symplectic matrix

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A real $2n \times 2n$ matrix $A \in \mathrm{M}_{2n}(\mathbb{R})$ is a *symplectic matrix* if $AJA^T = J$, where A^T is the transpose of A, and $J \in \mathrm{O}(2n)$ is the orthogonal matrix

$$J = \begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix}.$$

Here $\mathbf{I}_n \in \mathrm{M}_n(\mathbb{R})$ is the identity $n \times n$ matrix and $\mathbf{0} \in \mathrm{M}_n(\mathbb{R})$ is the zero $n \times n$ matrix.

Symplectic matrices satisfy the following properties:

1. The determinant of a symplectic matrix equals one.
2. With standard matrix multiplication, symplectic $2n \times 2n$ matrices form a group denoted by $\mathrm{Sp}(2n)$.
3. Suppose $\Psi = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, where A, B, C, D are $n \times n$ matrices. Then Ψ is symplectic if and only if
$$AD^T - BC^T = I, \quad AB^T = BA^T, \quad CD^T = DC^T.$$
4. If X and Y are real $n \times n$ matrices, then $U = X + iY$ is unitary if and only if $\begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}$ is symplectic.