orbifold

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Roughly, an orbifold is the quotient of a manifold by a finite group. For example, take a sheet of paper and add a small crease perpendicular to one side at the halfway point. Then, line up the two halves of the side. This may be thought of as the plane $\mathbb{R}^2$ modulo the group $\mathbb{Z}^2$. Now, let us give the definition.

Define a category $\mathcal{X}$: The objects are pairs (G, X), where G is a finite group acting effectively on a connected smooth manifold X. A morphism Φ between two objects (G', X') and (G, X) is a family of open embeddings $\phi : X' \to X$ which satisfy

- for each embedding $\phi \in \Phi$, there is an injective homomorphism $\lambda_\phi : G' \to G$ such that ϕ is λ_ϕ equivariant
- For $g \in G$, we have

$$\begin{aligned} g\phi &: X' \to X \\ g\phi &: x \mapsto g\phi(x) \end{aligned}$$

  and if $(g\phi)(X) \cap \phi(X) \neq \emptyset$, then $g \in \lambda_\phi(G')$.
- $\Phi = \{g\phi, g \in G\}$, for any $\phi \in \Phi$

Now, we define orbifolds. Given a paracompact Hausdorff space X and a nice open covering $\mathcal{U}$ which forms a basis for the topology on X, an orbifold structure $\mathcal{V}$ on X consists of

1. For $U \in \mathcal{U}$, $\mathcal{V}(U) = (G_U, \tilde{U}) \xrightarrow{\tau} U$ is a ramified cover $\tilde{U} \to U$ which identifies $\tilde{U}/G_U \cong U$
2. For $U \subset V \in \mathcal{U}$, there exists a morphism $\phi_{VU}(G_U, \tilde{U}) \to (G_V, \tilde{V})$ covering the inclusion
3. If $U \subset V \subset W \in \mathcal{U}$, $\phi_{WU} = \phi_{WV} \circ \phi_{VU}$

References:

[1] Kawasaki T., The Signature theorem for V-manifolds. Topology 17 (1978), 75-83. MR0474432 (57:14072)