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duality in mathematics

Canonical name	DualityInMathematics
Date of creation	2013-03-22 18:24:50
Last modified on	2013-03-22 18:24:50
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	51
Author	bci1 (20947)
Entry type	Topic
Classification	msc 51A10
Classification	msc 14F25
Classification	msc 55M05
Classification	msc 18-00
Synonym	categorical duality
Synonym	Poincaré duality
Synonym	polarity
Related topic	IndexOfCategoryTheory
Related topic	SerreDuality
Related topic	StoneSpace
Related topic	CompactQuantumGroup
Related topic	PoincareDuality
Related topic	Polarity2
Related topic	DualOfACoalgebraIsAnAlgebra
Related topic	GrassmanHopfAlgebrasAndTheirDualCoAlgebras
Related topic	PontryaginDuality
Related topic	LinearProgrammingProblem
Related topic	IdealInvertingInPruferRing
Related topic	IndexOfCategories

0.1 Duality in mathematics

The following is a mathematical topic entry on different types of *duality* encountered in different areas of mathematics; accordingly there is a string of distinct definitions associated with this topic rather than a single, general definition, although some of the linked definitions, that is, categorical duality, are more general than others.

0.1.1 Duality definitions in mathematics:

1. <http://planetmath.org/IndexOfCategoryTheoryCategoricaldualityandDualcategory>: reversing arrows
2. <http://planetmath.org/DualityPrinciple>Duality principle
3. Double duality
4. Triality
5. Self-duality
6. Duality functors, (for example the duality functor $Hom_k(-, k)$)
7. <http://planetmath.org/PoincareDuality>Poincaré duality/Poincaré isomorphism
8. Poincaré-Lefschetz duality, and Alexander-Lefschetz duality
9. Alexander duality: J. W. Alexander's duality theory (cca. 1915)
10. Serre duality : <http://planetmath.org/ProofOfRiemannRochTheoremexample>- in the proof of the Riemann-Roch theorem for curves.
11. Dualities in logic, example: <http://planetmath.org/IdealInvertingInPruferRingDeMorganDual>, Boolean algebra
12. Stone duality: Boolean algebras and Stone spaces
13. Dual numbers- as in an associative algebra; (almost synonymous with double)

14. Geometric dualities: dual polyhedron, dual of a planar graph, duality in order theory, the Legendre transformation -an application of the duality between points and lines; generalized Legendre, that is, the Legendre-Fenchel transformation.
15. Hamilton–Lagrange duality in theoretical mechanics and optics
16. <http://planetmath.org/DualSpace>Dual space
17. <http://planetmath.org/DoubleDualEmbedding>Dual space example
18. <http://planetmath.org/DualHomomorphism>Dual homomorphisms
19. <http://planetmath.org/Polarity2>Duality of Projective Geometry
20. Analytic dualities
21. <http://planetmath.org/DualOfACoalgebraIsAnAlgebra>Duals of an algebra/algebraic duality, for example, dual pairs of Hopf \ast -algebras and duality of cross products of C^\ast -algebras
22. <http://planetmath.org/GrassmanHopfAlgebrasAndTheirDualCoAlgebras>Tangled, or Mirror, duality: interchanging morphisms and objects
23. Duality as a homological mirror symmetry
24. Cohomology theory duals: de Rham cohomology \longleftrightarrow Alexander-Spanier cohomology
25. Hodge dual
26. <http://planetmath.org/CompactQuantumGroup>Duality of locally compact groups
27. <http://planetmath.org/PontryaginDuality>Pontryagin duality, for locally compact commutative topological groups and their linear representations
28. <http://planetmath.org/CompactQuantumGroup>Tannaka-Krein duality: for compact matrix pseudogroups and non-commutative topological groups; its generalization leads to quantum groups in Quantum theories; Tannaka’s theorem provides the means to reconstruct a compact

group G from its category of representations $\Pi(G)$; Krein's theorem shows which categories arise as a dual object to a compact group; the finite-dimensional representations of Drinfel'd's quantum groups form a braided monoidal category, whereas $\Pi(G)$ is a symmetric monoidal category.

29. Tannaka duality: an extension of Tannakian duality by Alexander <http://planetmath.org/AlexanderGrothendieckABiographyOfGrothendieck> to algebraic groups and Tannakian categories.
30. Contravariant dualities
31. Weak duality, <http://planetmath.org/LinearProgrammingProblemexample> : weak duality theorem in linear programming; dual problems in optimization theory
32. Dual codes
33. Duality in Electrical Engineering

0.1.2 Examples of duals:

1. a category \mathcal{C} and its dual \mathcal{C}^{op}
2. the category of Hopf algebras over a field is (equivalent to) the opposite category of affine group schemes over $\text{spec } k$
3. Dual Abelian variety
4. Example of <http://planetmath.org/DualSpaceSeparatesPoints> a dual space theorem
5. <http://planetmath.org/DualGroupOfGIsHomeomorphicToTheCharacterSpaceOfL1G> Example of Pontryagin duality
6. initial and final object
7. kernel and cokernel
8. limit and colimit
9. direct sum and product

References

- [1] S. Doplicher and J. Roberts. A new duality theory for compact groups.
Inventiones Mathematicae, 98:157–218, 1989.
- [2] André Joyal and Ross Street, An introduction to Tannaka duality and quantum groups, in Part II of Category Theory, Proceedings, Como 1990, eds. A. Carboni, M. C. Pedicchio and G. Rosolini, Lectures Notes in Mathematics No.1488, Springer, Berlin, 1991, 411-492.