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Lie derivative

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Let M be a smooth manifold, X a vector field on M, and T a tensor on M. Then the *Lie derivative* $\mathcal{L}_X T$ of T along X is a tensor of the same rank as T defined as

$$\mathcal{L}_X T = \frac{d}{dt} \left(\rho_t^*(T) \right) |_{t=0}$$

where ρ is the flow of X, and ρ_t^* is pullback by ρ_t.

The Lie derivative is a notion of directional derivative for tensors. Intuitively, this is the change in T in the direction of X.

If X and Y are vector fields, then $\mathcal{L}_X Y = [X, Y]$, the standard Lie bracket of vector fields.