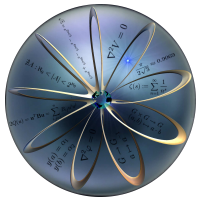

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Darboux's theorem (symplectic geometry)

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If (M, ω) is a $2n$-dimensional symplectic manifold, and $m \in M$, then there exists a neighborhood U of m with a coordinate chart

$$x = (x_1, \ldots, x_{2n}) : U \to \mathbb{R}^{2n},$$

such that

$$\omega = \sum_{i=1}^{n} dx_i \wedge dx_{n+i}.$$

These are called canonical or Darboux coordinates. On U, ω is the pullback by X of the standard symplectic form on $\mathbb{R}^{2n}$, so x is a symplectomorphism. Darboux's theorem implies that there are no local invariants in symplectic geometry, unlike in Riemannian geometry, where there is curvature.