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Frenet frame

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Entry type	Definition
Classification	msc 53A04
Synonym	moving trihedron
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Related topic	SpaceCurve
Defines	osculating plane
Defines	normal plane
Defines	rectifying plane
Defines	unit normal
Defines	unit tangent
Defines	binormal

Let $I \subset \mathbb{R}$ be an interval and let $\gamma : I \to \mathbb{R}^3$ be a parameterized space curve, assumed to be `http://planetmath.org/SpaceCurve`regular and free of points of inflection. We interpret $\gamma(t)$ as the trajectory of a particle moving through 3-dimensional space. The moving trihedron (also known as the Frenet frame, the Frenet trihedron, the repère mobile, and the moving frame) is an orthonormal basis of 3-vectors $T(t), N(t), B(t)$, defined and named as follows:

$$T(t) = \frac{\gamma'(t)}{\|\gamma'(t)\|}\,, \qquad \text{the unit tangent;}$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|}\,, \qquad \text{the unit normal;}$$

$$B(t) = T(t) \times N(t)\,, \qquad \text{the unit binormal.}$$

A straightforward application of the chain rule shows that these definitions are covariant with respect to reparameterizations. Hence, the above three vectors should be conceived as being attached to the point $\gamma(t)$ of the oriented space curve, rather than being functions of the parameter t.

Corresponding to the above vectors are 3 planes, passing through each point $\gamma(t)$ of the space curve. The *osculating plane* at the point $\gamma(t)$ is the plane spanned by $T(t)$ and $N(t)$; the *normal plane* at $\gamma(t)$ is the plane spanned by $N(t)$ and $B(t)$; the rectifying plane at $\gamma(t)$ is the plane spanned by $T(t)$ and $B(t)$.