volume of solid of revolution

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Let us consider a solid of revolution, which is generated when a planar domain D rotates about a line of the same plane. We chose this line for the x-axis, and for simplicity we assume that the boundaries of D are the mentioned axis, two ordinates $x = a,\ x = b\,(> a)$, and a continuous curve $y = f(x)$.

Between the bounds a anb b we fit a sequence of points $x_1,\, x_2,\, \ldots,\, x_{n-1}$ and draw through these the ordinates which divide the domain D in n parts. Moreover we form for every part the (maximal) inscribed and the (minimal) circumscribed rectangle. In the revolution of D, each rectangle generates a circular cylinder. The considered solid of revolution is part of the volume $V_>$ of the union of the cyliders generated by the circumscribed rectangles and at the same time contains the volume $V_<$ of the union of the cylinders generated by the inscribed rectangles.

Now it is apparent that

$$V_> = \pi[M_1^2(x_1 - a) + M_2^2(x_2 - x_1) + \ldots + M_n^2(b - x_{n-1})],$$

$$V_< = \pi[m_1^2(x_1 - a) + m_2^2(x_2 - x_1) + \ldots + m_n^2(b - x_{n-1})],$$

where $M_1,\, M_2,\, \ldots,\, M_n$ are the greatest and $m_1,\, m_2,\, \ldots,\, m_n$ the least values of the continuous function f on the `http://planetmath.org/Interval`intervals $[a,\, x_1],\ [x_1,\, x_2], \ldots, [x_{n-1},\, b]$. The volume V of the solid of revolution thus satisfies

$$V_< \leq V \leq V_>,$$

and this is true for any $x_1 < x_2 < \ldots < x_{n-1}$ of the interval $[a,\, b]$. The theory of the Riemann integral guarantees that there exists only one real number V having this property and that it is also the definition of the integral $\displaystyle\int_a^b \pi[f(x)]^2\,dx$. Therefore the volume of the given solid of revolution can be obtained from

$$V = \pi\int_a^b [f(x)]^2\,dx.$$

References

[1] E. Lindelöf: *Johdatus korkeampaan analyysiin.* Neljäs painos. Werner Söderström Osakeyhtiö, Porvoo ja Helsinki (1956).