proof of Heron's formula

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Let α be the angle between the sides b and c, then we get from the cosines law:

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}.$$

Using the equation

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

we get:

$$\sin\alpha = \frac{\sqrt{-a^4 - b^4 - c^4 + 2b^2c^2 + 2a^2b^2 + 2a^2c^2}}{2bc}.$$

Now we know:

$$\Delta = \frac{1}{2}bc\sin\alpha.$$

So we get:

$$\begin{aligned}\Delta &= \frac{1}{4}\sqrt{-a^4 - b^4 - c^4 + 2b^2c^2 + 2a^2b^2 + 2a^2c^2} \\ &= \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(a+c-b)(a+b-c)} \\ &= \sqrt{s(s-a)(s-b)(s-c)}.\end{aligned}$$

This is Heron's formula. $\square$