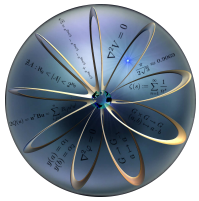

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proof of Apollonius theorem

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Let $b = \overline{CA}$, $a = \overline{BC}$, $c = \overline{AB}$, and $m = \overline{AM}$. Let $\angle CMA = \theta$, so that $\angle BMA = \pi - \theta$.

By the law of cosines, $b^2 = m^2 + \frac{a^2}{4} - am\cos\theta$ and $c^2 = m^2 + \frac{a^2}{4} - am\cos(\pi - \theta) = m^2 + \frac{a^2}{4} + am\cos\theta$, and adding gives

$$b^2 + c^2 = 2m^2 + \frac{a^2}{2}.$$

QED