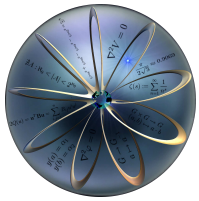

planetmath.org

Math for the people, by the people.

closed differential forms on a simply connected domain

Canonical name	ClosedDifferentialFormsOnASimplyConnectedDomain
Date of creation	2013-03-22 13:32:46
Last modified on	2013-03-22 13:32:46
Owner	paolini (1187)
Last modified by	paolini (1187)
Numerical id	14
Author	paolini (1187)
Entry type	Theorem
Classification	msc 53-00
Related topic	ClosedCurveTheorem
Related topic	PoincareLemma

Let $D \subset \mathbb{R}^2$ be an open set and let ω be a differential form defined on D.

Theorem 1 *If D is simply connected and ω is a closed differential form, then ω is an exact differential form.*

The proof of this result is a consequence of the following useful lemmas.

Lemma 1 *Let ω be a closed differential form and suppose that γ_0 and γ_1 are two regular homotopic curves in D (with the same end points). Then*

$$\int_{\gamma_0} \omega = \int_{\gamma_1} \omega.$$

Lemma 2 *Let ω be a continuous differential form. If given any two curves γ_0, γ_1 in D with the same end-points, it holds*

$$\int_{\gamma_0} \omega = \int_{\gamma_1} \omega,$$

then ω is exact.

See the Poincaré Lemma for a generalization of this result on n-dimensional manifolds.