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Hopf bundle

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Consider $S^3 \subset \mathbb{R}^4 = \mathbb{C}^2$. The structure of \mathbb{C}^2 gives a map $\mathbb{C}^2 - \{0\} \rightarrow \mathbb{C}P^1$, the complex projective line by the natural projection. Since $\mathbb{C}P^1$ is homeomorphic to S^2 , by restriction to S^3 , we get a map $\pi : S^3 \rightarrow S^2$. We call this the Hopf bundle.

This is a <http://planetmath.org/PrincipalBundle> principal S^1 -bundle, and a generator of $\pi_3(S^2)$. From the <http://planetmath.org/LongExactSequenceLocallyTrivial> exact sequence of the bundle:

$$\cdots \pi_n(S^1) \rightarrow \pi_n(S^3) \rightarrow \pi_n(S^2) \rightarrow \cdots$$

we get that $\pi_n(S^3) \cong \pi_n(S^2)$ for all $n \geq 3$. In particular, $\pi_3(S^2) \cong \pi_3(S^3) \cong \mathbb{Z}$.