proof of expected value of the hypergeometric distribution

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We will first prove a useful property of binomial coefficients. We know

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

This can be transformed to

$$\binom{n}{k} = \frac{n}{k}\frac{(n-1)!}{(k-1)!(n-1-(k-1))!} = \frac{n}{k}\binom{n-1}{k-1}. \tag{1}$$

Now we can start with the definition of the expected value:

$$E[X] = \sum_{x=0}^{n} \frac{x\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}.$$

Since for $x = 0$ we add a 0 in this we can say

$$E[X] = \sum_{x=1}^{n} \frac{x\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}.$$

Applying equation (**??**) we get:

$$E[X] = \frac{nK}{M}\sum_{x=1}^{n} \frac{\binom{K-1}{x-1}\binom{M-1-(K-1)}{n-1-(x-1)}}{\binom{M-1}{n-1}}.$$

Setting $l := x - 1$ we get:

$$E[X] = \frac{nK}{M}\sum_{l=0}^{n-1} \frac{\binom{K-1}{l}\binom{M-1-(K-1)}{n-1-l}}{\binom{M-1}{n-1}}.$$

The sum in this equation is 1 as it is the sum over all probabilities of a hypergeometric distribution. Therefore we have

$$E[X] = \frac{nK}{M}.$$