diffeotopy

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Defines	isotopic
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Let M be a manifold and $I = [0, 1]$ the closed unit interval. A smooth map $h\colon M \times I \to M$ is called a *diffeotopy* (on M) if for every $t \in I$:

$$h_t := h(-, t)\colon M \to M$$

is a diffeomorphism.

Two diffeomorphisms $f, g\colon M \to M$ are said to be *diffeotopic* if there is a diffeotopy $h\colon M \times I \to M$ such that

1. $h_0 = f$, and
2. $h_1 = g$.

Remark. Diffeotopy is an equivalence relation among diffeomorphisms. In particular, those diffeomorphisms that are diffeotopic to the identity map form a group.

Two points $a, b \in M$ are said to be *isotopic* if there is a diffeotopy h on M such that

1. $h_0 = id_M$, the identity map on M, and
2. $h_1(a) = b$.

Remark. If M is a connected manifold, then every pair of points on M are isotopic.

Pairs of isotopic points in a manifold can be generazlied to pairs of isotopic sets. Two arbitrary sets $A, B \subseteq M$ are said to be *isotopic* if there is a diffeotopy h on M such that

1. $h_0 = id_M$, and
2. $h_1(A) = B$.

Remark. One special example of isotopic sets is the isotopy of curves. In $\mathbb{R}^3$, curves that are isotopic to the unit circle are the trivial knots.