cohomology group theorem

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Related topic	HomologyTopologicalSpace
Related topic	ProofOfCohomologyGroupTheorem
Related topic	OmegaSpectrum
Related topic	ACRcomplex
Defines	conjugacy class or representation of π_1 into π
Defines	set of based homotopy classes of based maps

The following theorem involves Eilenberg-MacLane spaces in relation to cohomology groups for connected CW-complexes.

Theorem 0.1. *Cohomology group theorem for connected CW-complexes ([?]):*

Let $K(\pi, n)$ be Eilenberg-MacLane spaces for connected **http://planetmath.org/CWComplexDe** *complexes X, Abelian groups π and integers $n \geq 0$. Let us also consider the set of non-basepointed homotopy classes $[X, K(\pi, n)]$ of non-basepointed maps $\eta : X \to K(\pi, n)$ and the* **http://planetmath.org/GroupCohomology***cohomolgy groups $\overline{H}^n(X; \pi)$. Then, there exist the following* natural isomorphisms*:*

$$[X, K(\pi, n)] \cong \overline{H}^n(X; \pi), \tag{0.1}$$

0.1 Related remarks:

1. In order to determine all cohomology operations one needs only to compute the cohomology of all Eilenberg-MacLane spaces $K(\pi, n)$; (source: ref [?]);
2. When $n = 1$, and π is *non-Abelian*, one still has that $[X, K(\pi, 1)] \cong Hom(\pi_1(X), \pi)/\pi$, that is, the conjugacy class or representation of π_1 into π;
3. A derivation of this result based on the fundamental cohomology theorem is also attached.

References

[1] May, J.P. 1999. *A Concise Course in Algebraic Topology*, The University of Chicago Press: Chicago.,p.173.