surface of revolution

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Related topic	SurfaceOfRevolution
Related topic	PappussTheoremForSurfacesOfRevolution
Related topic	QuadraticSurfaces
Related topic	ConicalSurface
Related topic	Torus
Related topic	SolidOfRevolution
Related topic	LeastSurfaceOfRevolution
Related topic	ConeInMathbbR3
Defines	surface of revolution
Defines	axis of revolution
Defines	circle of latitude
Defines	meridian curve
Defines	0-meridian
Defines	cone of revolution
Defines	asymptote cone
Defines	catenoid

If a curve in $\mathbb{R}^3$ rotates about a line, it generates a *surface of revolution.* The line is called the *axis of revolution.* Every point of the curve generates a *circle of latitude.* If the surface is intersected by a half-plane beginning from the axis of revolution, the intersection curve is a *meridian curve.* One can always think that the surface of revolution is generated by the rotation of a certain meridian, which may be called the *0-meridian.*

Let $y = f(x)$ be a curve of the xy-plane rotating about the x-axis. Then any point $(x,\, y)$ of this 0-meridian draws a circle of latitude, parallel to the yz-plane, with centre on the x-axis and with the radius $|f(x)|$. So the y- and z-coordinates of each point on this circle satisfy the equation

$$y^2+z^2 \;=\; [f(x)]^2.$$

This equation is thus satisfied by all points $(x,\, y,\, z)$ of the surface of revolution and therefore it is the equation of the whole surface of revolution.

More generally, if the equation of the meridian curve in the xy-plane is given in the implicit form $F(x,\, y) = 0$, then the equation of the surface of revolution may be written

$$F(x,\, \sqrt{y^2+z^2}) \;=\; 0.$$

Examples.

When the catenary $y = a\cosh\frac{x}{a}$ rotates about the x-axis, it generates the *catenoid*

$$y^2+z^2 \;=\; a^2\cosh^2\frac{x}{a}.$$

The catenoid is the only surface of revolution being also a minimal surface.

The quadratic surfaces of revolution:

- When the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2} = 1$ rotates about the x-axis, we get the ellipsoid
$$\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2} \;=\; 1.$$
This is a *stretched ellipsoid*, if $a > b$, and a *flattened ellipsoid*, if $a < b$, and a sphere of radius a, if $a = b$.

- When the parabola $y^2 = 2px$ (with p the *latus rectum* or the parameter of parabola) rotates about the x-axis, we get the *paraboloid of revolution*
$$y^2+z^2 \;=\; 2px.$$

- When we let the conjugate hyperbolas and their common asymptotes $\frac{x^2}{a^2} - \frac{y^2}{b^2} = s$ (with $s = 1, -1, 0$) rotate about the x-axis, we obtain the *two-sheeted hyperboloid*

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} \;=\; 1,$$

the *one-sheeted hyperboloid*

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} \;=\; -1$$

and the *cone of revolution*

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} \;=\; 0,$$

which apparently is the common *asymptote cone* of both hyperboloids.

References

[1] Lauri Pimiä: *Analyyttinen geometria.* Werner Söderström Osakeyhtiö, Porvoo and Helsinki (1958).