tangent map

Canonical name	TangentMap
Date of creation	2013-03-22 14:06:19
Last modified on	2013-03-22 14:06:19
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	7
Author	matte (1858)
Entry type	Definition
Classification	msc 53-00
Synonym	push forward map
Synonym	pushforward
Synonym	pushforward map
Related topic	PullbackOfAKForm
Related topic	FlowBoxTheorem

Definition 1. *Suppose X and Y are smooth manifolds with tangent bundles TX and TY, and suppose $f : X \to Y$ is a smooth mapping. Then the* **tangent map** *of f is the map $Df \colon TX \to TY$ defined as follows: If $v \in T_x(X)$ for some $x \in X$, then we can represent v by some curve $c \colon I \to X$ with $c(0) = x$ and $I = (-1, 1)$. Now $(Df)(v)$ is defined as the tangent vector in $T(Y)$ represented by the curve $f \circ c \colon I \to Y$. Thus, since $(f \circ c)(0) = f(x)$, it follows that $(Df)(v) \in T_{f(x)}(Y)$.*

Properties

Suppose X and Y are a smooth manifolds.

- If id_X is the identity mapping on X, then $D\mathrm{id}_X$ is the identity mapping on TX.
- Suppose X, Y, Z are smooth manifolds, and f, g are mappings $f \colon X \to Y$, $g \colon Y \to Z$. Then
$$D(f \circ g) = (Df) \circ (Dg).$$
- If $f \colon X \to Y$ is a diffeomorphism, then the inverse of Df is a diffeomorphism, and
$$(Df)^{-1} = D(f^{-1}).$$

Notes

Note that if $f \colon X \to Y$ is a mapping as in the definition, then the tangent map is a mapping
$$Df \colon TX \to TY,$$
whereas the `http://planetmath.org/PullbackOfAKForm`pullback of f is a mapping
$$f^* \colon \Omega^k(Y) \to \Omega^k(X).$$
For this reason, the tangent map is also sometimes called the pushforward map. That is, a pullback takes objects from Y to X, and a pushforward takes objects from X to Y.

Sometimes, the tangent map of f is also denoted by f_*. However, the motivation for denoting the tangent map by Df is that if X and Y are open subsets in $\mathbb{R}^n$ and $\mathbb{R}^m$, then Df is simply the Jacobian of f.