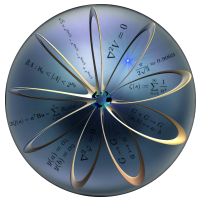

planetmath.org

Math for the people, by the people.

volume element

Canonical name	VolumeElement
Date of creation	2013-03-22 17:40:58
Last modified on	2013-03-22 17:40:58
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	5
Author	jirka (4157)
Entry type	Definition
Classification	msc 58A10
Classification	msc 53-00
Synonym	volume form
Synonym	volume measure
Defines	area element
Defines	area form
Defines	area measure
Defines	Euclidean volume element
Defines	Euclidean volume form
Defines	euclidean volume measure
Defines	surface area measure
Defines	surface area element
Defines	surface area form

If M is an n dimensional manifold, then a http://planetmath.org/DifferentialFormsdifferen n form that is never zero is called a *volume element* or a *volume form*. Usually one volume form is associated with the manifold. The volume element is sometimes denoted by dV, ω or vol_n. If the manifold is a Riemannian manifold with g, then the natural volume form is defined in local coordinates $x^1 \dots x^n$ by

$$dV := \sqrt{|g|} dx^1 \wedge \dots \wedge dx^n.$$

It is not hard to show that a manifold has a volume form if and only if it is orientable.

If the manifold is $\mathbb{R}^n$, then the usual volume element $dV = dx^1 \wedge dx^2 \wedge \dots \wedge dx^n$ is called the *Euclidean volume element* or *Euclidean volume form*. In this context, $\mathbb{C}^n$ is usually treated as $\mathbb{R}^{2n}$ unless stated otherwise.

When $n = 2$, then the form is frequently called the *area element* or *area form* and frequently denoted by dA. Furthermore, when the manifold is a submanifold of $\mathbb{R}^3$, then many authors will refer to a *surface area element* or *surface area form*.

When the context is measure theoretic, this form is sometimes called a *volume measure*, *area measure*, etc...

References

[1] Michael Spivak. , W.A. Benjamin, Inc., 1965.

[2] William M. Boothby. , Academic Press, San Diego, California, 2003.