K-distance set

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Let X be a set with metric d, $Y \subseteq X$, and $L = \{d(x,y) : x, y \in Y, x \neq y\}$. If $K := \#(L)$ is finite, Y is said to be a *K-distance set*.

Y is called a *maximal K-distance set* if and only if for all $x \in X \setminus Y$, there exists $y \in Y$ such that $d(x,y) \notin L$. That is, if anything is added to Y, it is no longer a K-distance set.

Y is called a *spherical K-distance set* if and only if Y is a K-distance set and every element of Y is a fixed distance r from some element c, so Y is a subset of the `http://planetmath.org/SphereMetricSpace`sphere centered at c with radius r.

For example, let $X = \mathbb{R}^2$ with $d =$ the box metric: $d(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ with x_i, y_i components of x, y, respectively. Let $Y = \{(0,0), (1,0), (2,0), (0,1), (1,1), (2$
Then $L = \{1, 2\}$, so $K = 2$, so Y is a 2-distance set.

Note: please do not confuse this definition of K-distance set with $\Delta_K(Y)$, the K-distance set of Y.