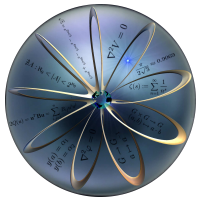

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long exact sequence (locally trivial bundle)

Canonical name	LongExactSequencelocallyTrivialBundle
Date of creation	2013-03-22 13:14:58
Last modified on	2013-03-22 13:14:58
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	6
Author	bwebste (988)
Entry type	Definition
Classification	msc 55Q05
Related topic	Fibration
Related topic	Fibration2
Related topic	HomotopyLiftingProperty

Let $\pi : E \to B$ is a locally trivial bundle, with fiber F. Then there is a long exact sequence of homotopy groups

$$\cdots \longrightarrow \pi_n(F) \xrightarrow{i_*} \pi_n(E) \xrightarrow{\pi_*} \pi_n(B) \xrightarrow{\partial_*} \pi_{n-1}(F) \longrightarrow \cdots$$

Here i_* is induced by the inclusion $i : F \hookrightarrow E$ as the fiber over the basepoint of B, and ∂_* is the following map: if $[\varphi] \in \pi_n(B)$, then φ lifts to a map of $(D^n, \partial D^n)$ into (E, F) (that is a map of the n-disk into E, taking its boundary to F), sending the basepoint on the boundary to the base point of $F \subset E$. Thus the map on $\partial D^n = S^{n-1}$, the $n-1$-sphere, defines an element of $\pi_{n-1}(F)$. This is $\partial_*[\varphi]$. The covering homotopy property of a locally trivial bundle shows that this is well-defined.