embedding

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Entry type	Definition
Classification	msc 57R40
Synonym	differential embedding
Defines	Whitney's theorem

Let M and N be manifolds and $f\colon M \to N$ a smooth map. Then f is an *embedding* if

1. $f(M)$ is a submanifold of N, and

2. $f\colon M \to f(M)$ is a diffeomorphism. (There's an abuse of notation here. This should really be restated as the map $g\colon M \to f(M)$ defined by $g(p) = f(p)$ is a diffeomorphism.)

The above characterization can be equivalently stated: $f\colon M \to N$ is an embedding if

1. f is an immersion, and

2. by abuse of notation, $f\colon M \to f(M)$ is a homeomorphism.

Remark. A celebrated theorem of Whitney states that every n dimensional manifold admits an embedding into $\mathbb{R}^{2n+1}$.