Mayer-Vietoris sequence

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Let X is a topological space, and $A, B \subset X$ are such that $X = \text{int}(A) \cup \text{int}(B)$, and $C = A \cap B$. Then there is an exact sequence of homology groups:

$$\cdots \longrightarrow H_n(C) \xrightarrow{i_* \oplus -j_*} H_n(A) \oplus H_n(B) \xrightarrow{j_* + i_*} H_n(X) \xrightarrow{\partial_*} H_{n-1}(C) \longrightarrow \cdots$$

Here, i_* is induced by the inclusions $i : B \hookrightarrow X$ and j_* by $j : A \hookrightarrow X$, and ∂_* is the following map: if x is in $H_n(X)$, then it can be written as the sum of a chain in A and one in B, $x = a + b$. $\partial a = -\partial b$, since $\partial x = 0$. Thus, ∂a is a chain in C, and so represents a class in $H_{n-1}(C)$. This is $\partial_* x$. One can easily check (by standard diagram chasing) that this map is well defined on the level of homology.