# convex functions lie above their supporting lines

| | |
|---|---|
| Canonical name | ConvexFunctionsLieAboveTheirSupportingLines |
| Date of creation | 2013-03-22 16:59:20 |
| Last modified on | 2013-03-22 16:59:20 |
| Owner | Andrea Ambrosio (7332) |
| Last modified by | Andrea Ambrosio (7332) |
| Numerical id | 5 |
| Author | Andrea Ambrosio (7332) |
| Entry type | Result |
| Classification | msc 52A41 |
| Classification | msc 26A51 |
| Classification | msc 26B25 |

Let $f : \mathbf{R} \to \mathbf{R}$ be a convex, twice differentiable function on $[a, b]$. Then $f(x)$ lies above its supporting lines, i.e. it's greater than any tangent line in $[a, b]$.

*Proof.* :

Let $r(x) = f(x_0) + f'(x_0)(x - x_0)$ be the tangent of $f(x)$ in $x = x_0 \in [a, b]$.

By Taylor theorem, with remainder in Lagrange form, one has, for any $x \in [a, b]$:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(\xi(x))(x - x_0)^2$$

with $\xi(x) \in [a, b]$. Then

$$f(x) - r(x) = \frac{1}{2} f''(\xi(x))(x - x_0)^2 \geq 0$$

since $f''(\xi(x)) \geq 0$ by convexity. $\square$