Hopf theorem

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In the following we will assume that the term "smooth" implies just C^1 (once continuously differentiable). By smooth homotopy we will that the homotopy mapping is itself continuously differentiable

Theorem. *Suppose that M is a connected, `http://planetmath.org/Orientation2`oriented smooth manifold without boundary of dimension m and suppose $f, g\colon M \to S^m$ are smooth mappings to the m-sphere. Then f and g are smoothly homotopic if and only if f and g have the same Brouwer degree.*

When M is not orientable, then we can always "flip" the orientation by following a closed loop on the manifold and one can then prove the following result.

Theorem. *Suppose that M is not orientable, connected smooth manifold without boundary of dimension m, and suppose $f, g\colon M \to S^m$ are smooth mappings to the m-sphere. Then f and g are smoothly homotopic if and only if f and g have the same degree mod 2.*

References

[1] John W. Milnor. . The University Press of Virginia, Charlottesville, Virginia, 1969.