exotic R4's

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If $n \neq 4$ then the smooth manifolds homeomorphic to a given topological n- manifold, M, are parameterized by some discrete algebraic invariant of M. In particular there is a unique smooth manifold homeomorphic to $\mathbb{R}^n$.

By contrast one may choose uncountably many open sets in $\mathbb{R}^4$, which are all homeomorphic to $\mathbb{R}^4$, but which are pairwise non-diffeomorphic.

A smooth manifold homeomorphic to $\mathbb{R}^4$, but not diffeomorphic to it is called an *exotic* $\mathbb{R}^4$.

Given an exotic $\mathbb{R}^4$, E, we have a diffeomorphism $E \times \mathbb{R} \to \mathbb{R}^5$. (As there is only one smooth manifold homeomorphic to $\mathbb{R}^5$). Hence exotic $\mathbb{R}^4$'s may be identified with closed submanifolds of $\mathbb{R}^5$. In particular this means the cardinality of the set of exotic $\mathbb{R}^4$'s is precisely continuum.

Historically, Donaldson's theorem led to the discovery of the Donaldson Freedman exotic $\mathbb{R}^4$.