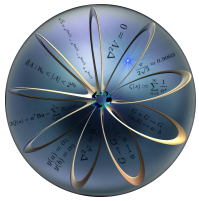

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pseudo-Riemannian manifold

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Defines	pseudo-Riemannian manifold

A *pseudo-Riemannian* manifold is a manifold M together with a http://planetmath.org/NonD degenerate, symmetric section g of $T^0_2(M)$ (2-covariant tensor bundle over M).

Unlike with a Riemannian manifold, g is not positive definite. That is, there exist vectors $v \in T_pM$ such that $g(v,v) \leq 0$.

A well known http://planetmath.org/SylvestersLawresult from linear algebra permits us to make a change of basis such that in the new base g is represented by a diagonal matrix with -1 or 1 elements in the diagonal. If there are i, -1 elements in the diagonal and j, 1, the tensor is said to have signature (i,j)

The signature will be invariant in every connected component of M, but usually the restriction that it be a global invariant is added to the definition of a pseudo-Riemannian manifold.

Unlike a Riemannian metric, some manifolds do not admit a pseudo-Riemannian metric.

Pseudo-Riemannian manifolds are crucial in Physics and in particular in General Relativity where space-time is modeled as a 4-pseudo Riemannian manifold with signature (1,3)[1].

Intuitively pseudo-Riemannian manifolds are generalizations of Minkowski's space just as a Riemannian manifold is a generalization of a vector space with a positive definite metric.

[1]also referred to as $(-+++)$