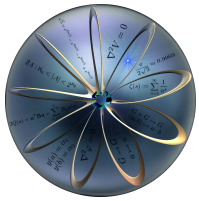

planetmath.org

Math for the people, by the people.

Sylvester's theorem

Canonical name	SylvestersTheorem
Date of creation	2013-03-22 13:59:36
Last modified on	2013-03-22 13:59:36
Owner	bbukh (348)
Last modified by	bbukh (348)
Numerical id	5
Author	bbukh (348)
Entry type	Theorem
Classification	msc 52C35
Classification	msc 51M04

For every finite collection of non-collinear points in Euclidean space, there is a line that passes through exactly two of them.

Proof. Consider all lines passing through two or more points in the collection. Since not all points lie on the same line, among pairs of points and lines that are non-incident we can find a point A and a line l such that the distance $d(A, l)$ between them is minimal. Suppose the line l contained more than two points. Then at least two of them, say B and C, would lie on the same side of the perpendicular from p to l. But then either $d(AB, C)$ or $d(AC, B)$ would be smaller than the distance $d(A, l)$ which contradicts the minimality of $d(A, l)$. □