Lusternik-Schnirelmann category

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Let X be a topological space. An important topological invariant of X called Lusternik-Schnirelmann category **cat** is defined as follows:

$$\mathrm{cat}(X) = \min\{\#(C) \colon \text{where } C \text{ are the coverings of } X \text{ by contractible open sets}\}.$$

If X is a manifold, $\mathrm{cat}(X)$ coincides with the minimal number of critical points among all smooth scalars maps $X \to \mathbb{R}$.

This is equivalent to saying that X has a covering $\{U_s\}$ such that it is posible to factor homotopically each $U_s \overset{i}{\hookrightarrow} X$ through $U_s \xrightarrow{a} * \xrightarrow{b} X$ i.e

$$i \simeq b \circ a.$$

This allows us to define another category, e.g.:

> We can ask about the minimal number of open sets U_s that cover X and are homotopically equivalent to S^1, say, the inclusion $U_s \overset{i}{\hookrightarrow} X$ and $U_s \xrightarrow{a} S^1 \xrightarrow{b} X$ are $i \simeq b \circ a$.

It is becoming standard to speak of the t-cat of X. This is related to the round complexity of the space.

References

[1] R.H. Fox, *On the Lusternik-Schnirelmann category*, Annals of Math. 42 (1941), 333-370.

[2] F. Takens, *The minimal number of critical points of a function on compact manifolds and the Lusternik-Schnirelmann category*, Invent. math. 6,(1968), 197-244.