examples of mapping class group

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An example of this concept is to take the 2-sphere S^2, then one can calculate that

$$\mathcal{M}(S^2) = 1,$$

but

$$\mathcal{M}^*(S^2) = \mathbb{Z}_2.$$

For the genus one orientable surface, i.e. the torus $T = S^1 \times S^1$, it is known that its (extended) mapping class group

$$\mathcal{M}^*(T) = GL_2(\mathbb{Z}),$$

but usually by the (non-extended) mapping class group, that is, the group of isotopy classes of homeomorphisms that preserve orientations (the Dehn's twists) is just

$$\mathcal{M}(T) = SL_2(\mathbb{Z}).$$

In these two examples we see that $\mathcal{M}^*$ is an extension of $\mathcal{M}$ by $\mathbb{Z}_2$, trivial for the 2-sphere and non trivial for the torus.

For the projective plane $\mathbb{R}P^2$ we have

$$\mathcal{M}(\mathbb{R}P^2) = \mathcal{M}^*(\mathbb{R}P^2) = 1$$

And what about the Klein bottle?

$$\mathcal{M}(K) = \mathbb{Z}_2$$

$$\mathcal{M}^*(K) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$