Sard's theorem

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Let $\phi : X^n \to Y^m$ be a smooth map on smooth manifolds. A *critical point* of ϕ is a point $p \in X$ such that the differential $\phi_* : T_pX \to T_{\phi(p)}Y$ considered as a linear transformation of real vector spaces has `http://planetmath.org/RankLinearMappingra` $< m$. A *critical value* of ϕ is the image of a critical point. A *regular value* of ϕ is a point $q \in Y$ which is not the image of any critical point. In particular, q is a regular value of ϕ if $q \in Y \setminus \phi(X)$.

Following Spivak [**?**], we say a subset V of Y^m *has measure zero* if there is a sequence of coordinate charts (x_i, U_i) whose union contains V and such that $x_i(U_i \cap V)$ has measure 0 (in the usual sense) in $\mathbb{R}^m$ for all i. With that definition, we can now state:

Sard's Theorem. Let $\phi : X \to Y$ be a smooth map on smooth manifolds. Then the set of critical values of ϕ has measure zero.

References

[Spivak] Spivak, Michael. *A Comprehensive Introduction to Differential Geometry.* Volume I, Third Edition. Publish of Perish, Inc. Houston, Texas. 1999.