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## vector field along a curve

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Let  $M$  be a differentiable manifold and  $\gamma : [a, b] \rightarrow M$  be a differentiable curve in  $M$ . Then a vector field along  $\gamma$  is a differentiable map  $\Gamma : [a, b] \rightarrow TM$ , the tangent bundle of  $M$ , which projects to  $\gamma$  under the natural projection  $\pi : TM \rightarrow M$ . That is, it assigns to each point  $t_0 \in [a, b]$  a vector tangent to  $M$  at the point  $\gamma(t_0)$ , in a continuous manner. A good example of a vector field along a curve is the speed vector  $\dot{\gamma}$ . This is the push-forward of the constant vector field  $\frac{d}{dt}$  by  $\gamma$ , i.e., at  $t_0$ , it is the derivation  $\dot{\gamma}(f) = \frac{d}{dt}(f \circ \gamma)|_{t=t_0}$ .