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## proof of cosines law

Canonical name	ProofOfCosinesLaw
Date of creation	2013-03-22 18:27:13
Last modified on	2013-03-22 18:27:13
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Last modified by	pahio (2872)
Numerical id	4
Author	pahio (2872)
Entry type	Proof
Classification	msc 51M04
Related topic	DerivationOfCosinesLaw

Let  $a, b, c$  be the sides of a triangle and  $\alpha, \beta, \gamma$  its angles, respectively. By the projection formula, one may write the equalities

$$\begin{cases} a = b \cos \gamma + c \cos \beta \\ b = c \cos \alpha + a \cos \gamma \\ c = a \cos \beta + b \cos \alpha. \end{cases}$$

Multiplying the equalities by  $a, -b$  and  $-c$ , respectively, they read

$$\begin{cases} a^2 = ab \cos \gamma + ca \cos \beta \\ -b^2 = -bc \cos \alpha - ab \cos \gamma \\ -c^2 = -ca \cos \beta - bc \cos \alpha. \end{cases}$$

Addition of these yields the sum equation

$$a^2 - b^2 - c^2 = -2bc \cos \alpha,$$

i.e.

$$a^2 = b^2 + c^2 - 2bc \cos \alpha,$$

which is the cosines law.