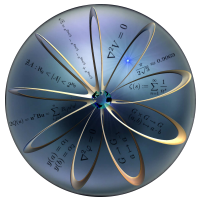

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comparison of common geometries

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In this entry, the most common models of the three most common two-dimensional geometries (http://planetmath.org/EuclideanGeometryEuclidean, http://planetmath.org/HyperbolicGeometryhyperbolic, and http://planetmath.org/Spheri will be considered.

The following abbreviations will be used in this entry:

- E^2 for the Euclidean plane (the most common model for two-dimensional Euclidean geometry);
- $\mathbb{H}^2$ for two-dimensional hyperbolic geometry;
- BK for the Beltrami-Klein model of $\mathbb{H}^2$;
- PD for the Poincaré disc model of $\mathbb{H}^2$;
- UHP for the upper half plane model of $\mathbb{H}^2$;
- S^2 for the unit sphere (the most common model for two-dimensional spherical geometry).

1 Comparison of Properties of the Models

property	E^2	BK	PD	UHP	
model has area when considered as a subset of a Euclidean space	no	yes	yes	no	
lines in model look like	lines	line segments	some line segments, some arcs of circles	some vertical rays, some semicircles	ci
lines have length when considered as a subset of a Euclidean space	no	yes	yes	yes for semicircles, no for vertical rays	
angles are preserved in model	yes	no	yes	yes	

2 Comparison of Properties of the Geometries

property	E^2	$\mathbb{H}^2$	
two distinct points determine a unique line	yes	yes	
parallel lines exist	yes	yes	
number of lines parallel to a given line and passing through a point not on the given line	1	∞	
space has infinite area with respect to its own geometry	yes	yes	
lines have infinite length	yes	yes	
number of `http://planetmath.org/Center8`centers of a circle	1	1	
angle sum Σ of triangles (in radians)	$\Sigma = \pi$	$0 < \Sigma < \pi$	
ASA holds	yes	yes	
SAS holds	yes	yes	
SSS holds	yes	yes	
AAS holds	yes	yes	`http://p`
AAA holds	no	yes	