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Poincaré duality

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If M is a compact, oriented, n -dimensional manifold, then there is a canonical (though non-<http://planetmath.org/NaturalTransformation>) isomorphism

$$D : H^q(M, \mathbb{Z}) \rightarrow H_{n-q}(M, \mathbb{Z})$$

(where $H^k(M, \mathbb{Z})$ is the k th homology group of M with integer coefficients and $H_k(M, \mathbb{Z})$ the k th <http://planetmath.org/DeRhamCohomology> cohomology group) for all q , which is given by cap product with a generator of $H_n(M, \mathbb{Z})$ (a choice of a generator here corresponds to an orientation). This isomorphism exists with coefficients in $\mathbb{Z}/2\mathbb{Z}$ regardless of orientation.

This isomorphism gives a nice interpretation to cup product. If X, Y are transverse submanifolds of M , then $X \cap Y$ is also a submanifold. All of these submanifolds represent homology classes of M in the appropriate dimensions, and

$$D^{-1}([X]) \cup D^{-1}([Y]) = D^{-1}([X \cap Y]),$$

where \cup is cup product, and \cap is intersection, not cap product.