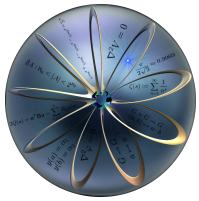

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symplectic manifold

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Defines	symplectic form
Defines	symplectomorphism
Defines	canonical transformation

Symplectic manifolds constitute the mathematical structure for modern Hamiltonian mechanics. Symplectic manifolds can also be seen as even dimensional analogues to contact manifolds.

Definition 1. *A* symplectic manifold *is a pair* (M, ω) *consisting of a smooth manifold* M *and a closed* `http://planetmath.org/DifferentialForms`*2-form* $\omega \in \Omega^2(M)$*, that is non-degenerate at each point. Then* ω *is called a* symplectic form *for* M*.*

Properties

1. Every symplectic manifold is even dimensional. This is easy to understand in view of the physics. In Hamilton equations, location and momentum vectors always appear in pairs.

2. A form $\omega \in \Omega^2(M)$ on a $2n$-dimensional manifold M is non-degenerate if and only if the n-fold product $\omega^n = \omega \wedge \cdots \wedge \omega$ is non-zero.

3. As a consequence of the last , every symplectic manifold is orientable.

Let (M, ω) and (N, η) be symplectic manifolds. Then a diffeomorphism $f\colon M \to N$ is called a *symplectomorphism* if $f^*\eta = \omega$, that is, if the symplectic form on N pulls back to the form on M.

Notes

A symplectomorphism is also known as a *canonical transformation*. This is mostly used in the mechanics literature.