asymptotes of graph of rational function

Canonical name	AsymptotesOfGraphOfRationalFunction
Date of creation	2013-03-22 15:09:34
Last modified on	2013-03-22 15:09:34
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Last modified by	eshyvari (13396)
Numerical id	10
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Entry type	Result
Classification	msc 51N99
Classification	msc 26C15
Classification	msc 26A09
Related topic	Polytrope

Let $f(x) = \frac{P(x)}{Q(x)}$ be a fractional expression where $P(x)$ and $Q(x)$ are polynomials with real coefficients such that their quotient can not be `http://planetmath.org/Divi` to a polynomial. We suppose that $P(x)$ and $Q(x)$ have no common zeros.

If the division of the polynomials is performed, then a result of the form

$$f(x) \;=\; H(x)+\frac{R(x)}{Q(x)}$$

is gotten, where $H(x)$ and $R(x)$ are polynomials such that

$$\deg R(x) < \deg Q(x)$$

The graph of the rational function f may have asymptotes:

1. Every zero a of the denominator $Q(x)$ gives a vertical asymptote $x = a$.
2. If $\deg H(x) < 1$ (i.e. 0 or $-\infty$) then the graph has the horizontal asymptote $y = H(x)$.
3. If $\deg H(x) = 1$ then the graph has the skew asymptote $y = H(x)$.

Proof of 2 and 3. We have $f(x) - H(x) = \dfrac{R(x)}{Q(x)} \to 0$ as $|x| \to \infty$.

Remark. Here we use the convention that the degree of the zero polynomial is $-\infty$.