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Euler line proof

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Let O the circumcenter of $\triangle ABC$ and G its centroid. Extend OG until a point P such that $OG/GP = 1/2$. We'll prove that P is the orthocenter H .

Draw the median AA' where A' is the midpoint of BC . Triangles OGA' and PGA are similar, since $GP = 2GO$, $AG = 2A'G$ and $\angle OGA' = \angle PGA$. Then $\angle OA'G = \angle PGA$ and $OA' \parallel AP$. But $OA' \perp BC$ so $AP \perp BC$, that is, AP is a height of the triangle.

Repeating the same argument for the other medians proves that P lies on the three heights and therefore it must be the orthocenter H .

The ratio is $OG/GH = 1/2$ since we constructed it that way.