4 surface bundles

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Four Kleinbottle bundles $K \subset M \to S^1$.

There are four because the extended mapping class group for the genus two, non orientable surface K the Klein bottle, is $\mathbb{Z}_2 \oplus \mathbb{Z}_2$.

This group is generated by a Dehn-twist τ about the unique two-sided curve in K and by the y-homeomorphism, both representing two isotopy classes of order two.

These bundles are

- $K \times S^1$, the trivial Cartesian product
- $K \times_\tau S^1$,
- $K \times_y S^1 = K \overset{\sim}{\times} I^O \cup_{(0,1)} M\ddot{o} \times S^1$,
- $K \times_{y\tau} S^1$.

Where $K \overset{\sim}{\times} I^O$ is the orientable twisted I-bundle over K, among the three I-bundles over K.The symbol $\cup_{(0,1)}$ is used to indicate that, the meridian in $\partial(M\ddot{o} \times S^1)$ is attached to the meridian of $\partial(K \overset{\sim}{\times} I^O)$, both 2-tori. $M\ddot{o}$ is the Möbius band.

Now, since those monodromies are periodic then they are also homeomorphic respectively to the Seifert fiber spaces

- $(NnI, 2|0) = K \times S^1$,
- $(NnI, 2|1) = (K \times S^1 \setminus \text{int}W) \cup_{(1,1)} W$,
- $(NnII, 2|0) = K \times_y S^1 = K \overset{\sim}{\times} I^O \cup_{(0,1)} M\ddot{o} \times S^1$ and
- $(NnII, 2|1) = (K \times_y S^1 \setminus \text{int}W) \cup_{(1,1)} W$

Where W is a solid torus in the space and $\cup_{(1,1)}$ is the Dehn surgery: meridian of ∂W to the longitude of $\partial(K \times S^1 \setminus \text{int}W)$.

The non trivial homeomorphisms were given by Per Orlik and Frank Raymond, in 1969.