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long exact sequence (of homology groups)

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If X is a topological space, and A and B are subspaces with $X \supset A \supset B$, then there is a long exact sequence:

$$\cdots \longrightarrow H_n(A, B) \xrightarrow{i_*} H_n(X, B) \xrightarrow{j_*} H_n(X, A) \xrightarrow{\partial_*} H_{n-1}(A, B) \longrightarrow$$

where i_* is induced by the inclusion $i : (A, B) \hookrightarrow (X, B)$, j_* by the inclusion $j : (X, B) \hookrightarrow (X, A)$, and ∂ is the following map: given $a \in H_n(X, A)$, choose a chain representing it. ∂a is an $(n-1)$ -chain of A , so it represents an element of $H_{n-1}(A, B)$. This is $\partial_* a$.

When B is the empty set, we get the long exact sequence of the pair (X, A) :

$$\cdots \longrightarrow H_n(A) \xrightarrow{i_*} H_n(X) \xrightarrow{j_*} H_n(X, A) \xrightarrow{\partial_*} H_{n-1}(A) \longrightarrow$$

The existence of this long exact sequence follows from the short exact sequence

$$0 \longrightarrow C_*(A, B) \xrightarrow{i_\#} C_*(X, B) \xrightarrow{j_\#} C_*(X, A) \longrightarrow 0$$

where $i_\#$ and $j_\#$ are the maps on chains induced by i and j , by the Snake Lemma.