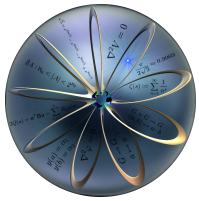

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tangent of conic section

Canonical name	TangentOfConicSection
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Classification	msc 51N20
Synonym	tangent of quadratic curve
Related topic	TangentLine
Related topic	TangentOfCircle
Related topic	TangentPlaneOfQuadraticSurface
Related topic	QuadraticInequality
Related topic	ConjugateDiametersOfEllipse
Related topic	ConjugateHyperbola
Related topic	QuadraticCurves
Related topic	EquationOfTangentOfCircle
Related topic	TangentOfHyperbola
Defines	polarising
Defines	polarizing
Defines	polarize
Defines	mixed term

The equation of every conic section (and the degenerate cases) in the rectangular (x, y)-coordinate system may be written in the form

$$Ax^2 + By^2 + 2Cxy + 2Dx + 2Ey + F = 0,$$

where A, B, C, D, E and F are constants and $A^2 + B^2 + C^2 > 0$.[1] (The $2Cxy$ is present only if the axes are not parallel to the coordinate axes.)

The equation of the *tangent line* of an ordinary conic section (i.e., circle, ellipse, hyperbola and parabola) in the point (x_0, y_0) of the curve is

$$Ax_0x + By_0y + C(y_0x + x_0y) + D(x + x_0) + E(y + y_0) + F = 0.$$

Thus, the equation of the tangent line can be obtained from the equation of the curve by *polarizing* it, i.e. by replacing

x^2 with x_0x, y^2 with y_0y, $2xy$ with $y_0x + x_0y$, $2x$ with $x + x_0$, $2y$ with $y + y_0$.

Examples: The of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$, the of the hyperbola $xy = \frac{1}{2}$ is $y_0x + x_0y = 1$.

[1] This is true also in any skew-angled coordinate system.