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Lagrange interpolation formula

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Defines	Lagrange polynomial

Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be n points in the plane ($x_i \neq x_j$ for $i \neq j$). Then there exists a unique polynomial $p(x)$ of degree at most $n-1$ such that $y_i = p(x_i)$ for $i = 1, \ldots, n$.

Such polynomial can be found using *Lagrange's interpolation formula*:

$$p(x) = \frac{f(x)}{(x-x_1)f'(x_1)}y_1 + \frac{f(x)}{(x-x_2)f'(x_2)}y_2 + \cdots + \frac{f(x)}{(x-x_n)f'(x_n)}y_n$$

where $f(x) = (x-x_1)(x-x_2)\cdots(x-x_n)$.

To see this, notice that the above formula is the same as

$$\begin{aligned} p(x) = y_1 &\frac{(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} + y_2\frac{(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)} \\ &+ \cdots + y_n\frac{(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})} \end{aligned}$$

and that for all x_i, every numerator except one vanishes, and this numerator will be identical to the denominator, making the overall quotient equal to 1. Therefore, each $p(x_i)$ equals y_i.