# order statistics

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| Canonical name | OrderStatistics |
| Date of creation | 2013-03-22 14:33:30 |
| Last modified on | 2013-03-22 14:33:30 |
| Owner | CWoo (3771) |
| Last modified by | CWoo (3771) |
| Numerical id | 8 |
| Author | CWoo (3771) |
| Entry type | Definition |
| Classification | msc 62G30 |

Let $X_1, \ldots, X_n$ be random variables with realizations in $\mathbb{R}$. Given an outcome $\omega$, order $x_i = X_i(\omega)$ in non-decreasing order so that

$$x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}.$$

Note that $x_{(1)} = \min(x_1, \ldots, x_n)$ and $x_{(n)} = \max(x_1, \ldots, x_n)$. Then each $X_{(i)}$, such that $X_{(i)}(\omega) = x_{(i)}$, is a random variable. Statistics defined by $X_{(1)}, \ldots, X_{(n)}$ are called *order statistics* of $X_1, \ldots, X_n$. If all the orderings are strict, then $X_{(1)}, \ldots, X_{(n)}$ are *the* order statistics of $X_1, \ldots, X_n$. Furthermore, each $X_{(i)}$ is called the $i$th order statistic of $X_1, \ldots, X_n$.

**Remark.** If $X_1, \ldots, X_n$ are iid as $X$ with probability density function $f_X$ (assuming $X$ is a continuous random variable), Let $\mathbf{Z}$ be the vector of the order statistics $(X_{(1)}, \ldots, X_{(n)})$ (with strict orderings), then one can show that the joint probability density function $f_{\mathbf{Z}}$ of the order statistics is:

$$f_{\mathbf{Z}}(\boldsymbol{z}) = n! \prod_{i=1}^{n} f_X(z_i),$$

where $\boldsymbol{z} = (z_1, \ldots, z_n)$ and $z_1 < \cdots < z_n$.