Lehmann-Scheffé theorem

Canonical name	LehmannScheffeTheorem
Date of creation	2013-03-22 16:31:59
Last modified on	2013-03-22 16:31:59
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	13
Author	CWoo (3771)
Entry type	Theorem
Classification	msc 62F10
Synonym	Lehmann-Scheffe theorem
Defines	complete statistic

A statistic $S(\boldsymbol{X})$ on a random sample of data $\boldsymbol{X} = (X_1, \ldots, X_n)$ is said to be a *complete statistic* if for any Borel measurable function g,

$$E(g(S)) = 0 \quad \text{implies} \quad P(g(S) = 0) = 1.$$

In other words, $g(S) = 0$ almost everywhere whenever the expected value of $g(S)$ is 0. If $S(\boldsymbol{X})$ is associated with a family $f(x \mid \theta)$ of probability density functions (or mass function in the discrete case), then completeness of S means that $g(S) = 0$ almost everywhere whenever $E_\theta(g(S)) = 0$ for every θ.

Theorem 1 (Lehmann-Scheffé)**.** *If $S(\boldsymbol{X})$ is a complete sufficient statistic and $h(\boldsymbol{X})$ is an unbiased estimator for θ, then, given*

$$h_0(s) = E(h(\boldsymbol{X})|S(\boldsymbol{X}) = s),$$

$h_0(S) = h_0(S(\boldsymbol{X}))$ is a uniformly minimum variance unbiased estimator of θ. Furthermore, $h_0(S)$ is unique almost everywhere for every θ.