classification of Platonic solids

Canonical name	ClassificationOfPlatonicSolids
Date of creation	2013-03-22 15:53:07
Last modified on	2013-03-22 15:53:07
Owner	mps (409)
Last modified by	mps (409)
Numerical id	5
Author	mps (409)
Entry type	Result
Classification	msc 51-00

Proposition.. *The regular tetrahedron, regular octahedron, regular icosahedron, cube, and regular dodecahedron are the only Platonic solids.*

Proof. Each vertex of a Platonic solid is incident with at least three faces. The interior angles incident with that vertex must sum to less than 2π, for otherwise the solid would be flat at that vertex. Since all faces of the solid have the same number of sides, this implies bounds on the number of faces which could meet at a vertex.

The interior angle of an equilateral triangle has measure $\frac{\pi}{3}$, so a Platonic solid could only have three, four, or five triangles meeting at each vertex. By similar reasoning, a Platonic solid could only have three squares or three pentagons meeting at each vertex. But the interior angle of a regular hexagon has measure $\frac{2\pi}{3}$. To avoid flatness a solid with hexagons as faces would thus have to have only two faces meeting at each vertex, which is impossible. For polygons with more sides it only gets worse.

Since a Platonic solid is uniquely determined by the number and kind of faces meeting at each vertex, there are at most five Platonic solids, with the numbers and kinds of faces listed above. But these correspond to the five known Platonic solids. Hence there are exactly five Platonic solids. $\square$