fundamental theorem of space curves

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Owner	rmilson (146)
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Author	rmilson (146)
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Informal summary. The curvature and torsion of a space curve are invariant with respect to Euclidean motions. Conversely, a given space curve is determined up to a Euclidean motion, by its curvature and torsion, expressed as functions of the arclength.

Theorem. Let $\boldsymbol{\gamma} : I \to \mathbb{R}$ be a regular, parameterized space curve, without points of inflection. Let $\kappa(t), \tau(t)$ be the corresponding curvature and torsion functions. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a Euclidean isometry. The curvature and torsion of the transformed curve $T(\boldsymbol{\gamma}(t))$ are given by $\kappa(t)$ and $\tau(t)$, respectively.

Conversely, let $\kappa, \tau : I \to \mathbb{R}$ be continuous functions, defined on an interval $I \subset \mathbb{R}$, and suppose that $\kappa(t)$ never vanishes. Then, there exists an arclength parameterization $\boldsymbol{\gamma} : I \to \mathbb{R}$ of a regular, oriented space curve, without points of inflection, such that $\kappa(t)$ and $\tau(t)$ are the corresponding curvature and torsion functions. If $\hat{\boldsymbol{\gamma}} : I \to \mathbb{R}$ is another such space curve, then there exists a Euclidean isometry $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\hat{\boldsymbol{\gamma}}(t) = T(\boldsymbol{\gamma}(t))$.