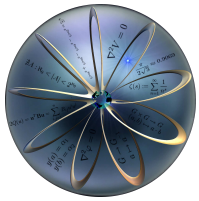

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similarity in geometry

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Synonym	similarity
Synonym	similitude
Related topic	Homothety
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Defines	similar
Defines	ratio of similarity
Defines	similitude ratio
Defines	line ratio

Two figures K and K' in a Euclidean plane or `http://planetmath.org/EuclideanVectorSpace` are *similar* iff there exists a bijection f from the set of points of K onto the set of points of K' such that, for any $P, Q \in K$, the ratio

$$\frac{P'Q'}{PQ}$$

of the lengths of the line segments $P'Q'$ and PQ is always the same number k, where $P' = f(P)$ and $Q' = f(Q)$.

The number k is called the *ratio of similarity* or the *line ratio* of the figure K' with respect to the figure K (N.B. the in which the figures are mentioned!). The similarity of K and K' is often denoted by

$$K' \sim K \quad (\text{or } \ K \sim K').$$

Examples

- All squares are similar.
- All cubes are similar.
- All circles are similar.
- All parabolas are similar.
- All sectors of circle with equal central angle are similar.
- All spheres are similar.
- All equilateral triangles are similar.

Nonexamples

- Not all rectangles are similar.
- Not all rhombi are similar.
- Not all rectangular prisms are similar.
- Not all ellipses are similar.

- Not all ellipsoids are similar.
- Not all triangles are similar.

Properties

- The corresponding angles (consisting of corresponding points) of two similar figures are equal.
- The lengths of any corresponding arcs of two similar figures are proportional in the ratio k.
- The areas of two similar regions are proportional in the ratio k^2 when k is the line ratio of the regions.
- The volumes of two similar solids are proportional in the ratio k^3 when k is the line ratio of the solids.

Remarks

- In any Euclidean space E, the `http://planetmath.org/Relation`relation of similarity (denoted $\sim$) on the set of figures in E is an equivalence relation.
- If one pair of corresponding line segments in the similar figures K and K' are equal, then all pairs of corresponding line segments are equal, i.e. the figures have also equal : They are `http://planetmath.org/Congruence`congruent ($K' \cong K$).