convex set

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Defines	polyconvex

Let S a subset of $\mathbb{R}^n$. We say that S is *convex* when, for any pair of points A, B in S, the segment $\overline{AB}$ lies entirely inside S.

The former statement is equivalent to saying that for any pair of vectors u, v in S, the vector $(1-t)u + tv$ is in S for all $t \in [0,1]$.

If S is a convex set, for any $u_1, u_2, \ldots, u_r$ in S, and any positive numbers $\lambda_1, \lambda_2, \ldots, \lambda_r$ such that $\lambda_1 + \lambda_2 + \cdots + \lambda_r = 1$ the vector

$$\sum_{k=1}^{r} \lambda_k u_k$$

is in S.

Examples of convex sets in the plane are circles, triangles, and ellipses. The definition given above can be generalized to any real vector space:

Let V be a vector space (over $\mathbb{R}$ or $\mathbb{C}$). A subset S of V is *convex* if for all points x, y in S, the line segment $\{\alpha x + (1-\alpha)y \mid \alpha \in (0,1)\}$ is also in S.

More generally, the same definition works for any vector space over an ordered field.

A *polyconvex set* is a finite union of compact, convex sets.

Remark. The notion of convexity can be generalized to an arbitrary partially ordered set: given a poset P (with partial ordering $\leq$), a subset C of P is said to be *convex* if for any $a, b \in C$, if $c \in P$ is between a and b, that is, $a \leq c \leq b$, then $c \in C$.