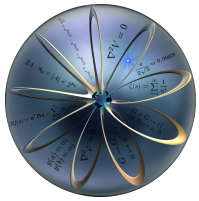

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H-space

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A topological space X is said to be an *H-space* (or *Hopf-space*) if there exists a continuous binary operation $\varphi\colon X \times X \to X$ and a point $p \in X$ such that the functions $X \to X$ defined by $x \mapsto \varphi(p, x)$ and $x \mapsto \varphi(x, p)$ are both homotopic to the identity map via homotopies that leave p fixed. The element p is sometimes referred to as an 'identity', although it need not be an identity element in the usual sense. Note that the definition implies that $\varphi(p, p) = p$.

Topological groups are examples of H-spaces.

If X is an H-space with 'identity' p, then the fundamental group $\pi_1(X, p)$ is abelian. (However, it is possible for the fundamental group to be non-abelian for other choices of basepoint, if X is not path-connected.)