cotangent bundle is a bundle

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Verifying the first criterion is simply a matter of writing it out:

$$(x^1,\dots,x^{2n}) \mapsto (\pi(x^1,\dots,x^{2n}), \phi_\alpha(x^1,\dots,x^n)) = ((x^1,\dots,x^n),(x^{n+1},\dots,x^{2n}))$$

This is obviously a homeomorphism.

As for the second criterion,

$$\begin{aligned}
\sum_{j=1}^{n} g_{\alpha\beta}{}^i_j(x^1,\dots,x^{2n})\phi_\beta{}^j(x^1,\dots,x^{2n}) &= \sum_{j=n}^{2n} \frac{\partial\big(\sigma_{\alpha\beta}(x^1,\dots x^n)\big)^i}{\partial x^j} x^{j+n} \\
&= \sigma'_{\alpha\beta}{}^{i+n}(x^1,\dots,x^{2n}) \\
&= \phi_\alpha{}^i(x^1,\dots,x^{2n})
\end{aligned}$$

The third criterion follows from the chain rule:

$$g_{\alpha\beta}{}^i_j g_{\beta\gamma}{}^j_k = \frac{\partial\big(\sigma_{\alpha\beta}(x^1,\dots x^n)\big)^i}{\partial x^j}\frac{\partial\big(\sigma_{\beta\gamma}(x^1,\dots x^n)\big)^j}{\partial x^k}$$