groupoid representation

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Let $q : E \longrightarrow M$ be a vector bundle, with E the *total space*, and M a smooth manifold. Then, consider the representation R_G of a group G as an action on a vector space V, that is, as a homomorphism $h : G \longrightarrow End(V)$, with $End(V)$ being the group of endomorphisms of the vector space V. The generalization of the group representation to general representations of groupoids then occurs somewhat naturally by considering the `http://planetmath.org/GroupoidAction`groupoid action on a vector bundle $E \longrightarrow M$.

Definition 0.1. Let $\mathcal{G}$ be a groupoid, and given a vector bundle $q : E \longrightarrow M$ consider the *frame groupoid*

$$\Phi(E) = s, t : \phi(E) \longrightarrow M,$$

with $\phi(E)$ being the set of all vector space isomorphisms $\eta : E_x \longrightarrow E_y$ over all pairs $(x, y) \in M^2$, also with the associated structure maps. Then, a general *representation* R_d of a groupoid $\mathcal{G}$ is defined as a homomorphism $R_d : \mathcal{G} \longrightarrow \Phi(E)$

Example 0.1: Lie groupoid representations

Definition 0.2. Let $\mathcal{G}_L = s, t : G_1 \longrightarrow M$ be a Lie groupoid. A *representation of a Lie groupoid* $\mathcal{G}_L = s, t : G_1 \longrightarrow M$ on a vector bundle $q : E \longrightarrow M$ is defined as a smooth homomorphism (or a functor) $\rho : \mathcal{G}_L \longrightarrow \Phi(E)$ of Lie groupoids over M.

Note: A *Lie groupoid representation* ρ thus yields a functor, $R : \mathcal{G}_L \longrightarrow$ **Vect**, with **Vect** being the category of vector spaces and $R(x) = E_x$ being the fiber at each $x \in M$, as well as an isomorphism $R(g)$ for each $g : x \to y$.

Example 0.2: Group representations If one restricts the vector bundle to a single vector space in **Definition 0.1** then one obtains a group representation, in the same manner as a groupoid that reduces to a group when its object space is reduced to a single object.