non-commutative structure

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Definition 0.1. Let $(C, \circ)$ be a structure consisting of a *class*, C, together with a *binary operation* $\circ$ defined for pairs of objects in C (or elements of C when the latter is a small class, i.e., a set). The structure– and the operation $\circ$– are said to be *noncommutative* if

$$a \circ b \neq b \circ a \tag{0.1}$$

for either at least some or all of the a, b pairs in C for which the operation is defined.

A structure that is noncommutative is also called sometimes a *non-Abelian structure*, although the latter term is, in general, more often used to specify `http://planetmath.org/NonAbelianTheorie` Abelian theories.

A binary operation that is not `http://planetmath.org/Commutative`commutative is said to be *non-commutative* (or *noncommutative*). Thus, a *noncommutative structure* can be alternatively defined as any structure whose binary operation is not `http://planetmath.org/Commutative`commu (that is, in the `http://planetmath.org/Commutative`commutative case one has

$$a \circ b = b \circ a \tag{0.2}$$

for all a, b pairs in C, and also that the operation $\circ$ is defined for all pairs in C).

An example of a commutative structure is the field of real numbers– with two commutative operations in this case– which are the addition and multiplication over the reals.

Remark 0.1. A commutative group is also called *Abelian*, whereas a category with structure that has commutative diagrams is not necessarily Abelian –unless it does satisfy the Ab1 to Ab6 axioms that define an Abelian category (or equivalently, if it has the properties specified in Mitchell's `http://planetmath.org/AlternativeDefinitionOfAnAbelianCategory`alternative definition of an Abelian category .)

An example of a non-commutative operation is the multiplication over $n \times n$ matrices. Another example of a *noncommutative algebra* is a general `http://planetmath.org/CCliffordAlgebra`Clifford algebra, which is of fundamental importance in the algebraic theory of observable quantum operators and also in quantum algebraic topology.