polygon

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Related topic	RegularPolygon
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Related topic	BasicPolygon
Related topic	Hexagon
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Defines	side
Defines	vertex
Defines	vertices
Defines	simple polygon
Defines	side-lines
Defines	ray
Defines	simple way
Defines	way
Defines	region
Defines	convex region
Defines	Jordan polygon
Defines	angles of a polygon
Defines	plane polygon
Defines	broken line

1 Definitions

We follow Forder [?] for most of this entry. The term polygon can be defined if one has a definition of an interval. For this entry we use betweenness geometry. A betweenness geometry is just one for which there is a set of points and a betweenness relation B defined. Rather than write $(a, b, c) \in B$ we write $a * b * c$.

1. If a and b are distinct points, the *line* ab is the set of all points p such that $p * a * b$ or $a * p * b$ or $a * b * p$. It can be shown that the line ab and the line ba are the same set of points.

2. If o and a are distinct points, a *ray* $[oa$ is the set of all points p such that $p = o$ or $o * p * a$ or $o * a * p$.

3. If a and b are distinct points, the *open interval* is the set of points p such that $a * p * b$. It is denoted by (a, b).

4. If a and b are distinct points, the *closed interval* is $(a, b) \cup \{a\} \cup \{b\}$, and denoted by $[a, b]$.

5. The *way* $a_1a_2 \ldots a_n$ is the finite set of points $\{a_1, \ldots, a_n\}$ along with the open intervals $(a_1, a_2), (a_2, a_3), \ldots, (a_{n-1}, a_n)$. The points $a_1, \ldots, a_n$ are called the *vertices* of the way, and the open intervals are called the *sides* of the way. A way is also called a *broken line*. The closed intervals $[a_1, a_2], \ldots, [a_{n-1}, a_n]$ are called the *side-intervals* of the way. The lines $a_1a_2, \ldots, a_{n-1}a_n$ are called the *side-lines* of the way. The way $a_1a_2 \ldots a_n$ is said to *join* a_1 to a_n. It is assumed that a_{i-1}, a_i, a_{i+1} are not collinear.

6. A way is said to be *simple* if it does not meet itself. To be precise, (i) no two side-intervals meet in any point which is not a vertex, and (ii) no three side-intervals meet in any point.

7. A *polygon* is a way $a_1a_2 \ldots a_n$ for which $a_1 = a_n$. Notice that there is no assumption that the points are coplanar.

8. A *simple polygon* is polygon for which the way is simple.

9. A *region* is a set of points not all collinear, any two of which can be joined by points of a way using only points of the region.

10. A region R is *convex* if for each pair of points $a, b \in R$ the open interval (a, b) is contained in R.

11. Let X and Y be two sets of points. If there is a set of points S such that every way joining a point of X to a point of Y meets S then S is said to *separate* X from Y.

12. If $a_1 a_2 \dots a_n$ is a polygon, then the *angles of the polygon* are $\angle a_n a_1 a_2$, $\angle a_1 a_2 a_3$, and so on.

Now assume that all points of the geometry are in one plane. Let P be a polygon. (P is called a *plane polygon.*)

1. A ray or line which does not go through a vertex of P will be called *suitable.*

2. An *inside point* a of P is one for which a suitable ray from a meets P an odd number of times. Points that are not on or inside P are said to be *outside* P.

3. Let $\{P_i\}$ be a set of polygons. We say that $\{P_i\}$ *dissect* P if the following three conditions are satisfied: (i) P_i and P_j do not have a common inside point for $i \neq j$, (ii) each inside point of P is inside or on some P_i and (iii) each inside point of P_i is inside P.

4. A *convex polygon* is one whose inside points are all on the same side of any side-line of the polygon.

2 Theorems

Assume that all points are in one plane. Let P be a polygon.

1. It can be shown that P separates the other points of the plane into at least two regions and that if P is simple there are exactly two regions. Moise proves this directly in [**?**], pp. 16-18.

2. It can be shown that P can be dissected into triangles $\{T_i\}$ such that every vertex of a T_i is a vertex of P.

3. The following theorem of Euler can be shown: Suppose P is dissected into $f > 1$ polygons and that the total number of vertices of these polygons is v, and the number of open intervals which are sides is e. Then
$$v - e + f = 1$$
.

A plane simple polygon with n sides is called an n-gon, although for small n there are more traditional names:

Number of sides	Name of the polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
10	decagon

A plane simple polygon is also called a *Jordan polygon*.

References

[1] K. Borsuk and W. Szmielew, *Foundations of Geometry*, North-Holland Publishing Company, 1960.

[2] H.G. Forder, *The Foundations of Euclidean Geometry*, Dover Publications, 1958.

[3] E.E. Moise, *Geometric Topology in Dimensions 2 and 3*, Springer-Verlag, 1977.