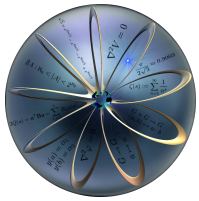

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Frobenius matrix norm

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Synonym	Euclidean matrix norm
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Synonym	Hilbert-Schmidt norm
Related topic	MatrixNorm
Related topic	MatrixPnorm
Related topic	VectorNorm
Related topic	VectorPnorm
Related topic	ShursInequality
Related topic	trace
Related topic	transpose
Related topic	Transpose
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Related topic	FrobeniusProduct

Let R be a ring with a valuation $|\cdot|$ and let $M(R)$ denote the set of matrices over R. The *Frobenius norm function* or *Euclidean matrix norm* is the norm function $||\cdot||_F : M(R) \to \mathbb{R}$ given by

$$||A||_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2},$$

where m and n respectively denote the number of rows and columns of A. Note A need not be square for this definition. A more concise (though) definition, in the case that $R = \mathbb{R}$ or $\mathbb{C}$, is

$$||A||_F = \sqrt{\text{trace}(A^*A)},$$

where A^* denotes the conjugate transpose of A.

Some :

- Denote the columns of A by A_i. A nice property of the norm is that

$$||A||_F^2 = ||A_1||_2^2 + ||A_2||_2^2 + \cdots + ||A_n||_2^2.$$

- Let A be a square matrix and let U be a unitary matrix of same size as A. Then $||A||_F = ||U^*AU||_F$ where U^* is the conjugate transpose of U.

- If AB is defined, then $||AB||_F \leq ||A||_F \, ||B||_F$.