3-manifold

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In this brief note we define and give instances of the notion of a 3-manifold.

A *3-manifold* is a Hausdorff topological space which is locally homeomorphic to the Euclidean space $\mathbb{R}^3$.

One can see from simple constructions the great variety of objects that indicate how they are worth to study.

First examples without boundary:

1. For example, with the Cartesian product we can get:
   - $S^2 \times S^1$
   - $\mathbb{R}P^2 \times S^1$
   - $T \times S^1$
   - $K \times S^1$

   where S^1 and S^2 are the 1- and 2-dimensional spheres respectively, T is a torus, K a Klein bottle, and $\mathbb{R}P^2$ is the 2-dimensional real projective space.

2. Also by the generalization of the Cartesian product: *fiber bundles*, one can build bundles E of the type

$$F \subset E \to S^1$$

   where F is any closed surface.

3. Or interchanging the roles, bundles as:

$$S^1 \subset E \to F$$

4. knots and links complements

For the second type it is known that for each *isotopy class* $[\phi]$ of maps $F \to F$ correspond to an unique bundle E_ϕ. Any homeomorphism $f : F \to F$ representing the isotopy class $[\phi]$ is called a *monodromy* for E_ϕ.

From the previous paragraph we infer that the *mapping class group* play a important role in the understanding at least for this subclass of objets.

For the third class above one can use an *orbifold* instead of a simple surface to get a class of 3-manifolds called *Seifert fiber spaces* which are a large class of spaces needed to understand the modern classifications for 3-manifolds.

References

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