# Helly's theorem

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Suppose $A_1, \ldots, A_m \subset \mathbb{R}^d$ is a family of convex sets, and every $d+1$ of them have a non-empty intersection. Then $\bigcap_{i=1}^m A_i$ is non-empty.

*Proof.* The proof is by induction on $m$. If $m = d+1$, then the statement is vacuous. Suppose the statement is true if $m$ is replaced by $m-1$. The sets $B_j = \bigcap_{i \neq j} A_i$ are non-empty by inductive hypothesis. Pick a point $p_j$ from each of $B_j$. By Radon's lemma, there is a partition of $p$'s into two sets $P_1$ and $P_2$ such that $I = (\operatorname{conv} P_1) \cap (\operatorname{conv} P_2) \neq \emptyset$. For every $A_j$ either every point in $P_1$ belongs to $A_j$ or every point in $P_2$ belongs to $A_j$. Hence $I \subseteq A_j$ for every $j$. $\square$