Morse complex

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Let M be a smooth manifold, and $u : M \to \mathbb{R}$ be a Morse function. Let $C_n^u(M)$ be a vector space of formal $\mathbb{C}$-linear combinations of critical points of u with index n. Then there exists a differential $\partial_n : C_n \to C_{n-1}$ based on the Morse flow making C_* into a chain complex called the *Morse complex* such that the homology of the complex is the singular homology of M. In particular, the number of critical points of u of index n on M is at least the n-th Betti number, and the alternating sum of the number of critical points of u is the Euler characteristic of M.