Euclidean distance

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Entry type	Definition
Classification	msc 53A99
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Synonym	Euclidean metric
Synonym	standard metric
Synonym	standard topology
Synonym	Euclidean
Synonym	canonical topology
Synonym	usual topology
Related topic	Topology
Related topic	BoundedInterval
Related topic	EuclideanVectorSpace
Related topic	DistanceOfNonParallelLines
Related topic	EuclideanVectorSpace2
Related topic	Hyperbola2
Related topic	CassiniOval

If $u = (x_1, y_1)$ and $v = (x_2, y_2)$ are two points on the plane, their *Euclidean distance* is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}. \tag{1}$$

Geometrically, it's the length of the segment joining u and v, and also the norm of the difference vector (considering $\mathbb{R}^n$ as vector space).

This distance induces a metric (and therefore a topology) on $\mathbb{R}^2$, called *Euclidean metric (on* $\mathbb{R}^2$*)* or *standard metric (on* $\mathbb{R}^2$). The topology so induced is called *standard topology* or *usual topology on* $\mathbb{R}^2$ and one basis can be obtained considering the set of all the open balls.

If $a = (x_1, x_2, \ldots, x_n)$ and $b = (y_1, y_2, \ldots, y_n)$, then formula **??** can be generalized to $\mathbb{R}^n$ by defining the Euclidean distance from a to b as

$$d(a, b) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}. \tag{2}$$

Notice that this distance coincides with absolute value when $n = 1$. Euclidean distance on $\mathbb{R}^n$ is also a metric (Euclidean or standard metric), and therefore we can give $\mathbb{R}^n$ a topology, which is called the standard (canonical, usual, etc) topology of $\mathbb{R}^n$. The resulting (topological and vectorial) space is known as *Euclidean space*.

This can also be done for $\mathbb{C}^n$ since as set $\mathbb{C} = \mathbb{R}^2$ and thus the metric on $\mathbb{C}$ is the same given to $\mathbb{R}^2$, and in general, $\mathbb{C}^n$ gets the same metric as R^{2n}.