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Smarandache geometries

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An axiom is said *smarandachely denied* (S-denied) if in the same space the axiom behaves differently (i.e., validated and invalided; or only invalidated but in at least two distinct ways).

A *Smarandache geometry* (SG) is a geometry which has at least one smarandachely denied axiom (1969).

Thus, as a particular case, Euclidean, Lobachevsky-Bolyai-Gauss, and Riemannian geometries may be united altogether, in the same space, by some SGs. These last geometries can be partially Euclidean and partially non-Euclidean.

The novelty of the SG is the fact that they introduce for the first time *the degree of negation in geometry*, similarly to the degree of falsehood in fuzzy or neutrosophic logic. For example an axiom can be denied in percentage of 30

Also SG are defined on multispaces, i.e. unions of Euclidean and non-Euclidean subspaces, or unions of distinct non-Euclidean spaces.

As an example of S-denying, a proposition ϕ, which is the conjunction of a set ϕ_i of propositions, can be *invalidated in many ways* if it is minimally unsatisfiable, that is, such that the conjunction of any proper subset of the ϕ_i is satisfied in a structure, but ϕ itself is not.

Many axioms have one of the following forms in the sequential logic:
Let $n, m \geqslant 1$ be two integers, also $A = A_1 \times A_2 \times \cdots \times A_n$ and $B = B_1 \times B_2 \times \cdots \times B_m$.
Let $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_m)$, also $P(x)$ and $Q(x)$ be relationships on x and $R(x, y)$ a relationship between x and y. Then:

a_1 $\forall x \in A$, $\exists$ and is unique $y \in B$, such that $R(x, y)$.
a_2 If $x \in A$ such that $P(x)$, then $Q(x)$.
a_3 $\forall x \in A$ one has $P(x)$.

One can invalidate in many ways the first class of axioms a_1 as follows:

1.1. $\exists x \in A$ and $\exists$ finitely many (at least two) $y \in B$ such that $R(x, y)$.
1.2. $\exists x \in A$ and $\exists$ infinitely many $y \in B$ such that $R(x, y)$.
1.3. $\exists x \in A$ such that $\nexists y \in B$ for which $R(x, y)$.

One can invalidate in many ways the second class of axioms a_2 as follows:

2.1. $\exists$ a unique $x \in A$ such that $P(x)$ but $\neg Q(x)$.
2.2. $\exists$ finitely many (more than one) $x \in A$ such that $P(x)$ but $\neg Q(x)$.
2.3. $\exists$ infinitely many $x \in A$ such that $P(x)$ but $\neg Q(x)$.
2.4. $\forall x \in A$ such that $P(x)$ one has $\neg Q(x)$.

One can invalidate in many ways the third class of axioms a_3 as follows:

3.1. $\exists x \in A$ and is unique such that $\neg P(x)$.

3.2. $\exists$ finitely many $x \in A$ such that $\neg P(x)$.
3.3. $\exists$ infinitely many $x \in A$ such that $\neg P(x)$.

Not all axioms can be smarandachely denied.

Here it is an example of what it means for *an axiom to be invalidated in multiple ways* [2]: As a particular axiom let's take Euclid's Fifth Postulate. In Euclidean or parabolic geometry a line has one parallel only through a given point. In Lobacevskian or hyperbolic geometry a line has at least two parallels through a given point. In Riemannian or elliptic geometry a line has no parallel through a given point. Whereas in Smarandache geometries there are lines which have no parallels through a given point and other lines which have one or more parallels through a given point (the fifth postulate is invalidated in many ways).

Therefore, the Euclid's Fifth Postulate (which asserts that there is only one parallel passing through an exterior point to a given line) can be invalidated in many ways, i.e. *Smarandachely denied*, as follows:

- first invalidation: there is no parallel passing through an exterior point to a given line;

- second invalidation: there is a finite number of parallels passing through an exterior point to a given line;

- third invalidation: there are infinitely many parallels passing through an exterior point to a given line.

References

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