



planetmath.org

Math for the people, by the people.

function differentiable at only one point

Canonical name	FunctionDifferentiableAtOnlyOnePoint
Date of creation	2013-03-22 15:48:16
Last modified on	2013-03-22 15:48:16
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	6
Author	matte (1858)
Entry type	Example
Classification	msc 57R35
Classification	msc 26A24
Related topic	FunctionContinuousAtOnlyOnePoint

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be the function

$$f(x) = \begin{cases} x, & \text{when } x \text{ is rational,} \\ -x, & \text{when } x \text{ is irrational.} \end{cases}$$

See <http://planetmath.org/FunctionContinuousAtOnlyOnePoint> this entry. Let  $g: \mathbb{R} \rightarrow \mathbb{R}$  be the function

$$g(x) = f(x)x.$$

Then  $g$  differentiable at 0, but everywhere else non-differentiable.

Indeed, since

$$\begin{aligned} g'(0) &= \lim_{h \rightarrow 0} \frac{f(h)h - f(0)0}{h} \\ &= \lim_{h \rightarrow 0} f(h) \\ &= 0 \end{aligned}$$

$g$  is differentiable at 0. If  $g$  would be continuous at  $x \neq 0$ , then  $f(x) = g(x)/x$  would be continuous at  $x$ . <http://planetmath.org/DifferentiableFunctionsAreContinuous> The result implies that  $g$  is non-differentiable away from the origin.