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exposed points are dense in the extreme  
points

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**Definition.** Let  $K \subset \mathbb{R}^n$  be a closed convex set. A point  $p \in K$  is called an *exposed* point if there is an  $n - 1$  dimensional hyperplane whose intersection with  $K$  is  $p$  alone.

**Theorem** (Straszewicz). *Let  $K \subset \mathbb{R}^n$  be a closed convex set. Then the set of exposed points is dense in the set of extreme points.*

For example, let  $C(p)$  denote the closed ball in  $\mathbb{R}^2$  of radius 1 and centered at  $p$ . Then take  $K$  to be the convex hull of  $C(-1, 0)$  and  $C(1, 0)$ . The points  $(-1, 1)$ ,  $(-1, -1)$ ,  $(1, 1)$ , and  $(1, -1)$  are extreme points, but they are not exposed points.