differentiable functions are continuous

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Proposition 1. *Suppose I is an open interval on $\mathbb{R}$, and $f\colon I \to \mathbb{C}$ is differentiable at $x \in I$. Then f is continuous at x. Further, if f is differentiable on I, then f is continuous on I.*

Proof. Suppose $x \in I$. Let us show that $f(y) \to f(x)$, when $y \to x$. First, if $y \in I$ is distinct to x, then

$$f(x) - f(y) = \frac{f(x) - f(y)}{x - y}(x - y).$$

Thus, if $f'(x)$ is the derivative of f at x, we have

$$\begin{aligned}
\lim_{y\to x} f(x) - f(y) &= \lim_{y\to x} \frac{f(x) - f(y)}{x - y}(x - y) \\
&= \lim_{y\to x} \frac{f(x) - f(y)}{x - y} \ \lim_{y\to x}(x - y) \\
&= f'(x)\ 0 \\
&= 0,
\end{aligned}$$

where the second equality is justified since both limits on the second line exist. The second claim follows since f is continuous on I if and only if f is continuous at x for all $x \in I$. □