Finsler geometry

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Let $\mathcal{M}$ be an n-dimensional differential manifold and let $\phi\colon T\mathcal{M} \to \mathbb{R}$ be a function $\phi(x, \xi)$ defined for $x \in \mathcal{M}$ and $\xi \in T_x\mathcal{M}$ such that $\phi(x, \cdot)$ is a possibly non symmetric norm on $T_x\mathcal{M}$. The couple $(\mathcal{M}, \phi)$ is called a Finsler space.

Let us define the ϕ-length of curves in $\mathcal{M}$. If $\gamma\colon [a, b] \to \mathcal{M}$ is a differentiable curve we define

$$\ell_\phi(\gamma) := \int_a^b \phi(\gamma'(t))\, dt.$$

So a natural geodesic distance can be defined on $\mathcal{M}$ which turns the Finsler space into a quasi-metric space (if $\mathcal{M}$ is connected):

$$d_\phi(x, y) := \inf\{\ell_\phi(\gamma) \colon \gamma \text{ is a differentiable curve } \gamma\colon [a, b] \to \mathcal{M} \text{ such that } \gamma(a) = x \text{ and } \gamma(b) = y\}.$$

Notice that every Riemann manifold $(\mathcal{M}, g)$ is also a Finsler space, the norm $\phi(x, \cdot)$ being the norm induced by the scalar product $g(x)$.

A finite dimensional Banach space is another simple example of Finsler space, where $\phi(x, \xi) := \|\xi\|$. Wulff Theorem is one of the most important theorems in this ambient space.