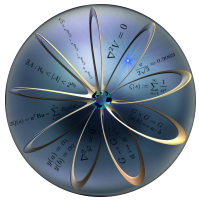

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normal bundle

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Let X be an immersed submanifold of M, with immersion $i : X \to M$. Then we can restrict the tangent bundle of M to N or more properly, take the pullback i^*TM. This, as an vector bundle over X should contain a lot of information about the embedding of X into M. But there is a natural injection $TX \to i^*TM$, and the subbundle which is the image of this only has information on the intrinsic properties of X, and thus is useless in obtaining information about the embedding of X into M. Instead, to get information on this, we take the quotient $i^*TM/TX = NX$, the normal bundle of X. The normal bundle is very strongly dependent on the immersion i. If E is any vector bundle on X, then E is the normal bundle for the embedding of X into E as the zero section.

The normal bundle determines the local geometry of the embedding of X into M in the following sense: In M, there exists an open neighborhood $U \supset X$ which is diffeomorphic to NX by a diffeomorphism taking X to the zero section.