ruled surface

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A straight line g moving continuously in space sweeps a *ruled surface*. Formally: A surface S in $\mathbb{R}^3$ is a ruled surface if it is connected and if for any point p of S, there is a line g such that $p \in g \subset S$.

Such a surface may be formed by using two auxiliary curves given e.g. in the parametric forms

$$\vec{r} \;=\; \vec{a}(t), \qquad \vec{r} \;=\; \vec{b}(t).$$

Using two parameters s and t we express the `http://planetmath.org/PositionVector`position vector of an arbitrary point of the ruled surface as

$$\vec{r} \;=\; \vec{a}(t) + s\,\vec{b}(t).$$

Here $\vec{r} = \vec{a}(t)$ is a curve on the ruled surface and is called or the of the surface, while $\vec{r} = \vec{b}(t)$ is the *director curve* of the surface. Every position of g is a *generatrix* or *ruling* of the ruled surface.

Examples

1. Choosing the z-axis ($\vec{r} = ct\vec{k},\;\; c \neq 0$) as the and the unit circle ($\vec{r} = \vec{i}\cos t + \vec{j}\sin t$) as the director curve we get the *helicoid* ("screw surface"; cf. the circular helix)

$$\vec{r} \;=\; ct\vec{k} + s\,(\vec{i}\cos t + \vec{j}\sin t) \;=\; \begin{pmatrix} s\,\cos t \\ s\,\sin t \\ ct \end{pmatrix}.$$

2. The equation

$$z \;=\; xy$$

presents a hyperbolic paraboloid (if we `http://planetmath.org/RotationMatrix`rotate the coordinate system 45 about the z-axis using the formulae $x = (x' - y')/\sqrt{2},\; y = (x' + y')/\sqrt{2},$ the equation gets the form $x'^2 - y'^2 = 2z$). Since the position vector of any point of the surface may be written using the parameters s and t as

$$\vec{r} \;=\; \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ t \end{pmatrix},$$

we see that it's a question of a ruled surface with rectilinear directrix and director curve.

3. Other ruled surfaces are for example all cylindrical surfaces (plane included), conical surfaces, `http://planetmath.org/QuadraticSurfaces`one-sheeted hyperboloid.