differential propositional calculus : appendix 1

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Note. The following Tables are best viewed in the Page Image mode.

Contents

0.1 Table A1. Propositional Forms on Two Variables

Table A1 lists equivalent expressions for the Boolean functions of two variables in a number of different notational systems.

Table A1. Propositional Forms on Two Variables

$\mathcal{L}_1$	$\mathcal{L}_2$		$\mathcal{L}_3$	$\mathcal{L}_4$	$\mathcal{L}_5$	$\mathcal{L}_6$
		$x =$	1 1 0 0			
		$y =$	1 0 1 0			
f_0	f_{0000}		0 0 0 0	$(\ )$	false	0
f_1	f_{0001}		0 0 0 1	$(x)(y)$	neither x nor y	$\neg x \land \neg y$
f_2	f_{0010}		0 0 1 0	$(x)\ y$	y without x	$\neg x \land y$
f_3	f_{0011}		0 0 1 1	(x)	not x	$\neg x$
f_4	f_{0100}		0 1 0 0	$x\ (y)$	x without y	$x \land \neg y$
f_5	f_{0101}		0 1 0 1	(y)	not y	$\neg y$
f_6	f_{0110}		0 1 1 0	$(x,\ y)$	x not equal to y	$x \neq y$
f_7	f_{0111}		0 1 1 1	$(x\ y)$	not both x and y	$\neg x \lor \neg y$
f_8	f_{1000}		1 0 0 0	$x\ y$	x and y	$x \land y$
f_9	f_{1001}		1 0 0 1	$((x,\ y))$	x equal to y	$x = y$
f_{10}	f_{1010}		1 0 1 0	y	y	y
f_{11}	f_{1011}		1 0 1 1	$(x\ (y))$	not x without y	$x \Rightarrow y$
f_{12}	f_{1100}		1 1 0 0	x	x	x
f_{13}	f_{1101}		1 1 0 1	$((x)\ y)$	not y without x	$x \Leftarrow y$
f_{14}	f_{1110}		1 1 1 0	$((x)(y))$	x or y	$x \lor y$
f_{15}	f_{1111}		1 1 1 1	$((\ ))$	true	1

0.2 Table A2. Propositional Forms on Two Variables

Table A2 lists the sixteen Boolean functions of two variables in a different order, grouping them by structural similarity into seven natural classes.

Table A2. Propositional Forms on Two Variables

$\mathcal{L}_1$	$\mathcal{L}_2$		$\mathcal{L}_3$	$\mathcal{L}_4$	$\mathcal{L}_5$	$\mathcal{L}_6$
		$x =$	1 1 0 0			
		$y =$	1 0 1 0			
f_0	f_{0000}		0 0 0 0	$(~)$	false	0
f_1	f_{0001}		0 0 0 1	$(x)(y)$	neither x nor y	$\neg x \land \neg y$
f_2	f_{0010}		0 0 1 0	$(x)~y$	y without x	$\neg x \land y$
f_4	f_{0100}		0 1 0 0	$x~(y)$	x without y	$x \land \neg y$
f_8	f_{1000}		1 0 0 0	$x~y$	x and y	$x \land y$
f_3	f_{0011}		0 0 1 1	(x)	not x	$\neg x$
f_{12}	f_{1100}		1 1 0 0	x	x	x
f_6	f_{0110}		0 1 1 0	$(x,~y)$	x not equal to y	$x \ne y$
f_9	f_{1001}		1 0 0 1	$((x,~y))$	x equal to y	$x = y$
f_5	f_{0101}		0 1 0 1	(y)	not y	$\neg y$
f_{10}	f_{1010}		1 0 1 0	y	y	y
f_7	f_{0111}		0 1 1 1	$(x~y)$	not both x and y	$\neg x \lor \neg y$
f_{11}	f_{1011}		1 0 1 1	$(x~(y))$	not x without y	$x \Rightarrow y$
f_{13}	f_{1101}		1 1 0 1	$((x)~y)$	not y without x	$x \Leftarrow y$
f_{14}	f_{1110}		1 1 1 0	$((x)(y))$	x or y	$x \lor y$
f_{15}	f_{1111}		1 1 1 1	$((~))$	true	1

0.3 Table A3. $\mathrm{E}\,f$ Expanded Over Differential Features $\{\mathrm{d}\,x, \mathrm{d}\,y\}$

Table A3. $\mathrm{E}\,f$ **Expanded Over Differential Features** $\{\mathrm{d}\,x, \mathrm{d}\,y\}$

	f	T_{11} $\mathrm{E}\,f\vert_{\mathrm{d}\,x\ \mathrm{d}\,y}$	T_{10} $\mathrm{E}\,f\vert_{\mathrm{d}\,x(\mathrm{d}\,y)}$	T_{01} $\mathrm{E}\,f\vert_{(\mathrm{d}\,x)}$
f_0	$(\ )$	$(\ )$	$(\ )$	$(\ )$
f_1	$(x)(y)$	$x\ y$	$x\ (y)$	$(x)\ y$
f_2	$(x)\ y$	$x\ (y)$	$x\ y$	$(x)(y)$
f_4	$x\ (y)$	$(x)\ y$	$(x)(y)$	$x\ y$
f_8	$x\ y$	$(x)(y)$	$(x)\ y$	$x\ (y)$
f_3	(x)	x	x	(x)
f_{12}	x	(x)	(x)	x
f_6	$(x,\ y)$	$(x,\ y)$	$((x,\ y))$	$((x,\ y)$
f_9	$((x,\ y))$	$((x,\ y))$	$(x,\ y)$	$(x,\ y)$
f_5	(y)	y	(y)	y
f_{10}	y	(y)	y	(y)
f_7	$(x\ y)$	$((x)(y))$	$((x)\ y)$	$(x\ (y)$
f_{11}	$(x\ (y))$	$((x)\ y)$	$((x)(y))$	$(x\ y)$
f_{13}	$((x)\ y)$	$(x\ (y))$	$(x\ y)$	$((x)(y)$
f_{14}	$((x)(y))$	$(x\ y)$	$(x\ (y))$	$((x)\ y$
f_{15}	$((\ ))$	$((\ ))$	$((\ ))$	$((\ ))$
`http://planetmath.org/FixedPoint`Fixed Point Total:		4	4	4

0.4 Table A4. $\mathrm{D} f$ Expanded Over Differential Features $\{\mathrm{d}\, x, \mathrm{d}\, y\}$

Table A4. $\mathrm{D} f$ **Expanded Over Differential Features** $\{\mathrm{d}\, x, \mathrm{d}\, y\}$

	f	$\mathrm{D} f\|_{\mathrm{d} x\ \mathrm{d} y}$	$\mathrm{D} f\|_{\mathrm{d} x (\mathrm{d} y)}$	$\mathrm{D} f\|_{(\mathrm{d} x) \mathrm{d} y}$	$\mathrm{D} f\|_{(\mathrm{d} x)(\mathrm{d} y)}$
f_0	$(\ )$	$(\ )$	$(\ )$	$(\ )$	$(\ )$
f_1	$(x)(y)$	$((x,\ y))$	(y)	(x)	$(\ )$
f_2	$(x)\ y$	$(x,\ y)$	y	(x)	$(\ )$
f_4	$x\ (y)$	$(x,\ y)$	(y)	x	$(\ )$
f_8	$x\ y$	$((x,\ y))$	y	x	$(\ )$
f_3	(x)	$((\ ))$	$((\ ))$	$(\ )$	$(\ )$
f_{12}	x	$((\ ))$	$((\ ))$	$(\ )$	$(\ )$
f_6	$(x,\ y)$	$(\ )$	$((\ ))$	$((\ ))$	$(\ )$
f_9	$((x,\ y))$	$(\ )$	$((\ ))$	$((\ ))$	$(\ )$
f_5	(y)	$((\ ))$	$(\ )$	$((\ ))$	$(\ )$
f_{10}	y	$((\ ))$	$(\ )$	$((\ ))$	$(\ )$
f_7	$(x\ y)$	$((x,\ y))$	y	x	$(\ )$
f_{11}	$(x\ (y))$	$(x,\ y)$	(y)	x	$(\ )$
f_{13}	$((x)\ y)$	$(x,\ y)$	y	(x)	$(\ )$
f_{14}	$((x)(y))$	$((x,\ y))$	(y)	(x)	$(\ )$
f_{15}	$((\ ))$	$(\ )$	$(\ )$	$(\ )$	$(\ )$

0.5 Table A5. $\mathrm{E}\,f$ Expanded Over Ordinary Features $\{x, y\}$

Table A5. $\mathrm{E}\,f$ **Expanded Over Ordinary Features** $\{x, y\}$

	f	$\mathrm{E}\,f\|_{x\ y}$	$\mathrm{E}\,f\|_{x(y)}$	$\mathrm{E}\,f\|_{(x)y}$	$\mathrm{E}\,f\|_{(x)(y)}$
f_0	$(\ )$	$(\ )$	$(\ )$	$(\ )$	$(\ )$
f_1	$(x)(y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$(\mathrm{d}\,x)\ \mathrm{d}\,y$	$(\mathrm{d}\,x)(\mathrm{d}\,y)$
f_2	$(x)\ y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$(\mathrm{d}\,x)(\mathrm{d}\,y)$	$(\mathrm{d}\,x)\ \mathrm{d}\,y$
f_4	$x\ (y)$	$(\mathrm{d}\,x)\ \mathrm{d}\,y$	$(\mathrm{d}\,x)(\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$
f_8	$x\ y$	$(\mathrm{d}\,x)(\mathrm{d}\,y)$	$(\mathrm{d}\,x)\ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$
f_3	(x)	$\mathrm{d}\,x$	$\mathrm{d}\,x$	$(\mathrm{d}\,x)$	$(\mathrm{d}\,x)$
f_{12}	x	$(\mathrm{d}\,x)$	$(\mathrm{d}\,x)$	$\mathrm{d}\,x$	$\mathrm{d}\,x$
f_6	$(x,\ y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$((\mathrm{d}\,x,\ \mathrm{d}\,y))$	$((\mathrm{d}\,x,\ \mathrm{d}\,y))$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$
f_9	$((x,\ y))$	$((\mathrm{d}\,x,\ \mathrm{d}\,y))$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$((\mathrm{d}\,x,\ \mathrm{d}\,y))$
f_5	(y)	$\mathrm{d}\,y$	$(\mathrm{d}\,y)$	$\mathrm{d}\,y$	$(\mathrm{d}\,y)$
f_{10}	y	$(\mathrm{d}\,y)$	$\mathrm{d}\,y$	$(\mathrm{d}\,y)$	$\mathrm{d}\,y$
f_7	$(x\ y)$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$((\mathrm{d}\,x)\ \mathrm{d}\,y)$	$(\mathrm{d}\,x\ (\mathrm{d}\,y))$	$(\mathrm{d}\,x\ \ \mathrm{d}\,y)$
f_{11}	$(x\ (y))$	$((\mathrm{d}\,x)\ \mathrm{d}\,y)$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$(\mathrm{d}\,x\ \ \mathrm{d}\,y)$	$(\mathrm{d}\,x\ (\mathrm{d}\,y))$
f_{13}	$((x)\ y)$	$(\mathrm{d}\,x\ (\mathrm{d}\,y))$	$(\mathrm{d}\,x\ \ \mathrm{d}\,y)$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$((\mathrm{d}\,x)\ \mathrm{d}\,y)$
f_{14}	$((x)(y))$	$(\mathrm{d}\,x\ \ \mathrm{d}\,y)$	$(\mathrm{d}\,x\ (\mathrm{d}\,y))$	$((\mathrm{d}\,x)\ \mathrm{d}\,y)$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$
f_{15}	$((\ ))$	$((\ ))$	$((\ ))$	$((\ ))$	$((\ ))$

0.6 Table A6. $\mathrm{D}\,f$ Expanded Over Ordinary Features $\{x, y\}$

Table A6. $\mathrm{D}\,f$ **Expanded Over Ordinary Features** $\{x, y\}$

	f	$\mathrm{D}\,f\|_{x\ y}$	$\mathrm{D}\,f\|_{x(y)}$	$\mathrm{D}\,f\|_{(x)y}$	$\mathrm{D}\,f\|_{(x)(y)}$
f_0	$(\ )$	$(\ )$	$(\ )$	$(\ )$	$(\ )$
f_1	$(x)(y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$
f_2	$(x)\ y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$
f_4	$x\ (y)$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$
f_8	$x\ y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$
f_3	(x)	$\mathrm{d}\,x$	$\mathrm{d}\,x$	$\mathrm{d}\,x$	$\mathrm{d}\,x$
f_{12}	x	$\mathrm{d}\,x$	$\mathrm{d}\,x$	$\mathrm{d}\,x$	$\mathrm{d}\,x$
f_6	$(x,\ y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$
f_9	$((x,\ y))$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$	$(\mathrm{d}\,x,\ \mathrm{d}\,y)$
f_5	(y)	$\mathrm{d}\,y$	$\mathrm{d}\,y$	$\mathrm{d}\,y$	$\mathrm{d}\,y$
f_{10}	y	$\mathrm{d}\,y$	$\mathrm{d}\,y$	$\mathrm{d}\,y$	$\mathrm{d}\,y$
f_7	$(x\ y)$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$
f_{11}	$(x\ (y))$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$
f_{13}	$((x)\ y)$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$
f_{14}	$((x)(y))$	$\mathrm{d}\,x\ \ \mathrm{d}\,y$	$\mathrm{d}\,x\ (\mathrm{d}\,y)$	$(\mathrm{d}\,x)\ \ \mathrm{d}\,y$	$((\mathrm{d}\,x)(\mathrm{d}\,y))$
f_{15}	$((\ ))$	$(\ )$	$(\ )$	$(\ )$	$(\ )$