vector projection

Canonical name	VectorProjection
Date of creation	2013-03-22 19:05:40
Last modified on	2013-03-22 19:05:40
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	13
Author	pahio (2872)
Entry type	Definition
Classification	msc 51N99
Classification	msc 51M04
Classification	msc 51F20
Related topic	Projection
Related topic	GramSchmidtOrthogonalization
Defines	vector component
Defines	scalar projection
Defines	scalar component

The principle used in the projection of line segment a line, which results a line segment, may be extended to concern the projection of a vector $\vec{u}$ on another non-zero vector $\vec{v}$, resulting a vector.

This projection vector, the so-called *vector projection* $\vec{u}_{\vec{v}}$ will be `http://planetmath.org/node/` to $\vec{v}$. It could have the `http://planetmath.org/Vector`length equal to $|\vec{u}|$ multiplied by the cosine of the inclination angle between the lines of $\vec{u}$ and $\vec{v}$, as in the case of line segment.

But better than that "inclination angle" is to take the `http://planetmath.org/node/6178`angle between the both vectors $\vec{u}$ and $\vec{v}$ which may also be obtuse or straight; in these cases the cosine is negative which is suitable to cause the projection vector $\vec{u}_{\vec{v}}$ to have the direction to $\vec{v}$ ($\vec{u}_{\vec{v}} \downarrow\uparrow \vec{v}$). In all cases we define the *vector projection* or the *vector component* of $\vec{u}$ along $\vec{v}$ as

$$\vec{u}_{\vec{v}} \;:=\; |\vec{u}|\cos(\vec{u},\vec{v})\,\vec{v}^{\circ} \tag{1}$$

where $\vec{v}^{\circ}$ is the unit vector having the `http://planetmath.org/node/6178`same direction as $\vec{v}$ (i.e., $\vec{v}^{\circ} \upuparrows \vec{v}$). For the that if $\vec{u} = \vec{0}$ and the angle is , then also the vector projection is the zero vector.

Using the expression for the `http://planetmath.org/node/6178`cosine of the angle between vectors and for the unit vector we thus have

$$\vec{u}_{\vec{v}} \;=\; |\vec{u}|\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\,\frac{\vec{v}}{|\vec{v}|}.$$

This is to

$$\vec{u}_{\vec{v}} \;=\; \frac{\vec{u}\cdot\vec{v}}{|\vec{v}||\vec{v}|}\,\vec{v}, \tag{2}$$

where the denominator is the scalar square of $\vec{v}$:

$$\vec{u}_{\vec{v}} \;=\; \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v} \tag{3}$$

One can also write from (1) the alternative form

$$\vec{u}_{\vec{v}} \;=\; (\vec{u}\cdot\vec{v}^{\circ})\,\vec{v}^{\circ}, \tag{4}$$

where the "coefficient" $\vec{u}\cdot\vec{v}^{\circ}$ of the unit vector $\vec{v}^{\circ}$ is called the *scalar projection* or the *scalar component* of $\vec{u}$ along $\vec{v}$.

Remark 1. The vector projection $\vec{u}_{\vec{v}}$ of $\vec{u}$ along $\vec{v}$ is sometimes denoted by $\text{proj}_{\vec{v}}\,\vec{u}$.

Remark 2. If one http://planetmath.org/DifferenceOfVectorssubtracts from $\vec{u}$ the vector component $\vec{u}_{\vec{v}}$, then one has another component of $\vec{u}$ such that the both components are orthogonal to each other (and their http://planetmath.org/SumVectorsum is $\vec{u}$); the orthogonality of the components follows from

$$(\vec{u}-\vec{u}_{\vec{v}})\cdot\vec{u}_{\vec{v}} \;=\; \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{u}\cdot\vec{v}-\left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)^2\vec{v}\cdot\vec{v} \;=\; 0.$$

Remark 3. The usual "component form"

$$\vec{u} \;=\; x\vec{i}+y\vec{j}+z\vec{k}$$

of vectors in the cartesian coordinate system of $\mathbb{R}^3$ that the http://planetmath.org/OrthogonalV vector components of $\vec{u}$ along the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ are

$$\vec{u}_{\vec{i}} \;=\; x\vec{i}, \quad \vec{u}_{\vec{j}} \;=\; y\vec{j}, \quad \vec{u}_{\vec{k}} \;=\; z\vec{k}$$

and the scalar components are x, y, z, respectively.