SSA

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SSA is a method for determining whether two triangles are congruent by comparing two sides and a non-inclusive angle. However, unlike SAS, SSS, ASA, and SAA, this does not prove congruence in all cases.

Suppose we have two triangles, $\triangle ABC$ and $\triangle PQR$. $\triangle ABC \cong ? \triangle PQR$ if $\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, and either $\angle BAC \cong \angle QPR$ or $\angle BCA \cong \angle QRP$.

Since this method does not prove congruence, it is more useful for disproving it. If the SSA method is attempted between $\triangle ABC$ and $\triangle PQR$ and fails for every ABC,BCA, and CBA against every PQR,QRP, and RPQ, then $\triangle ABC \ncong \triangle PQR$.

Suppose $\triangle ABC$ and $\triangle PQR$ the SSA test. The specific case where SSA fails, known as the ambiguous case, occurs if the congruent angles, $\angle BAC$ and $\angle QPR$, are acute. Let us illustrate this.

Suppose we have a right triangle, $\triangle XYZ$, with right angle $\angle XZY$. Let P and Q be two points on $\overleftrightarrow{XZ}$ equidistant from Z such that P is between X and Z and Q is not. Since $\angle XZY$ is right, this makes $\angle PZY$ right, and P,Q are equidistant from Z, thus $\overleftrightarrow{YZ}$ bisects P and Q, and as such, every point on that line is equidistant from P and Q. From this, we know Y is equidistant from P and Q, thus $\overline{YP} \cong \overline{YQ}$. Further, $\angle YXP$ is in fact the same angle as $\angle YXQ$, thus $\angle YXP \cong \angle YXQ$. Since $\overline{XY} \cong \overline{XY}$, $\triangle XYP$ and $\triangle XYQ$ clearly meet the SSA test, and yet, just as clearly, are not congruent. This results from $\angle YXZ$ being acute. This example also reveals the exception to the ambiguous case, namely $\triangle XYZ$. If R is a point on $\overleftrightarrow{XZ}$ such that $\overline{YR} \cong \overline{YZ}$, then $R \cong Z$. Proving this exception amounts to determining that $\angle XZY$ is right, in which case the congruency could be proven instead with SAA.

However, if the congruent angles are not acute, i.e., they are either right or obtuse, then SSA is definitive.