rectifiable current

Canonical name	RectifiableCurrent
Date of creation	2013-03-22 14:28:34
Last modified on	2013-03-22 14:28:34
Owner	paolini (1187)
Last modified by	paolini (1187)
Numerical id	4
Author	paolini (1187)
Entry type	Definition
Classification	msc 58A25
Defines	integral current
Defines	integral flat norm
Defines	integral flat chains

An m-dimensional *rectifiable current* is a current T whose action against a m-form ω can be written as

$$T(\omega) = \int_S \theta(x) \langle \xi(x), \omega(x) \rangle d\mathcal{H}^m(x)$$

where S is an m-dimensional bounded rectifiable set, ξ is an orientation of S i.e. $\xi(x)$ is a unit m-vector representing the approximate tangent plane of S at x for $\mathcal{H}^m$-a.e. $x \in S$ and, finally, $\theta(x)$ is an integer valued measurable function defined a.e. on S (called *multiplicity*). The space of m-dimensional rectifiable currents is denoted by $\mathcal{R}_m$.

An m-dimensional rectifiable current T such that the boundary ∂T is itself an $(m-1)$-dimensional rectifiable current, is called *integral current*. The space of integral currents is denoted by $\mathbf{I}_m$. We point out that the word "integral" refers to the fact that the multiplicity θ is integer valued.

Also notice that rectifiable and integral currents are not vector subspaces of the space of currents. In fact while the sum of two rectifiable currents is again a rectifiable current, the multiplication by a real number gives a rectifiable current only if the number is an integer.

The compactness theorem makes the space of integral currents a good space where geometric problems can be ambiented.

On rectifiable currents one can define an *integral flat norm*

$$\mathcal{F}(T) := \inf\{\mathbf{M}(A) + \mathbf{M}(B) \colon T = A + \partial B, \quad A \in \mathcal{R}_m,\ B \in \mathcal{R}_{m+1}\}.$$

The closure of the space $\mathcal{R}_m$ under the integral flat norm is called the space of *integral flat chains* and is denoted by $\mathcal{F}_m$.

As a consequence of the closure theorem, one finds that $\mathcal{R}_m = \{T \in \mathcal{F}_m \colon \mathbf{M}(T) < \infty\}$ where $\mathbf{M}$ is the mass norm of a current.