Newton and Cotes formulas

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Owner	drini (3)
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The usual way of numerically integrate a function, is to find a simpler function which approximates the given function and then integrating the interpolation function. That is, if we want to find $\int_a^b f(x)\,dx$, we find an approximating function $p(x)$ such that $f(x)$ and $p(x)$ be close (on some concept of distance) and then we say

$$\int_a^b f(x)\,dx \approx \int_a^b p(x)\,dx$$

The simplest approximation functions are polynomials. If we evaluate $f(x)$ at some points $x_0, x_1, \ldots, x_n$, we can use Lagrange's interpolating polynomial to find a polynomial $p(x)$ with degree n such that $p(x_j) = f(x_j)$ for $j = 0, 1, \ldots, n$.

Newton and Cotes' integration formulas are obtained when the $x_0, x_1, \ldots, x_n$ are sampled evenly over the interval, and then Lagrange interpolating polynomials are used to approximate the function.

The Newton and Cotes formulas for small values of n are given on the following table.

n	$\int p(x)$	**Name**
1	$\frac{h}{2}(f(x_0) + f(x_1))$	Trapezoidal rule
2	$\frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2))$	Simpson's rule
3	$\frac{3h}{8}(f(x_0) + 3f(x_1) + 3f(x_3) + f(x_3))$	Simpson's 3/8 rule
4	$\frac{2h}{45}(7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4))$	Milne's rule

recalling that $x_0, x_1, \ldots, x_n$ are evenly spaced on $[a, b]$.

Since the Simpson's rule is actually the Newton and Cotes formula for $n = 2$, the proof of Simpson's rule illustrates this method.