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some theorems on the axioms of order

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Let B be a betweenness relation on a set A .

Theorem 1.

If $(a, b, c) \in B$ and $(a, c, d) \in B$, then $(a, b, d) \in B$.

Theorem 2. For each pair of elements $p, q \in A$, we can define five sets:

1. $B_{*pq} := \{r \in A \mid (r, p, q) \in B\},$
2. $B_{p*q} := \{r \in A \mid (p, r, q) \in B\},$
3. $B_{pq*} := \{r \in A \mid (p, q, r) \in B\},$
4. $B_{pq} := B_{p*q} \cup \{q\} \cup B_{pq*},$ and
5. $B(p, q) := B_{*pq} \cup \{p\} \cup B_{pq}.$

Then

- (1) $B_{*pq} = B_{qp*}.$
- (2) $B_{p*q} = B_{q*p}.$
- (3) The intersection of any pair of the first three sets contains at most one element, either p or q .
- (4) Each of the sets can be partially ordered.
- (5) The partial order on B_{pq} and $B(p, q)$ extends that of the subsets.