line in plane

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Related topic	SlopeAngle
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Related topic	Slope
Related topic	AnalyticGeometry
Related topic	FanOfLines
Related topic	PencilOfConics
Defines	y-intercept
Defines	x-intercept
Defines	slope-intercept form

Equation of a line

Suppose $a, b, c \in \mathbb{R}$. Then the set of points (x, y) in the plane that satisfy

$$ax + by + c \ = \ 0,$$

where a and b can not be both 0, is an (infinite) *line*.

The value of y when $x = 0$, if it exists, is called the *y-intercept*. Geometrically, if d is the y-intercept, then $(0, d)$ is the point of intersection of the line and the y-axis. The y-intercept exists iff the line is not parallel to the y-axis. The *x-intercept* is defined similarly.

If $b \neq 0$, then the above equation of the line can be rewritten as

$$y = mx + d.$$

This is called the *slope-intercept form* of a line, because both the slope and the y-intercept are easily identifiable in the equation. The slope is m and the y-intercept is d.

Three finite points $(x_1,\, y_1)$, $(x_2,\, y_2)$, $(x_3,\, y_3)$ in $\mathbb{R}^2$ are collinear if and only if the following determinant vanishes:

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

Therefore, the line through distinct points $(x_1,\, y_1)$ and $(x_2,\, y_2)$ has equation

$$\begin{vmatrix} x_1 & x_2 & x \\ y_1 & y_2 & y \\ 1 & 1 & 1 \end{vmatrix} = 0,$$

or more simply

$$(y_1 - y_2)x + (x_2 - x_1)y + y_2x_1 - x_2y_1 = 0.$$

Line segment

Let $p_1 = (x_1,\, y_1)$ and $p_2 = (x_2,\, y_2)$ be distinct points in $\mathbb{R}^2$. The closed line segement generated by these points is the set

$$\{p \in \mathbb{R}^2 \mid p = tp_1 + (1 - t)p_2,\ 0 \leq t \leq 1\}.$$