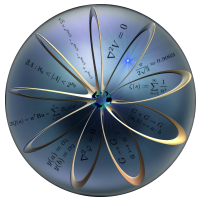

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orthonormal set

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Definition

An *orthonormal set* is a subset S of an inner product space, such that $\langle x, y \rangle = \delta_{xy}$ for all $x, y \in S$. Here $\langle \cdot, \cdot \rangle$ is the inner product, and δ is the Kronecker delta.

More verbosely, we may say that an orthonormal set is a subset S of an inner product space such that the following two conditions hold:

1. If $x, y \in S$ and $x \neq y$, then x is `http://planetmath.org/OrthogonalVector`orthogonal to y.
2. If $x \in S$, then the norm of x is 1.

Stated this way, the origin of the term is clear: an orthonormal set of vectors is both orthogonal and normalized.

Notes

Note that the empty set is orthonormal, as is a set consisting of a single vector of unit norm in an inner product space.

The columns (or rows) of a real orthogonal matrix form an orthonormal set. In fact, this is an example of an orthonormal basis.

Applications

A standard application is finding an orthonormal basis for a vector space, such as by Gram-Schmidt orthonormalization. Orthonormal bases are computationally simple to work with.