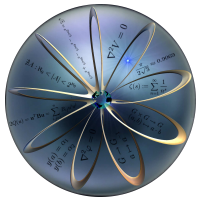

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similitude of parabolas

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Two parabolas need not be congruent, but they are always similar. Without the definition of parabola by focus and directrix, the fact turns out of the simplest equation $y = ax^2$ of parabola.

Let us take two parabolas

$$y \;=\; ax^2 \quad \text{and} \quad y \;=\; bx^2$$

which have the origin as common vertex and the y-axis as common axis. Cut the parabolas with the line $y = mx$ through the vertex. The first parabola gives

$$ax^2 = mx,$$

whence the abscissa of the other point of intersection is $\frac{m}{a}$; the corresponding ordinate is thus $\frac{m^2}{a}$. So, this point has the position vector

$$\vec{u} \;=\; \begin{pmatrix} \frac{m}{a} \\ \frac{m^2}{a} \end{pmatrix} \;=\; \frac{m}{a}\begin{pmatrix} 1 \\ m \end{pmatrix}$$

Similarly, the cutting point of the line and the second parabola has the position vector

$$\vec{v} \;=\; \begin{pmatrix} \frac{m}{b} \\ \frac{m^2}{b} \end{pmatrix} \;=\; \frac{m}{b}\begin{pmatrix} 1 \\ m \end{pmatrix}$$

Accordingly, those position vectors have the `http://planetmath.org/node/848`linear depencence

$$a\vec{u} \;=\; b\vec{v}$$

for all values of the slope m of the cutting line. This means that both parabolas are homothetic with respect to the origin and therefore also similar.