germ space

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Let X, Y be topological spaces and $x \in X$. Consider the set of all continuous functions

$$C(X,Y) = \{f : X \to Y \mid f \text{ is continuous}\}.$$

For any two functions $f, g : X \to Y$ we put

$$f \sim_x g$$

if and only if there exists an open neighbourhood $U \subseteq X$ of x such that

$$f_{|U} = g_{|U}.$$

The corresponding quotient set is called **the germ space** at $x \in X$ and we denote it by $G_x(X,Y)$.

More generally, if X, Y are topological spaces with $x \in X$, then consider the following set:

$$C_x(X,Y) = \{f : U \to Y \mid f \text{ is continuous and } U \text{ is an open neighbourhood of } x\}.$$

Again we define a relation on $C_x(X,Y)$. If $f : U \to Y$ and $g : U' \to Y$, then put

$$f \sim_x g$$

if and only if there exists and open neighbourhood $V \subseteq X$ of x such that $V \subseteq U \cap U'$ and

$$f_{|V} = g_{|V}.$$

The corresponding set is called **the generalized germ space** at $x \in X$ and we denote it by $G^*_x(X,Y)$.

Note that if $Y = \mathbb{R}$ or $Y = \mathbb{C}$ (or Y is any topological ring), then both $G_x(X,Y)$ and $G^*_x(X,Y)$ have a well-defined ring structure via pointwise addition and multiplication.