 1-\frac{x^2}{a^2},$$

i.e.

$$\frac{y^2}{b^2\left(1-\frac{x^2}{a^2}\right)}+\frac{z^2}{c^2\left(1-\frac{x^2}{a^2}\right)} \;=\; 1,$$

with the semiaxes

$$b_1 := b\sqrt{1-\frac{x^2}{a^2}}, \quad c_1 \;:=\; c\sqrt{1-\frac{x^2}{a^2}}.$$

The area of this ellipse is $\pi b_1 c_1$ (see area of plane region), and thus we have the function

$$A(x) \;:=\; \pi bc\left(1-\frac{x^2}{a^2}\right)$$

expressing the area cut of the ellipsoid by parallel planes. By the volume formula of the `http://planetmath.org/VolumeAsIntegral`parent entry we can calculate the volume of the ellipsoid as

$$V \;=\; \int_{-a}^{a} A(x)\,dx = \pi bc\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)\,dx \;=\; \pi bc\Big/_{x=-a}^{a}\left(x-\frac{x^3}{3a^2}\right) \;=\; \frac{4}{3}\pi abc.$$

The special case $a = b = c = r$ of a sphere is the well-known expression $\frac{4}{3}\pi r^3$.