Wulff theorem

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Definition 1 (Wulff shape)**.** *Let* $\phi\colon \mathbb{R}^n \to [0,+\infty)$ *be a non-negative, convex, coercive, positively* 1*-homogeneous function. We define the* Wulff shape *relative to* ϕ *as the set*

$$W_\phi := \{x \in \mathbb{R}^n \colon \langle x, y\rangle \leq 1 \text{ for all } y \text{ such that } \phi(y) \leq 1\}$$

(where $\langle \cdot, \cdot \rangle$ *is the Euclidean inner product in* $\mathbb{R}^n$*.)*

Theorem 1 (Wulff)**.** *Let* $\phi\colon \mathbb{R}^n \to [0,+\infty)$ *be a non-negative, convex, coercive,* 1*-homogeneous function. Given a regular open set* $D \subset \mathbb{R}^n$ *we consider the following anisotropic surface energy:*

$$F_\phi(D) = \int_{\partial D} \phi(\nu_D(x))\, d\sigma(x)$$

where $\nu_D(x)$ *is the outer unit normal to* ∂D*, and* σ *is the surface area on* ∂D*. Then, given any set* D *with the same volume as* W_ϕ*, i.e.* $|D| = |W_\phi|$*, one has* $F_\phi(D) \geq F_\phi(W_\phi)$*. Moreover if* $|D| = |W_\phi|$ *and* $F_\phi(D) = F_\phi(W_\phi)$ *then* D *is a translation of* W_ϕ *i.e. there exists* $v \in R^n$ *such that* $D = v + W_\phi$*.*