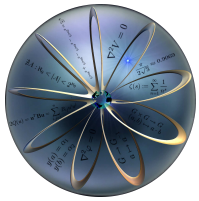

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probability problem

Canonical name	ProbabilityProblem
Date of creation	2013-03-22 19:11:21
Last modified on	2013-03-22 19:11:21
Owner	statsCab (25915)
Last modified by	statsCab (25915)
Numerical id	4
Author	statsCab (25915)
Entry type	Definition
Classification	msc 62-01

This is in response to the following request:

A parent particle divides into 0,1,or 2 particles with probabilities 1/4,1/2,1/4.it disappears after splitting.let Xn denotes the number of particles in n-th generations with X0=1.find P(X2¿0) and the probabilities that X1=2 given that X2=1.

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For my first entry I will try to answer the question.

Let p_0, p_1 and p_2 be the nonzero probabilities of dividing into 0, 1, or 2 particles, and let X_n denotes the number of particles at the n^{th} generation.

With $X_0 = 1$, find 1) $P(X_2 > 2)$ and 2) $P(X_1 = 2|X_2 = 1)$

1) After two generations there can be at most 2^2 particles so $P(X_2 > 2) = P(X_2 = 3) + P(X_2 = 4)$

$$P(X_2 = 4) = p_2^2$$

$$P(X_2 = 3) = 2p_1p_2^2$$

Note that if $X_2 = 3$, then $X_2 = 2$.

$$P(X_2 > 2) = p_2^2(1 + 2p_1)$$

Using your values I get 3/32.

2) From the definition of conditional probability

$$P(X_1|X_2) = \frac{P(X_1 \cap X_2)}{P(X_2)}$$

First

$$P(X_2 = 1) = p_1^2 + 2p_0p_1p_2$$

Why? To get to $X_2 = 1$, at $n = 1$ there are either one or two particles, if there is one particle it remains one at $n = 2$, and if there were two particles at $n = 1$, then one has to go to zero and the other one—this can happen two ways.

Finally $P(X_1 = 1 \cap X_2) = p_1p_2$.

$$P(X_1 = 2|X_2 = 1) = \frac{p_2}{p_1 + 2p_0p_2}$$

Using your values I get 2/3.

Now I have a question for you to think about. What happens in the long run, as $n \to \infty$?