betweenness relation

Canonical name	BetweennessRelation
Date of creation	2013-03-22 17:18:44
Last modified on	2013-03-22 17:18:44
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	6
Author	Mathprof (13753)
Entry type	Definition
Classification	msc 51G05
Synonym	axioms of order
Related topic	SomeTheoremsOnTheAxiomsOfOrder

1 Definition

Let A be a set. A ternary relation B on A is said to be a *betweenness relation* if it has the following properties:

O1 if $(a, b, c) \in B$, then $(c, b, a) \in B$; in other words, the set

$$B(b) = \{(a, c) \mid (a, b, c) \in B\}$$

is a `http://planetmath.org/Symmetric`symmetric relation for *each* b; thus, from now on, we may say, without any ambiguity, that b is *between* a and c if $(a, b, c) \in B$;

O2 if $(a, b, a) \in B$, then $a = b$;

O3 for each $a, b \in A$, there is a $c \in A$ such that $(a, b, c) \in B$;

O4 for each $a, b \in A$, there is a $c \in A$ such that $(a, c, b) \in B$;

O5 if $(a, b, c) \in B$ and $(b, a, c) \in B$, then $a = b$;

O6 if $(a, b, c) \in B$ and $(b, c, d) \in B$, then $(a, b, d) \in B$;

O7 if $(a, b, d) \in B$ and $(b, c, d) \in B$, then $(a, b, c) \in B$.