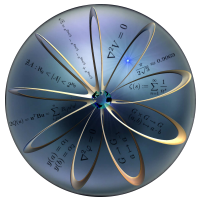

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polarity

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Related topic	PolaritiesAndForms
Defines	polarity
Defines	duality
Defines	correlation
Defines	pole
Defines	polar

Definition 1.

- *Given finite dimensional vector spaces V and W, a* duality *of the projective geometry $PG(V)$ to $PG(W)$ is an order-reversing bijection $f : PG(V) \to PG(W)$. If $W = V$ then we can refer to f as a correlation.*
- *A correlation of order 2 is called a* polarity.
- *The set of correlations and collineations $f : PG(V) \to PG(V)$ form a group denoted $P\Gamma L^*(V)$ with the operation of composition.*

Remark 2. *Dualities are determined by where they map collinear triples. Given a map define on the points of $PG(V)$ to the hyperplanes of $PG(W)$ which maps collinear triples to triples of hyperplanes which intersect in a codimension 2 subspace, this specifies a unique duality.*

Remark 3. *A polarity/duality necessarily interchanges points with hyperplanes. In this context points are called "poles" and hyperplanes "polars."*

An alternative definition of a duality is a projectivity (order-preserving map) $f : PG(V) \to PG(V^*)$.

Through the use of the fundamental theorem of projective geometry, dualities and polarities can be identified with non-degenerate sesquilinear forms. (See `http://planetmath.org/PolaritiesAndForms`Polarities and forms.)