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Hamiltonian vector field

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Let (M, ω) be a symplectic manifold, and $\tilde{\omega} : TM \rightarrow T^*M$ be the isomorphism from the tangent bundle to the cotangent bundle

$$X \mapsto \omega(\cdot, X)$$

and let $f : M \rightarrow \mathbb{R}$ is a smooth function. Then $H_f = \tilde{\omega}^{-1}(df)$ is the *Hamiltonian vector field* of f . The vector field H_f is <http://planetmath.org/SymplecticVectorFieldsyn> and a symplectic vector field X is <http://planetmath.org/node/6410Hamiltonian> if and only if the 1-form $\tilde{\omega}(X) = \omega(\cdot, X)$ is exact.

If T^*Q is the cotangent bundle of a manifold Q , which is naturally identified with the phase space of one particle on Q , and f is the Hamiltonian, then the flow of the Hamiltonian vector field H_f is the time flow of the physical system.