relative homology groups

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If X is a topological space, and A a subspace, then the inclusion map $A \hookrightarrow X$ makes $C_n(A)$ into a subgroup of $C_n(X)$. Since the boundary map on $C_*(X)$ restricts to the boundary map on $C_*(A)$, we can take the quotient complex $C_*(X, A)$,

$$\xleftarrow{\ \partial\ } C_n(X)/C_n(A) \xleftarrow{\ \partial\ } C_{n+1}(X)/C_{n+1}(A) \xleftarrow{\ \partial\ }$$

The homology groups of this complex $H_n(X, A)$, are called the *relative homology groups* of the pair (X, A). Under relatively mild hypotheses, $H_n(X, A) = H_n(X/A)$ where X/A is the set of equivalence classes of the relation $x \sim y$ if $x = y$ or if $x, y \in A$, given the quotient topology (this is essentially X, with A reduced to a single point). Relative homology groups are important for a number of reasons, principally for computational ones, since they fit into long exact sequences, which are powerful computational tools in homology.