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## Euler characteristic

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The term *Euler characteristic* is defined for several objects.

If  $K$  is a finite simplicial complex of dimension  $m$ , let  $\alpha_i$  be the number of simplexes of dimension  $i$ . The *Euler characteristic* of  $K$  is defined to be

$$\chi(K) = \sum_{i=0}^m (-1)^i \alpha_i.$$

Next, if  $K$  is a finite CW complex, let  $\alpha_i$  be the number of  $i$ -cells in  $K$ . The *Euler characteristic* of  $K$  is defined to be

$$\chi(K) = \sum_{i \geq 0} (-1)^i \alpha_i.$$

If  $X$  is a finite polyhedron, with triangulation  $K$ , a simplicial complex, then the *Euler characteristic* of  $X$  is  $\chi(K)$ . It can be shown that all triangulations of  $X$  have the same value for  $\chi(K)$  so that this is well-defined.

Finally, if  $C = \{C_q\}$  is a finitely generated graded group, then the *Euler characteristic* of  $C$  is defined to be

$$\chi(C) = \sum_{q \geq 0} (-1)^q \text{rank}(C_q).$$