volume as integral

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The http://planetmath.org/VolumeOfSolidOfRevolutionvolume of a solid of revolution can be obtained from

$$V\;=\;\int_a^b\pi[f(x)]^2\,dx,$$

where the integrand is the area of the intersection disc of the solid of revolution and a plane perpendicular to the axis of revolution at a certain value of x. This volume formula may be generalized to an analogous formula containing instead of the area $\pi[f(x)]^2$ a more general intersection area $A(t)$ obtained from a given solid by cutting it with a set of parallel planes determined by the parameter t on a certain axis. One must assume that the function $t\mapsto A(t)$ is continuous on an interval $[a,\,b]$ where a and b correspond to the "ends" of the solid. If the t-axis http://planetmath.org/AngleBetweenTwoLinesforms an angle ω with the normal line of those planes, then we have the volume formula of the form

$$V\;=\;\int_a^b A(t)\,dt\;\cos\omega.$$