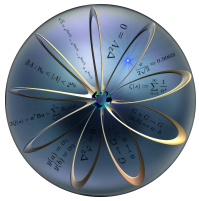

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multivariate gamma function (real-valued)

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Defines	gamma function (multivariate real)

The real-valued multivariate gamma function is defined by

$$\Gamma_m(a) = \int_{\mathfrak{S}} e^{-\operatorname{Tr} S} \, |S|^{a-\frac{1}{2}(m+1)} \, \mathrm{d}S, \tag{1}$$

where $\mathfrak{S}$ is the set of all $m \times m$ real, positive definite symmetric matrices, i.e.

$$\mathfrak{S} = \left\{ S \in \mathbb{R}^{m \times m} \mid S > 0, x^{\mathrm{T}} S x > 0 \, \forall \, x \in \mathbb{R}^{m \times 1} \setminus \{0\} \right\}. \tag{2}$$

The real-valued multivariate gamma function can also be expressed in terms of the gamma function as follows

$$\Gamma_m(a) = \pi^{\frac{1}{4}m(m-1)} \prod_{i=1}^{m} \Gamma\left(a - \frac{1}{2}(i-1)\right). \tag{3}$$

Reference

A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Statist.*, vol. 35, pp. 475-501, 1964.