



planetmath.org

Math for the people, by the people.

homotopy of maps

Canonical name	HomotopyOfMaps
Date of creation	2013-03-22 12:13:19
Last modified on	2013-03-22 12:13:19
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	12
Author	mathcam (2727)
Entry type	Definition
Classification	msc 55Q05
Synonym	homotopic maps
Related topic	HomotopyOfPaths
Related topic	HomotopyEquivalence
Related topic	ConstantFunction
Related topic	Contractible
Defines	homotopic
Defines	nullhomotopic

Let  $X, Y$  be topological spaces,  $A$  a closed subspace of  $X$  and  $f, g : X \rightarrow Y$  continuous maps. A *homotopy of maps* is a continuous function  $F : X \times [0, 1] \rightarrow Y$  satisfying

1.  $F(x, 0) = f(x)$  for all  $x \in X$
2.  $F(x, 1) = g(x)$  for all  $x \in X$
3.  $F(x, t) = f(x) = g(x)$  for all  $x \in A, t \in [0, 1]$ .

We say that  $f$  is homotopic to  $g$  relative to  $A$  and denote this by  $f \simeq g \text{ rel } A$ . If  $A = \emptyset$ , this can be written  $f \simeq g$ . If  $g$  is the constant map (i.e.  $g(x) = y$  for all  $x \in X$ ), then we say that  $f$  is *nullhomotopic*.