



Math for the people, by the people.

Seifert fiber space

Canonical name	SeifertFiberSpace
Date of creation	2013-03-22 15:50:53
Last modified on	2013-03-22 15:50:53
Owner	juanman (12619)
Last modified by	juanman (12619)
Numerical id	10
Author	juanman (12619)
Entry type	Definition
Classification	msc 57M50
Synonym	circle bundle
Related topic	3Manifolds

In the field of three dimensional manifolds there is a kind which can be considered as spaces which are fibered by <http://planetmath.org/Circlecircles>, that is, for each point in the space there is a unique circle (a homeomorph of S^1) which contains it.

A Seifert fiber space can be defined as a circle bundle over an orbifold, however these kind of 3-manifolds appeared much earlier than the concept of fiber-bundle.

To construct them one must begin learning how to fiber a solid torus. Naturally a solid torus $D^2 \times S^1$ is foliated by circles $p \times S^1$ where $p \in D^2$.

To get different fibered solid tori one must begin with a solid torus $D^2 \times S^1$, cut it along a meridian disk and twist the resulting solid cylinder by a rational angle $2\pi(a/b)$ (being a, b coprime) and reglue to obtain a new fibering by circles of the old torus, where the fibers all are still longitudinal circles but differs by the previous one in the fact that now only the center's disk is a trivial circle but any other point in the disk is in a circle which is b times longer than the central one.

It is possible construct an associated 2-manifold identifying each fiber to a point to get the so called **orbit surface**.

References

H. Seifert, *Topologie drei-dimensionaler gefaseter Räume*, Acta. Math. 60 (1933), 147 - 238.

M. Brin, *Seifert Fibered Spaces, Notes for a course given in the Spring of 1993*, on line at <ftp://ftp.math.binghamton.edu/pub/matt/seifert.pdf>