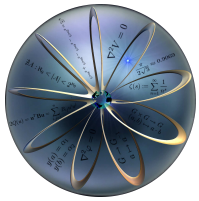

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normal section

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Normal sections

Let P be a point of a surface

$$F(x,\, y,\, z) = 0, \tag{1}$$

where F has the continuous first and partial derivatives in a neighbourhood of P. If one intersects the surface with a plane containing the surface normal at P, the intersection curve is called a *normal section*.

Normal curvatures

When the direction of the intersecting plane is varied, one gets different normal sections, and their `http://planetmath.org/CurvaturePlaneCurve`curvatures at P, the so-called *normal curvatures*, vary having a minimum value $\varkappa_1$ and a maximum value $\varkappa_2$. The arithmetic mean of $\varkappa_1$ and $\varkappa_2$ is called the *mean curvature* of the surface at P.

By the suppositions on the function F, examining the normal curvatures can without loss of generality be to the following: Examine the curvature of the normal sections through the origin, the surface given in the form

$$z = z(x,\, y), \tag{2}$$

where $z(x,\, y)$ has the continuous first and partial derivatives in a neighbourhood of the origin and

$$z(0,\, 0) = z'_x(0,\, 0) = z'_y(0,\, 0) = 0.$$

Indeed, one can take a new rectangular coordinate system with P the new origin and the normal at P the new z-axis; then the new xy-plane coincides with the tangent plane of the surface (1) at P. The equation (1) defines the function of (2).