Brouwer fixed point theorem

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Theorem Let $\mathbf{B} = \{x \in \mathbb{R}^n : \|x\| \leq 1\}$ be the closed unit ball in $\mathbb{R}^n$. Any continuous function $f : \mathbf{B} \to \mathbf{B}$ has a fixed point.

Notes

Shape is not important The theorem also applies to anything homeomorphic to a closed disk, of course. In particular, we can replace $\mathbf{B}$ in the formulation with a square or a triangle.

Compactness counts (a) The theorem is not true if we drop a point from the interior of $\mathbf{B}$. For example, the map $f(\vec{x}) = \frac{1}{2}\vec{x}$ has the single fixed point at 0; dropping it from the domain yields a map with no `http://planetmath.org/FixedPoint`fixed points.

Compactness counts (b) The theorem is not true for an open disk. For instance, the map $f(\vec{x}) = \frac{1}{2}\vec{x} + (\frac{1}{2}, 0, \ldots, 0)$ has its single fixed point on the boundary of $\mathbf{B}$.