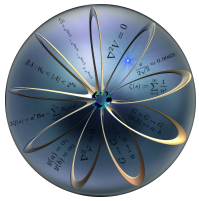

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mean square error

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Entry type	Definition
Classification	msc 62J10
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Synonym	MSE
Synonym	MVUE
Synonym	UMVU
Synonym	UMVUE
Synonym	uniformly minimum variance unbiased
Related topic	MeanSquareDeviation
Defines	minimum variance unbiased estimator
Defines	rms error
Defines	root-mean-square
Defines	root mean square
Defines	rms

The *mean square error* of an estimator $\hat{\theta}$ of a parameter θ in a statistical model is defined as:

$$\mathrm{MSE}(\hat{\theta}) \colon = \mathrm{E}\left[(\hat{\theta} - \theta)^2\right].$$

From the definition of the variance $\mathrm{Var}[X] = \mathrm{E}[X^2] - \mathrm{E}[X]^2$, we can express the mean square error in terms of the bias by expanding the right hand side above:

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}\left[\hat{\theta}\right] + \mathrm{Bias}(\hat{\theta})^2.$$

If $\hat{\theta}$ is an unbiased estimator, then its mean square error is identical to its variance: $\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}[\hat{\theta}]$. An unbiased estimator such that $\mathrm{MSE}(\hat{\theta})$ is a minimum value among all unbiased estimators for θ is called a *minimum variance unbiased estimator*, abbreviated *MVUE*, or *uniformly minimum variance unbiased estimator*, abbreviated *UMVU* estimator.

Example. Suppose $X_1, X_2, \ldots, X_n$ are iid random variables (n independent measurements of the radius of a coin, etc...) from a normal distribution $N(\mu, \sigma^2)$ (for example, μ would be the true radius of the coin, and σ^2 would be the error component of the measurements). Suppose $\overline{X}$ ($= \overline{X}_n$) is the sample mean. Then $\overline{X}$ is an unbiased estimator, so that

$$\mathrm{MSE}(\overline{X}) = \mathrm{Var}\left[\overline{X}\right] = \mathrm{Var}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n^2}\left(\sum_{i=1}^{n} \sigma^2\right) = \frac{\sigma^2}{n}.$$

Remark. The square root of MSE is called the "root mean square error", or *rms error* for short.