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homology sphere

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A compact n -manifold M is called a *homology sphere* if its homology is that of the n -sphere S^n , i.e. $H_0(M; \mathbb{Z}) \cong H_n(M; \mathbb{Z}) \cong \mathbb{Z}$ and is zero otherwise.

An application of the Hurewicz theorem and homological Whitehead theorem shows that any simply connected homology sphere is in fact homotopy equivalent to S^n , and hence homeomorphic to S^n for $n \neq 3$, by the higher dimensional equivalent of the Poincaré conjecture.

The original version of the Poincaré conjecture stated that every 3 dimensional homology sphere was homeomorphic to S^3 , but Poincaré himself found a counter-example. There are, in fact, a number of interesting 3-dimensional homology spheres.