non-Euclidean geometry

Canonical name	NonEuclideanGeometry
Date of creation	2013-03-22 13:54:51
Last modified on	2013-03-22 13:54:51
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	22
Author	Wkbj79 (1863)
Entry type	Definition
Classification	msc 51-00
Classification	msc 51M10
Related topic	Sphere
Related topic	ComparisonOfCommonGeometries
Defines	hyperbolic geometry
Defines	Bolyai-Lobachevski geometry
Defines	elliptic geometry
Defines	spherical geometry
Defines	semi-Euclidean geometry

A *non-Euclidean geometry* is a in which at least one of the axioms from Euclidean geometry fails. Within this entry, only geometries that are considered to be two-dimensional will be considered.

The most common non-Euclidean geometries are those in which the parallel postulate fails; `http://planetmath.org/Ie`i.e., there is not a unique line that does not intersect a given line through a point not on the given line. Note that this is equivalent to saying that the sum of the angles of a triangle is not equal to π radians.

If there is more than one such parallel line, the is called *hyperbolic* (or *Bolyai-Lobachevski*). In these of , the sum of the angles of a triangle is strictly in 0 and π radians. (This sum is not constant as in Euclidean geometry; it depends on the area of the triangle. See the entry regarding defect for more details.)

As an example, consider the disc $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ in which a point is similar to the Euclidean point and a line is defined to be a chord (excluding its endpoints) of the (`http://planetmath.org/Circle`circular) boundary. This is the Beltrami-Klein model for $\mathbb{H}^2$. It is relatively easy to see that, in this , given a line and a point not on the line, there are infinitely many lines passing through the point that are parallel to the given line.

If there is no parallel line, the is called *spherical* (or *elliptic*). In these of , the sum of the angles of a triangle is strictly in π and 3π radians. (This sum is not constant as in Euclidean geometry; it depends on the area of the triangle. See the entries regarding `http://planetmath.org/Defect`defect and area of a spherical triangle for more details.)

As an example, consider the surface of the `http://planetmath.org/Sphere`unit sphere $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ in which a point is similar to the Euclidean point and a line is defined to be a great circle. (Note that, when a sphere serves as a model of spherical geometry, its radius is typically assumed to be 1.) It is relatively easy to see that, in this , given a line and a point not on the line, it is impossible to find a line passing through the point that does not intersect the given line.

Note also that, in spherical geometry, two distinct points do not necessarily determine a unique line; however, two distinct points that are not antipodal always determine a unique line.

One final example of a non-Euclidean is *semi-Euclidean geometry*, in which the axiom of Archimedes fails.