frame groupoid

Canonical name	FrameGroupoid
Date of creation	2013-03-22 19:19:14
Last modified on	2013-03-22 19:19:14
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	29
Author	bci1 (20947)
Entry type	Definition
Classification	msc 55N33
Classification	msc 55N20
Classification	msc 55P10
Classification	msc 22A22
Classification	msc 20L05
Classification	msc 18B40
Classification	msc 55U40
Related topic	GroupAction
Related topic	VectorBundle
Related topic	GroupoidRepresentation4
Related topic	RepresentationsOfLocallyCompactGroupoids
Related topic	Functor
Related topic	FunctionalBiology
Defines	group representation
Defines	End(V)
Defines	group endomorphism
Defines	Lie groupoid representation
Defines	structure maps

Definition 0.1. Let $\mathcal{G}$ be a groupoid, defined as usual by a category in which all morphisms are invertible, with the *structure maps* $s, t : G_1 \longrightarrow G_0$, and $u : G_0 \longrightarrow G_1$. Given a vector bundle $q : E \longrightarrow G_0$, the *frame groupoid* is defined as

$$\Phi(E) = s, t : \phi(E) \longrightarrow G_0$$

, with $\phi(E)$ being the set of all vector space isomorphisms $\eta : E_x \longrightarrow E_y$ over all pairs $(x, y) \in {G_0}^2$, also with the usual conditions for the structure maps of the groupoid.

Definition 0.2. Let G be a group and V a vector space. A *group representation* is then defined as a homomorphism

$$h : G \longrightarrow End(V),$$

with $End(V)$ being the group of endomorphisms $e : V \longrightarrow V$ of the vector space V.

Note: With the notation used above, let us consider $q : E \longrightarrow G_0$ to be a vector bundle. Then, consider a *group representation*– which was here defined as the representation R_G of a group G *via* the group action on the vector space V, or as the homomorphism $h : G \longrightarrow End(V)$, with $End(V)$ being the group of endomorphisms of the vector space V. The generalization of group representations to the representations of groupoids then occurs naturally by considering the groupoid action on a vector bundle $q : E \longrightarrow G_0$. Therefore, the frame groupoid enters into the definition of `http://planetmath.org/GroupoidRepresentation4`groupoid representations.