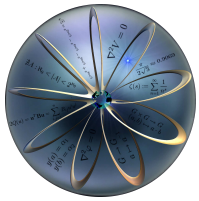

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some examples of universal bundles

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The universal bundle for a topological group G is usually written as $\pi : EG \to BG$. Any principal G-bundle for which the total space is contractible is universal; this will help us to find universal bundles without worrying about Milnor's construction of EG involving infinite joins.

- $G = \mathbb{Z}_2$: $E\mathbb{Z}_2 = S^\infty$ and $B\mathbb{Z}_2 = \mathbb{R}P^\infty$.
- $G = \mathbb{Z}_n$: $E\mathbb{Z}_n = S^\infty$ and $B\mathbb{Z}_n = S^\infty/\mathbb{Z}_n$. Here $\mathbb{Z}_n$ acts on S^∞ (considered as a subset of a separable complex Hilbert space) via multiplication with an n-th root of unity.
- $G = \mathbb{Z}^n$: $E\mathbb{Z}^n = \mathbb{R}^n$ and $B\mathbb{Z}^n = T^n$.
- More generally, if G is any discrete group then one can take BG to be any Eilenberg-Mac Lane space $K(G, 1)$ and EG to be its universal cover. Indeed EG is simply connected, and it follows from the lifting theorem that $\pi_n(EG) = 0$ for $n \geq 0$. This example includes the previous three and many more.
- $G = S^1$: $ES^1 = S^\infty$ and $BS^1 = \mathbb{C}P^\infty$.
- $G = SU(2)$: $ESU(2) = S^\infty$ and $BSU(2) = \mathbb{H}P^\infty$.
- $G = O(n)$, the n-th orthogonal group: $EO(n) = V(\infty, n)$, the manifold of frames of n orthonormal vectors in $\mathbb{R}^\infty$, and $BO(n) = G(\infty, n)$, the Grassmanian of n-planes in $\mathbb{R}^\infty$. The projection map is taking the subspace spanned by a frame of vectors.