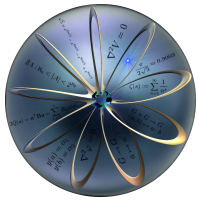

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alternate proof of parallelogram law

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Proof of this is simple, given the cosine law:

$$c^2 = a^2 + b^2 - 2ab\cos\phi$$

where a, b, and c are the lengths of the sides of the triangle, and angle ϕ is the corner angle opposite the side of length c.

Let us define the largest interior angles as angle θ. Applying this to the parallelogram, we find that

$$\begin{aligned} d_1^2 &= u^2 + v^2 - 2uv\cos\theta \\ d_2^2 &= u^2 + v^2 - 2uv\cos(\pi - \theta) \end{aligned}$$

Knowing that

$$\cos(\pi - \theta) = -\cos\theta$$

we can add the two expressions together, and find ourselves with

$$\begin{aligned} d_1^2 + d_2^2 &= 2u^2 + 2v^2 - 2uv\cos\theta + 2uv\cos\theta \\ d_1^2 + d_2^2 &= 2u^2 + 2v^2 \end{aligned}$$

which is the theorem we set out to prove.