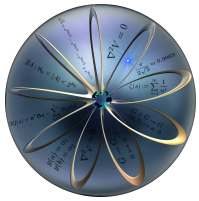

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tangent plane of quadratic surface

Canonical name	TangentPlaneOfQuadraticSurface
Date of creation	2013-03-22 14:58:48
Last modified on	2013-03-22 14:58:48
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Result
Classification	msc 51N20
Related topic	TangentOfConicSection
Related topic	QuadraticSurfaces

The common equation of all quadratic surfaces in the rectangular $(x,\ y,\ z)$-coordinate system is

$$Ax^2 + By^2 + Cz^2 + 2A'yz + 2B'zx + 2C'xy + 2A''x + 2B''y + 2C''z + D = 0 \tag{1}$$

where A, B, C, A', B', C', A'', B'', C'', D are constants and at least one of the six first is distinct from zero. The equation of the tangent plane of the surface, with $(x_0,\ y_0,\ z_0)$ as the point of tangency, is

$$Ax_0x+By_0y+Cz_0z+A'(z_0y+y_0z)+B'(x_0z+z_0x)+C'(y_0x+x_0y)+A''(x+x_0)+B''(y+y_0)+C''(z+z_0)+$$

This is said to be obtained from (1) by polarizing it.

Example. The tangent plane of the *elliptic paraboloid* $4x^2 + 9y^2 = 2z$ set in the point $(x_0,\ y_0,\ z_0)$ of the surface is $4x_0x + 9y_0y = z + z_0$, and especially in the point $(\frac{1}{2},\ \frac{1}{3},\ 1)$ it is $2x + 3y - z - 1 = 0$.