# properties of Poisson random variables

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**Proposition 1.** *If $X_1, X_2$ are independent Poisson random variables with parameters $\lambda_1, \lambda_2$, then $X_1+X_2$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.*

*Proof.* Let $X := X_1 + X_2$ and $\lambda := \lambda_1 + \lambda_2$, let us calculate the distribution function of $X$:

$$\begin{aligned}
F_X(x) &= P(X \leq x) = P(X_1 + X_2 \leq x) = \sum_{i=0}^{x} P(X_1 + X_2 = i) \\
&= \sum_{i=0}^{x}\sum_{j=0}^{i} P(X_1 = j \text{ and } X_2 = i - j) = \sum_{i=0}^{x}\sum_{j=0}^{i} P(X_1 = j)P(X_2 = i - j) \\
&= \sum_{i=0}^{x}\sum_{j=0}^{i} \frac{e^{-\lambda_1}\lambda_1^j}{j!}\frac{e^{-\lambda_2}\lambda_2^{i-j}}{(i-j)!} = \sum_{i=0}^{x}\sum_{j=0}^{i} \frac{e^{-\lambda}}{i!}\binom{i}{j}\lambda_1^j\lambda_2^{i-j} \\
&= \sum_{i=0}^{x} \frac{e^{-\lambda}}{i!}\sum_{j=0}^{i}\binom{i}{j}\lambda_1^j\lambda_2^{i-j} = \sum_{i=0}^{x}\frac{e^{-\lambda}}{i!}(\lambda_1 + \lambda_2)^i = \sum_{i=0}^{x}\frac{e^{-\lambda}}{i!}\lambda^i.
\end{aligned}$$

As a result, $X$ is a Poisson random variable with parameter $\lambda$. Notice that in the fifth equation, we used the assumption that $X_1$ and $X_2$ are independent. □

As a corollary, any sum of independent Poisson random variables is Poisson, with parameter the sum of the parameters from the independent random variables.

**Proposition 2.** *A Poisson random variable is infinitely divisible.*

*Proof.* Let $X$ be a Poisson random variable with parameter $\lambda$. Let $n$ be any positive integer. Let $X_1, \ldots, X_n$ be independent identically distributed Poisson random variables with parameter $\frac{\lambda}{n}$. Then the sum of these random variables is easily seen to be Poisson, with parameter $\lambda$, and is therefore identically distributed as $X$. □