planetmath.org

Math for the people, by the people.

mean curvature at surface point

Canonical name	MeanCurvatureAtSurfacePoint
Date of creation	2013-03-22 17:26:56
Last modified on	2013-03-22 17:26:56
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	8
Author	pahio (2872)
Entry type	Theorem
Classification	msc 53A05
Classification	msc 26B05
Classification	msc 26A24
Related topic	AdditionFormulasForSineAndCosine
Related topic	GaussianCurvature
Related topic	MeanCurvature
Defines	mean curvature

Let P be a point on the surface $F(x,\,y,\,z) = 0$ where the function F is twice continuously differentiable on a neighbourhood of P. Then the normal curvature $\varkappa_\theta$ at P is, by Euler's theorem, via the principal curvatures $\varkappa_1$ and $\varkappa_2$ as

$$\varkappa_\theta = \varkappa_1\cos^2\theta+\varkappa_2\sin^2\theta, \tag{1}$$

where θ is the http://planetmath.org/AngleBetweenTwoPlanesangle between the normal section plane corresponding $\varkappa_1$ and the normal section plane corresponding $\varkappa_\theta$. When we apply (1) by taking instead θ the angle $\theta+\frac{\pi}{2}$, we may write

$$\varkappa_{\theta+\frac{\pi}{2}} = \varkappa_1\sin^2\theta+\varkappa_2\cos^2\theta.$$

Adding this equation to (1) then yields

$$\frac{\varkappa_\theta+\varkappa_{\theta+\frac{\pi}{2}}}{2} = \frac{\varkappa_1+\varkappa_2}{2}.$$

The contents of this result is the

Theorem. The arithmetic mean of the http://planetmath.org/CurvaturePlaneCurvecurvat of two perpendicular normal sections has a value, which is equal to the arithmetic mean of the principal curvatures. This mean is called the *mean curvature* at the point in question.

References

[1] Ernst Lindelöf: *Differentiali- ja integralilasku ja sen sovellutukset II.* Mercatorin Kirjapaino Osakeyhtiö, Helsinki (1932).