geometric constructions by Euclid

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The *geometric constructions* using compass and straightedge consist of three fundamental tasks as given in Euclid's *The Elements* (in ancient Greek Στοιχεία, transliterated *Stoikheia*). These fundamental tasks are as follows:

1. Drawing a line through two given points.

2. Drawing a circle having a given point as its center and passing through another given point.

3. Setting a plane passing through three given non-collinear points, where one performs tasks based on the two preceding tasks.

Example. The usual task of drawing a circle with a given point as its center and with a given line segment as its radius (a fundamental task in many textbooks) can be to Euclid's fundamental tasks (one needs five circles!).

Remark. It can be proven that all geometric constructions with compass and straightedge are possible using *only* the compass. (See `http://planetmath.org/Eg`e.g. compass and straightedge construction of parallel line.)

In the text of Euclid, the constructions are not listed separately, but are combined with the theorems as propositions. A way to tell whether a proposition is a theorem or a construction is to go to the end of the proof and see if it ends with QED, in which case it is a theorem, or with QEF, in which case it is a construction. Note that QEF is an abbreviation for the Latin phrase *quod erat faciendum*, meaning 'which was to be done'.

Here is a list of the geometric constructions to be found in *The Elements*:

- I 1 Given a line segment, construct an equilateral triangle having that segment as a side.

- I 2 Given a point and a line segment, construct a line segment having the given point as an endpoint and equal in length to the given line segment.

- I 3 Given two line segments, produce a line segment whose length is the difference of the lengths of the two given line segments.

- I 9 Bisect a given angle.

- I 10 Bisect a given line segment.

- I 11 Given a line and a point on this line, construct a line orthogonal to the given line passing through the given point.

- I 12 Given a line and a point not on this line, construct a line orthogonal to the given line passing through the given point. (i.e. Find the projection of a point on a line.)

- III 1 Construct the center of a given circle.

If you are interested in seeing the rules for compass and straightedge constructions, click on the provided.

References

`http://www.physics.ntua.gr/Faculty/mourmouras/euclid/`Online edition of Euclid's *The Elements* in Greek prepared by D. E. Mourmouras.