Euclidean axiom by Hilbert

Canonical name	EuclideanAxiomByHilbert
Date of creation	2013-03-22 17:19:16
Last modified on	2013-03-22 17:19:16
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	9
Author	pahio (2872)
Entry type	Topic
Classification	msc 51M05
Classification	msc 51-01
Related topic	CorrespondingAnglesInTransversalCutting
Related topic	AxiomaticGeometry

In Hilbert's *Grundlagen der Geometrie* ('Foundations of Geometry'; the original edition in 1899) there is the following argumentation.

Let α be an arbitrary plane, a a line in α and A a point in α which lies outside a. If we draw in α a line c which passes through A and intersects a and then through A a line b such that the line c intersects the lines a, b with equal alternate interior angles ("unter gleichen Gegenwinkeln"), *then it follows easily from the theorem on the outer angles, that the lines a, b have no common point, i.e., in a plane α one can always draw otside a line a another line which does not intersect the line a.*

The Parallel Axiom reads now:

IV (). *Let a be an arbitrary line and A be a point outside a: then in the plane determined by a and A there exists at most one line which passes through A and does not intersect a.*

Explanation. *According the the preceding text and on grounds of the Parallel Axiom we realize, that there is one and only one line which passes through A and do not intersect a; that is called the parallel of a through A.*

The Parallel Axiom means the same as the following requirement:

When two lines a, b in a plane do not meet a third line c of the same plane, then also they do not meet each other.

The theorem on the outer angles is the following: *An outer angle of a triangle is greater than both non-adjacent angles of the triangle.* Using this one may indirectly justify the assertion in the first cited paragraph.

Introducing the Parallel Axiom simplifies the foundations and facilitates the construction of geometry significantly.

If we , then we obtain easily the following well-known fact:

Theorem 31. *If two parallels intersect a third line, then the corresponding angles and the alternate interior angles are congruent, and conversely: the `http://planetmath.org/GeometricCongruence`congruence of the corresponding or alternate interior angles implies that the lines are parallel.*

References

[1] D. Hilbert: *Grundlagen der Geometrie*. Neunte Auflage, revidiert und ergänzt von Paul Bernays. B. G. Teubner Verlagsgesellschaft, Stuttgart (1962).