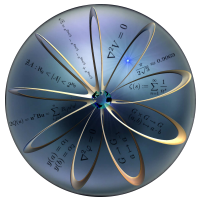

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symplectic vector space

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Classification	msc 53D05
Defines	symplectic vector space
Defines	linear symplectomorphism

A *symplectic vector space* (V, ω) is a finite dimensional real vector space V equipped with an alternating non-degenerate 2-tensor, i.e., a bilinear map $\omega \colon V \times V \to \mathbb{R}$ that satisfies the following properties:

1. Alternating: For all $v, w \in V$, $\omega(v, w) = -\omega(w, v)$.

2. Non-degenerate: If $\omega(v, w) = 0$ for all $w \in V$, then $v = 0$.

The tensor ω is called a for V.

A linear automorphism $T \in \mathrm{Aut}(V)$ is called *linear symplectomorphism* when $T^*\omega = \omega$, i.e.

$$\omega(Tv, Tw) = \omega(v, w) \quad \forall v, w \in W.$$

Linear symplectomorphisms of (V, ω) form a group (under composition of linear map) that is denoted by $\mathrm{Sp(V}, \omega)$.

One can show that a symplectic vector space is always even dimensional [**?**].

References

[1] D. McDuff, D. Salamon, *Introduction to Symplectic Topology*, Clarendon Press, 1997.