Levi-Civita connection

Canonical name	LeviCivitaConnection
Date of creation	2013-03-22 13:59:25
Last modified on	2013-03-22 13:59:25
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	7
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 53B05

On any Riemannian manifold $\langle M, g\rangle$, there is a unique torsion-free affine connection ∇ on the tangent bundle of M such that the covariant derivative of the metric tensor g is zero, i.e. g is covariantly constant. This condition can be also be expressed in terms of the inner product operation $\langle,\rangle\colon TM \times TM \to \mathbb{R}$ induced by g as follows: For all vector fields $X, Y, Z \in TM$, one has

$$X(\langle Y, Z\rangle) = \langle \nabla_X Y, Z\rangle + \langle Y, \nabla_X Z\rangle$$

and

$$\nabla_X Y - \nabla_Y X = [X, Y]$$

This connection is called the *Levi-Civita connection.*

In local coordinates $\{x_1, \ldots, x_n\}$, the `http://planetmath.org/Connection`Christoffel symbols Γ^i_{jk} are determined by

$$g_{i\ell}\Gamma^i_{jk} = \frac{1}{2}\left(\frac{\partial g_{j\ell}}{\partial x_k} + \frac{\partial g_{k\ell}}{\partial x_j} - \frac{\partial g_{jk}}{\partial x_\ell}\right).$$