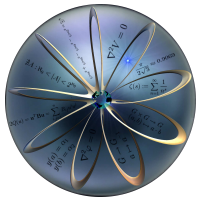

planetmath.org

Math for the people, by the people.

a line segment has at most one midpoint

Canonical name	ALineSegmentHasAtMostOneMidpoint
Date of creation	2013-03-22 17:17:37
Last modified on	2013-03-22 17:17:37
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	9
Author	Mathprof (13753)
Entry type	Theorem
Classification	msc 51G05

(this proof is not correct yet)

Theorem 1. *In an ordered geometry a line segment has at most one midpoint.*

Proof. Let $[p, q]$ be a closed line segment and suppose m and m' are midpoints. If $m : p : q$ then $[m, p] < [m, q]$ so m is not a midpoint. Similarly we cannot have $p : q : m$, so we have $p : m : q$. And also, $p : m' : q$. Suppose $m \neq m'$. Without loss of generality we can assume $p : m : m'$ and $m : m' : q$. But then $[p, m'] > [p, m] \cong [m, q] > [m', q]$ so that $[p, m'] \ncong [m', q]$, a contradiction. Hence $m = m'$. □