



planetmath.org

Math for the people, by the people.

random variable

Canonical name	RandomVariable
Date of creation	2013-03-22 11:53:10
Last modified on	2013-03-22 11:53:10
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	21
Author	mathcam (2727)
Entry type	Definition
Classification	msc 62-00
Classification	msc 60-00
Classification	msc 11R32
Classification	msc 03-01
Classification	msc 20B25
Related topic	DistributionFunction
Related topic	DensityFunction
Related topic	GeometricDistribution2
Defines	discrete random variable
Defines	continuous random variable
Defines	law of a random variable

If  $(\Omega, \mathcal{A}, P)$  is a probability space, then a **random variable** on  $\Omega$  is a measurable function  $X : (\Omega, \mathcal{A}) \rightarrow S$  to a measurable space  $S$  (frequently taken to be the real numbers with the standard measure). The *law* of a random variable is the probability measure  $PX^{-1} : S \rightarrow \mathbb{R}$  defined by  $PX^{-1}(s) = P(X^{-1}(s))$ .

A random variable  $X$  is said to be *discrete* if the set  $\{X(\omega) : \omega \in \Omega\}$  (i.e. the range of  $X$ ) is finite or countable. A more general version of this definition is as follows: A random variable  $X$  is discrete if there is a countable subset  $B$  of the range of  $X$  such that  $P(X \in B) = 1$  (Note that, as a countable subset of  $\mathbb{R}$ ,  $B$  is measurable).

A random variable  $Y$  is said to be *absolutely continuous* if it has a cumulative distribution function which is <http://planetmath.org/AbsolutelyContinuousFunction2> absolutely continuous.

Example:

Consider the event of throwing a coin. Thus,  $\Omega = \{H, T\}$  where  $H$  is the event in which the coin falls head and  $T$  the event in which falls tails. Let  $X$  = number of tails in the experiment. Then  $X$  is a (discrete) random variable.