planetmath.org

Math for the people, by the people.

cap product

Canonical name	CapProduct
Date of creation	2013-03-22 16:26:10
Last modified on	2013-03-22 16:26:10
Owner	Mazzu (14365)
Last modified by	Mazzu (14365)
Numerical id	9
Author	Mazzu (14365)
Entry type	Definition
Classification	msc 55N45
Defines	cap product

Let X be a topological space, $(C_*(X), \partial)$ the singular chain complex, and $(C^*(X;\mathbb{K}), \delta)$ the singular cochain complex in any coefficient group $\mathbb{K}$. We can define a bilinear pairing operation

$$\frown\colon C^i(X;\mathbb{K}) \times C_n(X) \to C_{n-i}(X), \quad (n \geq i)$$

in the following way: for each cochain $b \in C^i(X;\mathbb{K})$ and each chain $\sigma \in C_n(X)$ we define their *cap product* $b \frown \sigma$ as the unique $(n-i)$-singular chain such that

$$a(b \frown \sigma) = (a \smile b)(\sigma),$$

where $\smile\colon C^j(X;\mathbb{K}) \times C^h(X;\mathbb{K}) \to C^{j+h}(X;\mathbb{K})$ denotes the cup product. Combining the definition of cap product with the standard properties of cup product we obtain that

$$\partial(b \frown \xi) = (\partial b) \frown \xi + (-1)^{\dim(b)} b \frown \partial(\xi),$$

thus there is a corresponding operation in cohomology

$$\frown\colon H^i(X;\mathbb{K}) \otimes H_n(X) \to H_{n-i}(X), \quad (n \geq i)$$

that we also call *cap product*.