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affine parameter

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Given a geodesic curve, an *affine parameterization* for that curve is a parameterization by a parameter  $t$  such that the parametric equations for the curve satisfy the geodesic equation.

Put another way, if one picks a parameterization of a geodesic curve by an arbitrary parameter  $s$  and sets  $u^\mu = dx^\mu/ds$ , then we have

$$u^\mu \nabla_\mu u^\nu = f(s) u^\nu$$

for some function  $f$ . In general, the right hand side of this equation does not equal zero — it is only zero in the special case where  $t$  is an affine parameter.

The reason for the name “affine parameter” is that, if  $t_1$  and  $t_2$  are affine parameters for the same geodesic curve, then they are related by an affine transform, i.e. there exist constants  $a$  and  $b$  such that

$$t_1 = at_2 + b$$

Conversely, if  $t$  is an affine parameter, then  $at + b$  is also an affine parameter.

From this it follows that an affine parameter  $t$  is uniquely determined if we specify its value at two points on the geodesic or if we specify both its value and the value of  $dx^\mu/dt$  at a single point of the geodesic.