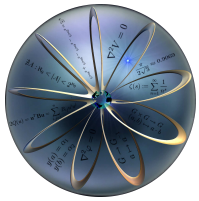

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cubically thin homotopy

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Defines	higher dimensional thin Homotopy

0.1 Cubically thin homotopy

Let u, u' be squares in X with common vertices.

1. A *cubically thin homotopy* $U : u \equiv^{\square}_{T} u'$ between u and u' is a `http://planetmath.org/Polyhedron`cube $U \in R^{\square}_{3}(X)$ such that

   - U is a homotopy between u and u',

     $$\text{i.e. } \partial^{-}_{1}(U) = u, \;\; \partial^{+}_{1}(U) = u',$$

   - U is rel. vertices of I^2,

     $$\text{i.e. } \partial^{-}_{2}\partial^{-}_{2}(U), \;\; \partial^{-}_{2}\partial^{+}_{2}(U), \;\; \partial^{+}_{2}\partial^{-}_{2}(U), \;\; \partial^{+}_{2}\partial^{+}_{2}(U) \text{ are constant,}$$

   - the faces $\partial^{\alpha}_{i}(U)$ are thin for $\alpha = \pm 1, \;\; i = 1, 2.$

2. The square u is *cubically T-equivalent* to u', denoted $u \equiv^{\square}_{T} u'$ if there is a cubically thin homotopy between u and u'.

This definition enables one to construct the homotopy double groupoid scheme $\boldsymbol{\rho}^{\square}_{2}(X)$, by defining a relation of cubically thin homotopy on the set $R^{\square}_{2}(X)$ of squares.

References

[1] K.A. Hardie, K.H. Kamps and R.W. Kieboom, A homotopy 2-groupoid of a Hausdorff space, *Applied Cat. Structures*, **8** (2000): 209-234.

[2] R. Brown, K.A. Hardie, K.H. Kamps and T. Porter, A homotopy double groupoid of a Hausdorff space, *Theory and Applications of Categories* **10**,(2002): 71-93.