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sine of angle of triangle

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The cosines law allows to express the cosine of an angle of triangle through the sides:

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}. \tag{1}$$

Substituting this to the "fundamental formula of trigonometry",

$$\sin^2\alpha + \cos^2\alpha \;=\; 1,$$

we can calculate as follows:

$$\begin{aligned}
\sin\alpha &= +\sqrt{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2} \\
&= \sqrt{\frac{(2bc)^2-(b^2+c^2-a^2)^2}{(2bc)^2}} \\
&= \frac{\sqrt{(2bc+b^2+c^2-a^2)(2bc-b^2-c^2+a^2)}}{2bc} \\
&= \frac{\sqrt{[(b+c)^2-a^2][a^2-(b-c)^2]}}{2bc} \\
&= \frac{\sqrt{(b+c+a)(b+c-a)(a+b-c)(a-b+c)}}{2bc}
\end{aligned}$$

Thus we have the beautiful formula

$$\sin\alpha \;=\; \frac{\sqrt{(-a+b+c)(a-b+c)(a+b-c)(a+b+c)}}{2bc}.$$

Substituting (1) similarly to the general formula for the sine of `http://planetmath.org/Goniom` angle

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}},$$

one can obtain the formula

$$\sin\frac{\alpha}{2} \;=\; \sqrt{\frac{(a-b+c)(a+b-c)}{4bc}}.$$