cubic spline interpolation

Canonical name	CubicSplineInterpolation
Date of creation	2013-03-22 13:40:25
Last modified on	2013-03-22 13:40:25
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Last modified by	yota (10184)
Numerical id	7
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Entry type	Definition
Classification	msc 65-01

Suppose we are given $N+1$ data points $\{(x_k, y_k)\}$ such that

$$a = x_0 < \cdots < x_N. \tag{1}$$

Then the function $S(x)$ is called a *cubic spline interpolation* if there exists N cubic polynomials $S_k(x)$ with coefficients $s_{k,i}$ $0 \le i \le 3$ such that the following hold.

1. $S(x) = S_k(x) = \sum_{i=0}^{3} s_{k,i}(x - x_k)^i \quad \forall x \in [x_k, x_{k+1}] \quad 0 \le k \le N-1$
2. $S(x_k) = y_k \quad 0 \le k \le N$
3. $S_k(x_{k+1}) = S_{k+1}(x_{k+1}) \quad 0 \le k \le N-2$
4. $S'_k(x_{k+1}) = S'_{k+1}(x_{k+1}) \quad 0 \le k \le N-2$
5. $S''_k(x_{k+1}) = S''_{k+1}(x_{k+1}) \quad 0 \le k \le N-2$

The set of points (**??**) are called the knots. The set of cubic splines on a fixed set of knots, forms a vector space for cubic spline addition and scalar multiplication.

So we see that the cubic spline not only interpolates the data $\{(x_k, y_k)\}$ but matches the first and second derivatives at the knots. Notice, from the above definition, one is free to specify constraints on the endpoints. One common end point constraint is $S''(a) = 0 \;\; S''(b) = 0$, which is called the natural spline. Other popular choices are the clamped cubic spline, parabolically terminated spline and curvature-adjusted spline. Cubic splines are frequently used in numerical analysis to fit data. Matlab uses the command spline to find cubic spline interpolations with not-a-knot end point conditions. For example, the following commands would find the cubic spline interpolation of the curve $4\cos(x) + 1$ and plot the curve and the interpolation marked with o's.

```
x = 0:2*pi;
y = 4*cos(x)+1;
 xx = 0:.001:2*pi;
yy = spline(x,y,xx);
plot(x,y,'o',xx,yy)
```