integral manifold

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In the following we will C^∞ when we say smooth.

Definition. Let M be a smooth manifold of dimension m and let Δ be a distribution of dimension n on M. Suppose that N is a connected submanifold of M such that for every $x \in N$ we have that $T_x(N)$ (the tangent space of N at x) is contained in Δ_x (the distribution at x). We can abbreviate this by saying that $T(N) \subset \Delta$. We then say that N is an *integral manifold* of Δ.

Do note that N could be of lower dimension then Δ and is not required to be a regular submanifold of M.

Definition. We say that a distribution Δ of dimension n on M is *completely integrable* if for each point $x \in M$ there exists an integral manifold N of Δ passing through x such that the dimension of N is equal to the dimension of Δ.

An example of an integral manifold is the integral curve of a non-vanishing vector field and then of course the span of the vector field is a completely integrable distribution.

References

[1] William M. Boothby. , Academic Press, San Diego, California, 2003.