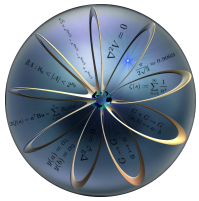

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spray space

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Entry type	Definition
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Synonym	Spray
Synonym	geodesic spray
Synonym	finsler spray
Defines	spray spaces

Take a fibred manifold $\pi\colon B \to X$. Choose a vector field S over B that satisfies $D\pi \circ S(y) = y$ for the Jacobian map $D\pi : TB \to B$ over all coordinate vectors $y = (y^1, \ldots, y^n) \in B$. A *spray* field $\mathbf{G}$ over B is a globally defined smooth vector field associated to the first jet bundle $J^1_B X$ of X that is given in local coordinates $x = (x^1, \ldots, x^n) \in B$ as

$$\mathbf{G} = y^i \frac{\partial}{\partial x^i} - G^i \frac{\partial}{\partial y^i}.$$

The *spray coefficients* $G^i(y)$ are second degree homogeneous functions which correspond up to nonlinear connections on M. Thus by $D\pi$ the integral curves of $\mathbf{G}$ must be of second order, and so given the constraints of the spray coefficients, satisfy $\ddot{c}^{ii} = 2G^i(\dot{c})$. Subsequently, the pair $(X, \mathbf{G})$ is called a *spray space*.

Example 1: Choose a system of second order quasilinear ordinary differential equations that satisfy

$$\ddot{c}^{ii} + 2G^i(\dot{c}) = 0$$

for a family of parameterized curves c, and let the system induce its corresponding *spray*. Then when c is also a Finsler geodesic in B with constant speed so that the covariant derivative gives $D_V V = 0$ along a vector field V, the corresponding autoparallels of the spray coefficients completely characterize a *path space* for B.