Cochran's theorem

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Defines	Fisher's theorem

Let $\mathbf{X}$ be multivariate normally distributed as $\boldsymbol{N_p(0, I)}$ such that

$$\mathbf{X}^{\mathrm{T}}\mathbf{X} = \sum_{i=1}^{k} Q_i,$$

where each

1. Q_i is a quadratic form
2. $Q_i = \mathbf{X}^{\mathrm{T}}\mathbf{B}_i\mathbf{X}$, where $\mathbf{B}_i$ is a p by p square matrix
3. $\mathbf{B}_i$ is positive semidefinite
4. $\mathrm{rank}(\mathbf{B}_i) = r_i$

Then any two of the following imply the third:

1. $\sum_{i=1}^{k} r_i = p$
2. each Q_i has a `http://planetmath.org/ChiSquaredRandomVariable`chi square distribution with r_i of freedom, $\chi^2(r_i)$
3. Q_i's are mutually independent

As an example, suppose ${X_1}^2 \sim \chi^2(m_1)$ and ${X_2}^2 \sim \chi^2(m_2)$. Furthermore, assume ${X_1}^2 \geq {X_2}^2$ and $m_1 > m_2$, then

$${X_1}^2 - {X_2}^2 \sim \chi^2(m_1 - m_2).$$

This corollary is known as *Fisher's theorem*.