tangent bundle

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Let M be a differentiable manifold. Let the tangent bundle TM of M be(as a set) the disjoint union $\coprod_{m \in M} T_m M$ of all the tangent spaces to M, i.e., the set of pairs

$$\{(m, x) | m \in M, x \in T_m M\}.$$

This naturally has a manifold structure, given as follows. For $M = \mathbb{R}^n$, $T\mathbb{R}^n$ is obviously isomorphic to $\mathbb{R}^{2n}$, and is thus obviously a manifold. By the definition of a differentiable manifold, for any $m \in M$, there is a neighborhood U of m and a diffeomorphism $\varphi : \mathbb{R}^n \to U$. Since this map is a diffeomorphism, its derivative is an isomorphism at all points. Thus $T\varphi : T\mathbb{R}^n = \mathbb{R}^{2n} \to TU$ is bijective, which endows TU with a natural structure of a differentiable manifold. Since the transition maps for M are differentiable, they are for TM as well, and TM is a differentiable manifold. In fact, the projection $\pi : TM \to M$ forgetting the tangent vector and remembering the point, is a vector bundle. A vector field on M is simply a section of this bundle.

The tangent bundle is functorial in the obvious sense: If $f : M \to N$ is differentiable, we get a map $Tf : TM \to TN$, defined by f on the base, and its derivative on the fibers.