category of groupoids

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1 Category of Groupoids

1.1 Properties

The category of groupoids, G_{pd}, has several important properties distinct from those of the category of groups,G_p, although it does contain the category of groups as a full subcategory. One such important property is that G_{pd} is *cartesian closed*. Thus, if J and K are two groupoids, one can form a groupoid $GPD(J, K)$ such that if G also is a groupoid then there exists a natural equivalence

$$G_{pd}(G \times J, K) \to G_{pd}(G, GPD(J, K))$$

.

Other important properties of G_{pd} are:

1. The category G_{pd} also has a unit interval object I, which is the groupoid with two objects $0, 1$ and exactly one arrow $0 \to 1$;

2. The groupoid I has allowed the development of a useful `http://planetmath.org/http://pla`
   Theory for groupoids that leads to analogies between groupoids and spaces or manifolds; effectively, groupoids may be viewed as "adding the spatial notion of a 'place' or location" to that of a group;

3. Groupoids extend the notion of invertible operation by comparison with that available for groups; such invertible operations also occur in the theory of inverse semigroups. Moreover, there are interesting relations beteen inverse semigroups and ordered groupoids. Such concepts are thus applicable to sequential machines and automata whose state spaces are semigroups. Interestingly, the category of finite automata, just like G_{pd} is also *cartesian closed*;

4. The category G_{pd} has a variety of types of morphisms, such as: quotient morphisms, retractions, covering morphisms, fibrations, universal morphisms, (in contrast to only the epimorphisms and monomorphisms of group theory);

5. A monoid object, $END(J) = GPD(J, J)$, also exists in the category of groupoids, that contains a maximal subgroup object denoted here as $AUT(J)$. Regarded as a group object in the category groupoids,

$AUT(J)$ is equivalent to a crossed module C_M, which in the case when J is a group is the traditional crossed module $J \to Aut(J)$, defined by the inner automorphisms.

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