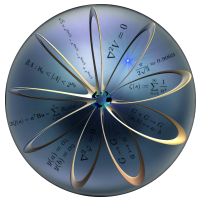

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Sorgenfrey line

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Synonym	Sorgenfrey topology
Defines	lower limit topology

The *Sorgenfrey line* is a nonstandard topology on the real line $\mathbb{R}$. Its topology is defined by the following base of half open intervals

$$\mathcal{B} = \{[a, b) \mid a, b \in \mathbb{R}, a < b\}.$$

Another name is *lower limit topology*, since a sequence x_α converges only if it converges in the standard topology and its limit is a limit from above (which, in this case, means that at most finitely many points of the sequence lie below the limit). For example, the sequence $(1/n)$ converges to 0, while $(-1/n)$ does not.

This topology is finer than the standard topology on $\mathbb{R}$. The Sorgenfrey line is first countable and separable, but is not second countable. It is therefore not metrizable.

References

[1] R. H. Sorgenfrey, *On the topological product of paracompact spaces*, Bulletin of the American Mathematical Society 53 (1947) 631–632. (This paper is `http://projecteuclid.org/euclid.bams/1183510809`available on-line from Project Euclid.)