Omega-spectrum

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Defines	equence of CW complexes
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Defines	Ω–spectrum
Defines	Omega spectrum
Defines	unit circle
Defines	cohomology group
Defines	category of spectra

This is a topic entry on Ω–spectra and their important role in reduced cohomology theories on CW complexes.

0.1 Introduction

In algebraic topology a *spectrum* $\mathbf{S}$ is defined as a sequence of topological spaces $[X_0; X_1; ...X_i; X_{i+1}; ...]$ together with http://planetmath.org/ClassesOfAlgebrasstructure mappings $S1 \bigwedge X_i \to X_{i+1}$, where $S1$ is the *unit circle* (that is, a circle with a unit radius).

0.2 Ω–spectrum

One can express the definition of an Ω–spectrum in terms of a sequence of CW complexes, $K_1, K_2, ...$ as follows.

Definition 0.1. Let us consider ΩK, the space of loops in a *CW* complex K called the *loopspace of* K, which is topologized as a subspace of the space K^I of all maps $I \to K$, where K^I is given the compact-open topology. Then, an Ω*–spectrum* $\{K_n\}$ is defined as a sequence $K_1, K_2, ...$ of CW complexes together with http://planetmath.org/WeakHomotopyEquivalenceweak homotopy equivalences (ϵ_n):

$$\epsilon_n : \Omega K_n \to K_{n+1},$$

with n being an integer.

An alternative definition of the Ω–spectrum can also be formulated as follows.

Definition 0.2. An Ω*–spectrum*, or *Omega spectrum*, is a spectrum $\mathbf{E}$ such that for every index i, the topological space X_i is fibered, and also the adjoints of the http://planetmath.org/ClassesOfAlge mappings are all weak equivalences $X_i \cong \Omega X_{i+1}$.

0.3 The Role of Ω-spectra in Reduced Cohomology Theories

A category of spectra (regarded as above as sequences) will provide a model category that enables one to construct a stable homotopy theory, so that the homotopy category of spectra is canonically defined in the classical manner. Therefore, for any given construction of an Ω–spectrum one is able to canonically define an associated cohomology theory; thus, one defines the http://planetmath.org/ProofOfCohomologyGroupTheoremcohomology groups of a CW-complex K associated with the Ω–spectrum $\mathbf{E}$ by setting the rule: $H^n(K; \mathbf{E}) = [K, E_n]$.

The latter set when K is a CW complex can be endowed with a group structure by requiring that $(\epsilon_n)* : [K, E_n] \to [K, \Omega E_{n+1}]$ is an isomorphism which defines the multiplication in $[K, E_n]$ induced by ϵ_n.

One can prove that if $\{K_n\}$ is a an Ω-spectrum then the functors defined by the assignments $X \longmapsto h^n(X) = (X, K_n)$, with $n \in \mathbb{Z}$ define a reduced cohomology theory on the category of basepointed CW complexes and basepoint preserving maps; furthermore, every reduced cohomology theory on CW complexes arises in this manner from an Ω-spectrum (the Brown representability theorem; p. 397 of [**?**]).

References

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