homotopy equivalence

Canonical name	HomotopyEquivalence
Date of creation	2013-03-22 12:13:22
Last modified on	2013-03-22 12:13:22
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	14
Author	matte (1858)
Entry type	Definition
Classification	msc 55P10
Related topic	HomotopyOfMaps
Related topic	WeakHomotopyEquivalence
Related topic	Contractible
Related topic	HomotopyInvariance
Related topic	ChainHomotopyEquivalence
Related topic	PathConnectnessAsAHomotopyInvariant
Related topic	TheoremOnCWComplexApproximationOfQuantumStateSpacesInQAT
Defines	homotopy equivalent
Defines	homotopically equivalent
Defines	homotopy type
Defines	strong homotopy equivalence

Definition Suppose that X and Y are topological spaces and $f : X \to Y$ is a continuous map. If there exists a continuous map $g : Y \to X$ such that $f \circ g \simeq id_Y$ (i.e. $f \circ g$ is http://planetmath.org/node/1584homotopic to the identity mapping on Y), and $g \circ f \simeq id_X$, then f is a *homotopy equivalence*. This homotopy equivalence is sometimes called *strong homotopy equivalence* to distinguish it from weak homotopy equivalence.

If there exist a homotopy equivalence between the topological spaces X and Y, we say that X and Y are *homotopy equivalent*, or that X and Y are of the same *homotopy type*. We then write $X \simeq Y$.

0.0.1 Properties

1. Any homeomorphism $f : X \to Y$ is obviously a homotopy equivalence with $g = f^{-1}$.
2. For topological spaces, homotopy equivalence is an equivalence relation.
3. A topological space X is (by definition) contractible, if X is homotopy equivalent to a point, i.e., $X \simeq \{x_0\}$.

References

[1] A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002. Also available http://www.math.cornell.edu/ hatcher/AT/ATpage.htmlonline.