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spectral radius

Canonical name	SpectralRadius
Date of creation	2013-03-22 13:13:58
Last modified on	2013-03-22 13:13:58
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	11
Author	Koro (127)
Entry type	Definition
Classification	msc 58C40
Defines	spectrum

If V is a vector space over $\mathbb{C}$, the spectrum of a linear mapping $T : V \to V$ is the set

$$\sigma(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{is not invertible}\},$$

where I denotes the identity mapping. If V is finite dimensional, the spectrum of T is precisely the set of its eigenvalues. For infinite dimensional spaces this is not generally true, although it is true that each eigenvalue of T belongs to $\sigma(T)$. The *spectral radius* of T is

$$\rho(T) = \sup\{|\lambda| : \lambda \in \sigma(T)\}.$$

More generally, the spectrum and spectral radius can be defined for Banach algebras with identity element: If $\mathcal{A}$ is a Banach algebra over $\mathbb{C}$ with identity element e, the spectrum of an element $a \in \mathcal{A}$ is the set

$$\sigma(a) = \{\lambda \in \mathbb{C} : a - \lambda e \text{is not invertible in} \mathcal{A}\}$$

The spectral radius of a is $\rho(a) = \sup\{|\lambda| : \lambda \in \sigma(a)\}$.