tilt curve

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The *tilt curves* (in German *die Neigungskurven*) of a surface

$$z = f(x,\, y)$$

are the curves on the surface which `http://planetmath.org/ConvexAngle`intersect orthogonally the level curves $f(x,\, y) = c$ of the surface. If the gravitation acts in direction of the negative z-axis, then a drop of water on the surface aspires to slide along a tilt curve. For example, since the level curves of the sphere $z = \pm\sqrt{r^2-x^2-y^2}$ are the "latitude circles", the tilt curves of the sphere are the "meridian circles". The tilt curves of a helicoid are circular helices.

If the tilt curves are projected on the xy-plane, the differential equation of those projection curves is

$$\frac{dy}{dx} = \frac{f'_y(x,\, y)}{f'_x(x,\, y)}. \tag{1}$$

Naturally, they also cut orthogonally (the projections of) the level curves.

Example. Let us find the tilt curves of the elliptic paraboloid

$$z = \frac{x^2}{a^2}+\frac{y^2}{b^2}.$$

The level curves are the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2} = c$. Now we have

$$f'_x(x,\, y) = \frac{\partial}{\partial x}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right) = \frac{2x}{a^2}, \quad f'_y(x,\, y) = \frac{\partial}{\partial y}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right) = \frac{2y}{b^2},$$

whence the differential equation of the tilt curves is

$$\frac{dy}{dx} = \frac{a^2}{b^2}\cdot\frac{y}{x}.$$

The separation of variables and the integration yield

$$\int\frac{dy}{y} = \frac{a^2}{b^2}\int\frac{dx}{x},$$

then

$$\ln|y| = \frac{a^2}{b^2}\ln|x|+\ln|C| = \ln(|C||x|^{a^2/b^2}),$$

and finally

$$y = C|x|^{a^2/b^2}. \tag{2}$$

Here, we may allow for C all positive and negative values. The curves (2) originate from the origin and continue infinitely far.

Remark. Given an arbitrary family of parametre curves on a surface

$$\vec{r} = (x(u,\, v),\ y(u,\, v),\ z(u,\, v))^\intercal$$

of $\mathbb{R}^3$, e.g. in the form

$$\frac{du}{dv} = f(u,\, v),$$

the family of its orthogonal curves on the surface has in the Gaussian coordinates u, v the differential equation

$$\frac{dv}{du} = -\frac{g_{11} + g_{12}f(u,\, v)}{g_{12} + g_{22}f(u,\, v)}, \tag{3}$$

where

$$g_{11} = \vec{r}\,'_u \cdot \vec{r}\,'_u, \quad g_{12} = \vec{r}\,'_u \cdot \vec{r}\,'_v, \quad g_{22} = \vec{r}\,'_v \cdot \vec{r}\,'_v$$

are the fundamental quantities E, F, G of Gauss, respectively.