solid of revolution

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Let $y = f(x)$ be a curve for x in an interval $[a, b]$ satisfying $f(x) > 0$ for x in (a, b). We may construct a corresponding solid of revolution, say $\mathcal{V} = \left\{(x, y, z) : x \in [a, b] \text{ and } y^2 + z^2 \leq f(x)^2\right\}$. Intuitively, it is the solid generated by rotating the surface $y \leq f(x)$ about the x-axis.

The interior of a surface of revolution is always a solid of revolution. These include

- the interior of a cylinder of radius $c > 0$ and height h with $f(x) = c$ for $0 \leq x \leq h$,
- the interior of a sphere of radius $R > 0$ with $f(x) = \sqrt{R^2 - x^2}$ for $-R \leq x \leq R$, and
- the interior of a (right, circular) cone of base radius $R > 0$ and height h with $f(x) = Rx/h$ for $0 \leq x \leq h$.

Let Γ be a simple closed curve with parametrization $\alpha(t) = (X(t), Y(t))$ for t in an interval $[a, b]$ satisfying $Y(t) \geq 0$ for t in $[a, b]$. By the Jordan curve theorem, we may choose the set of points, $\mathcal{S}$, "inside" the curve, i.e. let $\mathcal{S}$ be the bounded connected component of the two connected components found in $\mathbb{R}^2 \setminus \Gamma$. Another sort of solid of revolution is given by $\mathcal{V} = \{(x, y, z) : x = X(t) \text{ for some } t \text{ in } [a, b] \text{ and } y^2 + z^2 = s^2 \text{ for some } s \text{ such that } (x, s) \in \mathcal{S} \cup \Gamma\}$. Intuitively, it is the solid generated by rotating $\mathcal{S} \cup \Gamma$ about the x-axis.

Some examples of this sort of solid of revolution include

- the interior of a torus of minor radius $r > 0$ and major radius $R > r$ with $\alpha(t) = (r\cos t, r\sin t + R)$ for $0 \leq t \leq 2\pi$,
- a shell of a sphere with inner radius $r > 0$ and outer radius $R > r$ with

$$\alpha(t) = \begin{cases} (R\cos \pi t, R\sin \pi t) & \text{if } t \in [0, 1] \\ (r(1-t) + R(t-2), 0) & \text{if } t \in [1, 2] \\ (-r\cos \pi t, r\sin \pi t) & \text{if } t \in [2, 3] \\ (r(4-t) + R(t-3), 0) & \text{if } t \in [3, 4]. \end{cases}$$