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germ of smooth functions

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If x is a point on a smooth manifold M , then a *germ of smooth functions near x* is represented by a pair (U, f) where $U \subseteq M$ is an open neighbourhood of x , and f is a smooth function $U \rightarrow \mathbb{R}$. Two such pairs (U, f) and (V, g) are considered equivalent if there is a third open neighbourhood W of x , contained in both U and V , such that $f|_W = g|_W$. To be precise, a germ of smooth functions near x is an equivalence class of such pairs.

In more fancy language: the set \mathcal{O}_x of germs at x is the stalk at x of the sheaf \mathcal{O} of smooth functions on M . It is clearly an \mathbb{R} -algebra.

Germes are useful for defining the tangent space $T_x M$ in a coordinate-free manner: it is simply the space of all \mathbb{R} -linear maps $X : \mathcal{O}_x \rightarrow \mathbb{R}$ satisfying Leibniz' rule $X(fg) = X(f)g + fX(g)$. (Such a map is called an \mathbb{R} -linear derivation of \mathcal{O}_x with values in \mathbb{R} .)