Weibull random variable

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Synonym	Weibull distribution
Synonym	Rayleigh distribution

X is a *Weibull random variable* if it has a probability density function, given by

$$f_X(x) = \frac{\gamma}{\alpha}(\frac{x-\mu}{\alpha})^{\gamma-1}e^{-(\frac{x-\mu}{\alpha})^\gamma}$$

where $\alpha, \gamma, \mu \in \mathbb{R}$, $\alpha, \gamma > 0$ and $x \geq \mu$. α is the *scale parameter*, γ is the *shape parameter*, and μ is the *location parameter*.

Notation for X having a Weibull distribution is $X \sim \mathrm{Wei}(\alpha, \gamma, \mu)$. Usually, the location and scale parameters are dropped by the transformation

$$Y = \frac{X-\mu}{\alpha}$$

so that $Y \sim \mathrm{Wei}(\gamma) := \mathrm{Wei}(1, \gamma, 0)$. The resulting distribution is called the *standard Weibull*, or *Rayleigh distribution*:

$$f_X(x) = \gamma x^{\gamma-1}\exp(-x^\gamma)$$

: Given a standard Weibull distribution $X \sim \mathrm{Wei}(\gamma)$:

1. $\mathrm{E}[X] = \Gamma(\frac{\gamma+1}{\gamma})$, where Γ is the gamma function
2. Median $= (\ln 2)^{\frac{1}{\gamma}}$
3. Mode $= \begin{cases} (1-\frac{1}{\gamma})^{1/\gamma} & \text{if } \gamma > 1 \\ 0 & \text{otherwise} \end{cases}$
4. $\mathrm{Var}[X] = \Gamma(\frac{\gamma+2}{\gamma}) - \Gamma(\frac{\gamma+1}{\gamma})^2$
5. $X \sim \mathrm{Wei}(\alpha, \gamma, 0)$ iff $X^\gamma \sim \mathrm{Exp}(\alpha^\gamma)$, the exponential distribution with parameter α^γ

Remark. The Weibull distribution is often used to model reliability or lifetime of such as light bulbs.