covariance matrix

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Synonym	variance covariance matrix

Let $\mathbf{X} = (X_1, \ldots, X_n)^T$ be a random vector. Then the *covariance matrix* of $\mathbf{X}$, denoted by $\mathbf{Cov}(\mathbf{X})$, is $\{Cov(X_i, X_j)\}$. The diagonals of $\mathbf{Cov}(\mathbf{X})$ are $Cov(X_i, X_i) = Var[X_i]$. In matrix notation,

$$\mathbf{Cov}(\mathbf{X}) = \begin{pmatrix} Var[X_1] & \cdots & Cov(X_1, X_n) \\ \vdots & & \vdots \\ Cov(X_n, X_1) & \cdots & Var[X_n] \end{pmatrix}.$$

It is easily seen that $\mathbf{Cov}(\mathbf{X}) = \mathbf{Var}[\mathbf{X}]$ via

$$\begin{pmatrix} E[{X_1}^2] - E[X_1]^2 & \cdots & E[X_1 X_n] - E[X_1]E[X_n] \\ \vdots & & \vdots \\ E[X_n X_1] - E[X_n]E[X_1] & \cdots & E[{X_n}^2] - E[X_n]^2 \end{pmatrix} = \mathbf{E}\Big[\big(\mathbf{X} - \mathbf{E}[\mathbf{X}]\big)\big(\mathbf{X} - \mathbf{E}[\mathbf{X}]\big)^{\mathbf{T}}\Big].$$

The covariance matrix is symmetric and if the X_i's are independent, identically distributed (iid) with variance $\boldsymbol{\sigma}^2$, then

$$\mathbf{Cov}(\mathbf{X}) = \boldsymbol{\sigma}^2 \mathbf{I}.$$