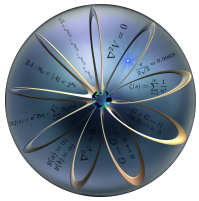

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circle of curvature

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Let a given plane curve γ have a definite `http://planetmath.org/CurvaturePlaneCurve`curvat κ in a point P of γ. The *circle of curvature* of γ in the point P is the circle which has the radius $\frac{1}{|\kappa|}$ and which has with the curve the common tangent in P and which in a neighbourhood of P is on the same side as the curve.

The radius ϱ of the circle of curvature is the *radius of curvature* of γ in P. The center of the circle of curvature is the *center of curvature* of γ in P.

When the curve γ is given in the parametric form

$$x = x(t), \quad y = y(t),$$

the coordinates of the center of curvature belonging to the point (x, y) of the curve are

$$\xi = x - \frac{(x'^2 + y'^2)y'}{x'y'' - x''y'}, \quad \eta = y + \frac{(x'^2 + y'^2)x'}{x'y'' - x''y'}.$$

Example. Since the curvature of the parabola $y = x^2$ in the origin is -2, the corresponding radius of curvature is $\frac{1}{2}$ and the center of curvature $(0, \frac{1}{2})$.

Furthermore, it is possible to define the circle of curvature without first knowing about curvature of the curve. (In fact, using this definition, one could reverse the procedure and define curvature as the radius of the circle of curvature.) We may define the circle of curvature a point P of γ as the unique circle passing through P which makes a second-order contact with γ at P.