proof of when is a point inside a triangle

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Owner	joen235 (18354)
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Author	joen235 (18354)
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Let $\mathbf{u} \in \mathbb{R}^2$, $\mathbf{v} \in \mathbb{R}^2$ and $\mathbf{0} \in \mathbb{R}^2$. Let's consider the convex hull of the set $T = \{\mathbf{u}, \mathbf{v}, \mathbf{0}\}$. By definition, the convex hull of T, noted coT, is the smallest convex set that contains T. Now, the triangle Δ_T spanned by T is convex and contains T. Then $coT \subseteq \Delta_T$. Now, every convex C set containing T must satisfy that $t\mathbf{u} + (1-t)\,\mathbf{v} \in C$, $t\mathbf{u} \in C$ and $t\mathbf{v} \in C$ for $0 \leq t \leq 1$ (at least the convex combination of the points of T are contained in C). This means that the boundary of Δ_T is contained in C. But then every convex combination of points of $\partial\Delta_T$ must also be contained in C, meaning that $\Delta_T \subseteq C$ for every convex set containing T. In particular, $\Delta_T \subseteq coT$.

Since the convex hull is exactly the set containing all convex combinations of points of T,

$$\Delta_T = coT = \left\{\mathbf{x} \in \mathbb{R}^2 : \mathbf{x} = \lambda\mathbf{u} + \mu\mathbf{v} + (1-\lambda-\mu)\,\mathbf{0}, 0 \leq \lambda, \mu, \leq 1, 0 \leq 1-\lambda-\mu \leq 1\right\}$$

we conclude that $\mathbf{x} \in \mathbb{R}^2$ is in the triangle spanned by T if and only if $\mathbf{x} = \lambda\mathbf{u} + \mu\mathbf{v}$ with $0 \leq \lambda, \mu, \leq 1$ and $0 \leq \lambda + \mu \leq 1$.