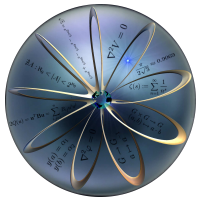

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Gauss-Bonnet theorem

Canonical name	GaussBonnetTheorem
Date of creation	2013-03-22 16:36:37
Last modified on	2013-03-22 16:36:37
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	9
Author	rspuzio (6075)
Entry type	Theorem
Classification	msc 53A05

(Carl Friedrich Gauss and Pierre Ossian Bonnet) Given a two-dimensional compact Riemannian manifold M with boundary, Gaussian curvature of points G and geodesic curvature of points g_x on the boundary ∂M, it is the case that

$$\int_M G\, dA + \int_{\partial M} g_x ds = 2\pi\chi(M),$$

where $\chi(M)$ is the Euler characteristic of the manifold, dA denotes the measure with respect to area, and ds denotes the measure with respect to arclength on the boundary. This theorem expresses a topological invariant in terms of geometrical information.