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Dehn surgery

Canonical name	DehnSurgery
Date of creation	2013-03-22 13:56:07
Last modified on	2013-03-22 13:56:07
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	4
Author	bwebste (988)
Entry type	Definition
Classification	msc 57M99

Let M be a smooth 3-manifold, and $K \subset M$ a smooth knot. Since K is an embedded submanifold, by the tubular neighborhood theorem there is a closed neighborhood U of K diffeomorphic to the solid torus $D^2 \times S^1$. We let U' denote the interior of U. Now, let $\varphi : \partial U \to \partial U$ be an automorphism of the torus, and consider the manifold $M' = M \backslash U' \coprod_{\varphi} U$, which is the disjoint union of $M \backslash U'$ and U, with points in the boundary of U identified with their images in the boundary of $M \backslash U'$ under φ.

It's a bit hard to visualize how this actually results in a different manifold, but it generally does. For example, if $M = S^3$, the 3-sphere, K is the trivial knot, and φ is the automorphism exchanging meridians and parallels (i.e., since $U \cong D^2 \times S^1$, get an isomorphism $\partial U \cong S^1 \times S^1$, and φ is the map interchanging to the two copies of S^1), then one can check that $M' \cong S^1 \times S^2$ ($S^3 \backslash U$ is also a solid torus, and after our automorphism, we glue the two solid tori, meridians to meridians, parallels to parallels, so the two copies of D^2 paste along the edges to make S^2).

Every compact 3-manifold can obtained from the S^3 by surgery around finitely many knots.