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## Euler-Maclaurin summation formula

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Let  $B_r$  be the  $r$ th Bernoulli number, and  $B_r(x)$  be the  $r$ th Bernoulli periodic function. For any integer  $k \geq 0$  and for any function  $f$  of class  $C^{k+1}$  on  $[a, b]$ ,  $a, b \in \mathbb{Z}$ , we have

$$\sum_{a < n \leq b} f(n) = \int_a^b f(t) dt + \sum_{r=0}^k \frac{(-1)^{r+1} B_{r+1}}{(r+1)!} (f^{(r)}(b) - f^{(r)}(a)) + \frac{(-1)^k}{(k+1)!} \int_a^b B_{k+1}(t) f^{(k+1)}(t) dt.$$