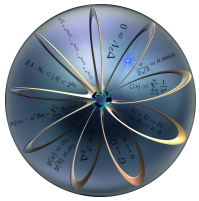

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intersection semilattice of a subspace arrangement

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Let $\mathcal{A}$ be a finite subspace arrangement in a finite-dimensional vector space V. The of $\mathcal{A}$ is the subspace arrangement $L(\mathcal{A})$ defined by taking the `http://planetmath.org/ClosureAxioms`closure of $\mathcal{A}$ under intersections. More formally, let

$$L(\mathcal{A}) = \Big\{ \bigcap_{H \in \mathcal{S}} H \mid \mathcal{S} \subset \mathcal{A} \Big\}.$$

`http://planetmath.org/Poset`Order the elements of $L(\mathcal{A})$ by reverse inclusion, and give it the structure of a join-semilattice by defining $H \vee K = H \cap K$ for all H, K in $L(\mathcal{A})$. Moreover, the elements of $L(\mathcal{A})$ are naturally graded by codimension. If $\mathcal{A}$ happens to be a central arrangement, its intersection semilattice is in fact a lattice, with the meet operation defined by $H \wedge K = \operatorname{span}(H \cup K)$, where $\operatorname{span}(H \cup K)$ is the subspace of V spanned by $H \cup K$.