geodesic completeness

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A Riemannian metric on a manifold M is said to be **geodesically complete** iff its geodesic flow is a complete flow, i.e. iff for every point $p \in M$ and every tangent vector $v \in T_pM$ at p the solution to the geodesic equation

$$\nabla_{\dot\gamma}\dot\gamma = 0$$

with initial condition $\gamma(0) = p$, $\dot\gamma(0) = v$ is defined for all time. The Hopf-Rinow theorem asserts that a Riemannian metric is complete if and only if the corresponding metric on M defined by

$$d(p,q) \colon = \inf\{L(c),\ c\colon [0,1] \to M,\ c(0) = p,\ c(1) = q\}$$

is a complete metric (i.e. Cauchy sequences converge). Here $L(c)$ denote the length of the smooth curve c, i.e.

$$L(c) \colon = \int_0^1 \|\dot c(t)\|_{c(t)}\, dt$$

For a proof of the Hopf-Rinow theorem see Milnor's monograph *Morse Theory* Princeton Annals of Math Studies **51** page 62.