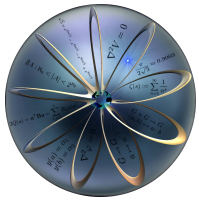

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momentum map

Canonical name	MomentumMap
Date of creation	2013-03-22 13:14:36
Last modified on	2013-03-22 13:14:36
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	4
Author	bwebste (988)
Entry type	Definition
Classification	msc 53D20
Synonym	moment map

Let (M, ω) be a symplectic manifold, G a Lie group acting on that manifold, $\mathfrak{g}$ its Lie algebra, and $\mathfrak{g}^*$ the dual of the Lie algebra. This action induces a map $\alpha : \mathfrak{g} \to \mathfrak{X}(M)$ where $\mathfrak{X}(M)$ is the Lie algebra of vector fields on M, such that $\exp(tX)(m) = \rho_t(m)$ where ρ is the flow of $\alpha(X)$. Then a *moment map* $\mu : M \to \mathfrak{g}^*$ for the action of G is a map such that

$$H_{\mu(X)} = \alpha(X).$$

Here $\mu(X)(m) = \mu(m)(X)$, that is, $\mu(m)$ is a covector, so we apply it to the vector X and get a scalar function $\mu(X)$, and $H_{\mu(X)}$ is its Hamiltonian vector field.

Generally, the moment maps we are interested in are equivariant with respect to the coadjoint action, that is, they satisfy

$$\mathrm{Ad}_g^* \circ \mu = \mu \circ g.$$