deformation retract

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Defines	strong deformation retract

Let X and Y be topological spaces such that $Y \subset X$. A *deformation retract* of X onto Y is a collection of mappings $f_t : X \to X$, $t \in [0,1]$ such that

1. $f_0 = id_X$, the identity mapping on X,
2. $f_1(X) \subseteq Y$,
3. Y is a retract of X via f_1 (that is, f_1 restricted to Y is the identity on Y)
4. the mapping $X \times I \to X$, $(x,t) \mapsto f_t(x)$ is continuous, where the topology on $X \times I$ is the product topology.

Of course, by condition 3, condition 2 can be improved: $f_1(X) = Y$.

A deformation retract is called a *strong deformation retract* if condition 3 above is replaced by a stronger form: Y is a retract of X via f_t for every $t \in [0,1]$.

Properties

- Let X and Y be as in the above definition. Then a collection of mappings $f_t : X \to X$, $t \in [0,1]$ is a deformation retract (of X onto Y) if and only if it is a `http://planetmath.org/HomotopyOfMaps`homotopy rel Y id$_X$ and some retraction r of X onto Y.

Examples

- If $x_0 \in \mathbb{R}^n$, then $f_t(x) = (1-t)x + tx_0$, $x \in \mathbb{R}^n$ shows that $\mathbb{R}^n$ deformation retracts onto $\{x_0\}$. Since $\{x_0\} \subset \mathbb{R}^n$, it follows that deformation retract is not an equivalence relation.
- The same map as in the previous example can be used to deformation retract any star-shaped set in $\mathbb{R}^n$ onto a point.
- we obtain a deformation retraction of $\mathbb{R}^n \backslash \{0\}$ onto the `http://planetmath.org/node/186`$(n-1)$-sphere S^{n-1} by setting

$$f_t(x) = (1-t)x + t\frac{x}{||x||},$$

  where $x \in \mathbb{R}^n \backslash \{0\}$, $n > 0$,

- The http://planetmath.org/node/3278Möbius strip deformation retracts onto the circle S^1.

- The 2-torus with one point removed deformation retracts onto two copies of S^1 joined at one point. (The circles can be chosen to be longitudinal and latitudinal circles of the torus.)

- The characters E,F,H,K,L,M,N, and T all deformation retract onto the charachter I, while the letter Q deformation retracts onto the letter O.