characteristic polynomial of a symplectic matrix is a reciprocal polynomial

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Theorem 1. *The characteristic polynomial of a symplectic matrix is a reciprocal polynomial.*

Proof. Let A be the symplectic matrix, and let $p(\lambda) = \det(A - \lambda I)$ be its characteristic polynomial. We wish to prove that

$$p(\lambda) = \pm\lambda^n p(1/\lambda).$$

By definition, $AJA^T = J$ where J is the matrix

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

Since A and J are symplectic matrices, their determinants are 1, and

$$\begin{aligned} p(\lambda) &= \det(AJ - \lambda J) \\ &= \det(AJ - \lambda AJA^T) \\ &= \det(-\lambda A)\det(J)\det(-\frac{1}{\lambda}J + JA^T) \\ &= \pm\lambda^n \det(A - \frac{1}{\lambda}I). \end{aligned}$$

as claimed. □