centre of mass of polygon

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Let $A_1A_2\ldots A_n$ be an http://planetmath.org/Polygon$n$-gon which is supposed to have a surface-density in all of its points, M the centre of mass of the polygon and O the origin. Then the position vector of M with respect to O is

$$\overrightarrow{OM} = \frac{1}{n}\sum_{i=1}^{n}\overrightarrow{OA_i}. \tag{1}$$

We can of course take especially $O = A_1$, and thus

$$\overrightarrow{A_1M} = \frac{1}{n}\sum_{i=1}^{n}\overrightarrow{A_1A_i} = \frac{1}{n}\sum_{i=2}^{n}\overrightarrow{A_1A_i}.$$

In the special case of the triangle ABC we have

$$\overrightarrow{AM} = \frac{1}{3}(\overrightarrow{AB}+\overrightarrow{AC}). \tag{2}$$

The centre of mass of a triangle is the common point of its medians.

Remark. An analogical result with (2) concerns also the tetrahedron $ABCD$,

$$\overrightarrow{AM} = \frac{1}{4}(\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}),$$

and any n-dimensional simplex (cf. the http://planetmath.org/Midpointmidpoint of line segment: $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$).