parallelism of two planes

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Two planes π and ϱ in the 3-dimensional Euclidean space are *parallel* iff they either have no common points or coincide, i.e. iff

$$\pi \cap \varrho \;=\; \varnothing \quad \text{or} \quad \pi \cap \varrho \;=\; \pi. \tag{1}$$

An http://planetmath.org/Equivalent3equivalent condition of the parallelism is that the normal vectors of π and ϱ are parallel.
The parallelism of planes is an equivalence relation in any set of planes of the space.

If the planes have the equations

$$A_1x+B_1y+C_1z+D_1 \;=\; 0 \quad \text{and} \quad A_2x+B_2y+C_2z+D_2 \;=\; 0, \tag{2}$$

the parallelism means the http://planetmath.org/Variationproportionality of the coefficients of the variables: there exists a k such that

$$A_1 \;=\; kA_2, \quad B_1 \;=\; kB_2, \quad C_1 \;=\; kC_2. \tag{3}$$

In this case, if also $D_1 \;=\; kD_2$, then the planes coincide.
Using vectors, the condition (3) may be written

$$\begin{pmatrix} A_1\\ B_1\\ C_1 \end{pmatrix} \;=\; k\begin{pmatrix} A_2\\ B_2\\ C_2 \end{pmatrix} \tag{4}$$

which equation utters the http://planetmath.org/MutualPositionsOfVectorsparallelism of the normal vectors.

Remark. The shortest distance of the parallel planes

$$Ax+By+Cz+D \;=\; 0 \quad \text{and} \quad Ax+By+Cz+E \;=\; 0$$

is obtained from the

$$d \;=\; \frac{|D-E|}{\sqrt{A^2+B^2+C^2}}, \tag{5}$$

as is easily shown by using http://planetmath.org/LagrangeMultiplierMethodLagrange multipliers (see http://planetmath.org/node/11604this entry).