



Math for the people, by the people.

connected sum

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Let M and N be two n -manifolds. Choose points $m \in M$ and $n \in N$, and let U, V be neighborhoods of these points, respectively. Since M and N are manifolds, we may assume that U and V are balls, with boundaries homeomorphic to $(n - 1)$ -spheres, since this is possible in \mathbb{R}^n . Then let $\varphi : \partial U \rightarrow \partial V$ be a homeomorphism. If M and N are oriented, this should be orientation preserving with respect to the induced orientation (that is, degree 1). Then the *connected sum* $M \sharp N$ is $M - U$ and $N - V$ glued along the boundaries by φ .

That is, $M \sharp N$ is the disjoint union of $M - U$ and $N - V$ modulo the equivalence relation $x \sim y$ if $x \in \partial U$, $y \in \partial V$ and $\varphi(x) = y$.