Szemerédi-Trotter theorem

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The number of incidences of a set of n points and a set of m lines in the real plane $\mathbb{R}^2$ is

$$I = O(n + m + (nm)^{\frac{2}{3}}).$$

Proof. Let's consider the points as vertices of a graph, and connect two vertices by an edge if they are adjacent on some line. Then the number of edges is $e = I - m$. If $e < 4n$ then we are done. If $e \geq 4n$ then by crossing lemma

$$m^2 \geq \mathrm{cr}(G) \geq \frac{1}{64} \frac{(I - m)^3}{n^2},$$

and the theorem follows.

Recently, Tóth[**?**] extended the theorem to the complex plane $\mathbb{C}^2$. The proof is difficult.

References

[1] Csaba D. Tóth. The Szemerédi-Trotter theorem in the complex plane. `http://www.arxiv.org/abs/math.CO/0305283`arXiv:CO/0305283, May 2003.