line through an intersection point

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Suppose that the lines

$$Ax + By + C = 0 \text{ and } A'x + B'y + C' = 0 \tag{1}$$

have an intersection point. Then for any real value of k, the equation

$$Ax + By + C + k(A'x + B'y + C') = 0 \tag{2}$$

represents a line passing through that point.

In fact, the of the equation (2) is 1, and therefore it represents a line; secondly, (2) is satisfied if both equations (1) are satisfied, and therefore the line passes through that intersection point.

Example. Determine the equation of the line passing through the point $(-5,\,2)$ and the intersection point of the lines $6x-7y+9=0$ and $5x+9y-3=0$.

The equation of a line through the common point of those lines is

$$6x - 7y + 9 + k(5x + 9y - 3) = 0. \tag{3}$$

We have to find such a value for k that also $(-5,\,2)$ lies on the line, i.e. that the equation (3) is satisfied by the values $x=-5,\ y=2$. So we get for determining k the equation

$$-35 - 10k = 0,$$

whence $k=-\frac{7}{2}$. Using this value in (3), multiplying the equation by 2 and simplifying, we obtain the sought equation

$$23x + 77y - 39 = 0.$$

This result would be obtained, of course, by first calculating the intersection point of the two given lines (it is $(-\frac{60}{89},\,\frac{63}{89})$) and then forming the equation of the line passing this point and the point $(-5,\,2)$, but then the calculations would have been substantially longer.

Note. It is apparent that no value of k allows the equation (2) to the line
$A'x+B'y+C'=0$ itself. Thus, if we had in the example instead the point $(-5,\,2)$ e.g. the point $(6,\,-3)$ of the line $5x+9y-3=0$, then we had the condition $66+0k=0$ which gives no value of k.

References

[1] K. Väisälä: *Algebran oppi- ja esimerkkikirja II.* Neljäs painos. Werner Söderström osakeyhtiö, Porvoo & Helsinki (1956).