proof of Erdös-Anning Theorem

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Let A, B and C be three non-collinear points. For an additional point P consider the triangle ABP. By using the triangle inequality for the sides PB and PA we find $-|AB| \leq |PB| - |PA| \leq |AB|$. Likewise, for triangle BCP we get $-|BC| \leq |PB| - |PC| \leq |BC|$. Geometrically, this means the point P lies on two hyperbola with A and B or B and C respectively as foci. Since all the lengths involved here are by assumption integer there are only $2|AB| + 1$ possibilities for $|PB| - |PA|$ and $2|BC| + 1$ possibilities for $|PB| - |PC|$. These hyperbola are distinct since they don't have the same major axis. So for each pair of hyperbola we can have at most 4 points of intersection and there can be no more than $4(2|AB| + 1)(2|BC| + 1)$ points satisfying the conditions.