loop theorem

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In the topology of 3-manifolds, **the loop theorem** is generalization of an ansatz discovered by Max Dehn (namely, Dehn's lemma), who saw that *if a continuous map from a 2-disk to a 3-manifold whose restriction to the boundary's disk has no singularities, then there exists another embedding whose restriction to the boundary's disk is equal to the boundary's restriction original map.*

The following statement called the loop theorem is a version from J. Stallings, but written in W. Jaco's book.

Let M be a three-manifold and let S be a connected surface in ∂M. Let $N \subset \pi_1(M)$ be a normal subgroup. Let $f\colon D^2 \to M$ be a **continuous map** *such that $f(\partial D^2) \subset S$ and $[f|\partial D^2] \notin N$.*
Then there exists an **embedding** $g\colon D^2 \to M$ *such that $g(\partial D^2) \subset S$ and $[g|\partial D^2] \notin N$,*

The proof is a clever construction due to C. Papakyriakopoulos about a sequence (a tower) of covering spaces. Maybe the best detailed presentation is due to A. Hatcher. But in general, accordingly to Jaco's opinion, *"... for anyone unfamiliar with the techniques of 3-manifold-topology and are here to gain a working knowledge for the study of problems in this ..., there is no better to start."*

References

W. Jaco, *Lectures on 3-manifolds topology*, A.M.S. regional conference series in Math 43.

J. Hempel, *3-manifolds*, Princeton University Press 1976.

A. Hatcher, *Notes on 3-manifolds*, available on-line.