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germ

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Defines Germ

Defines function germ.

Definition 1 (Germ). Let M and N be manifolds and $x \in M$. We consider all smooth mappings $f: U_f \to N$, where U_f is some open neighborhood of x in M. We define an equivalence relation on the set of mappings considered, and we put $f \sim_x g$ if there is some open neighborhood V of x with $f|_V = g|_V$. The equivalence class of a mapping f is called the $germ\ of\ f\ at\ x$, denoted by \overline{f} or, sometimes, $germ_x f$, and we write

$$\overline{f}:(M,x)\to(N,f(x))$$

Remark 1. Germs arise naturally in differential topolgy. It is very convenient when dealing with derivatives at the point x, as every mapping in a germ will have the same derivative values and properties in x, and hence can be identified for such purposes: every mapping in a germ gives rise to the same tangent vector of M at x.