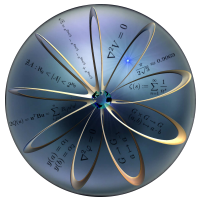

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DNA inequality

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Given Γ, a convex `http://planetmath.org/Curve`simple closed curve in the plane, and γ a closed curve contained in Γ, then $M(\Gamma) \leq M(\gamma)$ where M is the mean curvature function.

This was a conjecture due to S. Tabachnikov and was proved by Lagarias and Richardson of Bell Labs. The idea of the proof was to show that there was a way you could reduce a curve to the boundary of its convex hull so that if it holds for the boundary of the convex hull, then it holds for the curve itself.

Conjecture : *Equality holds iff* Γ *and* γ *coincide.*

It's amazing how many questions are still open in the Elementary Differential Geometry of curves and surfaces. Questions like this often serve as a great research opportunity for undergraduates. It is also interesting to see if you could extend this result to curves on surfaces:

Theorem : If Γ is a circle on S^2 , and γ is a closed curve contained in Γ then $M(\Gamma) \leq M(\gamma)$.

It is not known whether this result holds for Γ a simple closed convex curve on S^2.

It is known also that this inequality does not hold in the hyperbolic plane.