cup product

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Let X be a topological space and R be a commutative ring. The diagonal map $\Delta : X \to X \times X$ induces a chain map between singular cochain complexes:

$$\Delta^* : C^*(X \times X;\, R) \to C^*(X;\, R)$$

.

Let $h : C^*(X;\, R) \otimes C^*(X;\, R) \to C^*(X \times X;\, R)$

denote the chain homotopy equivalence associated with the Kunneth .

Given $\alpha \in C^p(X;\, R)$ and $\beta \in C^q(X;\, R)$ we define

$\alpha \smile \beta = \Delta^* h(\alpha \otimes \beta)$.

As Δ^* and h are chain maps, $\smile$ induces a well defined product on cohomology groups, known as the cup product. Hence the direct sum of the cohomology groups of X has the structure of a ring. This is called the cohomology ring of X.