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## Bregman divergence

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Synonym	Bregman distance

A *Bregman divergence*, or *Bregman distance*,  $B_F$  on a space  $\mathcal{X} \subseteq \mathbb{R}^d$  is defined for a strictly convex and differentiable function  $F : \mathcal{X} \rightarrow \mathbb{R}$  as

$$B_F(p, q) = F(p) - F(q) - \langle p - q, \nabla F(q) \rangle, \quad (1)$$

where

$$\langle p, q \rangle = p^T q$$

denotes the inner product, and

$$\nabla F(x) = [\frac{\partial F}{\partial x_1}, \dots, \frac{\partial F}{\partial x_d}]^T$$

the partial derivatives.

Choosing  $F(x) = \sum_{i=1}^d x_i^2$  yields the squared Euclidean distance  $B_{x^2}(p, q) = \|p - q\|^2$ , and choosing  $F(x) = \sum_{i=1}^d x_i \log x_i$  yields the relative entropy, called the Kullback-Leibler divergence.