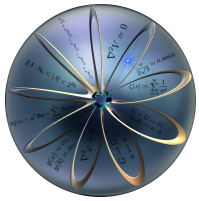

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weak homotopy equivalence

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Synonym	weak equivalence
Related topic	HomotopyEquivalence
Related topic	WeakHomotopyAdditionLemma
Related topic	ApproximationTheoremForAnArbitrarySpace
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Related topic	WhiteheadTheorem
Defines	weakly homotopy equivalent
Defines	weakly equivalent

A continuous map $f : X \to Y$ between path-connected based topological spaces is said to be a *weak homotopy equivalence* if for each $k \geq 1$ it induces an isomorphism $f_* : \pi_k(X) \to \pi_k(Y)$ between the kth homotopy groups. X and Y are then said to be *weakly homotopy equivalent.*

Remark 1. *It is* not *enough for $\pi_k(X)$ to be isomorphic to $\pi_k(Y)$ for all k. The definition requires these isomorphisms to be induced by a space-level map f.*

Remark 2. *More generally, two spaces X and Y are defined to be weakly homotopy equivalent if there is a sequence of spaces and maps*

$$X \to X_1 \leftarrow X_2 \to X_3 \leftarrow \cdots \to X_n \leftarrow Y$$

in which each map is a weak homotopy equivalence.