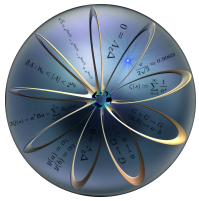

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triangular matrix

Canonical name	TriangularMatrix
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Classification	msc 65-00
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Defines	upper triangular
Defines	lower triangular
Defines	upper triangular matrix
Defines	lower triangular matrix
Defines	right triangular
Defines	right triangular matrix
Defines	left triangular
Defines	left triangular matrix

1 Triangular Matrix

Let n be a positive integer.

An *upper triangular matrix* is of the form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

An upper triangular matrix is sometimes also called *right triangular*.

A *lower triangular matrix* is of the form:

$$\begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & 0 \\ a_{31} & a_{32} & a_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

A lower triangular matrix is sometimes also called *left triangular*.

Note that upper triangular matrices and lower triangular matrices must be square matrices.

A *triangular matrix* is a matrix that is an upper triangular matrix or lower triangular matrix. Note that some matrices, such as the identity matrix, are both upper and lower triangular. A matrix is upper and lower triangular simultaneously if and only if it is a diagonal matrix.

Triangular matrices allow numerous algorithmic shortcuts in many situations. For example, if A is an $n \times n$ triangular matrix, the equation $Ax = b$ can be solved for x in at most n^2 operations.

In fact, triangular matrices are so useful that much computational linear algebra begins with factoring (or decomposing) a general matrix or matrices into triangular form. Some matrix factorization methods are the Cholesky factorization and the LU-factorization. Even including the factorization step, enough later operations are typically avoided to yield an overall time savings.

2 Properties

Triangular matrices have the following properties (“triangular” with either “upper” or “lower” uniformly):

- The inverse of a triangular matrix is a triangular matrix.
- The product of two triangular matrices is a triangular matrix.
- The determinant of a triangular matrix is the product of the diagonal elements.
- The eigenvalues of a triangular matrix are the diagonal elements.

The last two properties follow easily from the cofactor expansion of the triangular matrix.