locally Euclidean

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A locally Euclidean space X is a topological space that locally "looks" like $\mathbb{R}^n$. This makes it possible to talk about coordinate axes around X. It also gives some topological structure to the space: for example, since $\mathbb{R}^n$ is locally compact, so is X. However, the restriction does not induce any geometry onto X.

Definition Suppose X is a topological space. Then X is called *locally Euclidean* if for each $x \in X$ there is a neighbourhood $U \subseteq X$, a $V \subseteq \mathbb{R}^n$, and a homeomorphism $\phi : U \to V$. Then the triple (U, ϕ, n) is called a *chart* for X.

Here, $\mathbb{R}$ is the set of real numbers, and for $n = 0$ we define $\mathbb{R}^0$ as set with a single point equipped with the discrete topology.

Local dimension

Suppose X is a locally Euclidean space with $x \in X$. Further, suppose (U, ϕ, n) is a chart of X such that $x \in U$. Then we define the *local* of X at x is n. This is well defined, that is, the local dimension does not depend on the chosen chart. If (U', ϕ', n') is another chart with $x \in U'$, then $\psi \circ \phi^{-1} : \phi(U \cap U') \to \psi(U \cap U')$ is a homeomorphism between $\phi(U \cap U') \subseteq \mathbb{R}^n$ and $\psi(U \cap U') \subseteq \mathbb{R}^{n'}$. By Brouwer's theorem for the invariance of dimension (which is nontrivial), it follows that $n = n'$.

If the local dimension is constant, say n, we say that the dimension of X is n, and write $\dim X = n$.

Examples

- Any set with the discrete topology, is a locally Euclidean of dimension 0.
- Any open subset of $\mathbb{R}^n$ is locally Euclidean.
- Any manifold is locally Euclidean. For example, using a stereographic projection, one can show that the sphere S^n is locally Euclidean.
- The long line is locally Euclidean of dimension one. Note that the long line is not Hausforff. [**?**].

Notes

The concept locally Euclidean has a different meaning in the setting of Riemannian manifolds.

References

[1] L. Conlon, *Differentiable Manifolds: A first course*, Birkhäuser, 1993.