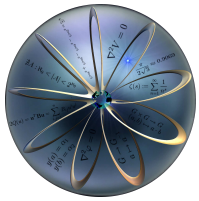

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area of surface of revolution

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A *surface of revolution* is a 3D surface, generated when an arc is rotated fully around a straight line.

The general surface of revolution is obtained when the arc is rotated about an arbitrary axis. If one chooses Cartesian coordinates, and specializes to the case of a surface of revolution generated by rotating about the x-axis a curve described by y in the interval $[a, b]$, its area can be calculated by the formula

$$A = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Similarly, if the curve is rotated about the y-axis rather than the x-axis, one has the following formula:

$$A = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

The general formula is most often seen with parametric coordinates. If $x(t)$ and $y(t)$ describe the curve, and $x(t)$ is always positive or zero, then the area of the general surface of revolution A in the interval $[a, b]$ can be calulated by the formula

$$A = 2\pi \int_a^b y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

To obtain a specific surface of revolution, translation or rotation can be used to move an arc before revolving it around an axis. For example, the specific surface of revolution around the line $y = s$ can be found by replacing y with $y-s$, moving the arc towards the x-axis so $y = s$ lies on it. Now, the surface of revolution can be found using one of the formulae above.

In this specific case, replacing y with $y = s$, the area of a surface of revolution is found using the formula

$$A = 2\pi \int_a^b (y - s) \sqrt{\left(\frac{dy}{dx}\right)^2}\, dy$$