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## hazard function

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Defines	cumulative hazard function

Let  $Y$  be a random variable with probability density function  $f_Y(y)$ . Then the *hazard function*  $h(y)$  is defined to be:

$$h(y) = \frac{f_Y(y)}{1 - F_Y(y)} = \frac{f_Y(y)}{S(y)},$$

where  $S(y)$  is the survivor function and  $Y$  is the survival time.

The hazard function is the rate of probability of death (non survival) is changing at time  $Y = y$ , given survival up to time  $y$ :

$$h(y) = \lim_{\Delta y \rightarrow 0} \frac{P(y \leq Y \leq y + \Delta y \mid Y > y)}{\Delta y}.$$

The *cumulative hazard function*,  $H(y)$  of  $Y$  is defined as

$$H(y) = \int_{-\infty}^y h(t) dt.$$

From this definition, we see that  $H(y) = -\ln S(y)$ .

**Examples.** The hazard functions for the three most widely used probability density functions for survival time are:

- The exponential distribution, with  $h(y) = \gamma$ .
- The Weibull distribution, with  $h(y) = \gamma y^{\gamma-1}$  using the standard Weibull distribution.
- The extreme-value distribution, with  $h(y) = \frac{1}{\beta} \exp(\frac{y-\alpha}{\beta})$ .