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multivariate gamma function (complex-valued)

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Defines	gamma function (multivariate complex)

The complex multivariate gamma function is defined as

$$\tilde{\Gamma}_m(a) = \int_{\mathfrak{A}} e^{-\text{Tr } A} |A|^{a-m} dA, \quad (1)$$

where \mathfrak{A} is the set of all $m \times m$ positive, complex-valued Hermitian matrices, i.e.

$$\mathfrak{A} = \{A \in \mathbb{C}^{m \times m} | A = A^H, A > 0\}. \quad (2)$$

It can also be expressed in terms of the gamma function as follows

$$\tilde{\Gamma}_m(a) = \pi^{\frac{1}{2}m(m-1)} \prod_{i=1}^m \Gamma(a - i + 1). \quad (3)$$

Reference

A. T. James, “Distributions of matrix variates and latent roots derived from normal samples,” *Ann. Math. Statist.*, vol. 35, pp. 475-501, 1964.